Correlation search for coherent pion emission in heavy ion collisions

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The methods allowing us to extract the coherent component of pion emission conditioned by the formation of a quasiclassical pion source in heavy ion collisions are suggested. They exploit a nontrivial modification of the quantum statistical and final state interaction effects on the correlation functions of like and unlike pions in the presence of the coherent radiation. The extraction of the coherent pion spectrum from $\pi^+\pi^-$ and $\pi^\pm\pi^\pm$ correlation functions and single-pion spectra is discussed in detail for large expanding systems produced in ultrarelativistic heavy ion collisions.

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I. INTRODUCTION

The hadronic observables, such as single- or multiparticle hadron spectra, play an important role in the studies of ultrarelativistic heavy ion collisions. However, these observables contain rather indirect information on the initial stage of the collision process since the particle interactions result in substantial stochastization and thermalization of a system during its evolution. Nevertheless, the final hadronic state can carry some residual signals of the earlier stages of the particle production process. A partial coherence of the produced pions is assumed to be one of the important examples.

The first systematic study of coherent processes in high energy hadron-nucleus ($h+A$) collisions was based on Glauber theory [1]. In this theory, the $h+A$ collision is considered as a process of subsequent scatterings of the projectile on separate nucleons of the nucleus; the projectile energies are assumed to be much higher than the inverse nucleus radius ($E_h \gg 1/R$), thus allowing to consider a linear projectile trajectory inside the nucleus (eikonal approximation). If the scattering process occurred with almost no recoil of the nucleus nucleons, i.e., with no *witnesses* of the individual scatterings, then the $h+A$ collision should be described by a coherent superposition of the elementary hadron-nucleon scattering amplitudes. Such a type of collision is called coherent scattering. Since the nucleus in coherent scattering does not change its state, it manifests itself just as a particle with some form factor. In the oscillator approximation, the nucleus form factor can be represented by a Gaussian: $\exp(-\mathbf{q}^2R^2/4)$. The coherent processes are essential only for small momenta transferred from the projectile hadron to the nucleus: $|\mathbf{q}|<1/R$. Then, one can neglect the recoil energy and consider the nucleus as a whole during the scattering process. There is a kinematic limitation of the minimal longitudinal momentum transfer, $|q_z|_{\min} \approx (M^2-m_h^2)/(2|\mathbf{p}_h|)$, required to produce a particle or a group of particles of the invariant mass M. The vanishing of $|q_z|_{\min}$ with the increasing energy explains why the coherent processes can take place only at high enough energies. It is worth noting that the total coherent cross section does not die out with the increasing energy (see., e.g., Ref. [2]).[1]

Typically, however, the transferred momenta are sufficient for substantial *recoil* effects and the excitation of the nucleus or its breakup. Then, due to a small *coherence length* $\sim 1/|\mathbf{q}|$, the nucleus does not participate in the collision as a whole and one can consider the $h+A$ collision as an incoherent superposition of elementary hadron-nucleon scatterings corresponding to random phases of the amplitudes of the latter. The resulting cross section is then given by the sum of the moduli squared of the amplitudes (probabilities) at each of the possible scattering points (unlike coherent scattering, when the individual amplitudes are summed first). As a result, the particles are produced in chaotic (incoherent) states.

Let us come back to the production of particles (e.g., pions) in the processes of nonelastic coherent scattering at small transferred momenta. Since the nucleus is not excited in these processes and manifests itself as a quasiclassical object, one can describe particle production using the quantum field model of interaction with a classical source [3]. It is well known that the interaction with a classical source results in the production of bosons in coherent states [4]. These states minimize the uncertainty relation and, so, are the closest to classical ones.[2] This is the main physical link between the processes of coherent scattering and particle production in coherent states.

In heavy ion collisions at high energies, the average multiplicities are quite high, e.g., several thousands of pions can be produced at maximal Relativistic Heavy Ion Collider (RHIC) energies. The inclusive particle spectra thus represent natural characteristics of these processes. A convenient way to account for the coherent properties of these processes consists of a model description of particle emission, rather than in detailed evaluation of the contributing amplitudes. The Gyulassy-Kauffmann-Wilson (GKW) model [8] is an example of such an approach. The model assumes that all

[1]We are grateful to V. L. Lyuboshitz for drawing our attention to this important point and for an interesting discussion.

[2]The *coherent states* have been introduced and studied in detail by Glauber [5]. The concept of coherent states was then applied to pion production in high energy processes in Refs. [6–8].

pions are radiated by classical currents (sources) which are produced in some space-time region during the collision process. The corresponding density matrix is constructed by averaging over the unobservable positions of the centers of individual sources. The pion spectra then effectively contain both chaotic and coherent components. In fact, the chaotic component dominates in the case of a large emission region, while, in the opposite limit of very small space-time extent of this region, almost all pions are produced in the coherent state. This seems to be rather general result: if the distances between the centers of pion sources are smaller than the typical wavelength of the quanta (the source size), the substantial overlap of the wave packets leads to the strong correlations (indistinguishability) between the phases in pion wave functions and, thus, to the coherence [9,10].

