Three-body calculation of the structure of ${}^{9}_{\Lambda}$Be

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The hypernucleus ${}^{9}_{\Lambda}$Be is investigated using the $\alpha+\Lambda+\alpha$ cluster model. The corresponding Faddeev equations are solved for different α-α and Λ-α interactions that describe both ^{8}Be and ${}^{5}_{\Lambda}$He energy spectra. For the ground state $J^{\pi}=\frac{1}{2}^{+}$ and excited states $\frac{3}{2}^{+}$ and $\frac{5}{2}^{+}$ we include the Coulomb repulsion between the α's and calculate, in addition to the energy eigenvalues, the Coulomb energy, the α- and Λ-particle mean radius, the rms charge radius, the electric quadrupole moment (Q), as well as the magnetic dipole (μ) and octopole (μ_3) moments. To the best of our knowledge these observables have not yet been measured and, therefore, our calculation constitutes the first prediction. Structural differences between ^{9}Be and ${}^{9}_{\Lambda}$Be lead to values of Q and μ that have opposite sign. Unlike previous theoretical work we find only two degenerate negative parity resonances $\frac{1}{2}^{-}(\frac{3}{2}^{-})$ but, in addition, we get two degenerate positive parity resonances with $J^{\pi}=\frac{7}{2}^{+}(\frac{9}{2}^{+})$ at higher energy which, together with the bound states, complete the positive parity rotational band.

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I. INTRODUCTION

As pointed out by Feshbach [1] a number of years ago, one of the main purposes of hypernucleon physics is to study new many-body spectroscopy through the implant of an "impurity" in the condensed matter sense to unveil dynamical symmetries that are forbidden in ordinary nuclei by the Pauli principle. One nucleus where this phenomenon plays an important role is ${}^{9}_{\Lambda}$Be that, *vis-à-vis* ^{9}Be, presents, not only a very different spectra of bound states and resonances, but also different dynamical structure, possibly leading to surprising results for observables such as the rms charge radius, the quadrupole moment Q, and the magnetic moment μ. This investigation, together with a new search for resonances in the continuum, is the main goal of the present work that follows the traditional three-body α-cluster approach that has been used before by some of us to study ^{9}Be [2,3] and also by others [4,5] for ${}^{9}_{\Lambda}$Be. A review on this and related subjects may be found in Refs. [6,7].

Unlike ^{9}Be, which is typically a Boromean nucleus due to neither ^{5}He nor ^{8}Be being stable nuclei, ${}^{9}_{\Lambda}$Be may be thought as ${}^{5}_{\Lambda}$He$+\alpha$ due to the existence of ${}^{5}_{\Lambda}$He as a stable hypernucleus with $B_{\Lambda}\simeq$ 3.12 MeV. The Λ, being distinct from the other nucleons, can occupy the *s*-shell, unlike the fifth nucleon in ^{5}He. Therefore the Λ-α interaction is predominantly attractive in *s* wave, while in that same partial wave the n-α interaction is effectively repulsive. One immediate consequence of this is that the binding force that makes ^{9}Be a stable nucleus comes from the *p*-wave n-α interaction, while in ${}^{9}_{\Lambda}$Be comes from the *s* wave Λ-α interaction, possibly leading to structural differences in the observables associated with their respective bound state wave functions. Although the small binding of ${}^{5}_{\Lambda}$He remains an enigma in terms of *ab initio* calculations based on the ΛN interaction, our three-body model takes the conventional wisdom of assuming that both Λ-α and α-α interactions, no matter their origin or underlying concept, are fitted to the spectra of both ${}^{5}_{\Lambda}$He and ^{8}Be. Our calculations are based on the solution of the Faddeev equations [8] that have been modified to include the Coulomb force [9] between the α's. This is one major difference relative to the recent work by Oryu *et al.* [4].

In Sec. II we describe the Λ-α and α-α potential we use and in Sec. III we show the main results for the ground state and excited states of ${}^{9}_{\Lambda}$Be. In Sec. IV we present our main findings for the continuum states and how they may be interpreted *vis-à-vis* the existing data. Finally in Sec. V we present the conclusions.

