Di-Ω $(\Omega\Omega)_{J^{\pi}=0^{+}}$ production cross section calculation

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Cross sections of baryon+baryon$\rightarrow(\Omega\Omega)_{J^{\pi}=0^{+}}+X$ are studied using an effective Hamiltonian method. For low energy region, the γ production process dominates; its cross sections are of order of 0.3–1.6 μb for $p_{\Omega}=100$–400 MeV. There are also some strong interaction processes for forming di-Ω such as $\Omega+\Omega\rightarrow(\Omega\Omega)_{0^{+}}+\eta(\eta',\phi)$ and $\Omega+\Xi\rightarrow(\Omega\Omega)_{0^{+}}+K(K^{\star})$. But these processes only make contributions in the high momentum region, because their threshold energies are around (or greater than) 1 GeV. The cross sections for the pseudoscalar meson production processes are about 2.0–4.5 μb, and for the vector meson production processes the cross sections are one order larger than those of the pseudoscalar meson production cases. Besides the above processes, a two-step process is also discussed, in which the first step is $\Omega+N\rightarrow(N\Omega)_{J^{\pi}=2^{+}}+\pi$ (or γ) and the second step is $\Omega+(N\Omega)_{2^{+}}\rightarrow(\Omega\Omega)_{0^{+}}+N$. The result shows that the cross sections are quite large for both steps. It seems that some two-step processes might also be important.

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I. INTRODUCTION

For more than two decades, the search for dibaryons has remained one of the most exciting topics in hadron physics, and many works, both theoretical and experimental, have been devoted to this field [1–4]. It seems now that most physicists hold the opinion that probably such objects, if they exist, will be seen in the strange sector because of the comparatively long lifetime, which is due to the stability with respect to strong decay. This means the strange quarks must be included in addition to the *u* and *d* quarks. Therefore, one needs a method to calculate a six-quark system containing *u*, *d*, and *s*. Recently we have analyzed the symmetry properties of the two-baryon systems by projecting them onto a six-quark system and found that among all 280 two-baryon physical states, only six states have the highest orbital symmetry. They are $(\Delta\Delta)_{ST=03}$, $(\Delta\Delta)_{ST=30}$, $(\Delta\Sigma^{\star})_{ST=0(5/2)}$, $(\Delta\Sigma^{\star})_{ST=3(1/2)}$, $(\Xi^{\star}\Omega)_{ST=0(1/2)}$, and $(\Omega\Omega)_{ST=00}$. It is more remarkable that $(\Omega\Omega)_{00}$ is the only one which is stable with respect to strong decay. This has stimulated us to try to carry out a dynamical calculation for a six-*s*-quark system based on the chiral SU(3) quark model, which was extended by us in Refs. [5,6] from the well-known chiral quark model [7,8]. This approach enables us to treat the *s* quarks on the same footing as the *u* and *d* quarks. In the calculation, the system is described as two clusters, each of them consisting of three *s* quarks. Then by employing a resonating group method (RGM) calculation, the binding energy for $(\Omega\Omega)_{0^{+}}$ is found to be from several tens to 100 MeV, which is much larger than the binding energy of the deuteron; the mean-squared distance between the two Ω's is about 0.84 fm, which is much smaller than the distance between the nucleons in deuterium [9–11]. These peculiar features have aroused enormous interest among experimentalists. At the same time, a natural and important question one has to answer is how to produce the di-Ω system experimentally in sufficient quantity.

For formation of a system with baryon number equal to 2 and strangeness equal to -6, a straightforward way is to have Ω beams colliding with each other. Since that is not feasible in the near future, our attention is focused on heavy-ion collisions. The reasons for that are the following: First, a recent experiment at the SPS observed [12] a strong enhancement in the production of multistrange hadrons in nucleus-nucleus collisions. Such an enhancement increases with the strangeness content of the particle, up to a factor 20 for $\Omega+\overline{\Omega}$. Although the mechanism of the enhancement is not quite clear, the very fact encourages us to consider the search for di-Ω dibaryon $(\Omega\Omega)_{0^{+}}$ in heavy-ion collision experiments. Second, in heavy-ion collisions a substantial fraction of nucleons from both the target and projectile participate in the reaction. As a result, a ''fireball'' with high baryon number could be formed. The fireball could be a hadronic gas in thermodynamic equilibrium or the long-sought quark-gluon plasma. In the former case, the density effect favors a subsequent collision between the already formed Ω's, the production of which has already been observed experimentally. As for the quark-gluon plasma, the hadronization is very complicated and not understood; we therefore do not think any reliable estimates for the di-Ω production cross-section can be made. However, if the formation of the di-Ω occurs in a baryon-rich hadronic gas, then some simple baryon-baryon fusion processes should lead to the production of $(\Omega\Omega)_{0^{+}}$. In this paper, as a first approach, we have calculated the cross sections of these fusion reactions. They are $\Omega+\Omega\rightarrow(\Omega\Omega)_{0^{+}}+\gamma$, $\Omega+\Omega\rightarrow(\Omega\Omega)_{0^{+}}+\eta(\eta',\phi)$, and $\Omega+\Xi\rightarrow(\Omega\Omega)_{0^{+}}+K(K^{\star})$. Moreover, there is a very interesting two-step process: $\Omega+N\rightarrow(N\Omega)_{J^\pi=2^+}+\pi$ (or γ), followed by $\Omega+(N\Omega)_{2^+}\rightarrow(\Omega\Omega)_{0^+}+N$, which is distinguished by its very large cross section. Taking into account the possible density effect, this process might become an important mechanism for di-Ω production.

The paper is organized as follows: In Sec. II A we first discuss the electromagnetic process $\Omega+\Omega\rightarrow(\Omega\Omega)_{LSJ}+\gamma$. The calculation of strong interaction fusion processes with the emission of η, η', K, ϕ, and K^* respectively is presented in Sec. II B. Section II C is devoted to the calculation of the two-step process we have just mentioned. Finally in Sec. III our calculations are summarized and discussed.

II. CALCULATION METHOD

As we have already mentioned in the Introduction, the most direct and simple process for the di-Ω formation is through the fusion of two Ω's. This may happen in colliding beams of Ω or, more realistically, in heavy-ion collisions, where a baryon-rich fireball might be formed. In this section we are going to calculate these fusion processes. In what follows the Ω's and all other baryons and mesons are treated as hadrons, but the wave function of the internal relative motion between two Ω's as well as the binding energy of $(\Omega\Omega)_{0^+}$ is taken from the results of the chiral SU(3) quark model calculations.