Recently, the coherence of multipion radiation in high energy heavy ion collisions was studied within the GKW model in Ref. [11]. In the model, due to the longitudinal Lorentz contraction of the colliding nuclei, almost all pions produced with small transverse momenta $p_t < 1/R$ in central nucleus-nucleus collisions are emitted coherently, and their momentum spectra are determined by the system's space-time extent. Clearly, the coherence of pions can be destroyed by pion rescatterings. Nevertheless, the duration of hadron formation may happen to be long enough to allow a considerable part of the coherent pions to escape from the interaction zone without rescatterings [11]. However, as noted in Ref. [11], one can expect a strong suppression of the GKW mechanism of coherent pion production if quark-gluon plasma were created: the hadronization then occurs in a thermal quark-gluon system and hadrons are produced in the chaotic state only. Note that clear signals of the thermalization and collective flows, observed at CERN SPS and RHIC energies (see, e.g., Refs. [12,13], and references therein), point to strong rescattering effects and may reflect also the importance of the quark-gluon degrees of freedom.

The new physical phenomena, expected in RHIC and Large Hadron Collider experiments with heavy ions, are associated with the creation of quasimacroscopic, very dense and hot systems. In such systems, the deconfinement phase transition and the restoration of the chiral symmetry are likely to happen, possibly leading to creation of the new states of matter: quark-gluon plasma (QGP) and disoriented chiral condensate (DCC). In the latter case, another possibility for the coherent pion radiation (above the thermal background) appears. If the DCC were created at the chiral phase transition, a quasiclassical pion field $\vec{\pi}_{cl}$ forms the ground state of the system. The subsequent system decay is accompanied by a relaxation of the ground state to normal vacuum. Such a process can be described by the quantum field model of interaction with a classical source (see, e.g., Ref. [14]), and results in the coherent pion radiation. One of the general conditions of the ground state rearrangement and formation of the quasiclassical field is a large enough system volume [15]. Therefore, such a field could be generated in heavy ion collisions at sufficiently high energies provided the spontaneous chiral symmetry breaking via DCC formation takes place. The overpopulation of the (quasi)pion medium, making it close to the Bose-Einstein condensation point, can lead to the strengthening of the coherent component conditioned by the ground state (quasiparticle vacuum) decay [16]. Since the DCC appears relatively late (at the end of the hadronization stage), the coherent radiation could partially survive and be observed.

The coherent emission manifests itself in a most direct way in the inclusive correlation function $C(p,q)$ of two identical bosons in the region of very small $|\mathbf{q}|$; $p=(p_1+p_2)/2, q=p_1-p_2$. In case of only chaotic contribution, the intercept of the quantum statistical (QS) Bose-Einstein part of the correlation function $C_{QS}(p,0)=2$ [17] while, in the presence of the coherent radiation, $C_{QS}(p,0)<2$. Generally, the coherence means strong phase correlations of different radiation components. In Ref. [9], a simple quantum-mechanical model of the phase-correlated one-particle wave packets with different radiation centers has been considered. In such a case (corresponding to indistinguishable correlated emitting centers), the emission amplitude $A(p)$ averaged over the event ensemble is not equal to zero, $\langle A(p)\rangle \neq 0$, and the QS correlation function intercept $C_{QS}(p,0)<2$. In the second quantization representation (more adequate for processes of multiboson production), the analogous results take place for inclusive averages of the quantum field operators: $\langle a(p)\rangle \neq 0, C_{QS}(p,0)<2$, provided the radiation has a nonzero *coherent-state* component. The latter represents a superposition of the states of all possible boson numbers at fixed phase relations.

In practice, most of the correlation measurements are done with *charged* particles. However, charged bosons cannot form the usual coherent state since it obviously violates the superselection rule. To overcome this difficulty, the generalized concept of charge-constrained coherent states should be used [7,8,18]. Nevertheless, the correlations of charged bosons are usually described with the help of ordinary (not charge-constrained) coherent states [19,20] (see, however, Refs. [21,22]). Our treatment of two-pion correlations takes into account the restrictions imposed by the superselection rule and is based on the density matrix formalism.

The density matrix approach gives the possibility to describe, in a natural way, the chaotic radiation (the initial state then corresponding to a local-equilibrium statistical operator of quasiparticle excitations) and coherent emission (arising due to the interaction with a classical source). This approach can easily incorporate also the squeeze-state component of pion radiation [23], appearing due to the modification of the energy spectrum of quasipions as compared with that of free pions [24]. The density matrix formalism is also simply related with the Wigner function description of the multiparticle phase space and its evolution governed by the relativistic transport equation [25], representing very useful tools with a clear classical limit. Recent development of the classical current approach to multiparticle production [23,19] has made it closer to the density matrix formalism; particularly, the clasical current in momentum space has been shown to be mathematically identical with the coherent-state representation of the density matrix, the latter called "P" or Glauber-Sudarshan representation [5], see also Ref. [26].

In our approach, the superselection rule requires an averaging, in the density matrix, over all orientations of the quasiclassical pion source in the isospin space. As a consequence, the averaged pion field vanishes: $\langle a(p)\rangle=0$ whereas, for identical pions, the intercept $C_{QS}(p,0)$ is still less than 2. The correlations of nonidentical pions also appear to be sensitive to the presence of the quasiclassical source. This sensitivity arises due to properties of the generalized coherent states satisfying, after the averaging over all orientations of the quasiclassical source in isospin space, the superselection rule for charged particles. Due to isospin symmetry of the strong-interaction Hamiltonian, there are unique relations for the intercepts $C^{ij}_{QS}(p,0)$ of the pure QS correlation functions of two pions in various charge states $i,j=\pm,0$. For example, the coherence suppression of $C^{\pm\pm}$ determines the coherence enhancement of C^{+-}.