II. CLUSTER MODEL OF ${}^{9}_{\Lambda}$Be

Three- and four-body studies of ${}^{3}_{\Lambda}$H [10] and ${}^{4}_{\Lambda}$He [11] are now possible based on realistic NN and ΛN (ΛN-ΣN) interactions [12]. Nevertheless for $A>4$ one is still required to use quantum many-body methods based on mean field theory or attempt to describe the system in terms of fewer degrees of freedom by using clusters and effective interactions between them. For ${}^{9}_{\Lambda}$Be, given the small binding energy of ${}^{5}_{\Lambda}$He relative to the α-particle binding and the large energy gap between $\Lambda+{}^{4}$He and ${}^{4}_{\Lambda}$He$+n$, the natural choice is a three-body cluster model based on two α particles and the hyperon Λ. The difficulty now lies on the choice of α-α and Λ-α interactions that are required to map the low lying spectra of ^{8}Be and ${}^{5}_{\Lambda}$He and provide, in the case of α-α, a reasonable description of Pauli blocking between nucleons in separate clusters.

A number of α-α interactions already exist; some treat Pauli blocking by means of a short range repulsion, others by including Pauli forbidden states that are subsequently removed. Although the latter approach seems to lead to a better description of the spectra of ^{12}C as a bound system of three α particles [13], we have tried both methods in ^{9}Be and found no great differences. Therefore, in the present work,

TABLE I. α-α states (MeV) for all three α-α potentials in the absence of Coulomb repulsion.

	UIM	AB	CB
$L=0$	-1.34	-1.33	-1.27
$L=2$	1.30-i(0.29)	1.36-i(0.30)	1.49-i(0.44)
$L=4$	9.49-i(1.54)	9.58-i(1.18)	10.20-i(1.87)

we use, not only Ali-Bodmer (AB) [14] and Chien-Brown (CB) [15] potentials, but also one derived in the framework of the resonating group method (RGM) through the unitary-interpolation method (UIM) [16] that is nonlocal due to the exchange nature of the RGM core potential. The corresponding energy spectra is shown in Table I for all three potentials in the absence of the Coulomb force.

For the Λ-α interaction we use a number of effective potentials that have been developed over the years using different folding techniques or phenomenology. They are the Tang Herndon (TH) [17] that has a simple Gaussian shape, the Dalitz (DA), and the Deloff-type (DE) potentials that were derived by folding [18] based on Λ-N interactions with a hard core proposed by Dalitz *et al.* [19] and Deloff [20], and the Maeda-Schmid [21] (MS) that uses two Woods-Saxon phenomenological potentials to fit the Λ-α interaction also calculated by folding. All potentials reproduce the ${}^{5}_{\Lambda}$He binding energy reasonably well. A more complete comparison of these potentials may be found in Ref. [4]. Recently a more sofisticated Λ-α potential model was derived [22] that seems to favor the Dalitz phase shifts. Nevertheless, given the uncertainty on this issue, for lack of experimental information on Λ-α phases, we feel that the chosen four potentials span a sufficiently broad range of possibilities to accomodate reality.

Although there is sufficient theoretical support [6] for the existence of a three-body force in $\alpha+\Lambda+\alpha$ due to strong ΛN-ΣN coupling, we find no compelling reason at this time to include such force in our calculations, given the large theoretical uncertainty on the Λ-α interaction. The same may be said about including a spin-orbit force in Λ-α since recent calculations [23] indicate that such term is very small, leading to 80–200 KeV splitting between $\frac{3}{2}^{+}$ and $\frac{5}{2}^{+}$ states. This has been recently confirmed [24] by experimental work performed at Brookhaven National Laboratory where the splitting between the $\frac{3}{2}^{+}$ and $\frac{5}{2}^{+}$ states was measured to be $\simeq 31.4$ keV.

For the chosen α-α and Λ-α interactions we solve the bound state Faddeev equations [8] including the Coulomb repulsion between the α's. The general form of the equations was first proposed by Lehman *et al.* [9] and subsequently generalized by Cravo [3] to study the structure of ^{9}Be. The equations we use here are the same as in Ref. [3] and therefore we refer to this publication for details. The α-α interaction is taken as a local interaction in different α-α partial waves (s, d, and g waves) or as a finite rank operator in the case of the RGM-based potential [16]. The Λ-α interaction is always expanded as a sum of separable terms using the method developed by Koike [25]. In Table II we show the convergence of our calculation in terms of rank "r" for different Λ-α interactions, and compare the results with those obtained by Oryu *et al.* [4] for UIM potential when only Λ-α s waves are included. Since the Λ-αp and d waves give a large contribution, our calculations include all Λ-α partial waves up to $l\leqslant 2$. The f wave contribution is negligible (less than 15 keV for the binding energy). This is shown in Table III for the ground state of ${}^{9}_{\Lambda}$Be in the absence of Coulomb repulsion between the α's. The Λ-α p wave contribution can be as high as 2.5 MeV for DA potential with UIM between the α's, but depends strongly on Λ-α and α-α potentials one uses. For AB and CB α-α potentials the Λ-α p-wave contribution is considerably smaller (0.4–1.4 MeV), but is the biggest for DA that has the strongest short range repulsion of all four Λ-α potentials. The same may be said about the d-wave contribution. Although the Λ-α p and d waves are not well constrained by either theory or experiment, they always increase the binding energy of ${}^{9}_{\Lambda}$Be, independently of the Λ-α potential one uses, and for this reason they should be included so that one may compare the results with other variational calculations where the Λ-α potential is usually included in all partial waves. Furthermore the strengh of the Λ-α p-wave interaction may be associated with the possible existence of ^{9}Be-like states in ${}^{9}_{\Lambda}$Be. This is another reason why we have included Λ-α p waves in our calculation, in spite of the uncertainty associated with their strength.

