Pion-only, chiral light-front model of the deuteron

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We investigate the symptoms of broken rotational invariance, caused by the use of light front dynamics, for deuterons obtained using one- and two-pion-exchange potentials. A large mass splitting between states with $m=0$ and $m=1$ is found for the deuteron obtained from the one-pion-exchange potential. The size of the splitting is smaller when the chiral two-pion-exchange (TPE) potential is used. When the TPE potential constructed without chiral symmetry is used, the deuteron becomes unbound. These results arise from significant relativistic effects that are much larger than those of the Wick-Cutkosky model because of the presence of the tensor force.

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Recently, there has been much interest in effective field theories (EFT) for nuclear physics. In particular, the expansion of the nuclear forces about the chiral limit gives good results [1,2]. Attempts have been made to apply pionless EFTs to study deuteron properties [3–5]. However, it has been shown that pions are required to obtain a well-controlled theory [6].

It is not surprising that the pions are necessary. Indeed, the gross features of the deuteron are determined principally by the one-pion-exchange (OPE) potential [7], a consequence of EFT. However, the short-distance behavior of the naive OPE potential is singular, and must be regulated. Physically, this regulation is provided by the exchange of more massive mesons and multimeson exchanges. EFT tells us that these high-energy parts can be replaced by anything that has the correct form dictated by symmetry, and the low-energy properties of the deuteron will be left unaffected.

The expectation that pionic effects are very important is related to work by Friar, Gibson, and Payne (FGP) [8] who obtained nonrelativistic deuteron wave functions using only the OPE potential. In their calculations a pion-nucleon form factor replaces the high-energy physics. The only influence of chiral symmetry in the FGP model is the requirement that the pion-nucleon coupling should be of the $\boldsymbol{\tau}\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}$ form.

In this Brief Report, we build upon the FGP model by performing a relativistic calculation using light-front dynamics. In addition, we go beyond the OPE potential and include the two-pion-exchange (TPE) potential. Chiral symmetry is known to have profound implications for the TPE interaction, so that we will be able to study the effect that ignoring chiral symmetry has on the deuteron.

Our plan is to introduce a model Lagrangian for nuclear physics using only nucleons and pions. A Lagrangian that includes chiral symmetry [9] has been used to compute the OPE and TPE potentials for a new light-front nucleon-nucleon potential [9–11], which involves the exchange of six different mesons. We retain only the contributions arising from pionic exchanges. The resulting pion-only potentials are then used to see if the deuteron state is bound, and if so, to compute the binding energy. The final section is devoted to presenting a brief set of conclusions.

We consider a pion-only light-front nucleon-nucleon potential derived from a nuclear Lagrangian. This model is inspired by the nonrelativistic one-pion-exchange model used by Friar, Gibson, and Payne [8]. We generalize their model to form the basis of our pion-only light-front model, which includes relativity automatically. This pion-only model is essentially the same as the model presented in Ref. [11] restricted to pions only.

Our starting point is an effective nuclear Lagrangian [9,13,11,14] that incorporates a nonlinear chiral model for the pions. The Lagrangian is based on the linear representations of chiral symmetry used by Gursey [12]. It is invariant (in the limit where $m_\pi \to 0$) under chiral transformations.

The pion-only model prescribes the use of nucleons ψ and the π meson, which is a pseudoscalar isovector. The Lagrangian $\mathcal{L}$ is given by

$$\mathcal{L}=\frac{1}{4}f^2\,\mathrm{Tr}(\partial_\mu U\partial^\mu U^\dagger)+\frac{1}{4}m_\pi^2 f^2\,\mathrm{Tr}(U+U^\dagger-2)+\bar{\psi}[i\not\partial-UM]\psi, \tag{1}$$

where the bare mass of the nucleon is M and the pion is m_π. The unitary matrix U can be chosen to have one of the three forms U_i:

$$U_1 \equiv e^{i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}/f}, \tag{2}$$

$$U_2 \equiv \frac{1+i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}/2f}{1-i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}/2f}, \tag{3}$$

$$U_3 \equiv \sqrt{1-\boldsymbol{\pi}^2/f^2}+i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}/f, \tag{4}$$

which correspond to different definitions of the fields. Each of these definitions can be expanded to give

$$U=1+i\gamma_5\frac{\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{f}-\frac{\pi^2}{2f^2}+\mathcal{O}\left(\frac{\pi^3}{f^3}\right). \tag{5}$$

In this paper, we consider at most two pion-exchange potentials, so we take U to be defined by Eq. (5).

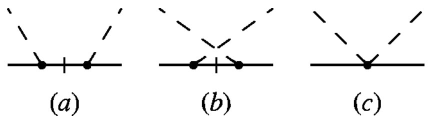

FIG. 1. The nonvanishing diagrams for pion-nucleon scattering at threshold: (a) $\mathcal{M}_U$, (b) $\mathcal{M}_X$, and (c) $\mathcal{M}_C$.