The coherence phenomena can be, however, masked by a number of effects suppressing the measured correlation functions. The most important among them are the decays of long-lived particles and resonances (e.g., Λ, K^0_s, η, η', . . .), the single- and two-track resolution and particle contamination. In Ref. [27], the method to discriminate between the effects of coherent radiation and decays of long-lived resonances has been proposed. The method assumes the simultaneous analysis of two- and three-particle correlation functions of identical pions. The practical utilization of the method is however difficult due to a low statistics of near-threshold three-pion combinations and the problem of the three-particle Coulomb interaction; also, one has to account for the superselection rule.[3] Therefore, in the present work we will restrict ourselves to the consideration of two-particle correlation functions.

In addition to QS, the correlations of particles with small relative velocities are also influenced by their final state interaction (FSI). The effect of the latter on two-particle correlations is well understood and introduces no principle problems. It is important that the correlations in different two-pion systems are influenced by the QS, FSI, and coherence effects in a different way. This offers a possibility to discriminate different effects suppressing the measured correlation functions and so to extract the coherent contribution using correlation functions of like and unlike pions measured at small relative momenta.

In the paper we study the influence of the coherent pion radiation on the behavior of pion inclusive spectra and two-pion correlation functions and, based on it, develop the methods for the extraction of the coherent component above the chaotic background. Despite the fact that we associate the coherent radiation with the formation of the DCC (as the most probable mechanism of the coherence in ultrarelativistic $A+A$ collisions), our results are rather general. Actually, they are based on the general properties of the coherent pion radiation: the quasiclassical nature of the coherent pion source and the constraints imposed by the charge superselection rule.

In Sec. II, we consider a general form of the density matrix of partially coherent pions, and calculate quantum statistical correlations of identical and nonidentical pions. In Sec. III, we set forth the density matrix formalism taking into account the decays of short-lived resonances and FSI of produced pions, and calculate the corresponding correlation functions. In Sec. IV, we discuss how to extract the coherent component of particle radiation from the two-pion correlation functions, particularly, in the case of large expanding systems produced in ultrarelativistic $A+A$ collisions. A short summary and conclusion are given in Sec. V.

[3] The latter problems are absent for neutral pions. However, sufficiently accurate measurements of neutral pion correlations are practically out of the present experimental possibilities.

II. QUANTUM STATISTICAL CORRELATIONS OF PARTIALLY COHERENT PIONS

It is well known that the description of the inclusive pion spectra and two-pion correlations is based on a computation of the following averages [8]:

$$
\begin{aligned}
\omega_{\mathbf{p}}\frac{d^3N_i}{d^3\mathbf{p}} &\equiv n_i(p)=\sum_{\alpha}|\mathcal{T}(in;p,\alpha)|^2=\langle a_i^\dagger(p)a_i(p)\rangle,\\
\omega_{\mathbf{p}_1}\omega_{\mathbf{p}_2}\frac{d^6N_{ij}}{d^3\mathbf{p}_1 d^3\mathbf{p}_2} &\equiv n_{ij}(p_1,p_2)\\
&=\sum_{\alpha}|\mathcal{T}(in;p_1,p_2,\alpha)|^2\\
&=\langle a_i^\dagger(p_1)a_j^\dagger(p_2)a_i(p_1)a_j(p_2)\rangle,\\
C^{ij}(p,q)=n_{ij}(p_1,p_2)/n_i(p_1)n_j(p_2), &\quad \omega_{\mathbf{p}_i}=\sqrt{m^2+\mathbf{p}_i^2},
\end{aligned}
\tag{1}
$$

where $\mathcal{T}(in;p,\alpha)$ is the normalized invariant production amplitude. The summation is done over all quantum numbers α of other produced particles, including integration over their momenta; $a_i^\dagger(p)$ and $a_i(p)$ are, respectively, the creation and annihilation operators of asymptotically free pions $i=\pm,0$; the bracket $\langle\ \dots\ \rangle$ formally corresponds to the averaging over some density matrix $|f\rangle\langle f|$. Special attention is required for the production of particles with nearby velocities which can be strongly influenced by particle interaction in the final state. In this section, we concentrate mainly on quantum statistical correlations ignoring, for a while, the effects of resonance decays and FSI.

Let us suppose that the density matrix ρ is a statistical operator describing the thermal hadronic system in a predecaying state on a hypersurface of the thermal freeze-out $\sigma_f: t=t_f(\mathbf{x})$. After the thermal freeze-out the system is out of local thermal equilibrium but still can be in a predecaying (interacting) state. In fact, the complete decay (neglecting the long-time scale forces) happens at some finite *asymptotic* times $t_{out}<\infty$. Then the formal solution of the Heisenberg equation for the pionic annihilation (creation) operators at this post-thermal freeze-out stage has the form[4]

[4] For a spacelike hypersurface σ_f [an example is $\sigma_f=t_f(\mathbf{x})=(\tau^2+x^2_{long})^{1/2}$ in the Bjorken hydrodynamic model with the proper expansion time τ], the use of the covariant Tomonaga-Schwinger formalism gives the same result with the substitution $t\rightarrow t(\mathbf{x})$.

$$a_{i,qm}(\mathbf{p},t_{out})=[a_{i,qm}(\mathbf{p},t_f)+d_i(\mathbf{p},t_f,t_{out})]e^{-i\omega_{\mathbf{p}}(t_{out}-t_f)}. \tag{2}$$

It formally corresponds to the sum of the general solution of the free (homogeneous) Heisenberg equation of motion for the pionic field (first term), and a particular solution of the complete (inhomogeneous) Heisenberg equation with a source (second term). The value $d_i(\mathbf{p},t_f,t_{out})$ depends on the actual form of the source term in the Heisenberg equation.