TABLE II. Convergence of the ground state binding energy (MeV) with rank "r" in the separable expansion of the s-wave Λ-α interaction. The α-α potential is UIM.

	TH	MS	DE	DA
$r=1$	-4.52	-6.02	-5.15	-5.72
$r=2$	-6.78	-7.01	-7.18	-7.43
$r=3$	-7.09	-7.02	-7.36	-7.71
$r=4$	-7.10	-7.02	-7.36	-7.49
Ref. [4]	-7.13	-7.03	-7.37	-7.47

TABLE III. Ground state energy (MeV) for different α-α and Λ-α potentials in the absence of the Coulomb repulsion between the α's. The "l" is the relative Λ-α orbital angular momentum.

		UIM	AB	CB
TH	$l\leqslant 0$	-7.10	-7.26	-7.32
	$l\leqslant 1$	-8.49	-7.65	-7.74
	$l\leqslant 2$	-8.57	-7.72	-7.82
MS	$l\leqslant 0$	-7.02	-7.56	-7.63
	$l\leqslant 1$	-8.95	-8.39	-8.47
	$l\leqslant 2$	-9.07	-8.49	-8.57
DE	$l\leqslant 0$	-7.36	-8.02	-8.10
	$l\leqslant 1$	-9.50	-9.02	-9.10
	$l\leqslant 2$	-9.66	-9.14	-9.21
DA	$l\leqslant 0$	-7.49	-8.51	-8.58
	$l\leqslant 1$	-9.95	-9.95	-9.92
	$l\leqslant 2$	-10.18	-10.04	-10.00

TABLE IV. $\frac{1}{2}^+$ binding energies (MeV) for different α-α and Λ-α interactions. The Coulomb repulsion has been included in all partial waves needed for convergence.

	UIM	AB	CB
TH	−5.96	−5.98	−6.02
MS	−6.50	−6.73	−6.75
DE	−7.08	−7.36	−7.37
DA	−7.74	−8.27	−8.19
Expt. [26]		-6.71	

III. STRUCTURE OF ${}^{9}_{\Lambda}$Be BOUND STATES

In the absence of a Λ-α spin-orbit force, two of the three bound states we find are degenerate ($J^\pi = \frac{3}{2}^+$ and $\frac{5}{2}^+$). The ground state, $J^\pi = \frac{1}{2}^+$, lies 2.3–3.0 MeV below the excited states. Our results are shown in Table IV for $J^\pi = \frac{1}{2}^+$ and in Table V for the excited states $\frac{3}{2}^+(\frac{5}{2}^+)$. The Coulomb interaction has been included up to partial waves $L=6$ for convergence. Experimental values are taken from Refs. [24,26]. The results indicate that the calculated binding energies depend more on the choice of Λ-α interaction than on the α-α potential one uses. This is expected given the lack of sufficient experimental information on ${}^{5}_{\Lambda}$He to constrain the Λ-α interaction. Although the calculated binding energies for the ground state and excited state span a wide range of energies, the excitation energy between the two states is reasonably independent of the combination of α-α and Λ-α potentials one uses, and in close agreement with the experimental excitation energy (3.1 MeV). The best results, compared to data, are obtained with MS Λ-α potential in combination with AB (or CB) α-α interaction.

It is perhaps worth mentioning at this time that the Coulomb energy is strongly dependent on the α-α interaction one uses, by comparing results in both Tables III and IV. While AB and CB potentials give rise to Coulomb energies on the order of $\simeq 1.8$ MeV, UIM leads to about 2.6 MeV. This is the result of the nonlocal character of the UIM interaction at short distances that allows the α particles to get closer to each other than with any of the other two potentials due to their repulsive cores at short distances in both s and d waves.