In the limit where $m_\pi \to 0$, the Lagrangian in Eq. (1) is invariant under the chiral transformation

$$\psi \to e^{i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{a}}\psi, \quad U \to e^{-i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{a}} U e^{-i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{a}}. \tag{6}$$

We use the Lagrangian to obtain the OPE and TPE light-front nucleon-nucleon potentials [9–11,13,14]. We find that there are three classes of TPE potentials. The first class, the TPE box diagrams, consists of diagrams where the pion lines do not cross. The second class, the TPE contact diagrams, consists of diagrams where at least one of the vertices is a two-pion contact vertex, as demanded by chiral symmetry. The third class is the TPE crossed diagrams, where the pion lines cross.

As discussed in Refs. [10,11,15,16], we have some freedom in deciding which TPE diagrams to include in our potential. In particular, we may neglect just the crossed diagrams, or both the crossed and contact diagrams. Although neglecting these diagrams may affect the exact deuteron binding energy calculated, we should find a partial restoration of rotational invariance as compared to using just the OPE potential.

We also want to keep the potentials chirally symmetric as well. Whereas rotational invariance of the potential is partially restored by including higher-order potentials, chiral symmetry is restored by including the $\pi\pi$ contact interaction graphs to the same order as the noncontact graphs.

Chiral symmetry tells us that for pion-nucleon scattering at threshold, the time-ordered graphs approximately cancel [9]. Furthermore, upon closer examination, we find that all the light-front time-ordered graphs for the scattering amplitude vanish except for the two graphs with instantaneous nucleons and the contact graph. These graphs are shown in Fig. 1. Using the Feynman rules, Refs. [9–11,13,14], and denoting the nucleon momentum by k and the pion momentum by q, we find that

$$\mathcal{M}_U = C\frac{\delta_{i,j} + i\epsilon_{i,j,k}\tau_k}{2(k^+ + q^+)} u(k')\gamma^+ u(k), \tag{7}$$

$$\mathcal{M}_X = C\frac{\delta_{i,j} - i\epsilon_{i,j,k}\tau_k}{2(k^+ - q^+)} u(k')\gamma^+ u(k), \tag{8}$$

$$\mathcal{M}_C = C\frac{-\delta_{i,j}}{M} u(k')u(k),p, \tag{9}$$

where the factors common to all amplitudes are denoted by C. The term in these equations proportional to τ_k is the famous Weinberg-Tomazowa term [17,18]. For threshold scattering (where $k^+ = M$ and $q^+ = m_\pi$), in the chiral limit, where $m_\pi \to 0$, the sum of $\mathcal{M}_U$, $\mathcal{M}_X$, and $\mathcal{M}_C$ vanishes [9,13,14].

TABLE I. Binding energies for deuterons calculated with several one-meson-exchange potentials and two different meson-nucleon form factors for the scalar-meson-only model. The coupling constant is $g^2/4\pi = 0.424$ and the meson mass is the same as the pion's mass.

OME potential	$\Lambda = 1.0$ GeV	$\Lambda = 1.915$ GeV
Nonrelativistic	2.2236 MeV	4.2539 MeV
Instantaneous	2.1581 MeV	4.0862 MeV
Light front, $m=1$	1.2027 MeV	2.2294 MeV

The fact that the amplitudes cancel only when the contact interaction is included demonstrates that chiral symmetry can have a significant effect on calculations. In terms of two-pion-exchange potentials, this result means that the contact potentials cancel strongly with both the iterated box potentials and the crossed potentials. This serves to reduce the strength of the total two-pion-exchange potential, which leads to more stable results.

However, in this paper we do not use the crossed graphs for the nucleon-nucleon potential, so we must come up with a prescription that divides the contact interaction into two parts that cancel the box and crossed diagrams separately. We do this by formally defining two new contact interactions, so that

$$\mathcal{M}_{C_U} \equiv \frac{M}{2(M+\lambda m_\pi)}\mathcal{M}_C, \tag{10}$$

$$\mathcal{M}_{C_X} \equiv \mathcal{M}_C - \mathcal{M}_{C_U}, \tag{11}$$

and the value of λ is of order unity. With these definitions, we find that at threshold and in the chiral limit,

$$\mathcal{M}_U + \mathcal{M}_{C_U} = 0, \tag{12}$$

$$\mathcal{M}_X + \mathcal{M}_{C_X} = 0. \tag{13}$$

Since Eqs. (10) and (11) give the correct results in the chiral limit for pion-nucleon scattering, we use it to factor the TPE potentials so that chiral symmetry can be approximately maintained while neglecting the crossed potentials.

