Q^2 independence of QF_2/F_1, Poincaré invariance, and the nonconservation of helicity

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A relativistic constituent quark model is found to reproduce the recent data regarding the ratio of proton form factors, $F_2(Q^2)/F_1(Q^2)$. We show that imposing Poincaré invariance leads to substantial violation of the helicity conservation rule, and an analytic result that the ratio $F_2(Q^2)/F_1(Q^2) \sim 1/Q$ for intermediate values of Q^2.

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I. INTRODUCTION

The recent discovery [1,2] that the ratio of proton form factors G_E/G_M falls linearly with Q^2, the ratio QF_2/F_1 reaches a constant value for $2 \leqslant Q^2 \leqslant 6$ GeV^2, and the plans [3] to make measurements up to $Q^2 = 9$ GeV^2, have focused attention on understanding nucleon structure. The constant nature of the ratio QF_2/F_1 contrasts with the prediction from perturbative QCD [4] that Q^2F_2/F_1 should be constant [5]. While this latter ratio could be achieved when experiments are pushed to higher values of Q^2, it is worthwhile to obtain a deeper understanding of the present results. In particular, the perturbative result is based on the notion that hadron helicity is conserved [6] in high momentum transfer exclusive processes, so that it becomes interesting to understand why this conservation does not seem to be applicable.

The qualitative nature of the experimental results were anticipated or reproduced by several model calculations [7–10], with that of Ref. [7] based on the work of Schlumpf [11–13] being the earliest. The implementation of relativity is an important feature of each of these calculations, so it is natural to seek an understanding of the form factors in terms of relativity. Our purpose here is to examine the model of Ref. [7], with the aim of highlighting the essential features that cause the ratio QF_2/F_1 to be constant.

We proceed by presenting definitions and kinematics relevant for a light-front analysis in Sec. II. The relevant features of our relativistic constituent quark model are displayed in Sec. III. The essential reason for the constant ratio QF_2/F_1 is discussed in Sec. IV, and elaborated upon numerically in Sec. V. The paper is concluded with a brief summary.

II. DEFINITIONS AND KINEMATICS

The electromagnetic current matrix element can be written in terms of two form factors taking into account current and parity conservations:

$$\langle N,\lambda' p'|J^\mu|N,\lambda p\rangle = \bar{u}_{\lambda'}(p')\left[F_1(Q^2)\gamma^\mu + \frac{\kappa F_2(Q^2)}{2M_N} i\sigma^{\mu\nu}(p'-p)_\nu\right]u_\lambda(p), \tag{1}$$

with momentum transfer $q^\mu = (p'-p)^\mu$, $Q^2 = -q^2$ and J^μ is taken as the electromagnetic current of free quarks. For $Q^2 = 0$ the form factors F_1 and κF_2 are, respectively, equal to the charge and the anomalous magnetic moment κ in units of e and e/M_N, and the magnetic moment is $\mu = F_1(0) + \kappa F_2(0) = 1 + \kappa$. The Sachs form factors are defined as

$$G_E = F_1 - \frac{Q^2}{4M_N^2}\kappa F_2 \quad \text{and} \quad G_M = F_1 + \kappa F_2. \tag{2}$$

The evaluation of the form factors is simplified by using the so-called Drell-Yan reference frame in which $q^+ = 0$ so that $Q^2 = q_\perp^2 q_1^2$. This means that the plus components of the nucleon momenta (and also those of the struck constituent quark) are not changed by the absorption of the incoming photon.

If light-front spinors for the nucleons are used, the form factors can be expressed simply in terms of the plus component of the current [14]:

$$F_1(Q^2) = \frac{1}{2P^+}\langle N,\uparrow|J^+|N,\uparrow\rangle$$

and

$$Q\kappa F_2(Q^2) = \frac{-2M_N}{2P^+}\langle N,\uparrow|J^+|N,\downarrow\rangle. \tag{3}$$

The form factors are calculated using the "good" component of the current, J^+, to suppress the effects of quark-pair terms.

It is worthwhile to compare the formalism embodied in Eq. (3) with the nonrelativistic quark model formalism in which G_E and G_M are the Fourier transforms of the ground state matrix elements of the quark charge $[\Sigma_{i=1,3} e_i \delta(\mathbf{r} - \mathbf{r_i})]$ and magnetization $[\Sigma_{i=1,3}(e_i/2m_i)\delta(\mathbf{r}-\mathbf{r_i})]$ density operators. At high momentum transfer one needs to account for the influence of the motion of the proton on its wave function. Since the charged quarks of the initial proton state (or final state, or both initial and final states) are moving, charge and magnetic effects are correlated in a manner consistent with relativity. It is necessary to maintain this relativistic connection, which is lost in the nonrelativistic quark model. The use of light-front dynamics, concomitant with Eq. (3), is a particularly convenient way to handle the motion of the initial and final states. This is because the proton wave function, a function of internal relative coordinates, is the same in any reference frame.

III. RELATIVISTIC CONSTITUENT QUARK MODEL OF THE NUCLEON

We study the form factors using relativistic constituent quark models in general, and starting with the model of Schlumpf [11–13], in particular. Such models have a long history [15–17], and many authors [18–34] have contributed to the necessary developments. Schlumpf's model is used because his power-law wave functions lead to a reasonably good description of the proton electromagnetic form factors, G_E and G_M, at all of the values of Q^2 where data were available as of 1992 [11–13]. This model uses the Bakamjian-Thomas (BT) construction, which implies the choice of a very specific model wave function [24,32]. The use of a definite model allows us to gain insight, but we also shall discuss the limitations of this approach.

