β decay of odd-A nuclei in the interacting boson-fermion model

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The energy levels, electromagnetic properties as well as the β-decay ratios are studied in the odd-mass isotopes of Rh and Pd in the neutron-proton interacting boson-fermion model.

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I. INTRODUCTION

A study of the β-decay of nuclei far from the line of stability has two reasons of interest: (i) to test the nuclear model by analyzing the existing data, because the β-decay rates are very sensitive to the details of the wave functions, and (ii) to provide reliable information for astrophysical applications.

In this paper we analyze the β decay from Rh to Pd isotopes of mass number $A=105,107,109$. Since the considered isotopes are in a mass region with transitional character, the proton-neutron interacting boson-fermion model (IBFM2) [1–3] should provide a suitable description of them. Precisely the Rh and Pd isotopes, which we analyze, can be considered to range from the U(5) to the O(6) limits of IBM2/IBFM2. The application of IBFM2 to the β decay was already proposed for nuclei in the region $52 \leqslant Z \leqslant 58$ [4], for Ru and Tc nuclei [5,6] and for β transition from even-even to odd-odd nuclei [7]. Some of the nuclei treated here were studied by Arias *et al.* [8], but they did not deal with β decay. In the present work we calculate β decay as well as energy levels and electromagnetic matrix elements.

II. THE IBFM2 MODEL

To describe an odd-A nucleus in the IBFM2, an odd nucleon is coupled to an even-even core of proton- and neutron-bosons. The Hamiltonian is written as

$$H = H^{\mathrm{B}} + H^{\mathrm{F}} + V^{\mathrm{BF}}, \tag{1}$$

where H^{B} is an IBM2 Hamiltonian [9],

$$\begin{aligned} H^{\mathrm{B}} = & + \epsilon_{d_\nu} n_{d_\nu} + \epsilon_{d_\pi} n_{d_\pi} + \kappa (Q_\nu^{\mathrm{B}} \cdot Q_\pi^{\mathrm{B}}) \\ & + \xi_1 ([d_\nu^\dagger d_\pi^\dagger]^{(1)} [\tilde{d}_\pi \tilde{d}_\nu]^{(1)}) \\ & + \frac{1}{2} \xi_2 [(d_\nu^\dagger s_\pi^\dagger - d_\pi^\dagger s_\nu^\dagger)(\tilde{d}_\nu s_\pi - \tilde{d}_\pi s_\nu)] \\ & + \frac{1}{2} \sum_{L=0,2,4} c_L ([d_\nu^\dagger d_\nu^\dagger]^{(k)} [\tilde{d}_\nu \tilde{d}_\nu]^{(k)}) \\ & + \frac{1}{2} \sum_{L=0,2,4} c_L ([d_\pi^\dagger d_\pi^\dagger]^{(k)} [\tilde{d}_\pi \tilde{d}_\pi]^{(k)}), \end{aligned} \tag{2}$$

where

$$Q_\nu^{\mathrm{B}} = d_\nu^\dagger s_\nu + s_\nu^\dagger \tilde{d}_\nu + \chi_\nu [d_\nu^\dagger \tilde{d}_\nu]^{(2)}, \tag{3}$$

$$Q_\pi^{\mathrm{B}} = d_\pi^\dagger s_\pi + s_\pi^\dagger \tilde{d}_\pi + \chi_\pi [d_\pi^\dagger \tilde{d}_\pi]^{(2)} \tag{4}$$

are the boson quadrupole operators. H^{F} is the Hamiltonian of the odd fermion,

$$H^{\mathrm{F}} = \sum_i \epsilon_i n_i, \tag{5}$$

and V^{BF} is the interaction between the bosons and the odd particle,

$$\begin{aligned} V^{\mathrm{BF}} = & \sum_{i,j} \Gamma_{ij} ([a_i^\dagger \tilde{a}_j]^{(2)} Q_\rho^{\mathrm{B}}) + \sum_{i,j} \Gamma'_{ij} ([a_i^\dagger \tilde{a}_j]^{(2)} Q_{\rho'}^{\mathrm{B}}) \\ & + \sum A_i n_i n_{d_\rho} + \sum A'_i n_i n_{d_{\rho'}} \\ & + \sum_{i,j} \Lambda_{ki}^j \{ :[[d_\rho^\dagger \tilde{a}_j]^{(k)} a_i^\dagger s_\rho]^{(2)} : [s_{\rho'}^\dagger \tilde{d}_{\rho'}]^{(2)} + \mathrm{H.c.} \} \\ & + BJL_\rho + B' JL_{\rho'}. \end{aligned} \tag{6}$$