We find that the results of our TPE calculation, which include approximate chiral symmetry have a very weak dependence on the value of λ, provided that $0 < \lambda < 1$. As such, we show only the results for $\lambda = 1$. These features indicate that we can incorporate approximate chiral symmetry without including crossed graphs simply by weighting each contact interaction graph with a factor of $M/[2(M+m_\pi)]$.

To summarize, the OPE potential breaks rotational invariance, but we can improve the rotational properties of the total potential by adding the TPE box potential. Since the TPE box potential breaks chiral symmetry, we must also include a TPE contact potential (with the appropriate weight-

TABLE II. Binding energies for deuterons calculated with several one-pion-exchange (OPE) potentials and two different pion-nucleon form factors for the pion-only model. The coupling constant is $g^2/4\pi=14.6$. A negative binding energy indicates that a state is unbound.

OPE potential	$\Lambda=1.01$ GeV	$\Lambda=1.9$ GeV
Nonrelativistic	2.2244 MeV	227.019 MeV
Light front, $m=0$	-0.0269 MeV	0.788 MeV
Light front, $m=1$	-0.0246 MeV	8.856 MeV

ing) to improve the chiral symmetry. We find that chiral symmetry is essential for producing stable results in the deuteron calculation.

To numerically calculate the bound states for these pion-only potentials, we must choose values for the potential parameters. As a starting point, we look for inspiration from the nonrelativistic one-pion-exchange model used by FGP [8]. The basic parameters of the model are the mass of the nucleon ($M=938.958$ MeV), the mass of the pion ($m_\pi=138.03$ MeV), and the pion decay constant ($f_0^2=0.079$), which correlate to a coupling constant of $g_\pi^2/4\pi=4M^2f_0^2/m_\pi^2=14.6228$ in our formalism.

The FGP model uses family of n-pole pion-nucleon form factors, with the form

$$F(q)=\left(\frac{\Lambda^2-m_\pi^2}{\Lambda^2+\boldsymbol{q}^2}\right)^n, \tag{14}$$

and the parameter Λ is fit to reproduce the deuteron binding energy for a given value of n. In this paper, we fix $n=1$. We can obtain the FGP model from our light-front nucleon-nucleon potential by considering only the OPE potential and performing a nonrelativistic reduction.

Based on our previous work with the Wick-Cutkosky model (where the mass splitting for states with different values of m was very small for lightly bound states [15]), one might think that the mass splitting will be small for the $m=0$ and $m=1$ states of the deuteron when calculated with the light-front OPE potential. However, the pseudoscalar coupling of the pion and the associated tensor force causes the deuteron wave function to have a high-momentum tail that falls off rather slowly, as $1/k^4$ [19]. This tail enhances the effects of the potential's relativistic components, such as the breaking of rotational invariance. As a consequence, the deuteron mass splitting is large in our light-front model.

To make sure that the mass splitting in the pion-only model is due to relativistic effects, we first consider a scalar version of this model, where we assume that the exchange meson has a scalar coupling to the nucleon. The scalar-meson-only potential is essentially a Wick-Cutkosky potential that incorporates a meson-nucleon form factor and spinor overlap factors. Table I shows the binding energies for deuterons calculated with two form factors for the scalar-meson-only nonrelativistic and light-front potentials, using the techniques detailed in Refs. [11,10]. The binding energies are of the same order of magnitude for all the potentials and the binding energies of the $m=0$ and $m=1$ light-front potentials are essentially degenerate. These results are the same as those for the Wick-Cutkosky model [16].

Now we consider the (pseudoscalar) pion-only model. The only difference between this potential and the scalar-meson-only potential is that the numerator contains spinor overlap factors of $\overline{u}i\gamma^5u$ instead of $\overline{u}u$. The pseudoscalar coupling generates a tensor force that is more sensitive to the relativistic components of the wave function than the scalar force.

Table II shows the binding energies for deuterons computed with the pion-only model for two form factors. The binding energies vary greatly depending on which potential is used. In fact, the light-front potentials do not bind the deuteron with the first form factor, and with the second form factor, the mass splitting between the $m=0$ and $m=1$ states is very large. Since the potentials are substantially different only in the high-momentum region, this result indicates that the pseudoscalar nature of the pion-exchange model makes it very sensitive to subtle changes in the potential's relativistic structure.

We can reduce the large mass splitting by adding TPE graphs to the OPE potential. We may include the TPE box graphs that we call the nonchiral-TPE (ncTPE) potential, or the sum of the TPE box graphs and the weighted TPE contact graphs, which we call the TPE potential. To check this stability and the restoration of rotational invariance of the pion-only model, we calculate the energy of the deuteron using the OPE, OPE+TPE, and OPE+ncTPE potentials.