We remind the reader about a few basic features of light-front treatments in the BT approach. The light-front formalism is specified by the invariant hypersurface $x^+=x^0+x^3=$ constant. The following notation is used: A four-vector A^μ is given by $A^\mu=(A^+,A^-,\mathbf{A}_\perp)$, where $A^\pm\equiv A^0\pm A^3$ and $\mathbf{A}_\perp=(A^1,A^2)$. Light-front momenta vectors are denoted by $\mathbf{p}=(p^+,\mathbf{p}_\perp)$, with $p^-=(p_\perp^2+m^2)/p^+$ for on-shell quarks. The three momenta $\mathbf{p}_i$ of the quarks can be transformed to the total and relative momenta to facilitate the separation of the center of mass motion as

$$\mathbf{P}=\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3,\quad \xi=\frac{p_1^+}{p_1^++p_2^+},\quad \eta=\frac{p_1^++p_2^+}{P^+},$$

$$\mathbf{k}_\perp=(1-\xi)\mathbf{p}_{1\perp}-\xi\mathbf{p}_{2\perp},\quad \mathbf{K}_\perp=(1-\eta)(\mathbf{p}_{1\perp}+\mathbf{p}_{2\perp})-\eta\mathbf{p}_{3\perp}. \tag{4}$$

It is also useful to consider the mass operator of a noninteracting system of total momentum P^μ:

$$M_0^2\equiv\sum_{i=1,3}p_i^-\quad P^+-P_\perp^2=\frac{K_\perp^2}{\eta(1-\eta)}+\frac{k_\perp^2+m^2}{\eta\xi(1-\xi)}+\frac{m^2}{1-\eta}, \tag{5}$$

where m is the light quark mass, taken as the same for up and down quarks.

One may express the proton wave function in the center of mass frame in which the individual momenta are given by

$$\mathbf{p}_{1\perp}=\mathbf{k}_\perp+\xi\mathbf{K}_\perp,\quad \mathbf{p}_{2\perp}=-\mathbf{k}_\perp+(1-\xi)\mathbf{K}_\perp,\quad \mathbf{p}_{3\perp}=-\mathbf{K}_\perp. \tag{6}$$

The use of light-front variables enables one to separate the center of mass motion from the internal motion. The internal wave function Ψ is therefore a function of the relative momenta $\mathbf{p}_i,\xi,\eta$. The internal wave function of the proton depends on these relative momenta. One can obtain the wave function in certain special frames via a kinematic boost. In particular, if the proton acquires a transverse momentum (is boosted) by the absorption of a photon of momentum $\mathbf{q}=(0,\mathbf{q}_\perp)$ by the third quark, the effects of the boost are obtained merely replacing the momenta $\mathbf{k}_\perp$ and $\mathbf{K}_\perp$ by

$$\mathbf{k}'_\perp=\mathbf{k}_\perp,\quad \mathbf{K}'_\perp=\mathbf{K}_\perp-\eta\mathbf{q}_\perp. \tag{7}$$

The mass operator, M'_0 of the boosted system is obtained by replacing $\mathbf{k}_\perp,\mathbf{K}_\perp$ by the variables of Eq. (7) so that

$$M_0'^2=M_0^2+\frac{-2\eta\mathbf{K}_\perp\cdot\mathbf{q}_\perp+\eta^2q_\perp^2}{\eta(1-\eta)}. \tag{8}$$

We now turn to the construction of the nonperturbative wave function Ψ. This is based on the attempt to construct a state, described in terms of the given light-front variables, which is also an eigenstate of angular momentum [21]. We take the proton wave function to be a product of an antisymmetric color wave function with a symmetric flavor-spin-momentum wave function Ψ. To understand the construction [21] of the relativistic wave function, it is worthwhile to start by considering the nonrelativistic model. Then

$$\Psi^{NR}=\frac{1}{\sqrt{2}}(\phi_\rho\chi_\rho+\phi_\lambda\chi_\lambda)\Phi, \tag{9}$$

where ϕ_ρ represents a mixed-antisymmetric and ϕ_λ a mixed-symmetric flavor wave function and $\chi_{\rho,\lambda}$ represents mixed-symmetric or antisymmetric spin wave functions (in terms of Pauli spinors). In the nonrelativistic model the wave function Φ depends on spatial variables only, and the computed form factors G_E and G_M will have the same dependence on Q^2.

The relativistic generalization of Eq. (9) is

$$\Psi(p_i)=u(p_1)u(p_2)u(p_3)\psi(p_1,p_2,p_3), \tag{10}$$

where p_i represents space, spin, and isospin indices: $p_i=\mathbf{p}_i s_i,\tau_i$ and repeated indices are summed over. The spinors u are canonical Dirac spinors:

$$u(p,s)=\frac{\not{p}+m}{\sqrt{E(p)+m}}\begin{pmatrix}\chi_s^{\text{Pauli}}\\0\end{pmatrix}, \tag{11}$$

with the isospin label suppressed.