A. Electromagnetic interaction process

$\Omega+\Omega\rightarrow(\Omega\Omega)_{LSJ}+\gamma$ is a electromagnetic interaction process; here L denotes the relative motion angular momentum between the two Ω's, S is the spin and J is the total angular momentum of the di-Ω system. As is well known, the electromagnetic vertex of particle with spin 3/2, after neglecting terms of higher order in momentum transfer, can be written down as

$$H_{em}=i\hat{e}\int\bar{\Psi}_\Omega(x)\gamma_\mu\Psi_\Omega(x)A_\mu(x)dx, \tag{1}$$

where Ψ_Ω is the wave function of Ω with spin $S=\frac{3}{2}$, satisfying the Rarita-Schwinger equation and $A_\mu(x)$ is the photon field. The S-matrix element from the initial state to the final state can be simplified by nonrelativistic decomposition:

$$\langle\vec{k}\lambda,\vec{p}\,'m'|H_{em}|\vec{p}m\rangle=\langle\psi_{m'}|\hat{H}^p_{em}|\psi_m\rangle, \tag{2}$$

where $\vec{p}$, m, and $\vec{p}\,'$, m' represent, respectively, the momenta and the spin components of the Ω in the initial and final states and $\vec{k}$ is the γ's momentum. ψ_m is the Pauli spinor of spin $S=\frac{3}{2}$ particle with momentum $\vec{p}=0$. $\hat{H}^p_{em}$ is the operator in momentum space:

$$\hat{H}^p_{em}=i(2\pi)^4\delta(p-p'-k)\left[\frac{1}{(2\pi)^{3/2}}\right]^3\frac{\hat{e}}{2M_\Omega}\frac{1}{\sqrt{2\omega_k}}\times\left(\vec{p}+\vec{p}\,'+i\frac{1}{3}\vec{k}\times\vec{\sigma}_\Omega\right)\cdot\vec{\varepsilon}_\lambda(\vec{k}), \tag{3}$$

where $\vec{\varepsilon}_\lambda(\vec{k})$ is the polarization vector of the photon with momentum $\vec{k}$ and $\vec{\sigma}_\Omega$ the spin operator of $S=\frac{3}{2}$. For the sake of convenience we have transformed Eq. (3) into coordinate space, and included the contribution arisen from the anomalous magnetic moment of Ω by adjusting by hand the coefficient of the third term on the right-hand side of the following equation:

$$\hat{H}^r_{em}=i(2\pi)^4\delta(E-E'-\omega_k)\left[\frac{1}{(2\pi)^{3/2}}\right]^3\frac{\hat{e}}{2M_\Omega}\frac{1}{\sqrt{2\omega_k}}\times\Bigg[-i\vec{\nabla}_{r'}\delta(\vec{r}-\vec{r}\,')e^{-i\vec{k}\cdot\vec{r}\,'}+i\vec{\nabla}_r\delta(\vec{r}-\vec{r}\,')e^{-i\vec{k}\cdot\vec{r}}+i\frac{1}{3}(\vec{k}\times\vec{\sigma}_\Omega)\delta(\vec{r}-\vec{r}\,')e^{-i\vec{k}\cdot\vec{r}}\Bigg]\cdot\vec{\varepsilon}_\lambda(\vec{k}). \tag{4}$$

For the process $\Omega+\Omega\rightarrow(\Omega\Omega)_{LSJ}+\gamma$, the initial state is a plane wave of two free Ω's and the wave function is denoted by $\Phi_{\Omega\Omega\mu_1\mu_2}$, where μ_1 and μ_2 are the spin components of the two Ω's respectively. The final state consists of a di-Ω $(\Omega\Omega)_{LSJ}$ and a photon. The wave function of the di-Ω $(\Omega\Omega)_{LSJ}$ is taken to be

$$\Phi_{LSJ}(\vec{r}_1,\vec{r}_2)=\frac{1}{(2\pi)^{3/2}}e^{i\vec{P}\cdot\vec{R}}(\varphi_L(\vec{r})\chi_s(1,2))_{JM}. \tag{5}$$

The exponential on the right-hand side of Eq. (5) is a plane wave for the center of mass motion and $\vec{R}$ is the center of mass coordinate. $\varphi_L(\vec{r})$ is the internal wave function of the di-Ω system, which is obtained from the RGM calculation on the quark level [10], and $\chi_s(1,2)$ represents the corresponding spin wave function. Taking into account the antisymmetrization between the two Ω's, the scattering matrix element of $\Omega+\Omega\rightarrow(\Omega\Omega)_{LSJ}+\gamma$ process has the following expression:

$$\langle\vec{k}\lambda,(\Omega\Omega)_{LSJ}|H_{em}|\Omega\Omega\mu_1\mu_2\rangle=\langle\Phi_{LSJ}|\hat{H}^r_{eff}|\Phi_{\Omega\Omega\mu_1\mu_2}\rangle, \tag{6}$$

where $\hat{H}^r_{eff}$ is an effective Hamiltonian operator defined as

$$\hat{H}^r_{eff}=i(2\pi)^4\delta(E-E'-\omega_k)\left[\frac{1}{(2\pi)^{3/2}}\right]^3\frac{\hat{e}}{2M_\Omega}\frac{1}{\sqrt{2\omega_k}}\times\sum_{j=1}^{2}\Bigg[-i\vec{\nabla}_{r_j'}\delta(\vec{r}_j-\vec{r}_j\,')e^{-i\vec{k}\cdot\vec{r}_j\,'}+i\vec{\nabla}_{r_j}\delta(\vec{r}_j-\vec{r}_j\,')e^{-i\vec{k}\cdot\vec{r}_j}+i\frac{1}{3}(\vec{k}\times\vec{\sigma}_\Omega)\delta(\vec{r}_j-\vec{r}_j\,')e^{-i\vec{k}\cdot\vec{r}_j}\Bigg]\cdot\vec{\varepsilon}_\lambda(\vec{k}), \tag{7}$$

where E and E' are the energies of the initial state and of the $(\Omega\Omega)_{LSJ}$ in the final state respectively. After integrating over the coordinate of the center of mass, we get the following expression:

$$\langle \vec{k}\lambda,(\Omega\Omega)_{LSJ}|H_{em}|\Omega\Omega\mu_1\mu_2\rangle = (2\pi)^4\delta(p-p'-k)T_{fi}, \tag{8}$$

with

$$T_{fi} = i\frac{1}{(2\pi)^6}\frac{\hat{e}}{2M_\Omega}\frac{1}{\sqrt{\omega_k}}\langle(\varphi_L(\vec{r})\chi_s(1,2))_{JM}| \times\sum_{j,l}\hat{O}_l(j)|e^{i\vec{p}\cdot\vec{r}}\chi_{\mu_1}(1)\chi_{\mu_2}(2)\rangle, \tag{9}$$

$$\hat{O}_1(1) = \overleftarrow{\nabla}_\lambda e^{-i\vec{k}\cdot\vec{r}/2}, \quad \hat{O}_2(1) = -e^{-i\vec{k}\cdot\vec{r}/2}\vec{\nabla}_\lambda,$$
$$\hat{O}_3(1) = i\tfrac{1}{3}\lambda k\sigma_{\Omega\lambda}e^{-i\vec{k}\cdot\vec{r}/2},$$
$$\hat{O}_1(2) = -\overleftarrow{\nabla}_\lambda e^{i\vec{k}\cdot\vec{r}/2}, \quad \hat{O}_2(2) = e^{i\vec{k}\cdot\vec{r}/2}\vec{\nabla}_\lambda,$$
$$\hat{O}_3(2) = i\tfrac{1}{3}\lambda k\ \sigma_{\Omega\lambda}e^{i\vec{k}\cdot\vec{r}/2}. \tag{10}$$