The modified annihilation operator $\tilde{d}$ is defined by $\tilde{d}_m = (-1)^m d_{-m}$. The symbols ρ and ρ' denote $\pi(\nu)$ and $\nu(\pi)$ if the odd fermion is a proton (a neutron). The creation operator of the odd particle is written as $a_{jm}^\dagger$, while the modified annihilation operator is defined as $\tilde{a}_{jm} = (-1)^{j-m} a_{j-m}$. The orbital dependence of the quadrupole and the exchange interactions are [10]

$$\Gamma_{i,j} = (u_i u_j - v_i v_j) Q_{i,j} \Gamma, \tag{7}$$

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TABLE I. IBM2 parameters taken from Ref. [11]. The unit is MeV except for the dimensionless χ_ν. The parameters $\chi_\pi=-0.3$ and $\xi_1=0.2$ MeV are fixed.

Odd nuclei	Core nucleus	ϵ_π	ϵ_ν	κ	χ_ν	c_0	c_2	c_4	ξ_2
^{105}Pd	^{104}Pd	0.860	0.920	−0.18	−0.48	−0.39	−0.13	0	0.05
^{105}Rh, ^{107}Pd	^{106}Pd	0.760	0.844	−0.16	−0.22	−0.45	−0.2	0.1	0.05
^{107}Rh, ^{109}Pd	^{108}Pd	0.716	0.828	−0.14	−0.10	−0.325	−0.25	−0.01	0.04
^{109}Rh	^{110}Pd	0.600	0.780	−0.125	0.0	−0.26	−0.29	−0.03	0.04

$$\Lambda_{k,i}^{j}=-\beta_{k,i}\beta j,k\left(\frac{10}{N_\rho(2j_k+1)}\right)^{1/2}\Lambda, \tag{8}$$

where

$$\beta_{i,j}=(u_iv_j+v_iu_j)Q_{i,j}, \tag{9}$$

$$Q_{i,j}=\langle l_i,\tfrac{1}{2},j_i||Y^{(2)}||l_j,\tfrac{1}{2},j_j\rangle. \tag{10}$$

III. CALCULATIONS

A. Hamiltonian and energy levels

We describe the even-even Pd core nuclei in the IBM2. In Table I we show the cores for the considered Rh and Pd isotopes and the IBM2 parameters, microscopically derived, from Ref. [11].

1. Rh isotopes

The proton single-particle energies for Rh isotopes calculated using a Woods-Saxon potential, whose parameters are suggested by Wyss [12], are shown in Table II.

The corresponding quasiparticle energies and occupation probabilities are obtained solving the BCS equations with a pairing gap $\Delta=12A^{-1/2}$ MeV. In Table III we present the parameter values in V^{BF} interaction potential.

The results of the calculations are shown in Fig. 1. The yrast and yrare levels of $I^\pi=7/2^+$, $9/2^+$, $5/2^+$ are well described. The observed levels with smaller spins, i.e., with $I^\pi=1/2^+$ and $I^\pi=3/2^+$ appear much lower than the calculated ones. This is systematically seen in all three isotopes. This effect is not a consequence of shortcoming of our parametrization. In the zeroth order the lowest levels with $I^\pi=1/2^+$, $3/2^+$ are based on the 4_1^+ core state and the standard IBFM2 interaction cannot sufficiently decrease their energy. The lowering of the low-spin states is the effect of the exchange interaction, which has a strong dependence on occupation probabilities of fermion configurations. Nevertheless, even when the occupation probabilities are calculated taking into account all orbits of the 28-50 shell, the resulting excitation energies of low-spin states are modified less than 30 keV in respect to the present calculation. The low-spin states are based on the $g_{9/2}$ configuration, with very small contributions of other fermion components from the 50-82 shell. As long as one wants to have a reasonable description of the ground state and a few excited states of higher spin, any calculation with a spherical or weakly deformed core and fermions in the single-j shell and the shell above (or below), with the Fermi level in the single-j shell, gives results similar to those in the present calculation. On the other hand, the coupling to a deformed core state(s) would sizeably decrease the excitation energy of low-spin states. This could be an evidence that wave functions of low-spin states of positive parity in Rh isotopes have sizable components based on fermion configurations coupled to the intruder core state, as it is proposed for some states in Pd isotopes, too (see Sec. III C). Therefore these levels cannot be described appropriately in this approach and their wave functions are not realistic enough to give a satisfactory description of the electromagnetic decays.