Next we proceed to characterize these two states in terms of α-particle and Λ-particle point rms radius, charge radius, electric quadrupole moment, and magnetic dipole and octopole moments. Although most of these observables have not been measured for ${}^{9}_{\Lambda}$Be, they provide a better understanding of the structure of this hypernucleus and allow a comparison with ^{9}Be. All calculations include the Coulomb repulsion between the α's.

TABLE V. Same as in Table IV for $\frac{3}{2}^+(\frac{5}{2}^+)$ binding energies (MeV).

	UIM	AB	CB
TH	−3.44	−3.25	−3.17
MS	−4.05	−3.97	−3.80
DE	−4.63	−4.59	−4.41
DA	−5.18	−5.35	−5.05
Expt. [24]		−3.67	

TABLE VI. α-particle rms radius in fermis (point particle) for all α-α and Λ-α potentials.

	$\frac{1}{2}^+$			$\frac{3}{2}^+(\frac{5}{2}^+)$		
	UIM	AB	CB	UIM	AB	CB
TH	1.85	1.89	1.87	1.88	1.95	1.95
MS	1.82	1.84	1.82	1.80	1.84	1.83
DE	1.80	1.82	1.80	1.77	1.80	1.80
DA	1.83	1.82	1.81	1.80	1.80	1.81

In Tables VI and VII we list both the α-particle and the Λ-particle rms radius calculated in the point-particle approximation. Although both values are slightly bigger for the excited state $\frac{3}{2}^+(\frac{5}{2}^+)$ than for the ground state $\frac{1}{2}^+$, they are comparable and almost independent of the choice of potentials. Therefore we can claim that $\overline{r_\alpha} \simeq 1.83$ fm and $\overline{r_\Lambda} \simeq 2.15$ fm. Comparing with ^{9}Be [3] we find that $\overline{r_\alpha}$ is about the same, but that $\overline{r_n} \simeq 3.1$ fm. Therefore one may conclude that the neutron in ^{9}Be moves well outside the average location of the α particles relative to the center of mass of the system, while the Λ, in ${}^{9}_{\Lambda}$Be, shares its position with the α's. This gives a certain credibility to hypernuclear models that take ${}^{5}_{\Lambda}$He as a cluster.

Next we show in Table VIII the values for the charge radius of ${}^{9}_{\Lambda}$Be including the contribution of the α particle form factor. The result is also almost independent of the potential choice or state, leading to a value for $r_c \simeq 2.48$ fm. This value compares with $r_c = 2.48$ fm for ^{9}Be calculated with AB potential. This clearly indicates that the size of both nuclei is about the same from the view point of the α subclusters but that the neutron in ^{9}Be sticks much further out than the Λ in ${}^{9}_{\Lambda}$Be. This is somehow expected from our knowledge of Λ-α and n-α interactions, but it is nevertheless interesting to know that, under the combined effect of the two α's, the neutron in ^{9}Be still behaves as a neutron halo, while the Λ remains close to either ^{4}He clusters. In addition we find no evidence of contraction of the ^{8}Be core due to the substitution of the neutron by the Λ-

TABLE VII. Same as in Table VI for the Λ-particle rms radius in fermis (point particle).

	$\frac{1}{2}^+$			$\frac{3}{2}^+(\frac{5}{2}^+)$		
	UIM	AB	CB	UIM	AB	CB
TH	2.14	2.21	2.18	2.16	2.24	2.22
MS	2.16	2.18	2.15	2.13	2.16	2.16
DE	2.12	2.14	2.11	2.09	2.11	2.11
DA	2.17	2.14	2.14	2.16	2.14	2.15

TABLE VIII. Same as in Table VI for the charge radius in fm, including the effect of the α particle form factor.

		$\frac{1}{2}^+$			$\frac{3}{2}^+(\frac{5}{2}^+)$	
	UIM	AB	CB	UIM	AB	CB
TH	2.49	2.52	2.51	2.51	2.57	2.57
MS	2.47	2.49	2.47	2.45	2.48	2.48
DE	2.46	2.47	2.45	2.43	2.46	2.45
DA	2.48	2.47	2.46	2.46	2.46	2.46

particle as pointed out by others. This may indicate that the Pauli repulsion between the α's is strong enough to block any additional binding effect created by the Λ particle.