With the formulas thus derived, the the cross section can be calculated using the standard procedure:

$$\sigma_{fi} = \frac{(2\pi)^{10}}{v_{rel}}\frac{1}{16}\frac{1}{4\pi} \times\sum_{\lambda=\pm 1}\sum_M\sum_{\mu_1\mu_2}\int\delta(p-p'-k)|T_{fi}|^2 d^3k d^3p' d\Omega_p. \tag{11}$$

v_{rel} in Eq. (11) is the relative velocity between the two Ω's in the initial state, $\vec{k}$, $\vec{p}$, and $\vec{p}\,'$ are the momentum of the photon, that of the relative motion of initial state, and that of the di-Ω system in the final state, respectively. Of particular interest is the process which leads to the formation of di-Ω in its ground state: $\Omega+\Omega\to(\Omega\Omega)_{0^+}+\gamma$. The result for this particular process is given explicitly in the following:

$$\sigma_{fi} = \frac{\pi}{4}\frac{\hat{e}^2}{M_\Omega}\frac{1}{p}\frac{kE_{0^+}}{E_{0^+}+k}D, \tag{12}$$

where

$$D = \sum_{l_1,l_2,l_3}A_{l_1l_2}A_{l_3l_2} + 5\sum_{l_1}A_{l_1}A_{l_1} \tag{13}$$

and

$$A_{l_1l_2} = (1+(-1)^{1+l_1})[A^{(1)}_{l_1l_2}+A^{(2)}_{l_1l_2}], \tag{14}$$

$$A^{(1)}_{l_1l_2} = \frac{1}{\sqrt{4\pi}}i^{l_1+l_2+1}\frac{2l_1+1}{\sqrt{2l_2+1}}C^{l_20}_{10l_10}C^{l_21}_{11l_10} \times\int\varphi_0'(r)j_{l_1}\left(\frac{1}{2}kr\right)j_{l_1}(pr)r^2dr, \tag{15}$$

$$A^{(2)}_{l_1l_2} = \frac{1}{\sqrt{4\pi}}(-1)^{l_1}\frac{2l_1+1}{\sqrt{2l_2+1}}C^{l_20}_{10l_10}C^{l_21}_{11\ l_10}p \times\int\varphi_0(r)j_{l_1}\left(\frac{1}{2}kr\right)j_{l_2}(pr)r^2dr, \tag{16}$$

$$A_{l_1} = (1+(-1)^{1+l_1})\ A^{(3)}_{l_1}, \tag{17}$$

$$A^{(3)}_{l_1} = \frac{1}{\sqrt{4\pi}}(-1)^{l_1}\frac{1}{3}\sqrt{2l_1+1}k \times\int\varphi_0(r)j_{l_1}\left(\frac{1}{2}kr\right)j_{l_1}(pr)r^2dr, \tag{18}$$

where E_{0^+} is the energy of the $(\Omega\Omega)_{0^+}$ system, $\varphi_0(r)$ is the radial wave function of the two Ω's relative motion, and $\varphi_0'(r) = d\varphi_0(r)/dr$. Both E_{0^+} and $\varphi_{0^+}(r)$ are obtained from the RGM calculation using the chiral SU(3) quark model [10]. For a given initial momentum p, the value k is fixed based on energy-momentum conservation; the total cross section then can be calculated.

Similar calculations can also be carried out if the di-Ω is in the p or d state.

B. Strong interaction process

The di-Ω system can also be formed through strong interaction with the emission of mesons. There are five possible processes: namely, the three pseudoscalar meson production processes: $\Omega+\Omega\to(\Omega\Omega)_{0^+}+\eta$, $\Omega+\Omega\to(\Omega\Omega)_{0^+}+\eta'$, and $\Omega+\Xi\to(\Omega\Omega)_{0^+}+K$ and the two vector meson production processes $\Omega+\Omega\to(\Omega\Omega)_{0^+}+\phi$ and $\Omega+\Xi\to(\Omega\Omega)_{0^+}+K^\star$. It should be noted that the emission of pions is strongly suppressed because Ω is a three-strange-quark system. The vertex functions of the baryon-pseudoscalar meson interactions can be expressed as

$$H_{\Omega\Omega\eta(\eta')} = ig_{\Omega\Omega\eta(\eta')}\int\bar{\Psi}_\Omega(x)\gamma_5\Psi_\Omega(x)\phi_{\eta(\eta')}(x)dx \tag{19}$$

and

$$H_{\Xi\Omega K} = i\frac{g_{\Xi\Omega K}}{M_P}\int\bar{\Psi}_{\Omega\mu}(x)\left(-i\frac{\partial}{\partial x_\mu}\vec{\phi}_K(x)\right) \cdot\vec{\tau}_{\Omega\Xi}\Psi_\Xi(x)dx + \text{H.c.} \tag{20}$$

The corresponding vertex functions for the vector meson production are

$$H_{\Omega\Omega\phi} = ig_{\Omega\Omega\phi}\int\bar{\Psi}_\Omega(x)\gamma_\mu\phi_{\phi\mu}(x)\Psi_\Omega(x)dx + i\frac{f_{\Omega\Omega\phi}}{2M_P}\int\bar{\Psi}_\Omega(x)\sigma_{\mu\nu}\partial_\nu\phi_{\phi\mu}(x)\Psi_\Omega(x)dx \tag{21}$$

and

$$H_{\Xi\Omega K^\star}=ig_{\Xi\Omega K^\star}\int \overline{\Psi}_{\Omega\mu}(x)\gamma_5\vec{\phi}_{K^\star\mu}(x)\cdot\vec{\tau}_{\Omega\Xi}\Psi_{\Xi}(x)dx + i\frac{f_{\Xi\Omega K^\star}}{M_P}\int \overline{\Psi}_{\Omega\mu'}(x)\gamma_\mu\gamma_5\left(\frac{\partial}{\partial x_{\mu'}}\vec{\phi}_{K^\star\mu}(x)\cdot\vec{\tau}_{\Omega\Xi}\right)\times\Psi_{\Xi}(x)dx+\text{H.c.} \qquad (22)$$

In Eqs. (19)–(22), Ψ_Ξ is the wave function of Ξ with spin $S=\frac{1}{2}$ and Ψ_Ω the wave function of Ω with spin $S=\frac{3}{2}$. Here ϕ is the wave function of meson and M_P is a scale mass which usually can be taken as the proton's mass. $\vec{\tau}_{\Omega\Xi}$ is a transition operator in the flavor space. $g_{\Omega\Omega\eta(\eta')}$, $g_{\Xi\Omega K}$, $g_{\Omega\Omega\phi}$, $f_{\Omega\Omega\phi}$, $g_{\Xi\Omega K^\star}$, and $f_{\Xi\Omega K^\star}$ are the corresponding coupling constants. Now taking the process of η production as an example, a brief explanation for the calculation formulas is given as follows: When terms of high order in momentum transfer are neglected, the vertex function of $\Omega\to\Omega+\eta$ can be expressed as

$$\langle\vec{p}\,'m',\vec{k}|H_{\Omega\Omega\eta}|\vec{p}m\rangle=\langle\psi_{m'}|\hat{H}_{\Omega\Omega\eta}|\psi_m\rangle \qquad (23)$$