The completely symmetric nature of the space-spin-flavor wave function is preserved by using [21]

$$\psi(p_1,p_2,p_3)=\Phi[\overline{u}(p_1)\Gamma\overline{u}_2^T(p_2)\overline{u}_3(p_3)u_N(0)+\overline{u}(p_1)\Gamma^{\mu,\alpha}\overline{u}_2^T(p_2)\overline{u}_3(p_3)\tilde{\Gamma}^{\nu,\alpha}u_N(0)g_{\mu\nu}] \tag{12}$$

$$\Gamma\equiv-\frac{1}{\sqrt{2}}\frac{1+\beta}{2}\gamma_5Ci\tau_2,\quad \Gamma^{\mu,\alpha}\equiv\frac{1}{\sqrt{6}}\gamma^\mu Ci\tau_\alpha\tau_2,$$
$$\tilde{\Gamma}^{\nu,\alpha}\equiv\frac{1+\beta}{2}\gamma^\nu\gamma_5\tau_\alpha, \tag{13}$$

where the charge-conjugation matrix $C \equiv i\gamma^2\gamma^0 = -i\gamma_5\sigma_2$. Note that if one takes the nonrelativistic limit of Eq. (12) by taking $\mathbf{p}_i \rightarrow (m, \mathbf{0}_\perp)$, one gets Eq. (9) for the spin-isospin dependence.

The general properties (based on symmetries) of the wave function of Eq. (12) are now specified. This wave function is also expressed in terms of relative momentum variables. Thus our wave function maintains the remarkable feature about the light-front approach that one is able to write the wave function of the proton in a manner that is independent of the inertial reference frame. This wave function is also an eigenstate of the spin operator, that is defined in terms of the Pauli-Lubanski vector W and the nucleon mass operator M. The squares of W and M are two Casimir operators of the generators of the Poincaré group. A nice discussion of the relevant formalism is presented in Ref. [33]. The feature that the wave function is an eigenstate of spin is the essential ingredient that leads to the numerical results presented below. See Ref. [34] for a critique of the procedure.

A. Wave function Φ

All that remains prior to numerical evaluation is to specify the wave function Φ. This must be chosen as a function of M_0 to fulfill the requirements of spherical and permutation symmetries. We take this to be of a power-law form:

$$\Phi(M_0) = \frac{N}{(M_0^2+\beta^2)^\gamma}, \tag{14}$$

which depends on two free parameters, the constituent quark mass and the confinement scale parameter β. Schlumpf's parameters are $\beta = 0.607$ GeV, $\gamma = 3.5$, and the constituent quark mass, $m = 0.267$ GeV. It is worthwhile to review Schlumpf's motivation for these parameters [11]. The power falloff is chosen to reproduce the high Q^2 behavior of the wave functions. In particular, the data for $G_M(Q^2)$ [35] was well described up to a value of $Q^2 = 30$ GeV^2, with $Q^4 G_M(Q^2)$ approximately constant for Q^2 greater than about 10 GeV^2. The form factor (QF_2) is an integral over four perpendicular components of momentum variables times the product of the wave functions, which varies as a power of perpendicular momentum variables as to the power of $-2\gamma = -7$. At high Q^2, one finds roughly that $QF_2 \sim Q^4/Q^7$, so $G_M \sim 1/Q^4$. Thus the power γ is fixed by the high Q^2 behavior of G_M. The remaining parameter β is determined mainly by the charge radius of the proton. At low Q^2, the value of β controls the momentum of the quarks. The size of the system is (via the uncertainty principle) of order $(\pi/\beta) \approx 1$ fm. Schlumpf's work shows that these parameters lead to an excellent description of many baryonic properties [11–13].

IV. THE ESSENTIAL EFFECT

The form factors of Eq. (3) are obtained by computing the matrix elements of the current operator J^+ between the initial and final states given by the wave function of Eqs. (10)–(14). The relative momenta of the initial and final wave functions are given by Eqs. (6)–(8). Here we take the quarks to be elementary particles, so that the operator J^+ is the operator γ^+ times the charge of the quarks. The definition of the calculation is now complete, and we could proceed immediately to obtain numerical results. However, the necessary computations may be simplified by making a unitary transformation that replaces the Dirac spinors of Eq. (10) by light-front spinors:

$$u_L(p,\lambda) = \frac{\not{p}+m}{\sqrt{2p^+}}\gamma^+ \begin{pmatrix} \chi_\lambda^{\text{Pauli}} \\ 0 \end{pmatrix}. \tag{15}$$

The use of these spinors simplifies the evaluation of the matrix elements of γ^+ because [5]

$$\overline{u_L}(p^+,\mathbf{p}',\lambda')\gamma^+ u_L(p^+,\mathbf{p},\lambda) = 2\delta_{\lambda\lambda'}p^+. \tag{16}$$

One then uses the completeness relation $1 = \Sigma_\lambda u_L(p,\lambda)\overline{u_L}(p,\lambda)/2m$, for each of the three quarks in Eq. (10), to obtain the light-front representation for the wave function:

$$\Psi(p_i) = u_L(p_1,\lambda_1)u_L(p_2,\lambda_2)u_L(p_3,\lambda_3)\psi_L(p_i,\lambda_i), \tag{17}$$

$$\psi_L(p_i,\lambda_i) \equiv [\overline{u_L}(\mathbf{p}_1,\lambda_1)u(\mathbf{p}_1,s_1)][\overline{u_L}(\mathbf{p}_2,\lambda_2)u(\mathbf{p}_2,s_2)] \times [\overline{u_L}(\mathbf{p}_3,\lambda_3)u(\mathbf{p}_3,s_3)]\psi(p_1,p_2,p_3). \tag{18}$$