Further understanding is obtained by calculating the nonvanishing electric and magnetic moments. The electric quadrupole moment Q is zero for the $\frac{1}{2}^+$ state due to Wigner Echart theorem, but for $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states we show in Table IX the corresponding results for different potential models. The quadrupole moment, as expected, is large but of negative sign: on the average we get $Q \simeq -5.28$ e fm^2 for the $\frac{3}{2}^+$ state and $Q \simeq -7.55$ e fm^2 for the $\frac{5}{2}^+$ state. Here we observe an important structural difference relative to ^{9}Be that carries a positive quadrupole moment whose experimental value is 5.3 ± 0.3 e fm^2 [27]. This is a direct consequence of the underlying differences in the dominant partial waves that bind $^9_\Lambda$Be and ^{9}Be as three-body cluster nuclei. In the $\frac{3}{2}^+$ state of $^9_\Lambda$Be the dominant channel in the wave function involves the two α's in relative orbital angular momentum $L = 2$ while the Λ carries orbital angular momentum $l = 0$ relative to the center of mass of the α's. Since the matrix element for Q involves $J = J_z = \frac{3}{2}$, there are two possible contributions to Q coming from $(L=2, M=2)$ together with Λ spin projection $m_\Lambda = -\frac{1}{2}$ and $(L=2, M=1)$ with $m_\Lambda = \frac{1}{2}$. The well known angular distribution function $Y_{22}(\hat{R})$ for the $\vec{R}$ vector between the α's is an elongated torus revolving around the z axis but sitting on the xy plane, leading to a large negative contribution to the quadrupole moment which is not compensated by the smaller but positive contribution from $Y_{21}(\hat{R})$. Comparing with ^{9}Be we find instead that the dominant $\frac{3}{2}^-$ wave function components are $L=0$ and $L=2$ with $l=1$ for the neutron orbital angular momentum relative to the center of mass of the α's. The $L=0$ contribution to Q is zero and from $L=2$ we have three terms: $(L=2, M=0)$ with $m_j = \frac{3}{2}$, $(L=2, M=1)$ with $m_j = \frac{1}{2}$, and $(L=2, M=2)$ with $m_j = -\frac{1}{2}$ where $m_j = m + m_n$ such that m is the z component of $\vec{l}$ and m_n is the neutron spin projection. The angular distribution function for $Y_{20}(\hat{R})$, which stretches along the z axis gives rise to a large positive contribution to Q that dominates the other two, leading to a calculated value of 4.79 e fm^2 [3] for the AB α-α potential. Therefore one important conclusion is that $^9_\Lambda$Be is an oblate nucleus while ^{9}Be is prolate. This behavior is further enhanced in $^9_\Lambda$Be $\frac{5}{2}^+$ state because $J_z = \frac{5}{2}$ implies the contribution of $(L=2, M=2)$ with $m_\Lambda = \frac{1}{2}$ alone, leading to a single large negative contribution from $Y_{22}(\hat{R})$.

TABLE IX. Electric quadrupole moment Q in e fm^2 of the $\frac{3}{2}^+$ and $\frac{5}{2}^+$ excited states for all α-α and Λ-α potentials. For comparison we also show the calculated quadrupole moment of ^{9}Be $\frac{3}{2}^-$ state [3].

		$\frac{3}{2}^+$			$\frac{5}{2}^+$	
	UIM	AB	CB	UIM	AB	CB
TH	−5.53	−5.98	−6.00	−7.89	−8.54	−8.57
MS	−5.06	−5.27	−5.26	−7.24	−7.54	−7.52
DE	−4.91	−5.08	−5.05	−7.02	−7.26	−7.21
DA	−5.07	−5.07	−5.12	−7.24	−7.25	−7.32
^{9}Be		4.79				

TABLE X. Magnetic moment μ (in μ_N) of the $\frac{3}{2}^+$ and $\frac{5}{2}^+$ excited states for all α-α and Λ-α potentials. We also show, for comparison, the calculated magnetic moment of ^{9}Be [3] and, in parentheses, the corresponding value one gets when the intrinsic magnetic moment of the neutron is the same as Λ and one neglects the contribution from the neutron form factor.

		$\frac{3}{2}^+$			$\frac{5}{2}^+$	
	UIM	AB	CB	UIM	AB	CB
TH	1.26	1.25	1.25	0.38	0.37	0.37
MS	1.26	1.26	1.26	0.38	0.38	0.38
DE	1.27	1.27	1.27	0.39	0.39	0.39
DA	1.27	1.27	1.27	0.39	0.39	0.39
^{9}Be		−1.15	(−0.15)			

Since the ground state of $^9_\Lambda$Be is $\frac{1}{2}^+$ the dominant channels involve $l = L = 0$. Therefore the magnetic moment $\mu = \mu_\Lambda$ for such state. Nevertheless the same sign change effect relative to ^{9}Be takes place in the magnetic moment μ of $^9_\Lambda$Be excited states that are shown in Table X: again, on the average, we find $\mu \simeq 1.26$ μ_N for the $\frac{3}{2}^+$ state and $\mu = 0.38$ μ_N for the $\frac{5}{2}^+$ state. Although the Λ-particle and the neutron have magnetic moments of the same sign, once embeded in ^{8}Be, one gets magnetic moments of opposite sign for reasons similar to the ones explained for Q. Even if we make the intrinsic magnetic moment of the neutron $\mu_n = \mu_\Lambda$ in ^{9}Be we get a smaller but negative value as shown in Table X in parenthesis. The difference between this new result and the value obtained for $^9_\Lambda$Be is solely due to structural differences between the two nuclei.