and

$$\hat{H}_{\Omega\Omega\eta}=ig_{\Omega\Omega\eta}\left(\frac{1}{2\pi}\right)^{9/2}\frac{1}{6M_\Omega}(2\pi)^4\times\delta(p-p'-k)\frac{1}{\sqrt{2\omega}}\vec{\sigma}_\Omega\cdot(\vec{p}\,'-\vec{p}), \qquad (24)$$

where $\vec{p},m$ and $\vec{p}\,',m'$ are, respectively, the momenta and spin components of the Ω in the initial and final states and $\vec{k}$ is the momentum of the meson η . Using the same method described in Sec. II A, the operator $\hat{H}_{\Omega\Omega\eta}$ in momentum space can be easily transformed into coordinate space. The scattering amplitude T^{η}_{fi} for the reaction $\Omega+\Omega\to(\Omega\Omega)_{0^+}+\eta$ can be written down as follows:

$$\langle\vec{k},(\Omega\Omega)_{0^+}|H_{\Omega\Omega\eta}(1)+H_{\Omega\Omega\eta}(2)|\Omega\Omega\mu_1\mu_2\rangle=(2\pi)^4\delta(P_i-P_f-k)T^{\eta}_{fi}, \qquad (25)$$

where P_i is the four-momentum of the initial state, P_f and k are the corresponding momenta of $(\Omega\Omega)_{0^+}$ and η meson in the final state, respectively, and

$$T^{\eta}_{fi}=ig_{\Omega\Omega\eta}\left(\frac{1}{2\pi}\right)^6(4\pi)^2\frac{1}{6}\sqrt{\frac{5}{3}}\frac{1}{M_\Omega}\frac{1}{\sqrt{\omega_k}}\times\sum_{l_1l_2\lambda}B_{l_1l_2}C^{1\lambda}_{3/2\mu_1,3/2\mu_2}(Y_{l_1}(\hat{p})Y_{l_2}(\hat{k}))_{1\lambda}, \qquad (26)$$

with

$$B_{l_1l_2}=\sqrt{\frac{1}{4\pi}}(1-(-1)^{l_1})\sqrt{2l_1+1}C^{l_20}_{l_10,10}k\times\int\varphi_0(r)j_{l_1}\left(\frac{1}{2}kr\right)j_{l_1}(pr)r^2dr. \qquad (27)$$

The corresponding total cross section is obtained as

$$\sigma^{\eta}_{fi}=g^2_{\Omega\Omega\eta}\frac{5\pi}{72}\frac{1}{M_\Omega}\frac{kE_{0^+}}{p(E_{0^+}+\omega_k)}\sum_{l_1l_2}B^2_{l_1l_2}. \qquad (28)$$

In Eqs. (25)–(28), $|\Omega\Omega\mu_1\mu_2\rangle$ is the antisymmetrized wave function of the two Ω's in the initial state and $g_{\Omega\Omega\eta}$ is the coupling constant between the Ω and η, which is fixed by using the experimental value of the $NN\pi$ coupling constant $g_{NN\pi}/\sqrt{4\pi}=3.70$ and the SU(3) quark model. $\vec{p}$ denotes the relative momentum between the two Ω's of the initial state and $\vec{k}$ the relative momentum between the $(\Omega\Omega)_{0^+}$ and the meson of the final state . $\varphi_0(r)$ is the internal wave function and E_{0^+} the energy of the $(\Omega\Omega)_{0^+}$, both obtained from the chiral SU(3) quark model calculation [10].

The formula for the η' production cross section is similar to that for the η production with only the coupling constant $g_{\Omega\Omega\eta}$ replaced by $g_{\Omega\Omega\eta'}$. For another three processes, the formulas of their cross sections are given as follows:

$$\sigma^{\phi}_{fi}=\frac{\pi}{2}M_\Omega\frac{kE_{0^+}}{p(E_{0^+}+\omega_k)}\left[\frac{5}{18}\left(\frac{g_{\Omega\Omega\phi}}{M_\Omega}+\frac{f_{\Omega\Omega\phi}}{M_P}\right)^2\times\sum_{l_1l_2}(B_{l_1l_2})^2+g^2_{\Omega\Omega\phi}\sum_{l}B^2_l\right], \qquad (29)$$

for the process $\Omega+\Omega\to(\Omega\Omega)_{J^\pi=0^+}+\phi$, where $B_{l_1l_2}$ is taken from Eq. (27), and

$$B_l=\sqrt{\frac{1}{4\pi}}(1+(-1)^l)\sqrt{2l+1}\int\varphi_0(r)j_l\left(\frac{1}{2}kr\right)j_l(pr)r^2dr, \qquad (30)$$

$$\sigma^{K}_{fi}=g^2_{\Xi\Omega K}\frac{2\pi}{3}\frac{\mu_{\Xi\Omega}}{M_P^2}\frac{kE_{0^+}}{p(E_{0^+}+\omega_k)}\sum_{l_1l_2}(B^K_{l_1l_2})^2, \qquad (31)$$

and

$$\sigma^{K^\star}_{fi}=\frac{\pi}{8\mu_{\Xi\Omega}}\left(g^2_{\Xi\Omega K^\star}+\frac{16\mu^2_{\Xi\Omega}}{M_P^2}f^2_{\Xi\Omega K^\star}-\frac{32\mu_{\Xi\Omega}}{9M_P}g_{\Xi\Omega K^\star}f_{\Xi\Omega K^\star}\right)\frac{kE_{0^+}}{p(E_{0^+}+\omega_k)}\sum_{l_1l_2}(B^K_{l_1l_2})^2, \qquad (32)$$

for the processes $\Omega+\Xi^{-,0}\to(\Omega\Omega)_{J^\pi=0^+}+K^{0,+}(K^{\star0,+})$ with

$$\mu_{\Xi\Omega}=\frac{M_\Omega M_\Xi}{(M_\Omega+M_\Xi)} \qquad (33)$$

and

$$B^K_{l_1 l_2}=\sqrt{\frac{1}{4\pi}}\sqrt{2l_1+1}C^{l_2 0}_{l_1 0,10}k\int \varphi_0(r)\times j_{l_1}\left(\frac{M_\Omega}{M_\Omega+M_\Xi}kr\right)j_{l_1}(pr)r^2dr. \tag{34}$$

C. Two-step process

When considering the production of $(\Omega\Omega)_{0^+}$ in heavy-ion collisions, a two-step process is of particular interest. As is known in heavy-ion collisions, a fireball will be formed immediately after the collision of two heavy-ions. The radius of the fireball is around 10 fm. If the mean free path of the produced particle inside the fireball is smaller than the radius of the fireball, a secondary collision is possible. In particular, if the fireball is baryon rich, the Ω is first captured by a nucleon, which is abundant in the fireball. The newly formed system $(N\Omega)_{LSJ=022}$ is a weakly bound state with binding energy of about 5 MeV [11]. The di-Ω $(\Omega\Omega)_{0^+}$ could be produced through a subsequent three-body exchange reaction: $\Omega+(N\Omega)_{022}\to(\Omega\Omega)_{0^+}+N$. To be more precise, there are two channels in the first step: $N+\Omega\to(N\Omega)_{022}+\gamma$ and $N+\Omega\to(N\Omega)_{022}+\pi$. The corresponding interaction vertices are

$$H^\Omega_{em}=i\hat{e}\int \overline{\Psi}_\Omega(x)\gamma_\mu\Psi_\Omega(x)A_\mu(x)dx, \tag{35}$$

$$H^N_{em}=i\hat{e}\int \overline{\Psi}_N(x)\gamma_\mu\Psi_N(x)A_\mu(x)dx, \tag{36}$$

and

$$H_{NN\pi}=ig_{NN\pi}\int \overline{\Psi}_N(x)\gamma_5\Psi_N(x)\phi_\pi(x)dx. \tag{37}$$

