Evidence of multifractality and constant specific heat in hadronic collisions at high energies

Dipak Ghosh, Argha Deb, Prabhat Bandyopadhyay, Mitali Mondal, Swarnapratim Bhattacharyya, Jayita Ghosh, and Kanchan Kumar Patra

Nuclear and Particle Physics Research Centre, Physics Department, Jadavpur University, Kolkata-700 032, India

(Received 17 July 2001; published 7 June 2002)

We analyzed the experimental data on the charged pion multiplicity distribution in p-AgBr interactions at 200 GeV/c and π^--AgBr interactions at 200 and 350 GeV/c in terms of multifractals. We calculate the values of generalized dimension D_q for $q=2$, 3, 4, and 5 with the help of the method proposed by Takagi. Also from the knowledge of D_q, the multifractal specific heat is calculated for different sets of interactions. It has been observed that the value of specific heat is more or less constant over the entire range of energies. This observation supports the constant specific heat approximation model and suggests that this parameter may come out as an essential characteristic of the hadron-hadron and hadron-nucleus interactions.

DOI: 10.1103/PhysRevC.65.067902 PACS number(s): 13.85.Hd, 25.80.−e

The investigation of multiplicity fluctuation in particle production has recently entered into a new phase. The study of multiparticle production in high-energy collisions has revealed self-similar properties [1] with respect to the resolution in phase space, conjectured by Bialas and Peschanski [2]. These analyses contain two types of approaches: one is to open up the possibility of more stringent tests on the dynamical model describing multiparticle production, while the other is to provide a strong hint of the possible existence of fractal properties in such production processes. The recent investigations of multiparticle production processes at high energies indicate fractality of (pseudo)rapidity distribution of produced particles [3,4]. Such a behavior of the spectra can also be a signature of a phase transition to quark matter [5]. Thus the search for fractality helps us to study both geometrical and dynamical properties of hadronization process. The numerous experimental evidences of the intermittent behavior have been obtained from hadron-hadron, hadron-nucleus, and nucleus-nucleus reactions. The elucidation of these experimental data is still a real problem. Multifractal interpretation of the data [4,6], in particular, considered from a view of thermodynamic point [4,7], may be followed to search for the phase-transition-like phenomena. It is known, on the other hand, that some simple thermodynamic approximations could be applicable to the multiparticle production as well. Bershadskii [8] reported to notice that the constant specific heat (CSH) approximation, widely used in the ordinary thermodynamics, is also applicable to the multifractal data. They studied for two sets of data corresponding to different type of reactions and observed that the specific heat has approximately the same value, which is in agreement with their prediction. This indicates that there is enough scope to study the constant specific heat approximation in other high-energy interactions.

Takagi [9] proposed the following method for studying multifractality of multiparticle production process. A single event contains n hadrons distributed in the interval $\eta_{\min}<\eta<\eta_{\max}$. The hadron multiplicity n changes from event to event according to the distribution $P_n(\eta)$, where $\eta=\eta_{\max}-\eta_{\min}$. The full rapidity interval of length η is divided into m bins of equal size $\delta\eta=\eta/m$. The multiplicity distribution for a single bin $P_n(\delta\eta)$ for $n=0,1,2,3,\ldots$, where we assume that the inclusive rapidity distribution $dn/d\eta$ is constant and $P_n(\delta\eta)$ is independent of the location of the bin. Hadrons produced in Ω independent events are distributed in Ωm bins of size $\delta\eta$. Let n_{ij} be the multiplicity of hadrons in the jth bin of the ith event and N be the total number of hadrons produced in these Ω events. The normalized density P_{ij} is defined by considering the theory of multifractals [10],

$$P_{ij}=n_{ij}/N.$$

This is of course also true when $N\to\infty$. Then one has to consider the quantity

$$T_q(\delta\eta)=\ln\sum_{j=1}^{\Omega}\sum_{i=1}^{m}P_{ij}^q \quad \text{for } q>0,$$

which behaves like a linear function of the logarithm of the "resolution" $R(\delta\eta)$, q is the order number,

$$T_q(\delta\eta)=A_q+B_q\ln R(\delta\eta),$$

where A_q and B_q are constants independent of $\delta\eta$. If such a behavior is observed for a considerable range of $R(\delta\eta)$, a generalized dimension may be determined as

$$D_q=B_q/(q-1). \tag{1}$$

Now evaluating the double sum of P_{ij}^q for sufficiently large Ω, Takagi [9] expects a linear relation,

$$\ln\langle n^q\rangle=A_q+(B_q+1)\ln R(\delta\eta),$$

and suggested that $\langle n\rangle$ would be a better choice of the "resolution" $R(\delta\eta)$ because $dn/d\langle n\rangle$ is flat by definition [10]. Choosing $R(\delta\eta)=\langle n\rangle$ one has

$$\ln\langle n^q\rangle=A_q+(B_q+1)\ln\langle n\rangle, \tag{2}$$

a simple linear relation between $\ln\langle n^q\rangle$ and $\ln\langle n\rangle$. The generalized dimension D_q can be obtained from the slope values.