Finally, in Table XI we show the octopole moment μ_3 as a result of our calculations for different force models: for the $\frac{3}{2}^+$ state one may still estimate an average $\mu_3 \simeq -0.35\, \mu_N$ fm^2 but for the $\frac{5}{2}^+$ state the calculated μ_3 span a wide range of values between $-0.38 \leqslant \mu_3 \leqslant 0.17 \mu_N$ fm^2. Both values are much smaller than those obtained for ^{9}Be as shown in the same table. The difference reveals again structural differences between the two nuclei.

IV. RESONANCES IN $^9_\Lambda$Be

Much like what is found in ^{9}Be and already experimentally observed in $^9_\Lambda$Be, this hypernucleus presents a number

TABLE XI. Same as in Table X for the magnetic octopole moment μ_3 (μ_N fm^2) of the $\frac{3}{2}^+$ and $\frac{5}{2}^+$ excited states.

	$\frac{3}{2}^+$			$\frac{5}{2}^+$		
	UIM	AB	CB	UIM	AB	CB
TH	−0.39	−0.43	−0.43	0.17	0.23	0.23
MS	−0.34	−0.35	−0.35	−0.07	−0.06	−0.05
DE	−0.32	−0.33	−0.33	−0.14	−0.13	−0.13
DA	−0.31	−0.31	−0.31	−0.37	−0.39	−0.38
^{9}Be		6.01	(2.01)			

of resonances in the continuum, whose location, width, and spin assignment is not always well established. Therefore we proceed to study the resonances of the $\alpha+\Lambda+\alpha$ system in the framework of the present potential model calculations.

The method we use to search for resonances requires the solution of the three-body Faddeev equations in continuum for three-body complex energies. It is known [28] that the use of contour rotation for the momentum variable integration in the kernel of the integral equations allows the analytic continuation to complex energy Z in the fourth quadrant of the E plane. This is a very powerfull method that is applicable even above three-body breakup threshold. Nevertheless this procedure is not well established for two-variable integral equations that emerge whenever the potentials are local and breaksdown in the presence of the Coulomb force.

Therefore, in order to expedite this procedure we expand the local potentials AB and CB in a separable form using the method of Ref. [25] and neglect Coulomb between the α particles. From the technical view point our calculation differs from Ref. [4] in that we use a different separable expansion method based on orthogonal polynomials, while they use the Ernst, Shakin, and Thaler (EST) expansion that requires the numerical solution of two-body equations. This together with a different contour rotation method [28] leads to a very stable numerical procedure for the calculation of resonances. As for the accuracy of the separable expansions for AB and CB potentials, we repeated the calculations for the ground state and excited state and compare with the results obtained in Tables IV and V. On the average we find that the finite rank representation of the AB t matrix leads to an accuracy of $\leqslant 30$ keV for the ground state and $\leqslant 2$ keV for the excited state.

In the present work we find two pairs of resonances: one below breakup threshold and one above breakup threshold.

TABLE XII. Position and width of the $\frac{1}{2}^-(\frac{3}{2}^-)$ resonance (MeV) for different α-α and Λ-α potentials in the absence of the Coulomb repulsion.

	UIM		AB		CB	
	E	$\Gamma/2$	E	$\Gamma/2$	E	$\Gamma/2$
TH	−2.00	0.62	−1.98	0.59	−2.03	0.68
MS	−1.69	0.56	−1.65	0.55	−1.67	0.64
DE	−1.89	0.58	−1.84	0.57	−1.86	0.66
DA	−1.92	0.41	−1.89	0.40	−1.87	0.47

TABLE XIII. Position and width of the $\frac{7}{2}^+(\frac{9}{2}^+)$ resonance (MeV) for different α-α and Λ-α potentials in the absence of the Coulomb repulsion.