We emphasize that the wave functions of Eqs. (13) and (17) represent the same quantity. We have simply used a unitary transformation that simplifies the calculation of the matrix element of the operator γ^+. The coefficients of the unitary transformation are the same as a transformation introduced by Melosh [36]. For us, there is no dynamical significance in this. The notation $\langle\lambda_i|\mathcal{R}_M^\dagger(\mathbf{p}_i)|s_i\rangle_i \equiv \overline{u_L}(\mathbf{p}_i,\lambda_i)u(\mathbf{p}_i,s_i)$, with $|\lambda_i\rangle, |s_i\rangle$ as Pauli spinors, is simply a convenient abbreviation. Since canonical Dirac spinors are well known [given in Eq. (11)], the light-front spinors are also conventional and are given in Eq. (15), it is a matter of algebra to show, for example, that

$$\langle\lambda_3|\mathcal{R}_M^\dagger(\mathbf{p}_3)|s_3\rangle = \overline{u_L}(\mathbf{p}_3,\lambda_3)u(\mathbf{p}_3,s_3) = \langle\lambda_3|\left[\frac{m+(1-\eta)M_0+i\boldsymbol{\sigma}\cdot(\mathbf{n}\times\mathbf{p}_3)}{\sqrt{[m+(1-\eta)M_0]^2+p_{3\perp}^2}}\right]|s_3\rangle. \tag{19}$$

The net result of this is that the relativistic spin effect is to replace the Pauli spinors of Eq. (9) with Melosh rotation operators acting on the very same Pauli spinors [21]. The spin-wave function of the ith quark is given by

$$|\uparrow\mathbf{p}_i\rangle \equiv \mathcal{R}_M^\dagger(\mathbf{p}_i)\begin{pmatrix}1\\0\end{pmatrix} \quad \text{and} \quad |\uparrow\mathbf{p}_i\rangle \equiv \mathcal{R}_M^\dagger(\mathbf{p}_i)\begin{pmatrix}0\\1\end{pmatrix}. \tag{20}$$

This means that, for example, the spin-wave function χ_ρ is replaced by a momentum-dependent spin-wave function $|\chi_\rho^{\rm rel}\mathbf{p}_i\rangle$,

$$|\chi_\rho^{\rm rel}\mathbf{p}_i\rangle=\frac{1}{\sqrt{2}}|\uparrow\mathbf{p}_1\downarrow\mathbf{p}_2-\downarrow\mathbf{p}_1\uparrow\mathbf{p}_2\rangle|\uparrow\mathbf{p}_3\rangle\equiv|\chi_0^{\rm rel}(\mathbf{p}_1,\mathbf{p}_2)\rangle|\uparrow\mathbf{p}_3\rangle, \tag{21}$$

for a spin $+1/2$ proton. The term $\chi_0^{\rm rel}$ represents the relativistic generalization spin-0 wave function of the quark pair labeled by (1,2). Note that the momenta $\mathbf{p}_i$ are to be expressed in terms of the relative variables $\mathbf{k}_\perp$, $\mathbf{K}_\perp$, $\mathbf{k}'_\perp$, and $\mathbf{K}'_\perp$ of Eqs. (4) and (7). We shall see that the important relativistic effect is contained in the difference between the spinors of Eq. (20) and Pauli spinors.

The next step is to simplify the calculation by using the symmetry of the wave function under interchange of particle labels to replace the quark current operator by three times that operator acting only on the third quark. The average charge of the third quark of the mixed-symmetric flavor wave function vanishes. This means that the second term of Eq. (12) does not contribute to proton electromagnetic form factors. The first term involves a mixed-antisymmetric wave function, so the third quark carries the spin of the proton, s_3, of Eq. (19). However, the spin of the light-front spinor λ_3 can either be the same or different from s_3. The weighting of the terms can be understood readily by examining the two terms of the Melosh transformation of Eq. (19), and considering the arguments of the final state wave function. The effect of the boost is incorporated simply by using Eq. (7) in the argument of the final state wave function. Thus if large momentum $Q\gg\beta$ is involved, both terms of the Melosh rotation (19) can be expected to have similar magnitudes. In particular, the term involving M_0 and the spin-flip term $\boldsymbol{\sigma}\cdot(\mathbf{n}\times\mathbf{p}_3)$ each gain a magnitude of order Q. The precise terms are given in the following section. Thus spin-flip and non-spin-flip terms of Eq. (19) are comparable.

The importance of the spin-flip term has immediate implications for our problem. There is clearly a substantial amplitude for a spin-up valence quark ($s_3=+1/2$) to carry a negative light-front spin ($\lambda_3=-1/2$). The spin of the struck quark need not be the same as that of the proton. Therefore, we cannot expect the hadron helicity selection rule to apply if we use a wave function such as that of Eqs. (10)–(14), which is constructed to be an eigenstate of spin. Helicity conservation does not occur. This same conclusion has been obtained by Ralston and co-workers [37] using the light-front basis, based on the presence of nonzero orbital angular momentum. Although our spatial wave function Φ corresponds to quarks moving in relative *s* states, the use of Dirac spinors in Eq. (10) gives the same helicity nonconservation as that of Ref. [37]. In particular, the lower components of the Dirac spinors contain the nonzero orbital angular momentum present in the light-front basis. Thus we support the statements of Ref. [37] that helicity nonconservation is an important effect. We also note that Braun *et al.* [38] argue that soft nonfactorizable terms in the wave function, with helicity structure similar to ours, are important at intermediate values of Q^2. In our model, there is no basis for expecting hadronic helicity conservation because the nonperturbative wave function is a mixture of different light-front-spin states, with the mixture expressed precisely in Eqs. (17) and (18).