The case with $q=1$ can be obtained by taking an appropriate limit [11]. The value of information dimension D_1 can also be determined from a new and simple relation suggested by Takagi [9],

$$\langle n\ln n\rangle/\langle n\rangle=C_1+D_1\ln\langle n\rangle, \tag{3}$$

where C_1 is a constant.

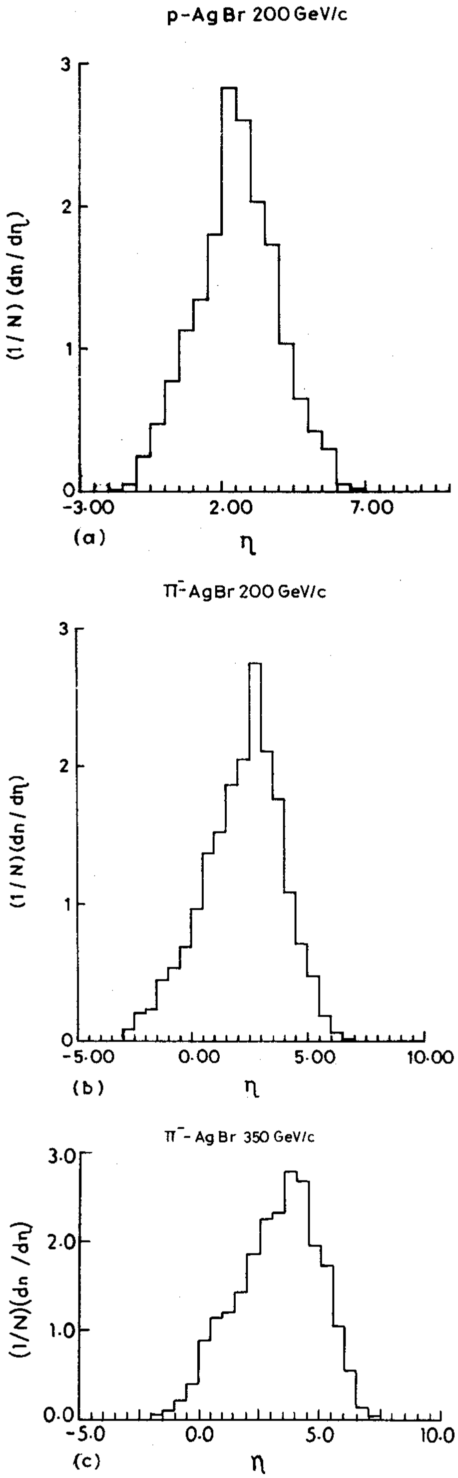

FIG. 1. Plot of pseudorapidity distribution of the produced particles in p-AgBr interactions at 200 GeV/c and π^--AgBr interactions at 200 and 350 GeV/c, as shown in (a)–(c).

If the q dependence of D_q satisfies the condition $D_q > D_{q'}$ for $q < q'$ then the spectra is said to exihibit multifractality.

Considering the thermodynamic point of view, Bershadskii [8] has suggested that the following relation may be obtained if one assumes constant specific heat approximation:

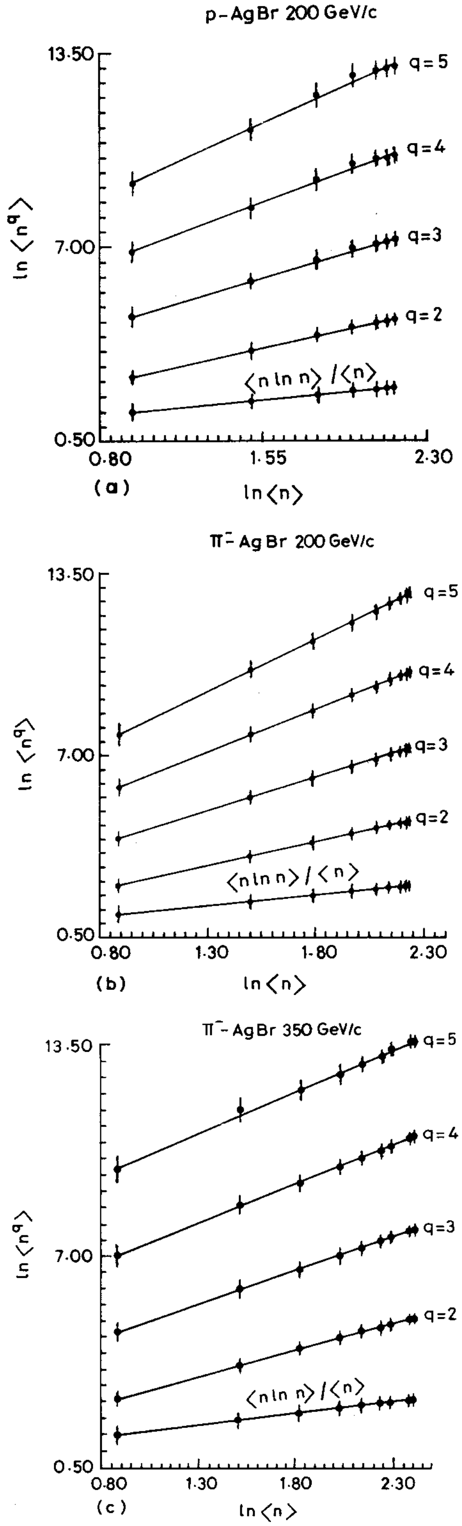

FIG. 2. Plot of $\langle n \ln n\rangle/\langle n\rangle$ against $\ln\langle n\rangle$ and plot of $\ln\langle n^q\rangle$ against $\ln\langle n\rangle$ for $q=2$, 3, 4, and 5 for p-AgBr interactions at 200 GeV/c and π^--AgBr interactions at 200 and 350 GeV/c with best linear fits, as shown in (a)–(c).

$$D_q \approx (a-c) + c\,\frac{\ln q}{(q-1)}, \tag{4}$$

where a and c are some constants. c is known as the CSH.

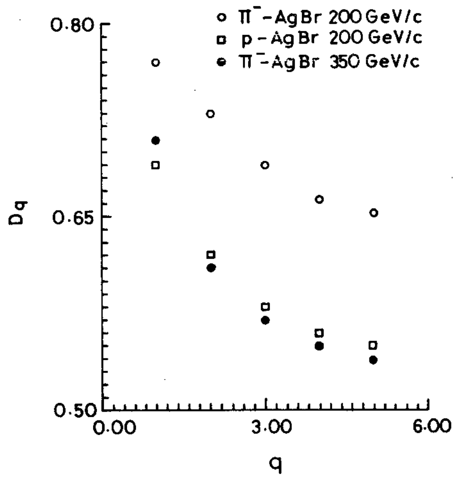

FIG. 3. Plot of the generalized dimension D_q against q for p-AgBr interactions at 200 GeV/c and π^--AgBr interactions at 200 and 350 GeV/c.

In this Brief Report we intend to study the multifractality of pion multiplicity distribution in case of p-AgBr interactions at 200 GeV/c and π^--AgBr interactions at 200 and 350 GeV/c following the method proposed by Takagi [9] and we will also find out the specific heat c with the help of Eq. (4) using generalized dimension D_q for different q's for the different data sets.

The data have been obtained from emulsion plates exposed horizontally to proton beams at Fermilab with 200 GeV/c and π^- beam at Fermilab with 200 GeV/c and at CERN with 350 GeV/c incident energies. The details of scanning and measurement including selection of events are described in our earlier papers [12]. The nuclear emulsion covers 4π geometry and provides very good accuracy, even less than 0.1 mrad, in angle measurement of produced particles with respect to the projectile beam axis due to high spatial resolution, and thus, is suitable as a detector for the study of fluctuations in fine resolution intervals of the pseudorapidity phase space.

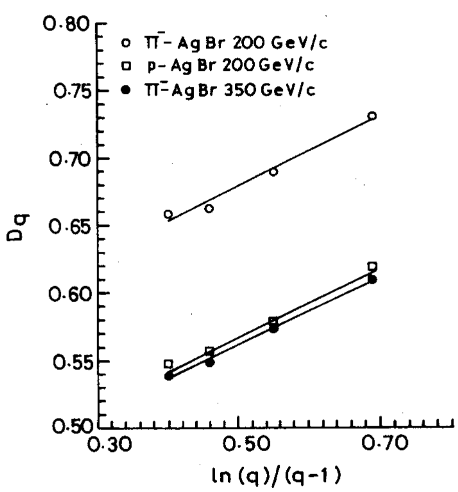

FIG. 4. Plot of generalized dimension D_q against $\ln q/(q-1)$ for p-AgBr interactions at 200 GeV/c and π^--AgBr interactions at 200 and 350 GeV/c. The straight lines are drawn for comparison with CSH approximation (4).