Using the same method outlined in the previous section, the corresponding reaction amplitudes for these two processes, namely, T^{em}_{fi} and T^π_{fi}, can be obtained:

$$\langle\vec{k}\lambda,(N\Omega)_{022\mu}|H^\Omega_{em}+H^N_{em}|N\Omega\mu_1\mu_2\rangle =(2\pi)^4\delta(P_i-P_f-k)T^{em}_{fi} \tag{38}$$

and

$$\langle\vec{k},(N\Omega)_{022\mu}|H_{NN\pi}|N\Omega\mu_1\mu_2\rangle =(2\pi)^4\delta(P_i-P_f-k)T^\pi_{fi}. \tag{39}$$

For calculating the production cross section of the $(N\Omega)_{LSJ=022}$, the binding energy and the internal wave function of $(N\Omega)_{LSJ=022}$ are also taken from the result of the chiral SU(3) quark model calculation [11]. As for the second step reaction, one should note that there are three different channels for a three-body reaction: $\Omega_1+(N_2\Omega_3)_{022}$, $N_2+(\Omega_1\Omega_3)_{000}$, and $\Omega_3+(N_2\Omega_1)_{022}$. The wave functions of these three different channels are denoted by Φ_α, with $\alpha=1,2,3$, respectively:

$$\Phi_1=\Psi_{\vec{p}}(\vec{r}_1-\vec{R}_{23})\varphi_{(N\Omega)_{L=0}}(\vec{r}_{23})\chi_{\Omega\mu_1}(1)\chi_{N\Omega,2\mu_{23}}(2,3), \tag{40}$$

$$\Phi_2=\Psi_{\vec{p}}(\vec{r}_2-\vec{R}_{13})\varphi_{(\Omega\Omega)_{L=0}}(\vec{r}_{13})\chi_{N\mu_2}(2)\chi_{\Omega\Omega,0}(1,3), \tag{41}$$

and

$$\Phi_3=\Psi_{\vec{p}}(\vec{r}_3-\vec{R}_{21})\varphi_{(N\Omega)_{L=0}}(\vec{r}_{21})\chi_{\Omega\mu_3}(3)\chi_{N\Omega,2\mu_{21}}(2,1), \tag{42}$$

where $\chi_{N\mu}$ $(\chi_{\Omega\mu})$ is the spin wave function of single nucleon N (Ω) and $\chi_{N\Omega,2\mu}(\chi_{\Omega\Omega,0})$ is the spin wave function of the two-baryon system $N\Omega$ $(\Omega\Omega)$ with spin 2 (0). Here $\vec{r}$ represents the coordinate of the ith baryon, $\vec{r}_{ij}=\vec{r}_i-\vec{r}_j$ and $\vec{R}_{ij}$ is the center of mass coordinate of the two- baryon system (ij). $\Psi_{\vec{p}}(\vec{r}_k-\vec{R}_{ij})$ is the wave function of the relative motion between the kth baryon and the two-baryon system (ij). Here $\varphi_{(N\Omega)_{L=0}}(\vec{r})$ and $\varphi_{(\Omega\Omega)_{L=0}}(\vec{r})$ are the internal wave functions of the $N\Omega$ and $\Omega\Omega$ systems, respectively.

According to the AGS equation derived from the multichannel theory of a three-body collision [13], the reaction amplitude for the channel α to β is

$$F_{\beta\alpha}=-(2\pi)^2\mu_\beta\langle\Phi_\beta|U_{\beta\alpha}|\Phi_\alpha\rangle, \tag{43}$$

where μ_β is the reduced mass of channel β and $U_{\beta\alpha}$ the transition operator for channel α to β:

$$U_{\beta\alpha}=V_{\alpha\beta}+V_{\beta\gamma}G_0V_{\alpha\gamma}+\cdots. \tag{44}$$

$V_{\alpha\beta}$ in Eq. (44) is the interaction between the two baryons α and β, and G_0 the Green function, $G_0^{-1}=E+i\epsilon-H_0$. Since we only need some qualitative information concerning the cross sections of the reaction, therefore as the lowest order approximation, one may take simply,

$$U_{\beta\alpha}=V_{\alpha\beta}. \tag{45}$$

For the case interesting to us, we have $V_{\alpha\beta}=V_{12}=V_{N\Omega}$. After antisymmetrizing the two Ω's in the three-baryon system, the wave functions of the initial and final states Φ_i and Φ_f can be expressed as

$$\Phi_i=\sqrt{\frac{1}{2}}[\Phi_1-\Phi_3] \tag{46}$$

and

$$\Phi_f=\Phi_2. \tag{47}$$

Then the reaction amplitude F_{fi} and the corresponding cross section σ are given as

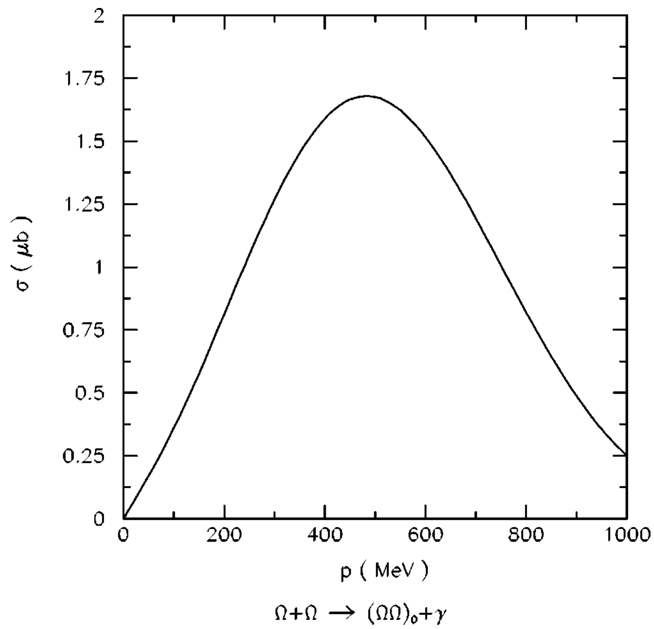

FIG. 1. The cross sections of $\Omega+\Omega\rightarrow(\Omega\Omega)_{0^+}+\gamma$.

$$F_{fi}=-\sqrt{\frac{1}{2}}(2\pi)^2\mu_f[\langle\Phi_2|V_{12}|\Phi_1\rangle-\langle\Phi_2|V_{32}|\Phi_3\rangle]$$
$$=-\sqrt{2}(2\pi)^2\mu_f\langle\Phi_2|V_{12}|\Phi_1\rangle, \tag{48}$$

where Φ_1, Φ_2, and Φ_3 are taken from Eqs. (40)–(42) and

$$\sigma=\frac{v_2}{v_1}\frac{1}{4\times5}\sum_{\mu_1\mu_2\mu_{23}}\int|F_{fi}|^2d\Omega_p, \tag{49}$$

where v_1 is the relative velocity between Ω and $(N\Omega)_{022}$, v_2 the relative velocity between N and $(\Omega\Omega)_{000}$ in the final state, and factors 4 and 5 are the statistical weights of the initial system. To carry out the cross section calculation, we need to know the interaction $V_{N\Omega}$ between the N and Ω as well as the internal wave functions for both $N\Omega$ and $\Omega\Omega$ systems, $\varphi_{(N\Omega)_{L=0}}(\vec{r})$ and $\varphi_{(\Omega\Omega)_{L=0}}(\vec{r})$. All of these are taken from Refs. [10,11].