If the value of Q^2 becomes asymptotically large, the effects of the nonperturbative wave function may disappear and perturbative effects, that do respect helicity conservation, could take over. But for the present, helicity nonconservation occurs and it will be worthwhile to consider the implications of this nonconservation.

V. PROTON FORM FACTORS

We obtain the form factors by using the wave function of Eq. (18) in Eq. (3). Our result can be expressed as

$$F_1(Q^2)=\int\frac{d^2q_\perp d\xi}{\xi(1-\xi)}\frac{d^2K_\perp d\eta}{\eta(1-\eta)}\tilde{\Phi}^\dagger(M'_0)\tilde{\Phi}(M_0)\times\langle\chi_0^{\rm rel}(\mathbf{p}'_1,\mathbf{p}'_2)|\chi_0^{\rm rel}(\mathbf{p}_1,\mathbf{p}_2)\rangle\langle\uparrow\mathbf{p}'_3|\uparrow(\mathbf{p}_3)\rangle, \tag{22}$$

$$Q\kappa F_2(Q^2)=2M_N\int\frac{d^2q_\perp d\xi}{\xi(1-\xi)}\frac{d^2K_\perp d\eta}{\eta(1-\eta)}\tilde{\Phi}^\dagger(M'_0)\tilde{\Phi}(M_0)\times\langle\chi_0^{\rm rel}(\mathbf{p}'_1,\mathbf{p}'_2))|\chi_0^{\rm rel}(\mathbf{p}_1,\mathbf{p}_2)\rangle\langle\uparrow\mathbf{p}'_3|\downarrow(\mathbf{p}_3)\rangle. \tag{23}$$

The value of M'_0 is obtained by using Eq. (8), and

$$\tilde{\Phi}(M_0)\equiv\sqrt{\frac{E_3E_{12}E_1}{M_0}}\Phi(M_0), \tag{24}$$

$$E_1=\sqrt{\frac{k_\perp^2+m^2}{4\xi(1-\xi)}},\qquad E_{12}=\frac{K_\perp^2+4E_1^2+\eta^2M_0^2}{2\eta M_0},\qquad E_3=M_0-E_{12}. \tag{25}$$

Numerical evaluations of these equations for large values of Q^2 were presented in Ref. [7]. Our aim here is to understand the essential features of the different Q^2 dependence of F_1 and F_2. The expressions for the form factors differ only by the presence of the last factor $\langle\uparrow\mathbf{p}'_3|\downarrow\mathbf{p}_3\rangle$, or $\langle\uparrow\mathbf{p}'_3|\uparrow\mathbf{p}_3\rangle$, with F_1 depending on the non-spin-flip term and F_2 on the spin-flip term [39]. To proceed we evaluate these overlaps, using the Melosh transformations with $\mathbf{p}_{3\perp}=-\mathbf{K}_\perp$, and $\mathbf{p}'_{3\perp}=-\mathbf{K}_\perp+\eta\mathbf{q}_\perp$. One computes the product of the matrices $\mathcal{R}_M(\mathbf{p}'_3)\mathcal{R}_M^\dagger(\mathbf{p}_3)$. The upper diagonal element (non-spin-flip term) appears in the expression for F_1, and the upper off-diagonal element (spin-flip term proportional to $\boldsymbol{\sigma}$) determines F_2. The evaluation is simplified by realizing that integration over $d^2K_\perp$ causes terms linear in the component of $\mathbf{K}$, which are perpendicular to $\mathbf{q}$ in order to vanish.

To be definite, we take $\mathbf{q}_\perp$ to lie along the *x* direction, so that we find

$$\langle\uparrow\mathbf{p}_3'|\uparrow\mathbf{p}_3\rangle=\frac{[m+(1-\eta)M_0][m+(1-\eta)M_0']+K_\perp^2-\eta QK_x}{\sqrt{\{[m+(1-\eta)M_0]^2+\mathbf{K}_\perp^2\}\{[m+(1-\eta)M_0']^2+(\mathbf{K}_\perp-\eta\mathbf{q}_\perp)^2\}}}$$

$$\langle\uparrow\mathbf{p}_3'|\downarrow\mathbf{p}_3\rangle=\frac{\eta Q[m+(1-\eta)M_0]+(1-\eta)(M_0'-M_0)K_x}{\sqrt{\{[m+(1-\eta)M_0]^2+\mathbf{K}_\perp^2\}\{[m+(1-\eta)M_0']^2+(\mathbf{K}_\perp-\eta\mathbf{q}_\perp)^2\}}}. \tag{26}$$

We may understand the qualitative nature of the ratio $[QF_2(Q^2)]/[F_1(Q^2)]$ using the notion that the value of $Q=\sqrt{\mathbf{q}_\perp^2}$ can be much larger than the typical momenta, of order of $\beta=560$ MeV, which appear in the wave function. Then for $Q\gg\beta$, we may approximate Eq. (8) by

$$M_0'\approx Q\sqrt{\frac{\eta}{1-\eta}}, \tag{27}$$

and take the terms of the bracketed expressions of Eq. (26), which are proportional to Q as dominant. Using Eq. (27) in Eq. (26), and keeping only the terms proportional to Q, defines an approximation (valid for very large values of Q^2) to the form factors that we denote as $F_{1,2}^{As}$. Then we see that each of F_1^{As} and QF_2^{As} contains an explicit factor of Q, and

$$\frac{Q\kappa F_2^{As}}{F_1^{As}}\approx 2M_N\frac{\langle\eta[m+(1-\eta)M_0]+\sqrt{\eta(1-\eta)}K_x\rangle}{\langle -K_x+[m+(1-\eta)M_0]\sqrt{\eta/(1-\eta)}\rangle}, \tag{28}$$

where the expectation value symbols abbreviate the operation of multiplying by the remaining factors of Eqs. (22) and (23) (without approximation) and performing the necessary six-dimensional integral. The terms η, M_0, K_x can be anticipated to have an expectation value independent of Q^2, so the ratio is anticipated to be constant.