TABLE I. Multifractal specific heat (c) for p-AgBr interactions at 200 GeV/c and π^--AgBr interactions at 200 and 350 GeV/c, π^+p interactions at 250 GeV/c, and p-Em interactions at 800 GeV/c.

Interactions	Energy at GeV/c	Multifractal specific heat (c)	Reference
p-AgBr	200	0.25	This expt.
π^--AgBr	200	0.26	This expt.
π^+p	250	0.26	[8]
π^--AgBr	350	0.24	This expt.
p-emulsion $N_g=1.2$	800	0.25	[8]
p-emulsion $N_g=0$	800	0.25	[8]

We have calculated $\langle n \ln n\rangle/\langle n\rangle$ and $\ln\langle n^q\rangle$ in different pseudorapidity intervals. In order to do that we have considered central symmetric overlapping bins ($\eta_c-\Delta\eta<\eta_c<\eta_c+\Delta\eta$) around the peak ($\eta_c$) of η distributions. The pseudorapidity distribution for our data sets in the whole phase space in p-AgBr interaction at 200 GeV/c and π^--AgBr interactions at 200 and 350 GeV/c are represented in the Figs. 1(a), 1(b), and 1(c), respectively. The bin size ($2\Delta\eta$) increases in steps of 0.5. $\eta_c=2.25$ for p-AgBr interactions at 200 GeV/c and $\eta_c=2.75$ for π^--AgBr interactions at 200 GeV/c and $\eta_c=3.75$ for π^--AgBr interactions at 350 GeV/c. The bin size dependence of multiplicity moments for different bins of pionization phase space is observed.

$\ln\langle n^q\rangle$ are plotted against $\ln\langle n\rangle$ for $q=2$, 3, 4, and 5 and $\langle n \ln n\rangle/\langle n\rangle$ is plotted against $\ln\langle n\rangle$ as shown in the Figs. 2(a), 2(b), and 2(c) for different pseudorapidity intervals of our experimental data. It has been observed that our experimental data show the expected excellent linear behavior in the above log-log plot and now there is no problem to find out the slopes of the linear fits. We have determined the generalized dimensions D_q from the slopes of such a linear fit with the help of Eqs. (1) and (2). In Fig. 3 we plot D_q against q for different energy data sets. It is observed that in each case D_q decreases monotonically with q, which reveals multifractality of multiplicity distributions for all data. Further, the values of generalized dimensions D_q are plotted as a function of $\ln(q)/(q-1)$ to extract the specific heat as shown in Fig. 4. For our data, multifractal specific heats (c) are calculated from the slope values of the best fitted straight lines in Fig. 4. The Table I presents the values of multifractal specific heats (c) for different data sets. It also includes the results of other data [8]. It is extremely interesting to note that specific heats (c) remain almost constant over the entire energy range 200–800 GeV/c of hadron-nucleus interactions.

Thus this analysis clearly indicates the multifractal nature of multiplicity distributions of our data and strongly supports the constant multifractal specific heat approximation as proposed by Bershadskii [8]. The multifractal specific heat (c) seems to be an interesting and challenging parameter that may turn out as an universal characteristic of multiparticle production in hadron-hadron and hadron-nucleus interactions.

The authors express their gratitude to Professor W. Lock of CERN and Professor J. J. Lord of University of Washington, USA for providing us with the emulsion plates for this analysis. Support from the Department of Physics, Jadavpur University, and the University Grants Commission of India under their COSIST program is gratefully acknowledged.

[1] B. Buschbeck and P. Lipa, Mod. Phys. Lett. A **4**, 1871 (1989).
[2] A. Bialas and R. Peschanski, Nucl. Phys. **B273**, 703 (1986).
[3] A. Bialas, Nucl. Phys. **A525**, 345 (1991).
[4] R. Peschanski, Mod. Phys. Lett. A **6**, 3681 (1991).
[5] Ph. Brax and R. Peschanski, Phys. Lett. B **253**, 225 (1991).
[6] R. C. Hwa and M. T. Nazirov, Phys. Rev. Lett. **69**, 741 (1992).
[7] E. A. De Wolf and I. M. Dremin *et al.*, Phys. Rep. **270**, 1 (1996).
[8] A. Bershadskii, Eur. Phys. J. A **2**, 223 (1998).
[9] Fujio Takagi, Phys. Rev. Lett. **72**, 32 (1994).
[10] A. Bialas and M. Gazdzicki, Phys. Lett. B **252**, 483 (1990).
[11] H. G. E. Hentschel and I. Procaccia, Physica D **8**, 435 (1983).
[12] D. Ghosh *et al.*, Phys. Rev. D **40**, 3712 (1992); **40**, 3113 (1994); Mod. Phys. Lett. A **8**, 32233 (1993).