	UIM		AB		CB	
	E	$\Gamma/2$	E	$\Gamma/2$	E	$\Gamma/2$
TH	1.29	0.14	0.59	0.06	1.66	0.20
MS	0.51	0.06	0.06	0.03	1.09	0.12
DE	−0.06	0.04	−0.51	0.02	0.51	0.09
DA	−0.30	0.04	−0.64	0.02	0.29	0.09

The first one is $\frac{1}{2}^-(\frac{3}{2}^-)$ while the second one is $\frac{7}{2}^+(\frac{9}{2}^+)$. The results are shown in Table XII and XIII, respectively. The $\frac{1}{2}^-(\frac{3}{2}^-)$ resonance is located just above the Λ-α threshold and has a large decay width. Nevertheless, unlike the work in Ref. [4], we find no negative parity states above breakup threshold, but unveil instead a positive parity state $\frac{7}{2}^+(\frac{9}{2}^+)$ that relies on the α-α g-wave interaction. This last resonance has not been found before by other continuum calculations but, together with the $\frac{1}{2}^+$ bound state and the $\frac{3}{2}^+(\frac{5}{2}^+)$ excited state, complete the even parity rotational band. These three energy states become five in the presence of a Λ-α spin-orbit force and are in some sense related to the five positive parity states observed in ^{9}Be in this same energy region [29]. On the contrary, the absence of a large number of negative parity states in ${}^{9}_{\Lambda}$Be is twofold: (a) a direct result of the Pauli principle that allows the Λ to stay predominantly in s-state while keeping the fifth neutron in ^{9}Be in a p state; (b) proper coupling to the decay channel $\alpha+\Lambda+\alpha$ and correct location of all underlying branch cuts of the α-α system in the complex energy plane. We have tried to search for $\frac{5}{2}^-(\frac{7}{2}^-)$ resonances corresponding to total orbital angular momentum $\mathbf{L}=3$ but could not find any. In Table XIV we show the excitation energy E_x relative to the $\frac{1}{2}^+$ ground state for both $\frac{1}{2}^-(\frac{3}{2}^-)$ and $\frac{7}{2}^+(\frac{9}{2}^+)$ resonances. We find that $5.7<E_x<8.3$ for the negative parity state, while $8.3\leqslant E_x\leqslant 10.3$ for the positive parity resonance. If we consider, for example, the MS Λ-α potential together with CB α-α interaction the $\frac{1}{2}^-(\frac{3}{2}^-)$ resonance corresponds to $\simeq 6.9$ MeV excitation and the $\frac{7}{2}^+(\frac{9}{2}^+)$ to $\simeq 9.7$ MeV.

So far the most accepted understanding of the level structure of ${}^{9}_{\Lambda}$Be is the work of Yamada *et al.* [30], where a variational $(\alpha+3N+N)$ + Λ cluster model that includes

TABLE XIV. $\frac{1}{2}^-(\frac{3}{2}^-)$ and $\frac{7}{2}^+(\frac{9}{2}^+)$ excitation energies (MeV) for different Λ-α and α-α potentials.

	$\frac{1}{2}^-(\frac{3}{2}^-)$			$\frac{7}{2}^+(\frac{9}{2}^+)$		
	UIM	AB	CB	UIM	AB	CB
TH	6.57	5.74	5.79	9.86	8.31	9.48
MS	7.38	6.84	6.90	9.58	8.55	9.66
DE	7.77	7.30	7.35	9.60	8.63	9.72
DA	8.26	8.15	8.13	9.88	9.40	10.29

α-particle breakup modes is applied to achieve a unified understanding of this hypernucleus in the energy region up to 20-MeV excitation. They find a very rich spectra of rotational bands of which we can only find the lowest one made up of $\frac{1}{2}^+, \frac{3}{2}^+ (\frac{5}{2}^+)$, and $\frac{7}{2}^+ (\frac{9}{2}^+)$ states. The two negative parity bands are absent from our calculation except for the lowest state $\frac{1}{2}^- (\frac{3}{2}^-)$ in the first band that carries total orbital angular momentum $\mathbf{L}=1$. Two reasons may explain such disagreement: (a) lack of internal structure of the α particle in our calculation; (b) lack of appropriate coupling to the continuum in their calculation. As shown in the study of ^{9}Be nucleus [29], it is the coupling to the continuum, through the appropriate location of the underlying analytical structure of two-body bound states and resonances in the kernel of the Faddeev equations, that ultimately decides the position and width of three-body resonances. If the width is large one may not be able to find it numerically, and, even if we do, it may not show up experimentally.