The decay of the system at this stage, $t_f<t<t_{out}$, can be accompanied by the coherent pion radiation due to the modification of hadron properties in a hot and dense hadronic environment or due to some peculiarities of the phase transition from QGP to hadron gas, e.g., the formation of DCC. In both cases, almost noninteracting quasiparticle excitations could be formed above a rearranged ground state (''condensate'').

In the systems containing the DCC, the appearance of the quasiclassical pion field $\vec{\pi}_{cl}$ (corresponding to the density of virtual pionic excitations of the quasipionic vacuum) at the thermal stage is usually described in the mean field approximation as $\pi_{i,cl}(x)=\pi_i(x)-\pi_{i,qm}(x)$, where the field $\pi_{i,qm}(x)$ corresponds to the quasipion quantum excitations above the temporary vacuum background $\pi_{i,cl}(x)$ (the order parameter). Assuming the isotopic symmetry of the Lagrangian such as in the sigma model (see, e.g., Ref. [28]), we have $\pi_{i,cl}(x)=e_i\pi_{cl}(x)$, where $\mathbf{e}$ is a randomly oriented unit vector, $\mathbf{e}^2=1$, in the three-dimensional isospin space. Then, for each $\mathbf{e}$ orientation of the quasipionic vacuum at the thermal freeze-out, the free quasipions $\vec{\pi}_{qm}$ are distributed according to the Gibbs local-equilibrium density matrix $\rho_{\mathbf{e}}$ above the quasipionic vacuum. After the thermal freeze-out, when the decay of such a thermal system happens, the quasipion masses approach the usual free particle values and the condensate (the temporary *disoriented* vacuum) tends to relax back to the normal vacuum by emitting physical pions in coherent states—the vacuum for quasiparticles becomes a coherent state for free particles. The latter process is similar to particle radiation by a classical source.

Then the ''source'' term in Eq. (2) takes on the form

$$d_i(\mathbf{p},t_f,t_{out})=d_{i,qm}(\mathbf{p},t_f,t_{out})+e_i d_{coh}(\mathbf{p},t_f,t_{out}),$$

$$e_0=\cos\theta,\quad e_{\pm}=\frac{\sin\theta}{\sqrt{2}}e^{\pm i\phi}, \tag{3}$$

where $d_{i,qm}(\mathbf{p},t_f,t_{out})$ and $e_i d_{coh}(\mathbf{p},t_f,t_{out})$ are q- and c-value quantities, respectively. While the total number of pions of momentum $\mathbf{p}$ radiated by a classical source is fixed by $|d_{coh}(\mathbf{p},t_f,t_{out})|^2$, the distribution of radiating pions in isospace is determined by the orientation of the vector $\mathbf{e}$; we suppose $\mathbf{e}$ is independent of x. We further assume that the quasipion masses at the thermal freeze-out are near the physical mass, $m_i(t_f)\simeq m_{out}\equiv m$, neglecting a possible mass shift which can generate squeeze-state components in particle radiation.[5] We will neglect the rescatterings at the post-thermal freeze-out stage, i.e., put $d_{i,qm}(\mathbf{p},t_f,t_{out})\approx 0$, and approximately describe the production of coherent pions at this stage by the quantum field model of the interaction with a classical source [3]. Then, there is a well known linear relationship between the annihilation (creation) operators diagonalizing the pion field Hamiltonian at the times t_f and t_{out} $(i=\pm,0)$:

$$a_{i,qm}(\mathbf{p},t_{out})=[a_{i,qm}(\mathbf{p},t_f)+e_i d_{coh}(\mathbf{p},t_f,t_{out})]e^{-i\omega_{\mathbf{p}}(t_{out}-t_f)}, \tag{4}$$

where the c-value quantity $d_{coh}(\mathbf{p},t_f,t_{out})$ depends on a mechanism and the rate of the classical field decay.[6]

The operators $a_i(p)$ of the asymptotic free pion field (with the origin of the time coordinate shifted to the point t_f) are connected with the operators $a_{i,qm}(\mathbf{p},t)$ taken at the *asymptotic* times t_{out} by the relation [30]

$$a_i(p)=\sqrt{p_0}e^{ip_0(t_{out}-t_f)}a_{i,qm}(\mathbf{p},t_{out}),\quad p_0=\omega_{\mathbf{p}}. \tag{5}$$

Equations (4) and (5) allow to calculate the mean values of the asymptotic operators $a_i(p)$ and $a_i^\dagger(p)$ for each $\mathbf{e}$ orientation of the quasipion vacuum applying the thermal Wick theorem to the operators $a_{i,qm}(\mathbf{p},t_f)$ and $a^\dagger_{i,qm}(\mathbf{p},t_f)$. The Gaussian form of the statistical operator $\rho_{\mathbf{e}}$ guarantees that $\langle a_{i,qm}(\mathbf{p},t_f)\rangle_{\mathbf{e}}=0$ for any fixed isospin orientation $\mathbf{e}$ of the quasiparticle vacuum. Then,

$$\begin{aligned}&\langle a_i^\dagger(p_1)a_j^\dagger(p_2)a_i(p_1)a_j(p_2)\rangle_{\mathbf{e}}\\&\quad=\langle a_i^\dagger(p_1)a_i(p_1)\rangle_{\mathbf{e}}\langle a_j^\dagger(p_2)a_j(p_2)\rangle_{\mathbf{e}}\\&\qquad+\delta_{ij}[\langle a_i^\dagger(p_2)a_i(p_1)\rangle_{\mathbf{e}}\langle a_i^\dagger(p_1)a_i(p_2)\rangle_{\mathbf{e}}\\&\qquad-\langle a_i^\dagger(p_1)\rangle_{\mathbf{e}}\langle a_i^\dagger(p_2)\rangle_{\mathbf{e}}\langle a_i(p_1)\rangle_{\mathbf{e}}\langle a_i(p_2)\rangle_{\mathbf{e}}].\end{aligned} \tag{6}$$