TABLE II. Single-particle energies (MeV) of the proton orbitals in Rh.

Orbit	^{105}Rh	^{107}Rh	^{109}Rh
$g_{9/2}$	0.00	0.00	0.00
$d_{5/2}$	5.75	5.79	5.83
$g_{7/2}$	6.54	6.45	6.37
$s_{1/2}$	8.18	8.26	8.33
$d_{3/2}$	8.72	8.76	8.78
$h_{11/2}$	8.91	8.92	8.93

2. Pd isotopes

The neutron single-particle energies for Pd isotopes are shown in Table IV. Those for $N>50$ are taken from Ref. [16], except the energies of the $g_{7/2}$ orbit which are a little lowered. The energy of $g_{9/2}$ is fixed to -5 MeV.

The corresponding quasiparticle energies and occupation probabilities are obtained solving the BCS equations with the pairing strength $G=23A^{-1}$ MeV.

The parameter values used in V^{BF} are shown in Table V, while the results of the calculations are shown in Fig. 2. In the Pd isotopes the levels with $I^\pi=1/2^+$, $3/2^+$, $5/2^+$, and $7/2^+$ are generally well described. The structure of $5/2^+$ and $7/2^+$ levels is of special interest for this work because β transitions from $7/2^+$ ground states of parent Rh nuclei occur

TABLE III. Parameters in the boson-fermion interaction (MeV) for Rh.

Isotope	Γ	Γ'	A_ρ	$A_{\rho'}$	Λ	B
^{105}Rh	0	0.4	−0.2	0	0.65	0
^{107}Rh	0	0.4	−0.2	0	0.60	0
^{109}Rh	0	0.5	−0.2	0	0.45	0

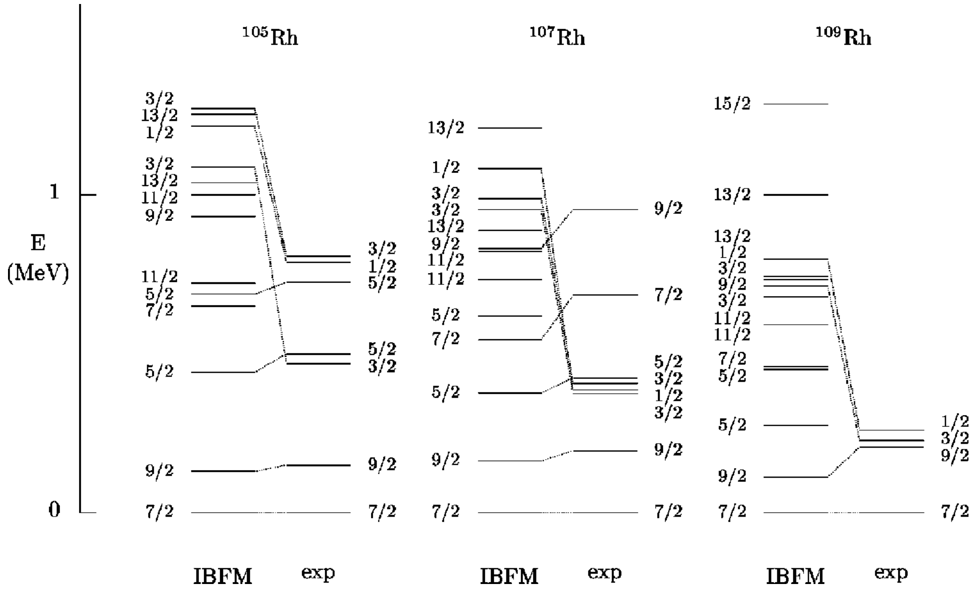

FIG. 1. Comparison between the calculated (IBFM) and the experimental (exp) energy levels of positive parity in ^{105,107,109}Rh. The experimental data are taken from Refs. [13–15].

mainly to these levels. In odd neutron nuclei in this mass region, at low energies, systematically two $7/2^+$ levels appear. Since the work of Casten *et al.* [17] it is known that the third $7/2^+$ level is not present at low energy in ^{109}Pd and the only exception might be the ^{107}Pd, where below 0.4 MeV of excitation energy three $7/2^+$ levels are reported [14]. According to Ref. [17] one of these $7/2^+$ assignments may be incorrect or incomplete (i.e., corresponds to an unresolved doublet in the charged particle spectra). In the present work we have supposed that it is the case of the level at 0.367 MeV (not shown in Fig. 2), which is not populated in the β decay.