The exact model calculation and the approximation (28) are compared in Fig. 1. Equation (28) qualitatively reproduces the constant nature of the ratio and its value. Thus the constant nature of the ratio is understood from the properties of the Melosh transformation, which here embodies the relativistic effects.

Equation (28) represents a simple quick argument that gives a constant ratio. But this is only a rough approximation because each of $F_{1,2}$ is overpredicted by about 40%. Numerical work shows that neglecting the terms proportional to K_x in both the numerator and denominator of Eq. (28) leads to a different approximation:

$$\frac{Q\kappa F_2^{As}}{F_1^{As}}\approx\frac{\langle\eta[m+(1-\eta)M_0]\rangle}{\langle[m+(1-\eta)M_0]\sqrt{\eta(1-\eta)}\rangle}, \tag{29}$$

which, as shown in Fig. 2 leads to an even better reproduction of the model results for F_1 and F_2.

Equation (29) is a better approximation because the terms involving K_x cancel against terms involving the difference between M_0' and its approximation (27). Thus it seems that values of Q^2 going up to 20 GeV^2 are not large enough to allow one to completely neglect other terms, and therefore also not large enough to extract the asymptotic behavior.

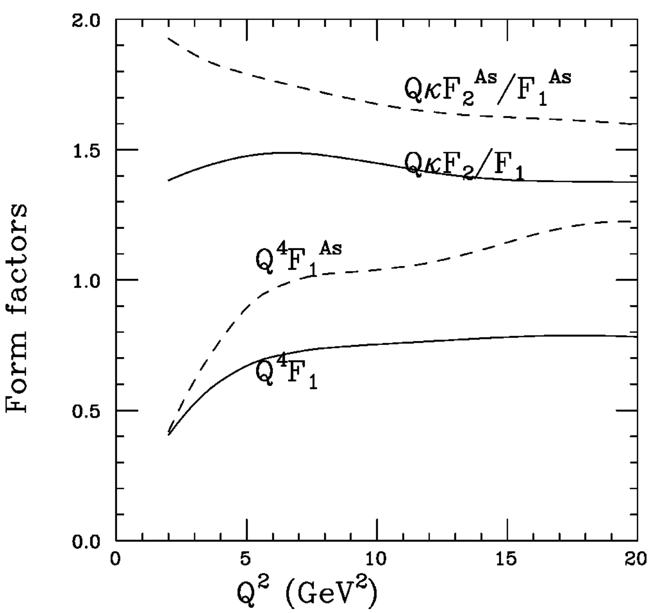

FIG. 1. Model calculation of Eqs. (22), (23), and (26) (solid lines) vs the approximation Eq. (28) (dashed lines).

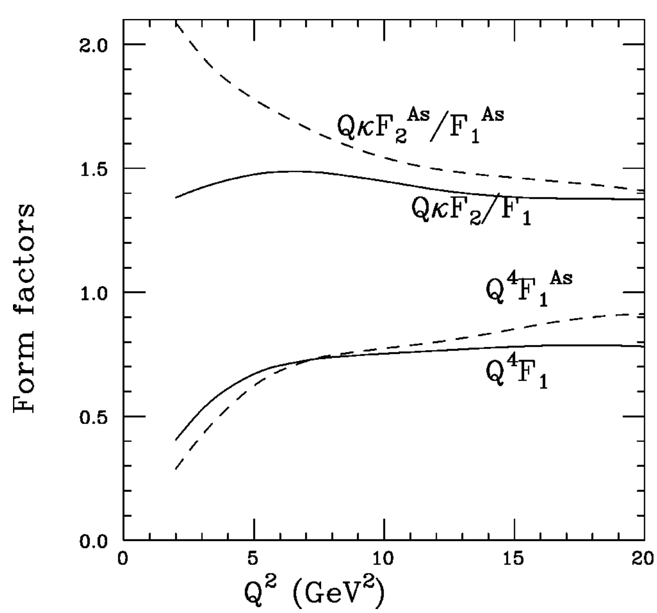

FIG. 2. Model calculation of Eqs. (22),(23), and (26) (solid lines) solid vs the approximation Eq. (29) (dashed lines).

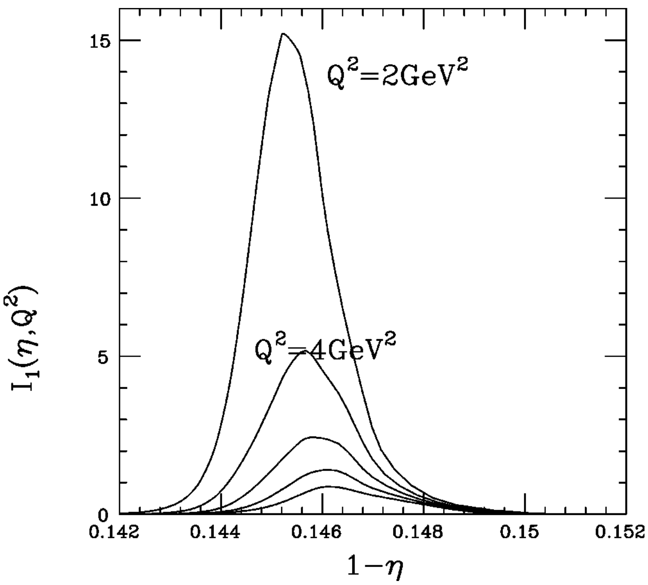

FIG. 3. Important region of integration for F_1, Eq. (30). The curves show the derivative of I_1 for values of $Q^2=2,4,6,8,10$ GeV^2, with the larger values occurring for the smaller values of Q^2.