As far as we know, the experimental evidence for the spectra of $^9_\Lambda$Be comes from Refs. [31,32]. In Ref. [31] only two peaks are found above the $\alpha+\Lambda+\alpha$ threshold. The second peak, at about 20 MeV above breakup threshold, is clearly out of the domain of applicability of our calculation due to lack of structure in the α particle. The first peak, also centered at about 5 MeV above breakup, may not correspond to any of the states we have found because of both location and width; the $\frac{1}{2}^- (\frac{3}{2}^-)$ has the right width but is located very close to the breakup threshold once we add about 1.8 MeV for the Coulomb energy, while the $\frac{7}{2}^+ (\frac{9}{2}^+)$ has the correct position but is too narrow to correspond to the experimental peak. Instead the experimental work of Ref. [32] finds two resonance peaks corresponding to excitation energies of $E_x = 6$ and 10 MeV above the ground state, which may correspond to the states we find. The first one corresponds to the $\frac{1}{2}^- (\frac{3}{2}^-)$ state (1^-), while the second to the $\frac{7}{2}^+ (\frac{9}{2}^+)$ state (4^+). In Ref. [32] this last state has been denoted as 3^- in accordance with the theoretical predictions of Ref. [30] but, as we mentioned above, we find no additional negative parity resonance to justify such assignment.

V. CONCLUSIONS

In the present work we study $^9_\Lambda$Be as a cluster of two α particles and a Λ and calculate the structure of bound states and resonances. As already found by others [4] we confirm the existence of three even parity bound state $\frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+$ strongly associated with the $L=0$ $(\frac{1}{2}^+)$ and $L=2$ $(\frac{3}{2}^+, \frac{5}{2}^+)$ rotational states in ^{8}Be. In the absence of Λ-α spin-orbit force, the $(\frac{3}{2}^+)$ and $(\frac{5}{2}^+)$ states are degenerate. Depending on the combination of Λ-α and α-α interactions one uses, the energy difference between the two states lies between 2.3 MeV and 3.0 MeV. This compares with the experimental excitation energy of $\simeq 3.1$ MeV.

In the framework of this model we predict that the electric quadrupole moments of the $\frac{3}{2}^+$and $\frac{5}{2}^+$ states are negative. The calculated $Q(\frac{3}{2}^+) \simeq -5.28$ e fm^2 while $Q(\frac{5}{2}^+) \simeq -7.55$ e fm^2. Therefore $^9_\Lambda$Be is an oblate nucleus much like ^{8}Be, while ^{9}Be is prolate with an experimental $Q = 5.3 \pm 0.3$ e fm^2. The same sign change takes place if one compares the magnetic moments of both $^9_\Lambda$Be and ^{9}Be nuclei. While ^{9}Be $\frac{3}{2}^-$ ground state has a negative $\mu = -1.18\,\mu_N$, we predict that $\mu(\frac{3}{2}^+) \simeq 1.26\,\mu_N$ and $\mu(\frac{5}{2}^+) \simeq 0.38\,\mu_N$. The $\frac{1}{2}^+$ ground state has $\mu = \mu_\Lambda$. It should be extremely interesting to find out if shell-model calculations of $^9_\Lambda$Be and ^{9}Be would reproduce this sign change for both Q and μ.

In addition, we calculate the charge radius for all $^9_\Lambda$Be states and find it to be the same as in ^{9}Be. Therefore we do not find any contraction of the system due to the strong Λ-α s-wave attraction, but find instead that the Λ shares its location with the α's $(\overline{r_\Lambda} \simeq \overline{r_\alpha} \simeq 2$ fm) while the fifth neutron in ^{9}Be sticks much further out $(\overline{r_n} \simeq 3$ fm).

In the resonance region we locate the even parity states $(\frac{7}{2}^+, \frac{9}{2}^+)$ corresponding to an excitation energy $E_x \simeq 9.6$ MeV above the ground state, that complete the ^{8}Be-like rotational band. Given the similarity between the even parity states in $^9_\Lambda$Be and ^{8}Be we find it plausible to assume these to be ^{8}Be-like states as discussed in Refs. [7,30].

Close to breakup threshold we find a single pair of negative parity states $(\frac{1}{2}^-, \frac{3}{2}^-)$ that depend strongly on the strengh of the Λ-α s-wave interaction. Therefore we cannot denote this state as a ^{9}Be-like resonance but instead as part of a genuine hypernuclear band.

Given the strengh of the Λ-α effective p-wave interaction we cannot find ^{9}Be-like states independently of the Λ-α interaction we choose. If they exist they have such a large width that we cannot find them numerically in our three-body cluster model. Presumably they were found by Yamada *et al.* [30] due to lack of coupling to the continuum in their calculation.

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