B. Electromagnetic properties

The electromagnetic transition operators are

$$T^{(E2)}=e^{\rm B}_{\pi}Q^{\rm B}_{\pi}+e^{\rm B}_{\nu}Q^{\rm B}_{\nu}+\sum_{i,j} e'_{i,j}[a_i^{\dagger}\tilde{a}_j]^{(E2)}, \tag{11}$$

where

$$e'_{i,j}=-\frac{1}{\sqrt{5}}(u_iu_j-v_iv_j)\langle i||r^2Y^{(2)}||j\rangle. \tag{12}$$

For $M1$,

$$T^{(M1)}=\sqrt{\frac{3}{4\pi}}\left(g^{\rm B}_{\pi}L^{\rm B}_{\pi}+g^{\rm B}_{\nu}L^{\rm B}_{\nu}+\sum_{i,j} e^{(1)}_{i,j}[a_i^{\dagger}\tilde{a}_j]^{(1)}\right), \tag{13}$$

where

$$e^{(1)}_{i,j}=-\frac{1}{\sqrt{3}}(u_iu_j+v_iv_j)\langle i||g_l\mathbf{l}+g_s\mathbf{s}||j\rangle. \tag{14}$$

The boson effective charges $e^{\rm B}_{\pi}=0.12\ e$ b and $e^{\rm B}_{\nu}=0.10\ e$ b are taken from Ref. [11]. For the odd fermion, the effective charges 0.5 e and 1.5 e are taken for the neutrons and the protons, respectively. The boson g factors $g^{\rm B}_{\pi}=1.25\mu_N$ and $g^{\rm B}_{\nu}=-0.10\mu_N$ are taken from Ref. [11], where they have been calculated microscopically. The fermion g factors are the Schmidt values with spin factors quenched by a factor 0.6.

In Table VI the results of the calculations are shown. In ^{105}Rh the calculated values are consistent with the experimental data except for $B(M1;\ 9/2^+_1\rightarrow 7/2^+_1)$ for which the calculated value is somewhat smaller than the experimental one. The good agreement between the calculated and experimental values for the magnetic moment of $7/2^+_1$ shows that the wave function for this level is right and, therefore, can be taken as accurate for the calculation of β decay. In ^{105}Pd the calculated values are complexively consistent with the experimental ones. The behavior of the measured magnetic moments are reproduced but not the magnitude. In the isotopes with $A\geqslant 107$ there are only a few experimental data. The

TABLE IV. Single-particle energies (MeV) of the neutron orbitals in Pd, taken from Ref. [13]. The energy of $g_{7/2}$ has been changed.

Orbit	^{105}Pd	^{107}Pd	^{109}Pd
$g_{9/2}$	−5.000	−5.000	−5.000
$d_{5/2}$	−0.053	−0.045	−0.038
$g_{7/2}$	1.000	1.000	0.95
$s_{1/2}$	2.019	2.078	2.065
$h_{11/2}$	2.546	2.539	2.533
$d_{3/2}$	3.007	2.977	2.948

TABLE V. Parameters in the boson-fermion interaction (MeV) for Pd.

Isotope	Γ	Γ'	A_{ρ}	$A_{\rho'}$	Λ	B
^{105}Pd	0.10	1.10	−0.15	0	0.62	0
^{107}Pd	0.10	1.30	−0.15	0	0.57	0
^{109}Pd	0.10	1.35	−0.25	0	0.50	0.02