Here

$$\langle a_i^\dagger(p_1)a_i(p_2)\rangle_{\mathbf{e}}=\langle a_i^\dagger(p_1)a_i(p_2)\rangle_{ch}+\langle a_i^\dagger(p_1)\rangle_{\mathbf{e}}\langle a_i(p_2)\rangle_{\mathbf{e}}, \tag{7}$$

where the irreducible (thermal) part of the two-operator average

$$\langle a_i^\dagger(p_1)a_i(p_2)\rangle_{ch}=\sqrt{p_{10}p_{20}}\langle a^\dagger_{i,qm}(\mathbf{p}_1,t_f)a_{i,qm}(\mathbf{p}_2,t_f)\rangle_{\mathbf{e}} \tag{8}$$

[5]Squeeze-state components can arise also in a strongly inhomogeneous thermal boson system for particles with wavelengths larger than the system's homogeneity lengths [29]. Below we will assume the pion Compton wavelength to be much smaller than the typical system lengths of homogeneity (e.g., hydrodynamical lengths) at the thermal freeze-out hypersurface σ_f.

[6]It follows, from the continuity of the complete field $\pi_i(x)$ and its derivative at $t=t_f$, that, for a fast freeze-out ($t_{out}-t_f\to 0$), the quantity $\mathrm{d}_{coh}(\mathbf{p},t_f,t_{out})$ is directly associated with the strength of the pion condensate. On the other hand, an adiabatically slow switch-off of the classical source yields $\mathrm{d}_{coh}(\mathbf{p},t_f,t_{out})\approx 0$ [3].

does not depend on $\mathbf{e}$[7] and

$$\langle a_i(p)\rangle_{\mathbf{e}}=e_i d(p)\equiv e_i\sqrt{p_0}\mathrm{d}_{coh}(\mathbf{p},t_f,t_{out}). \tag{9}$$

One can introduce the one-particle Wigner function [25]

$$f_{\mathbf{e},i}(x,p)=(2\pi)^{-3}\int d^4q'\,\delta(q'\cdot p)e^{iq'x}\times\langle a_i^{\dagger}(p+q'/2)a_i(p-q'/2)\rangle_{\mathbf{e}}, \tag{10}$$

satisfying the relation

$$p_\mu\partial^\mu f_{\mathbf{e},i}(x,p)=0 \tag{11}$$

and describing the phase-space density of the noninteracting pions at $t\geqslant t_{out}$ or, in covariant formalism, at $t\geqslant\sigma_{out}=t_{out}(\mathbf{x})$; here σ_{out} is a space-time hypersurface where the interactions are ''switched off'' and particles can be considered as free. From Eq. (10), we get

$$\langle a_i^{\dagger}(p_1)a_i(p_2)\rangle_{\mathbf{e}}=\int_{\sigma_{out}}d\sigma_\mu p^\mu f_{\mathbf{e},i}(x,p)e^{-iqx},$$
$$q=p_1-p_2,\quad p=(p_1+p_2)/2. \tag{12}$$

Using Eqs. (7), (9), and (12), one can split the Wigner function into the chaotic (ch) and coherent (coh) components:

$$f_{\mathbf{e},i}(x,p)=f_{ch}(x,p)+|e_i|^2 f_{coh}(x,p). \tag{13}$$

Integrated over σ_{out}, these components determine the operator averages $\langle a_i^{\dagger}(p_1)a_i(p_2)\rangle_{ch}$ and $\langle a_i^{\dagger}(p_1)\rangle_{\mathbf{e}}\langle a_i(p_2)\rangle_{\mathbf{e}}$, respectively:

$$\langle a_i^{\dagger}(p_1)a_i(p_2)\rangle_{ch}=\int_{\sigma_{out}}d\sigma_\mu p^\mu e^{-iq\cdot x}f_{ch}(x,p),$$
$$\langle a_i^{\dagger}(p_1)\rangle_{\mathbf{e}}\langle a_i(p_2)\rangle_{\mathbf{e}}=|e_i|^2 d^*(p_1)d(p_2)=|e_i|^2\int_{\sigma_{out}}d\sigma_\mu p^\mu e^{-iq\cdot x}f_{coh}(x,p). \tag{14}$$

We suppose that the system has zero *average* charge and calculate the observables averaging over the random orientation of the quasipion vacuum in the isospin space $[d\Omega(\mathbf{e})=d\cos\theta d\phi]$:

$$\langle\ \dots\rangle\equiv Sp(\ \dots\rho)=(4\pi)^{-1}\int d\Omega(\mathbf{e})\langle\ \dots\rangle_{\mathbf{e}}\equiv(4\pi)^{-1}\int d\Omega(\mathbf{e})Sp(\ \dots\rho_{\mathbf{e}}). \tag{15}$$

[7]Such a dependence could take place if the mass shift were nonzero and dependent on the $\mathbf{e}$ orientation of the quasipion vacuum.