Now it is appropriate to note that in parallel with the formation of $(N\Omega)_{LSJ=022}$, the scattering between N and Ω can also lead to the formation of $(N\Omega)_{LSJ=011}$, which decays rapidly via strong interaction to $(\Lambda\Xi)_{LSJ=011}$. Therefore, whenever a simulation of the whole process is considered, this effect should be taken into consideration.

III. RESULTS AND DISCUSSIONS

We are now going to summarize the numerical calculations for all fusion reactions discussed in the preceding sections, which can occur in the baryon-rich hot matter formed in heavy-ion collisions. In the calculations the nonrelativistic approximation was adopted.

First, let us turn to the electromagnetic processes. Figure 1 shows the cross sections of the reaction $\Omega+\Omega\rightarrow(\Omega\Omega)_{0^+}+\gamma$. The general trend is the following: The cross section increases with energy in the low energy region and reaches the maximum at $p\simeq500$ MeV in the center of mass system. It then goes down with energy. As indicated in Fig. 1, the cross section rises from $\sigma=0.3$ to 1.6 μb for $p=100$–400 MeV. Roughly speaking, the cross section of this reaction is of order μb and, if compared with the corresponding deuteron production cross section, is about 100–12 times smaller for the incoming particle momentum p in the region from 100 to about 200 MeV; this is because the binding energy of $(\Omega\Omega)_{0^+}$ is about 50 times larger than that of the deuteron's. The di-Ω can also be produced in one of its excited states and then disintegrates to the ground state via cascade radiation. For estimating the contributions from the excited state of di-Ω, we have also calculated the cross sections for the processes $\Omega+\Omega\rightarrow(\Omega\Omega)_{LSJ=110}+\gamma$ and $\Omega+\Omega\rightarrow(\Omega\Omega)_{LSJ=202}+\gamma$. The results have shown that the cross sections of these two processes are even smaller than that of the direct $(\Omega\Omega)_{0^+}$ production process. For instance, the cross section of $\sigma_{\Omega+\Omega\rightarrow(\Omega\Omega)_{LSJ=202}+\gamma}$ is about 50–10 times smaller than that of the direct $(\Omega\Omega)_{0^+}$ production in the region from $p=250$ to 300 MeV. That means that the contribution of the excited states in forming the $(\Omega\Omega)_{0^+}$ can be neglected.

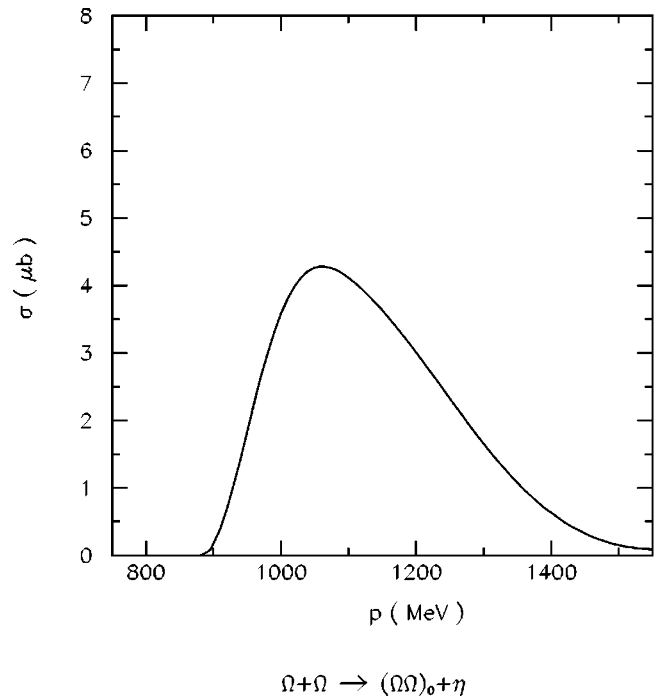

FIG. 2. The cross sections of $\Omega+\Omega\rightarrow(\Omega\Omega)_{0^+}+\eta$.

$(\Omega\Omega)_{0^+}$ can also be produced through strong interaction with the emission of meson. The calculated cross sections of $\Omega+\Omega\rightarrow(\Omega\Omega)_{0^+}+\eta(\eta')$, $\Omega+\Xi\rightarrow(\Omega\Omega)_{0^+}+K$ (including K^0 and K^+ productions), $\Omega+\Omega\rightarrow(\Omega\Omega)_{0^+}+\phi$, and $\Omega+\Xi\rightarrow(\Omega\Omega)_{0^+}+K^\star$ (including $K^{\star0}$ and $K^{\star+}$ productions) are plotted in Figs. 2–6. For these reactions, the threshold energies are all close to or greater than 1 GeV in the center of mass system, namely, 877 MeV for η production, 1259 MeV for η', 1103 MeV for K, 1310 MeV for ϕ, and 1412 MeV for $K^\star$. Therefore, only those Ω's produced in the heavy-ion collisions with very high energies can contribute to the

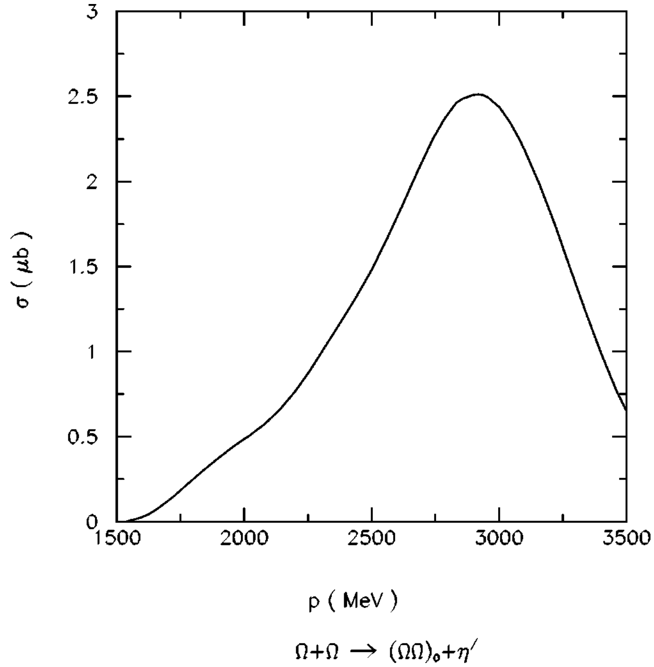

FIG. 3. The cross sections of $\Omega+\Omega\rightarrow(\Omega\Omega)_{0^{+}}+\eta'$.