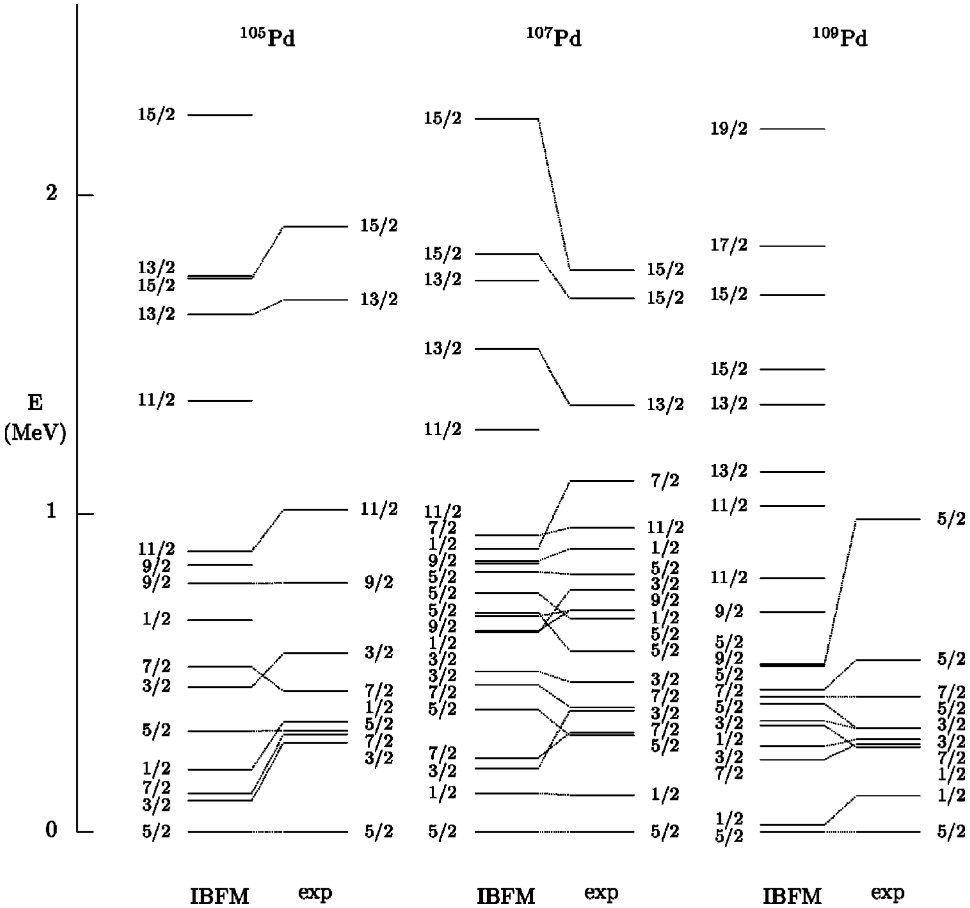

FIG. 2. Comparison between the calculated (IBFM) and the experimental (exp) energy levels of positive parity in ^{105,107,109}Pd. The experimental data are taken from Refs. [13–15].

calculated and experimental data are consistent except for Rh isotopes, where the experimental data concern the $1/2_1^+$, $3/2_1^+$ levels for which the theoretical wave functions are not accurate, as mentioned above.

C. β decay

The Fermi $\Sigma_k t^{\pm}(k)$ and the Gamow-Teller $\Sigma_k t^{\pm}(k)\sigma(k)$ transition operators [18] can be expressed in the framework of IBFM2. They can be constructed by the transfer operators [3,4,10,18]

$$A_m^{\dagger(j)}=\zeta_j a_{jm}^{\dagger}+\sum_{j'}\zeta_{jj'}s^{\dagger}[\tilde{d}a_{j'}^{\dagger}]_m^{(j)}(\Delta n_j=1,\ \Delta N=0),\tag{15}$$

$$B_m^{\dagger(j)}=\theta_j s^{\dagger}\tilde{a}_{jm}+\sum_{j'}\theta_{jj'}[d^{\dagger}\tilde{a}_{j'}]_m^{(j)}(\Delta n_j=-1,\ \Delta N=1).\tag{16}$$

The former creates a fermion, while the latter annihilates a fermion simultaneously creating a boson. Either operator increases the quantity n_j+2N by one unit. The conjugate operators are

$$\begin{aligned}\tilde{A}_m^{(j)}&=(-1)^{j-m}\{A_{-m}^{\dagger(j)}\}^{\dagger}\\&=\zeta_j^{*}\tilde{a}_{jm}+\sum_{j'}\zeta_{jj'}^{*}s[d^{\dagger}\tilde{a}_{j'}]_m^{(j)}(\Delta n_j=-1,\ \Delta N=0),\end{aligned}\tag{17}$$

$$\begin{aligned}\tilde{B}_m^{(j)}&=(-1)^{j-m}\{B_{-m}^{\dagger(j)}\}^{\dagger}\\&=-\theta_j^{*}sa_{jm}^{\dagger}-\sum_{j'}\theta_{jj'}^{*}[\tilde{d}a_{j'}^{\dagger}]_m^{(j)}(\Delta n_j=1,\ \Delta N=-1),\end{aligned}\tag{18}$$

where the asterisks mean complex conjugate. These decrease the quantity n_j+2N by one unit.