The *observable* pion field is related to the ensemble of events only, so the corresponding complete averages of the asymptotically free operators vanish, for example, $\langle a_{\pi^+}(p)\rangle=(4\pi)^{-1}\int d\Omega(\mathbf{e})\langle a_{\pi^+}(p)\rangle_{\mathbf{e}}=0$. The averages of these operators also vanish for charge-constrained coherent pion states $|c\rangle$, the states of a fixed electric charge and isospin—so called generalized coherent states [7,8,18]. This means that the density matrix ρ can be represented as a weighted sum of the projection operators $|c\rangle\langle c|$ of these states.

To illustrate this statement, let us consider a simple artificial case of only two sorts of oppositely charged bosons in one mode. Then the usual coherent states $|\alpha_\lambda\rangle$, $\lambda=\pm$, are

$$|\alpha_\lambda\rangle=\exp(-\tfrac{1}{2}|\alpha_\lambda|^2)\sum_{n=0}^{\infty}\frac{\alpha_\lambda^n}{(n!)^{1/2}}|n_\lambda\rangle,\quad a_\lambda|\alpha_\lambda\rangle=\alpha_\lambda|\alpha_\lambda\rangle,$$
$$|n_\lambda\rangle=(n!)^{-1/2}(a_\lambda^{\dagger})^n|0_\lambda\rangle,\quad [a_\lambda,a_{\lambda'}^{\dagger}]=\delta_{\lambda\lambda'},$$
$$\alpha_\pm=|\alpha|e^{\pm i\phi}. \tag{16}$$

These states represent superpositions of the states with different charges and so violate the superselection rule. The charge-constrained coherent state $|c_0\rangle$ of charged quanta with a zero total charge may be obtained by projecting this state out from the charge-unconstrained two-component coherent state $|\alpha_+\rangle|\alpha_-\rangle$ [18]:

$$|c_0\rangle=\frac{1}{2\pi}\int_0^{2\pi}d\phi|\alpha_+\rangle|\alpha_-\rangle=\exp(-|\alpha|^2)\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!}|n_+\rangle|n_-\rangle. \tag{17}$$

One may see that the zero charge state $|c_0\rangle$ represents a superposition of the states with the same charges (with equal numbers of particles and antiparticles) and thus satisfies the superselection rule. Similarly, the density matrix

$$\hat{\rho}=\frac{1}{2\pi}\int_0^{2\pi}d\phi|\alpha_+\rangle|\alpha_-\rangle\langle\alpha_+|\langle\alpha_-|=\exp(-2|\alpha|^2)\times\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\frac{|\alpha|^{n_1+n_2+n_3+n_4}}{(n_1!)^{1/2}(n_2!)^{1/2}(n_3!)^{1/2}(n_4!)^{1/2}}\times\delta_{n_1-n_2,n_3-n_4}|n_{1,+}\rangle|n_{2,-}\rangle\langle n_{3,+}|\langle n_{4,-}| \tag{18}$$

describes the mixture of the charge-constrained coherent states $|c_n\rangle$:

$$\hat{\rho}=\sum_{n=-\infty}^{\infty}|c_n\rangle\langle c_n|, \tag{19}$$

where $|c_n\rangle$ is the coherent state of charge ''n'':

$$|c_n\rangle=\exp(-|\alpha|^2)\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\delta_{n_1-n_2,n}\times\frac{|\alpha|^{n_1+n_2}e^{i\phi(n_1-n_2)}}{(n_1!)^{1/2}(n_2!)^{1/2}}|n_{1,+}\rangle|n_{2,-}\rangle. \quad (20)$$

While, in our example, the system described by the density matrix $\hat{\rho}$ does not have a definite charge, the average charge is equal to zero:

$$Sp[\hat{\rho}(a_+^\dagger a_+ - a_-^\dagger a_-)]=0. \quad (21)$$

Note that the expectation values of the annihilation operators in the corresponding coherent states are nonzero, $\langle\alpha_\lambda|a_\lambda|\alpha_\lambda\rangle=\alpha_\lambda$, while $Sp(\hat{\rho}a_\lambda)=0$.

Continuing the discussion of coherent pion production, we will assume the density matrix $\rho_{\mathbf{e}}$ of a Gaussian-type in terms of the quasiparticle annihilation (creation) operators $a_{i,qm}(\mathbf{p},t_f)$, related to the free particle operators according to Eqs. (4) and (5). Then, similar to the above example, this density matrix can be expressed through the projection operators on the usual charge-unconstrained coherent states of the free pion field. Averaging $\rho_{\mathbf{e}}$ over all directions of the isovector $\mathbf{e}$ according to Eq. (15), we finally get the density matrix ρ in the form of a weighted sum of the projection operators on the charge-constrained coherent states describing, in agreement with the superselection rule, the system of a fixed average charge.[8]