$(\Omega\Omega)_{0^{+}}$ production. Furthermore, from the plots one sees that the cross sections of pseudoscalar meson productions are of the order of 2.0–4.5 μb or about several times larger than that of the corresponding electromagnetic process. This is because the large momentum transfer associated with the huge meson mass reduces the value of the overlapping integral containing two Bessel functions $[j_l(\frac{1}{2}kr)j_l(pr)]$ in Eq. (27). As can be seen from the plots, the cross sections for vector meson production generally are about an order larger

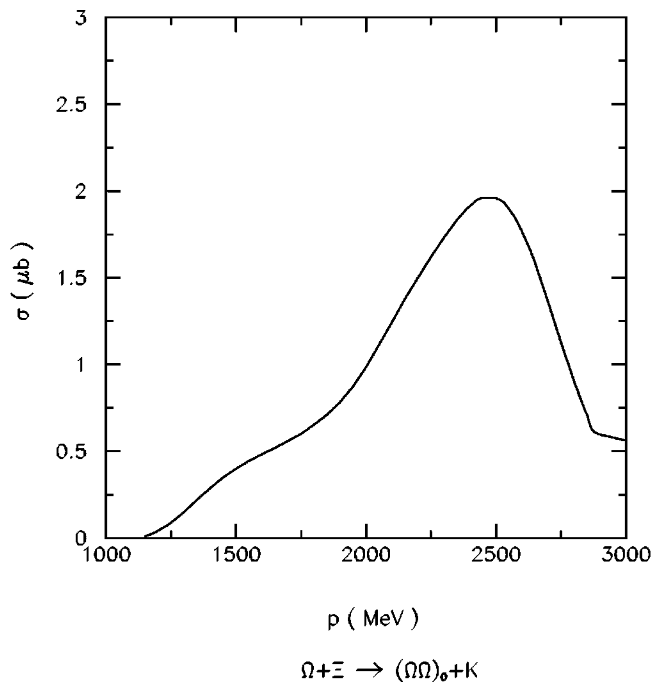

FIG. 4. The cross sections of $\Omega+\Xi\rightarrow(\Omega\Omega)_{0^{+}}+K$.

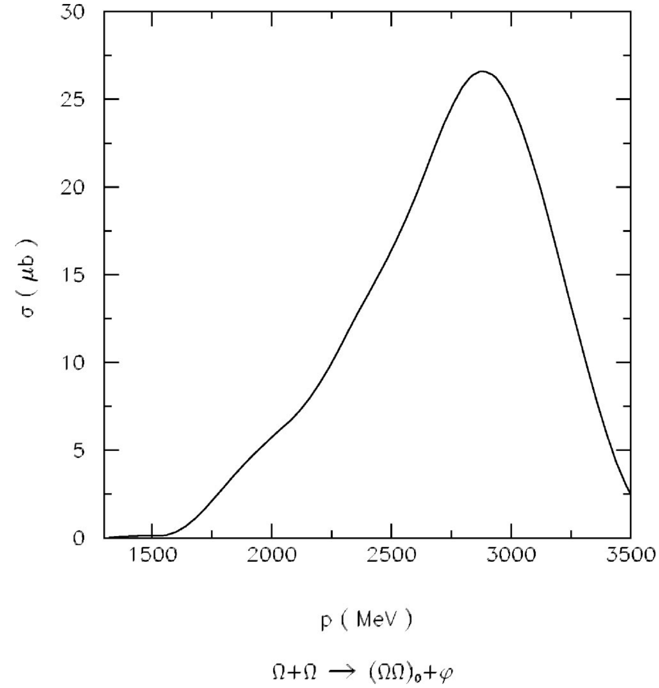

FIG. 5. The cross sections of $\Omega+\Omega\rightarrow(\Omega\Omega)_{0^{+}}+\phi$.

than those for the pseudoscalar meson productions. This conclusion is obtained for the coupling constants $g_{\Omega\Omega\phi}$, $f_{\Omega\Omega\phi}$, and $g_{\Omega\Xi K*}$ to be fixed according to the N-N scattering experimental values of $g_{NN\rho}/\sqrt{4\pi}=0.59$ and $f_{NN\rho}/\sqrt{4\pi}=4.82$ (from Nijmegen model D [14]) and the SU(3) quark model. Since the calculations are based on the nonrelativistic approximation and there are still uncertainties as to how to determine the values of the coupling constants, so that what one can say is that possibly reactions with vector meson emission are important in the high energy region. As it is

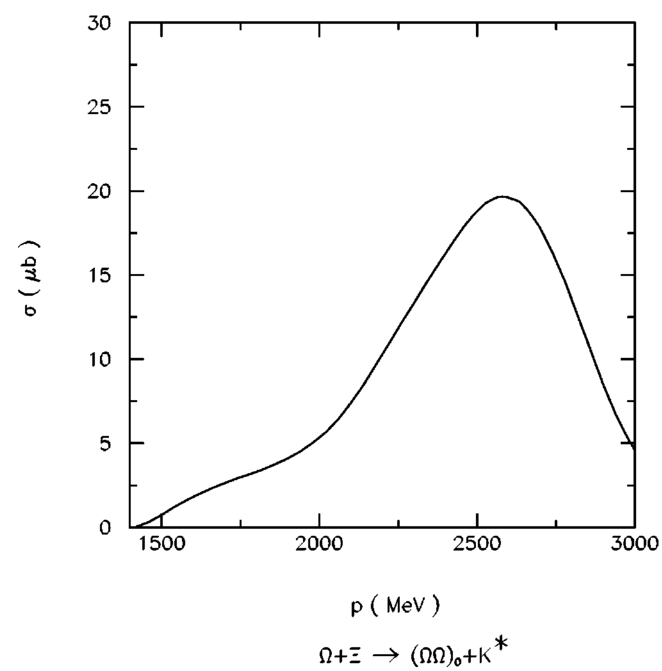

FIG. 6. The cross sections of $\Omega+\Xi\rightarrow(\Omega\Omega)_{0^{+}}+K^{\star}$.

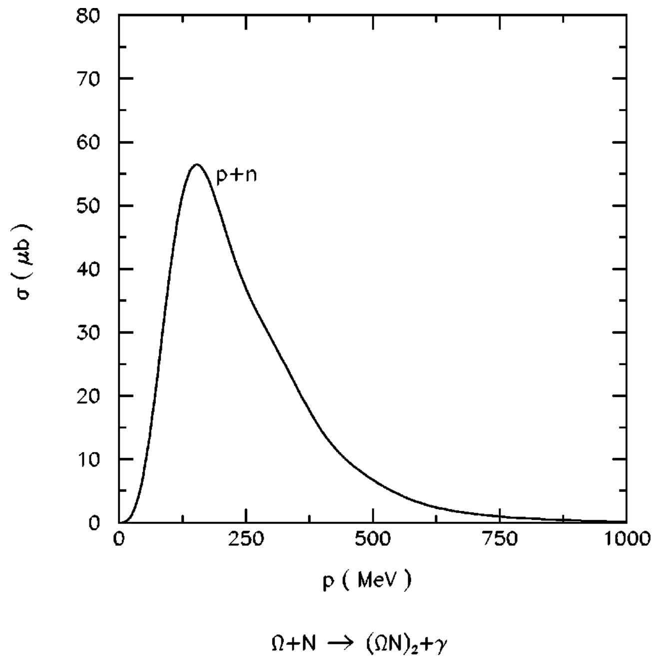

FIG. 7. The cross sections of $\Omega+N\rightarrow(N\Omega)_{022}+\gamma$.