The IBFM image of the Fermi $\Sigma_k \mathbf{t}^{\pm}(k)$ and the Gamow-Teller transition operator $\Sigma_k t^{\pm}(k)\boldsymbol{\sigma}(k)$ are written as

$$O^{\mathrm{F}}=\sum_j -\sqrt{2j+1}[P_{\pi}^{(j)}P_{\nu}^{(j)}]^{(0)},\tag{19}$$

$$O^{\mathrm{GT}}=\sum_{j'j}\eta_{j'j}[P_{\pi}^{(j')}P_{\nu}^{(j)}]^{(1)},\tag{20}$$

where

$$\begin{aligned}\eta_{j'j}&=-\frac{1}{\sqrt{3}}\langle l'\tfrac{1}{2};j'||\boldsymbol{\sigma}||l\tfrac{1}{2};j\rangle\\&=-\delta_{l'l}\sqrt{2(2j'+1)(2j+1)}W(lj'\tfrac{1}{2}1;\tfrac{1}{2}j).\end{aligned}\tag{21}$$

The transfer operator $P_{\rho}^{(j)}$ are chosen from Eqs. (15)–(18) depending on the nuclei. In the present case,

TABLE VI. Electromagnetic moments and transitions.

Nucleus	Observable	Cal.	Exp.
^{105}Rh	$B(E2;\ 9/2_1^+\to 7/2_1^+)$ (e^2 b^2)	0.164	>0.22
	$B(E2;\ 3/2_1^+\to 7/2_1^+)$	0.031	>0.0062
	$B(E2;\ 5/2_2^+\to 5/2_1^+)$	0.000	$>0.000\,62$
	$B(E2;\ 5/2_2^+\to 3/2_1^+)$	0.047	$>0.000\,18$
	$B(E2;\ 5/2_2^+\to 9/2_1^+)$	0.057	$>0.000\,08$
	$B(E2;\ 5/2_2^+\to 7/2_1^+)$	0.040	$>3.50\times 10^{-6}$
	$B(E2;\ 1/2_1^+\to 5/2_1^+)$	0.052	$>0.000\,12$
	$B(E2;\ 1/2_1^+\to 3/2_1^+)$	0.0002	>0.001
	$\mu(7/2_1^+)$ (μ_N)	4.628	4.452 (10)
	$B(M1;\ 9/2_1^+\to 7/2_1^+)$ (μ_N^2)	0.0019	>0.03
	$B(M1;\ 5/2_2^+\to 5/2_1^+)$	0.057	$>2.1\times 10^{-5}$
	$B(M1;\ 5/2_2^+\to 3/2_1^+)$	0.308	$>8.0\times 10^{-6}$
	$B(M1;\ 5/2_2^+\to 7/2_1^+)$	0.195	>0.0005
	$B(M1;\ 1/2_1^+\to 3/2_1^+)$	2.068	>0.002
^{105}Pd	$Q(5/2_1^+)(e$ b)	0.499	0.660 (11)
	$B(E2;\ 3/2_1^+\to 5/2_1^+)$	0.0063	0.0029 (11)
	$B(E2;\ 5/2_2^+\to 3/2_1^+)$	0.0110	<0.9
	$B(E2;\ 5/2_2^+\to 5/2_1^+)$	0.0051	0.0044 (6)
	$B(E2;\ 1/2_1^+\to 3/2_1^+)$	0.0474	<0.028
	$B(E2;\ 1/2_1^+\to 5/2_1^+)$	0.0124	0.0103 (6)
	$B(E2;\ 7/2_2^+\to 5/2_1^+)$	0.129	0.044 (18)
	$B(E2;\ 9/2_1^+\to 7/2_2^+)$	0.0151	<0.0044
	$B(E2;\ 9/2_1^+\to 5/2_1^+)$	0.0327	0.059 (9)
	$\mu(5/2_1^+)$	-0.885	-0.642 (3)
	$\mu(5/2_2^+)$	0.482	0.95 (10)
	$\mu(3/2_1^+)$	0.148	-0.074 (13)
	$B(M1;\ 3/2_1^+\to 5/2_1^+)$	0.083	0.027 (11)
	$B(M1;\ 5/2_2^+\to 3/2_1^+)$	0.0270	0.0205 (27)
	$B(M1;\ 5/2_2^+\to 5/2_1^+)$	0.0299	0.0286 (36)
	$B(M1;\ 1/2_1^+\to 3/2_1^+)$	0.0003	0.0269 (18)
	$B(M1;\ 7/2_2^+\to 5/2_1^+)$	0.0535	0.116 (43)
	$B(M1;\ 9/2_1^+\to 7/2_2^+)$	0.038	<0.232
^{107}Rh	$B(E2;\ 3/2_1^+\to 7/2_1^+)$	0.0160	0.000 48 (6)
^{107}Pd	$B(E2;\ 1/2_1^+\to 5/2_1^+)$	0.0024	0.001 75 (21)
^{109}Rh	$B(E2;\ 3/2_1^+\to 7/2_1^+)$	0.0374	0.000 054 (2)
	$B(E2;\ 1/2_1^+\to 3/2_1^+)$	0.0125	0.56 (37)
	$B(M1;\ 1/2_1^+\to 3/2_1^+)$	1.8385	0.001 04 (27)
^{109}Pd	$B(E2;\ 1/2_1^+\to 5/2_1^+)$	0.0033	0.004 11 (56)