The expressions for pion spectra in Eq. (1) thus contain the averaging over the direction of the isovector $\mathbf{e}$. As a result, the single-pion spectra are independent of pion charges $i=\pm,0$:

$$\omega_{\mathbf{p}}\frac{d^3N_i}{d^3\mathbf{p}}=(4\pi)^{-1}\int d\Omega(\mathbf{e})\int d\sigma_\mu p^\mu f_{\mathbf{e},i}(x,p)$$
$$=\int d\sigma_\mu p^\mu f(x,p),$$
$$f(x,p)=f_{ch}(x,p)+\tfrac{1}{3}f_{coh}(x,p), \quad (22)$$

where we have used the equality $(4\pi)^{-1}\int d\Omega(\mathbf{e})|e_i|^2=1/3$. Note that the coherent part of the single-pion spectrum is

$$\omega_{\mathbf{p}}\frac{d^3N_{coh}}{d^3\mathbf{p}}\equiv\omega_{\mathbf{p}}\frac{d^3N}{d^3\mathbf{p}}G(p)\equiv\omega_{\mathbf{p}}\frac{d^3N_{ch}}{d^3\mathbf{p}}D(p)=\frac{1}{3}\int d\sigma_\mu p^\mu f_{coh}(x,p)=\frac{1}{3}|d(p)|^2, \quad (23)$$

where the functions $G(p)$ and $D(p)$ measure the coherent fraction:

$$G(p)=\frac{D(p)}{1+D(p)}\equiv\frac{d^3N_{coh}/d^3\mathbf{p}}{d^3N/d^3\mathbf{p}}=\frac{\frac{1}{3}\int d\sigma_\mu p^\mu f_{coh}(x,p)}{\int d\sigma_\mu p^\mu f(x,p)},$$

$$D(p)\equiv\frac{d^3N_{coh}/d^3\mathbf{p}}{d^3N_{ch}/d^3\mathbf{p}}=\frac{\frac{1}{3}\int d\sigma_\mu p^\mu f_{coh}(x,p)}{\int d\sigma_\mu p^\mu f_{ch}(x,p)}. \quad (24)$$

The coherence influences also the quantum statistical (without FSI) correlation functions:

$$C_{QS}^{ij}(p,q)=\frac{(4\pi)^{-1}\int d\Omega(\mathbf{e})\langle a_i^\dagger(p_1)a_j^\dagger(p_2)a_i(p_1)a_j(p_2)\rangle_{\mathbf{e}}}{\left((4\pi)^{-1}\int d\Omega(\mathbf{e})\langle a_i^\dagger(p_1)a_i(p_1)\rangle_{\mathbf{e}}\right)\left((4\pi)^{-1}\int d\Omega(\mathbf{e})\langle a_j^\dagger(p_2)a_j(p_2)\rangle_{\mathbf{e}}\right)}. \quad (25)$$

Taking into account Eqs. (6)–(9) and (24) and the equalities $p_{1,2}=p\pm q/2$, we get

$$C_{QS}^{ij}(p,q)=1+(9\langle|e_ie_j|^2\rangle-1-\delta_{ij})G(p_1)G(p_2)+\delta_{ij}\langle\cos(qx_{12})\rangle'$$
$$=[1+D(p_1)]^{-1}[1+D(p_2)]^{-1}\{1+D(p_1)+D(p_2)+9\langle|e_ie_j|^2\rangle D(p_1)D(p_2)$$
$$+\delta_{ij}\langle\cos(qx_{12})\rangle'_{ch}[1+\mathcal{D}(p_1,p_2)+\mathcal{D}(p_2,p_1)]\}, \quad (26)$$

[8]We do not consider here the squeeze states of the density matrix conditioned by a possible mass shift of quasiparticles. Note, however, that charged pions have no squeeze-state components anyway [23].

where the quasi-average $\langle \cos(qx_{12})\rangle' \equiv \langle \cos[q(x_1-x_2)]\rangle'$ is defined as

$$\langle \cos(qx_{12})\rangle' = \frac{\int d^3\sigma_\mu(x_1)d^3\sigma_\nu(x_2)p^\mu p^\nu f(x_1,p)f(x_2,p)\cos(qx_{12})}{\int d^3\sigma_\mu(x_1)d^3\sigma_\nu(x_2)p_1^\mu p_2^\nu f(x_1,p_1)f(x_2,p_2)} \tag{27}$$

and similarly, with the substitution $f\to f_{ch}$, the quasi-average $\langle \cos(qx_{12})\rangle'_{ch}$; the function

$$\mathcal{D}(p_1,p_2)=\frac{\frac{1}{3}\int d\sigma_\mu p^\mu f_{coh}(x,p)e^{-iq\cdot x}}{\int d\sigma_\mu p^\mu f_{ch}(x,p)e^{-iq\cdot x}} = \frac{\frac{1}{3}d^*(p_1)d(p_2)}{\langle a_i^\dagger(p_1)a_i(p_2)\rangle_{ch}},$$

$$\mathcal{D}(p,p)=D(p). \tag{28}$$

Note that

$$\langle \cos(qx_{12})\rangle' = G(p_1)G(p_2) + \frac{1+\mathcal{D}(p_1,p_2)+\mathcal{D}(p_2,p_1)}{[1+D(p_1)][1+D(p_2)]}\langle \cos(qx_{12})\rangle'_{ch}$$
$$= \frac{1+\mathcal{D}(p_1,p_2)+\mathcal{D}(p_2,p_1)+\mathcal{D}(p_1,p_2)\mathcal{D}(p_2,p_1)}{[1+D(p_1)][1+D(p_2)]} \times \langle \cos(qx_{12})\rangle'_{ch}. \tag{29}$$