We find that $\Lambda=1.9$ GeV gives a reasonable range of binding energies, as Table III shows. The mass splitting and the difference in the percentage of the D-state wave function both decrease when TPE diagrams are included. The D-state

TABLE III. The binding energy for the $m=0$ and $m=1$ states, the difference of those energies (Δ), and percentages of the wave function in the D state and in the $J=1$ state for the $m=0$ and $m=1$ states, calculated with several potentials using a pion-nucleon form factor with $\Lambda=1.9$ GeV. A negative binding energy indicates that a state is unbound.

Potential	Binding energy (MeV)			D state (%)		$J=1$ (%)	
	$m=0$	$m=1$	Δ	$m=0$	$m=1$	$m=0$	$m=1$
OPE	0.7884	8.8561	-8.0677	4.09	12.27	99.99	99.78
OPE+TPE	0.6845	2.5606	-1.8761	3.98	8.08	99.98	99.88
OPE+ncTPE	-0.0107	-0.0087	-0.0020	0.15	0.66	100.00	100.00

probability increases with the binding energy, and it is consistent (given the range of values the probability and the binding energy take) with the value of 6% reported in Ref. [8] for the FGP model.

Table III also shows that numerically the states are *almost* angular momentum eigenstates, since the percentage of each state with $J=1$ is almost 100%. This occurs even though the eigenstates of the light-front Hamiltonian are not, in principle, eigenstates of the angular momentum. However, this does not mean that rotational invariance is unbroken. Rotations relate the different m states, and from the mass splittings, we see that the states do not transform correctly.

Table III also shows the effects of using the ncTPE potential. As mentioned earlier, this potential is greater in magnitude than the TPE potential, and it unbinds the deuteron. (Strictly speaking, this indicates only that the binding energy is very small or zero; the uncertainty in the binding energy calculation increases as the binding energy approaches zero.) Because the ncTPE potential does not produce a bound deuteron, we cannot determine what effect it has on the rotational properties of the state.

Finally, we performed the same calculation using two more pion-nucleon form factors, one with $\Lambda=2.1$ GeV and another with $\Lambda=2.9$ GeV and $n=2$. The results are qualitatively the same as those in Table III, demonstrating that these results are robust [10].

In this paper, a light-front pion-only model was used to investigate the effects that relativity and chiral symmetry have on the deuteron. We used OPE, OPE+TPE, and OPE +ncTPE potentials to calculate the binding energy and wave function for the $m=0$ and $m=1$ states of the deuteron. We find that the splitting between the $m=0$ and $m=1$ states is smaller for the OPE+TPE potential as compared to what the OPE potential gives. We also find that chiral symmetry must be included to obtain sensible results when using two-pion-exchange potentials.

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[1] S. Weinberg, Nucl. Phys. **B363**, 3 (1991).
[2] C. Ordonez, L. Ray, and U. van Kolck, Phys. Rev. C **53**, 2086 (1996).
[3] D. B. Kaplan, M. J. Savage, and M. B. Wise, Phys. Lett. B **424**, 390 (1998).
[4] U. van Kolck, Nucl. Phys. **A645**, 273 (1999).
[5] J. W. Chen, G. Rupak, and M. J. Savage, Nucl. Phys. **A653**, 386 (1999).
[6] S. R. Beane, P. F. Bedaque, M. J. Savage, and U. van Kolck, nucl-th/0104030 (unpublished).
[7] Torleif Ericson and Wolfram Weise, *Pions and Nuclei* (Clarendon, Oxford, UK, 1988).
[8] J. L. Friar, B. F. Gibson, and G. L. Payne, Phys. Rev. C **30**, 1084 (1984).
[9] G. A. Miller, Phys. Rev. C **56**, 2789 (1997).
[10] J. R. Cooke, nucl-th/0112029 (unpublished).
[11] J. R. Cooke and G. A. Miller, Phys. Rev. C (submitted), nucl-th/0112037.
[12] F. Gursey, Nuovo Cimento **16**, 230 (1960).
[13] G. A. Miller, Phys. Rev. C **56**, 8 (1997).
[14] G. A. Miller and R. Machleidt, Phys. Rev. C **60**, 035202 (1999).
[15] J. R. Cooke, G. A. Miller, and D. R. Phillips, Phys. Rev. C **61**, 064005 (2000).
[16] J. R. Cooke and G. A. Miller, Phys. Rev. C **62**, 054008 (2000).
[17] B. H. Brandsden and R. G. Moorhouse, *The Pion-Nucleon System* (Princeton University Press, Princeton, NJ, 1973).
[18] S. L. Adler and R. F. Dashen, *Current Algebras* (Benjamin, New York, 1968).
[19] R. Machleidt, K. Holinde, and C. Elster, Phys. Rep. **149**, 1 (1987).