$$P_\pi^{(j')}=\tilde{A}_\pi^{(j')}, \tag{22}$$

$$P_\nu^{(j)}=\tilde{B}_\nu^{(j)}. \tag{23}$$

The squares of the β-decay matrix elements are

$$\langle M_{\rm F}\rangle^2=\frac{1}{2I_i+1}|\langle I_f||O^{\rm F}||I_i\rangle|^2, \tag{24}$$

$$\langle M_{\rm GT}\rangle^2=\frac{1}{2I_i+1}|\langle I_f||O^{\rm GT}||I_i\rangle|^2, \tag{25}$$

from which the ft value is calculated by

$$ft=\frac{6163}{\langle M_{\rm F}\rangle^2+(G_{\rm A}/G_{\rm V})^2\langle M_{\rm GT}\rangle^2} \tag{26}$$

in units of second where $(G_{\rm A}/G_{\rm V})^2=1.59$.

Now we estimate the coefficients η_j, $\eta_{jj'}$, θ_j, $\theta_{jj'}$ appearing in Eqs. (15)–(18), following the formulation of Ref. [3]:

$$\zeta_j=u_j\frac{1}{K_j'}, \tag{27}$$

$$\zeta_{jj'}=-v_j\beta_{j'j}\left(\frac{10}{N(2j+1)}\right)^{1/2}\frac{1}{KK_j'}, \tag{28}$$

$$\theta_j=\frac{v_j}{\sqrt{N}}\frac{1}{K_j''}, \tag{29}$$

$$\theta_{jj'}=u_j\beta_{j'j}\left(\frac{10}{2j+1}\right)^{1/2}\frac{1}{KK_j''}, \tag{30}$$

where N is N_π or N_ν, depending on the transfer operator, and K, K_j', K_j'' are determined by

$$K=\left(\sum_{jj'}\beta_{jj'}^2\right)^{1/2}, \tag{31}$$

and the conditions

$$\sum_{\alpha J}\langle \text{odd};\,\alpha J||A^{\dagger j}||\text{even};\,0_1^+\rangle^2=(2j+1)u_j^2, \tag{32}$$

$$\sum_{\alpha J}\langle \text{even};\,0_1^+||B^{\dagger j}||\text{odd};\,\alpha J\rangle^2=(2j+1)v_j^2. \tag{33}$$

For Rh, because the odd proton is a hole in respect to the boson core, u_j and v_j are interchanged in Eqs. (27)–(33). In Fig. 3 we present $\log_{10}ft$ values for β^- decay of ^{105,107,109}Rh. The overall features are reasonably well described. This is correct for transitions to $5/2_1^+$ and $7/2_1^+$ where both the trend and the magnitude are well explained as a function of mass number. An increase of $\log_{10}ft$ values for the transitions to $7/2_1^+$ follows a development of its fermion contributions to the wave functions from an almost pure $g_{7/2}$ (for ^{105}Pd) to a predominant $d_{5/2}$ structure (^{109}Pd). The $\log_{10}ft$ values to $5/2_1^+$, in all three isotopes, are higher than to $5/2_2^+$, due to small contribution of $g_{7/2}$ in respect to the predominant $d_{5/2}$ configuration in ground states in Pd isotopes. The behavior of the $\log_{10}ft$ values for these transitions to $5/2_1^+$, as a function of mass number, is therefore governed more by core structure than by the fermion components. In fact going toward mid shell, core nuclei become more deformed and therefore many small contributions to the transition matrix elements, coming from the boson part of the transition operator, can be important.