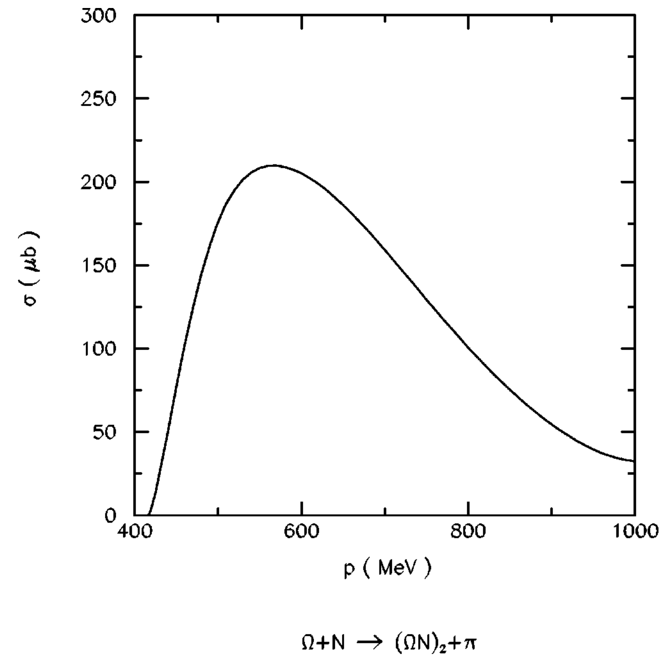

FIG. 8. The cross sections of $\Omega+N\rightarrow(N\Omega)_{022}+\pi$.

known that the production rate of Ξ is 4–5 times higher than the rate of Ω in the heavy-ion collision processes [12], the production processes with emission of K and K^* from $\Omega+\Xi\rightarrow(\Omega\Omega)_{0^+}+K(K^*)$ should be more important for forming di-Ω $(\Omega\Omega)_{0^+}$. Nevertheless, all the reactions we have just mentioned suffer from too high thresholds so that practically the contributions from these reactions to the production of $(\Omega\Omega)_{0^+}$ may be not so significant in the present experiments.

Combining the photon emission and the strong interaction processes discussed above, we can see that in the low momentum region the production cross sections of $(\Omega\Omega)_{0^+}$ are quite small through these simple one-step processes. As a matter of fact, the di-Ω production rate in relativistic heavy-ion collisions was estimated by Pal *et al.* in Ref. [15] with only the γ and η productions taken into account. The numerical result for the $(\Omega\Omega)_{0^+}$ rate is about 3×10^{-7} per event at the RIHC energy of $\sqrt{s}=130A$ GeV. But when the momentum of Ω in the heavy-ion collision processes increases to about 2 GeV, the production rate of $(\Omega\Omega)_{0^+}$ could be enhanced by two to three orders.

Among all processes calculated in this paper, the two-step reaction is of particular interest not only because of the low energy needed for this reaction but also the comparatively large cross sections . The calculated results are depicted in Figs. 7, 8, and 9. It is noted that in the first step the production cross section of $(N\Omega)_{022}$ with radiation increases rapidly with momentum and reaches the maximum at $p\leqslant200$ MeV in the center of mass system. The value of the cross section is of the order 50 μb. If compared with the Ω-Ω to $(\Omega\Omega)_{000}$ production cross section, the former is about several tens times larger than the latter at $p=200$ MeV. If the initial momentum p increases further to about 420 MeV, the π emission process prevails. The cross section of $\sigma_{N+\Omega\rightarrow(N\Omega)_{022}+\pi}$ can increase to 170 μb for $p=500$ MeV. Taking into account that in the baryon-rich fireball the nucleons are much more abundant than the Ω's, the formation of $(N\Omega)_{022}$ is predominant, provided the density of fireball is sufficiently high. Moreover, the high density also favors the second step reaction to take place subsequently: $\Omega+(N\Omega)_{022}\rightarrow(\Omega\Omega)_{000}+N$. The remarkable feature of this reaction is the very large cross section. As shown in Fig. 9, when the momentum of the initial system is $p=50$–100 MeV, the cross section is $\sigma_{ex}\simeq50$–20 mb, the largest among all reactions we have calculated so far. There-

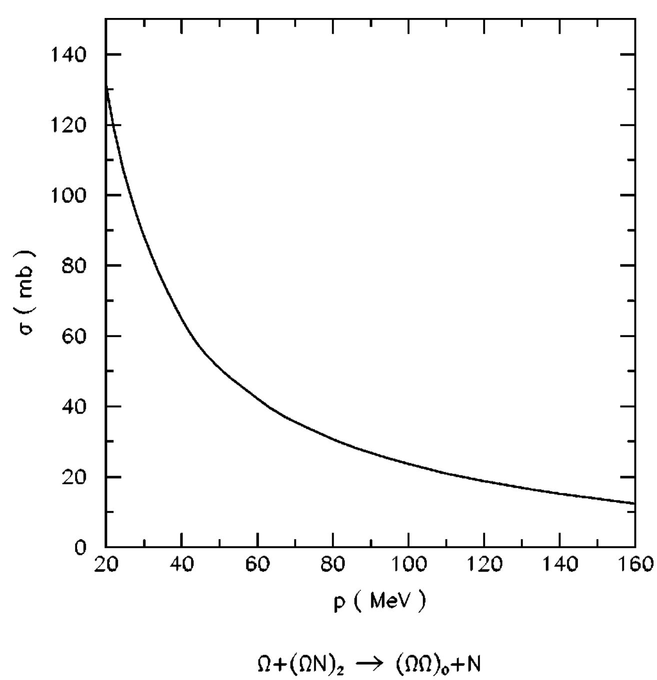

FIG. 9. The cross sections of $\Omega+(N\Omega)_{022}\rightarrow(\Omega\Omega)_{0^+}+N$

fore, it is expected that the two-step reaction might be the dominant mechanism for $(\Omega\Omega)_{0^+}$ production in heavy-ion collisions. This speculation, however, depends critically on the behavior and evolution of the fireball formed in the heavy-ion collisions. Therefore, the cross section estimates given here are an attempt towards a more detailed simulation study.

We believe the processes discussed in this paper are the most important fusion reactions for $(\Omega\Omega)_{0+}$ production in heavy-ion collisions, especially the two-step reaction. The cross sections are of the order 100–10 μb. That means the search for $(\Omega\Omega)_{0^+}$ in heavy-ion collisions is not an easy, but far from being an impossible experiment.

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