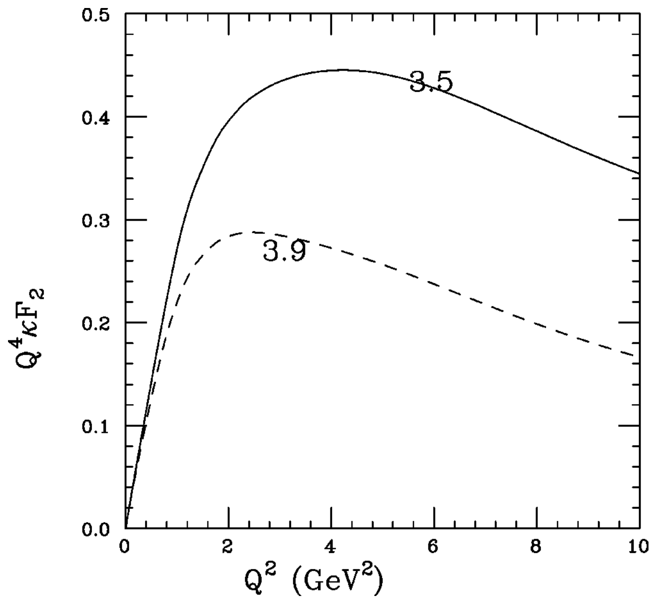

FIG. 5. The effect of varying the parameter γ that governs the power of the falloff of the wave function of Eq. (14). The curves for F_2 are labeled by the value of $\gamma=3.5$ that is the correct model value (solid) or $\gamma=3.9$ (dashed).

This feature of not reaching asymptotic values of Q^2 may be understood by examining the dependence of the integrands of Eqs. (22) and (23) on the value of η. We may write

$$F_{1,2}(Q^2)=\int_0^1 d\,\eta I_{1,2}(\eta,Q^2), \tag{30}$$

and determine the important regions by examining $I_{1,2}(\eta,Q^2)$. As shown in Figs. 3 and 4 the important contributions occur for a very narrow band of values close to $1-\eta=x_3=0.145$. The sharp peaking is maintained for all of the values of Q^2 considered here, and is a central reason for the qualitative success of the approximations (28) and (29). The small factor $1-\eta$ multiplies the large factor Q appearing in Eq. (27), and suppresses the dominance of the terms proportional to Q. The integrands peak at $x_3=0.15$, a small value (compared to 0.33, expected if each quark were to carry the same momentum) that indicates the presence of

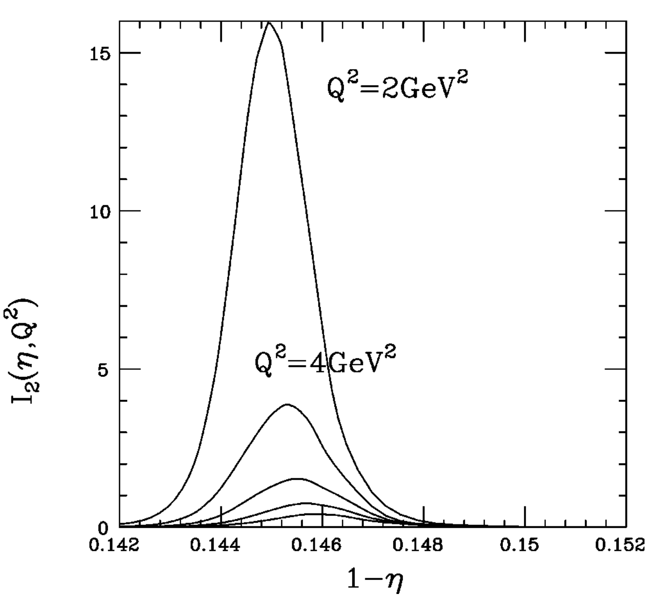

FIG. 4. Important region of integration for F_2, Eq. (30). The curves show the derivative of I_1 for values of $Q^2=2,4,6,8,10$ GeV^2, with the larger values occurring for the smaller values of Q^2.