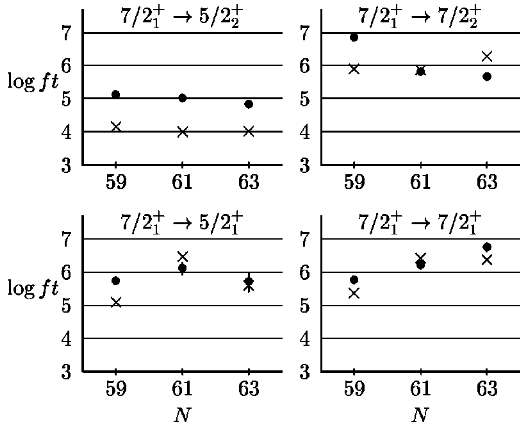

FIG. 3. Comparison of $\log_{10}ft$ values in the decays ${}_{45}\mathrm{Rh}_{N+1} \to {}_{46}\mathrm{Pd}_N$. The experimental data are presented by ● while the calculated values are shown by ×.

We notice that once the wave functions are determined in IBFM2 calculations of energy levels, the $\log_{10} ft$ values are obtained in a parameter-free calculation. In fact, in constrast to shell model calculations of $\log_{10} ft$ values, we do not use any additional normalization.

Both the experimental data and our calculations give an almost constant slowly decreasing $\log_{10} ft$ value for the transitions to $5/2_2^+$ as a function of mass number, but the theoretical ft is by a factor 10 smaller. The calculated wave functions of $5/2_2^+$ have predominant $g_{7/2}$ components that slowly decrease going towards ^{109}Pd, which explains the trend. The magnitude, however, suggests that $5/2_2^+$ may contain appreciable amount of components outside the model space. In fact, it is known [11,19] that even Pd isotopes, in this mass region, have intruder 0^+ states and therefore components based on fermion configurations coupled to the intruder 0^+ can alter transition matrix elements in a sizeable way. Evidently, the higher the level is, more intruder components outside the model space it can have. This could be the case also for the transition to the $7/2_2^+$ where our calculations fail to reproduce the trend of $\log_{10} ft$ as a function of mass number.

In addition we notice that our calculations reproduce reasonably well other known $\ln ft$ values for these nuclei. In ^{107}Pd the calculated $\ln ft$ for $7/2_1^+ \to 5/2_3^+$ and $7/2_1^+ \to 5/2_4^+$ are 6.41 and 5.09, respectively, while the experimental data are 6.10 (± 0.10) and 5.30 (± 0.10). In ^{109}Pd the calculated $\ln ft$ $7/2_1^+ \to 5/2_3^+$ is 4.87 and the experimental value is 5.49 (± 0.08).

IV. CONCLUSIONS

We have carried out a systematic analysis of energy levels and electromagnetic properties of a sequence of odd-A Rh and Pd isotopes in IBFM2. The main goal was to obtain realistic wave functions for calculation of β^- decays from ground states of parent Rh nuclei into respective Pd nuclei. The agreement between calculations and experiment for energy levels and electromagnetic properties is generally good, except for levels with small spins in Rh. The available set of experimental data of $\log_{10} ft$ values, as well as its dependence on the mass numbers, is well described in this approach, except for β decays into $5/2_2^+$ and $7/2_2^+$. The observed discrepancies might be attributed to the components in the wave functions deriving from coupling of neutron configurations to the intruder levels of the respective cores. We especially point out the fact that once the wave functions have been obtained in IBFM2 fit of energy levels, the $\log_{10} ft$ values have been obtained in parameter-free calculations without any normalization of the theoretical results.

We can conclude that the proton-neutron interacting boson-fermion model is appropriate for calculations of β-decay properties in odd nuclei, and therefore, our aim is to extend this kind of calculations to other mass regions.

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