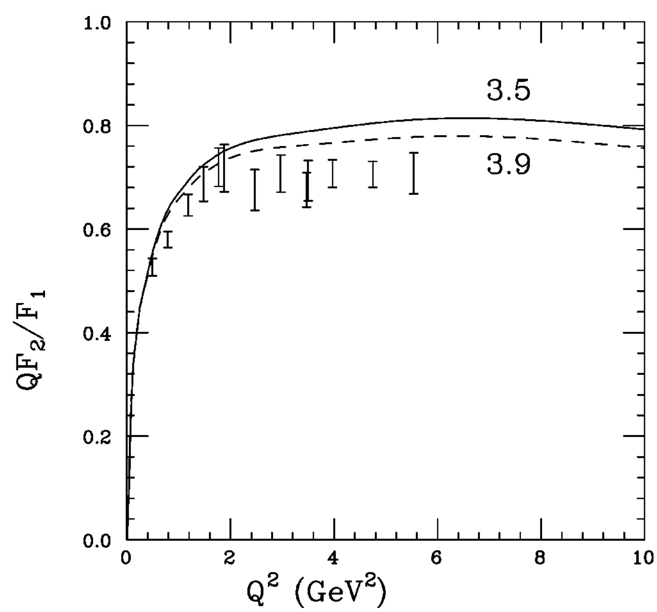

FIG. 6. The effect of varying the parameter γ that governs the power of the falloff of the wave function of Eq. (14). The curves for $Q\ F_2/F_1$ are labeled by the value of $\gamma=3.5$ (solid) or $\gamma=3.9$ (dashed). The data for $2\leqslant Q^2\leqslant 3.5$ GeV^2 are from Ref. [1], and those for $3.5\leqslant Q^2\leqslant 5.5$ GeV^2 are from Ref. [2].

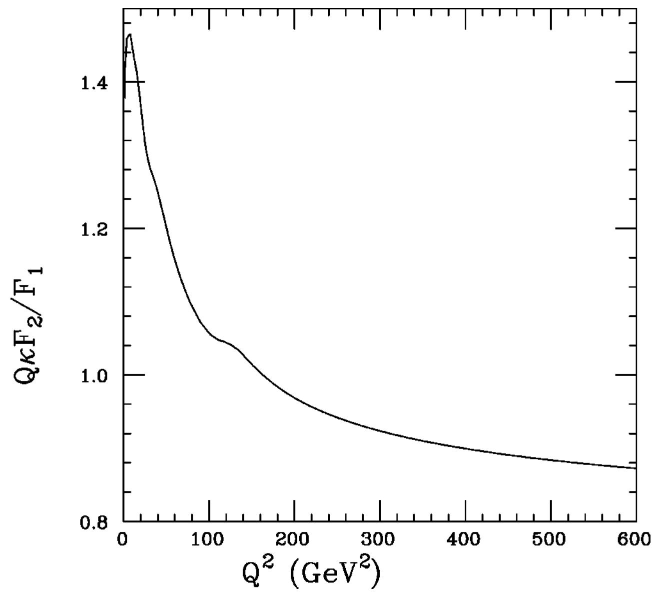

FIG. 7. High Q^2 behavior of $Q\kappa F_2/F_1$.

end-point corrections, and a corresponding difficulty of using simple arguments to extract asymptotic properties of form factors.

We also find that the computed value of the ratio $Q\kappa F_2/F_1$ is remarkably independent of the parameters of the model. For example, Fig. 5 shows that a 10% increase in the value of γ, Eq. (14), causes about a 50% decrease in the computed values of F_2, but Fig. 6 shows only a 5% change in the ratio.

The solid curve of Fig. 6 represents our final results and predictions for values of Q^2 that are not yet measured. The dashed curve of Fig. 6 is closer to the data for the ratio, but it corresponds to values of F_2 at high Q^2, which are much smaller than the data of Ref. [40]. Thus the original model of Schlumpf seems to account for both F_1 and F_2. There is a small disagreement ($\sim 15\%$) with the data for the ratio, which cannot be fixed simply by varying the parameters. This small degree of disagreement between the model and the data is remarkable because so many plausible effects, such as configuration mixing involving both quark and gluon degrees of freedom and a nonperturbative Q^2 variation of the constituent quark masses [41] are ignored. Pion cloud effects [42] are surely present, but these do not seem to be significant for values of Q^2 greater than about 2 GeV^2.

We also study the behavior of the ratio $Q\kappa F_2/F_1$ for very large values of Q^2; see Fig. 7. The constant nature of the ratio seen in previous figures is actually the result of a broad maximum occurring near $Q^2 \approx 10$ GeV^2. The ratio falls for asymptotic values of Q^2, but not as quickly as expected [5] from perturbative QCD, $Q\kappa F_2/F_1 \sim 1/Q$.

VI. SUMMARY AND DISCUSSION

We have seen how a simple relativistic constituent quark model accounts for both G_M and QF_2/F_1 for values of Q^2 between 2 and 5.5 GeV^2. The most relevant ingredient in the model is its attempt to use a wave function that is Poincaré invariant. In such wave functions the helicity conservation rule is not satisfied because the nonperturbative wave function is a mixture of different spin states, as defined in Eqs. (17) and (18) and explained in Sec. IV. This feature leads to an approximate analytic understanding, embodied in Eqs. (28) and (29), that $QF_2(Q^2)$ and $F_1(Q^2)$ have the same variation with Q^2. The predicted value of the ratio $QF_2(Q^2)/F_1(Q^2) \approx 0.8$ for the values of Q^2 up to 20 GeV^2, and drops slowly for larger values.

The present data set extends to $Q^2 = 5.5$ GeV^2, and its measurement of the nonconservation of helicity has implications for other exclusive processes involving protons. Hadronic helicity conservation should not be relevant if we consider proton-proton scattering at high momentum transfers, up to $-t = 5.5$ GeV^2. This means large values of various analyzing powers can be expected. Perhaps the most interesting mystery in proton-proton scattering is the large value of A_{NN} observed in 90° proton scattering at $s \approx 20$ GeV^2 [43,44]. This corresponds to $-t \approx 7-10$ GeV^2. If the present measurements are extended to values of Q^2 such as these, and if the constant nature of the ratio $QF_2(Q^2)/F_1(Q^2)$ is maintained, one could be able to seek an explanation of the large value of the analyzing power in terms of the nonperturbative proton wave function.

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