Calculating the averages

$$\langle |e_ie_j|^2\rangle = (4\pi)^{-1}\int d\Omega(\mathbf{e})|e_ie_j|^2, \tag{30}$$

$$\langle |e_0|^4\rangle = \frac{1}{5}, \quad \langle |e_\pm|^4\rangle = \langle |e_+e_-|^2\rangle = \frac{2}{15}, \quad \langle |e_0e_\pm|^2\rangle = \frac{1}{15}, \tag{31}$$

we get for the intercepts of the QS correlation functions,

$$C_{QS}^{++}(p,0)=2-\frac{4}{5}G^2(p), \quad C_{QS}^{00}(p,0)=2-\frac{1}{5}G^2(p),$$
$$C_{QS}^{+-}(p,0)=1+\frac{1}{5}G^2(p), \quad C_{QS}^{+0}(p,0)=1-\frac{2}{5}G^2(p). \tag{32}$$

Particularly, it follows from Eqs. (32) that the decay of the quasipion vacuum suppresses the correlation functions of identical charged pions and enhances the one of nonidentical charged pions, the latter effect being by a factor of 4 smaller. For $G^2(p)=1$, the intercepts in Eqs. (32) coincide with those found in Ref. [31] in the case of a strong pion condensation. Our results, however, differ from the intercepts found in the model [21,22] of pion emission in a pure quantum state, the charge-constrained coherent state. They are recovered only for large average numbers of coherent pions. One can then replace the canonical ensemble corresponding to the pure quantum state with a fixed charge, by the grand canonical one, described by the density matrix of the ensemble with a fixed *average* charge. For ultrarelativistic $A+A$ collisions, the inclusive description based on the grand canonical ensemble is a fairly adequate approach, allowing to built explicitly the density matrix for a mixture of thermal and charge-constrained coherent radiations and make some calculations analytically.

One can check that the intercepts, as well as the QS correlation functions at any q, satisfy the relation [32]

$$C_{QS}^{++}+C_{QS}^{+-}=C_{QS}^{00}+C_{QS}^{+0}. \tag{33}$$

This relation follows from the assumed isotopically unpolarized pion emission. It is valid also for the complete correlation functions (with FSI), except for the region of very small $|\mathbf{q}|$ where the correlation functions of charged pions are strongly affected by the isospin nonconserving Coulomb interaction.

Note that the correlation functions, as well as their QS parts, satisfy the usual normalization condition $C(p,q)\to 1$ at large $|\mathbf{q}|$ provided that the coherent part of the Wigner density vanishes with the increasing $|p\pm q/2|$ faster than the chaotic one, i.e., $G(p\pm q/2)\to 0$ at large $|\mathbf{q}|$.

To get some insight into a possible behavior of the relative coherent contribution $G(p)$, consider the situation when the system decays during a rather short time, $t_{out}-t_f\to 0$, and the partial (at a fixed $\mathbf{e}$) average of the pion annihilation operator has a simple Gaussian form:

$$\langle a_i(p)\rangle_{\mathbf{e}} \sim \exp(-R_{coh}^2\mathbf{p}^2). \tag{34}$$

According to Eq. (14), the corresponding Wigner density

$$f_{coh}(x,p)\sim \exp(-2R_{coh}^2\mathbf{p}^2-\mathbf{x}^2/2R_{coh}^2), \tag{35}$$

so the parameter R_{coh} determines not only the spectrum, but also the characteristic radius of the region of the instantaneous coherent pion emission in accordance with the minimized uncertainty relation $\Delta x\Delta p=\hbar/2$. Let us assume a similar Gaussian parametrization of the chaotic component of the Wigner density in the nonrelativistic momentum region:

$$f_{ch}(x,p) \sim \exp(-2R_T^2\mathbf{p}^2 - \mathbf{x}^2/2R_{ch}^2), \tag{36}$$

where $R_T \equiv (4mT)^{-1/2}$ measures the characteristic size of the single-pion emitter (heat de Broglie length) and $R_{ch} \geqslant R_T$ is the characteristic radius of the region of the chaotic pion emission. In the considered rare gas limit, we then get the correlator

$$\langle \cos(qx_{12}) \rangle'_{ch} = \exp(-R^2\mathbf{q}^2), \tag{37}$$

where $R = (R_{ch}^2 - R_T^2)^{1/2} \approx R_{ch}$ represents (in the absence of the coherent contribution) the usual interferometry radius. The coherent fraction $G(p) = D(p)/[1+D(p)]$ and

$$D(p) = \frac{d^3N_{coh}/d^3\mathbf{p}}{d^3N_{ch}/d^3\mathbf{p}} \equiv \frac{\frac{1}{3}\int d\sigma_\mu p^\mu f_{coh}(x,p)}{\int d\sigma_\mu p^\mu f_{ch}(x,p)} \sim \exp[-2(R_{coh}^2 - R_T^2)\mathbf{p}^2]. \tag{38}$$

We see that $G(p) \to 0$ at large $|\mathbf{p}|$ on a reasonable condition $R_{coh} > R_T$.

In fact, since the quasiclassical (coherent) pion emission is conditioned by the decay of a thermal system, one may expect the effective radius for the coherent radiation, R_{coh}, close to that for the thermal emission, R_{ch}. Generally, in dynamical models, the effective radius varies with the momentum $\mathbf{p}$ and characterizes the size of the homogeneity region—the region of a substantial density of the pions emitted at the freeze-out time with three-momenta in the vicinity of $\mathbf{p}$. In this case, both the coherent and chaotic radii practically coincide with the homogeneity length of the system. Assuming $R_{coh} \approx R_{ch}$, we have $\mathcal{D}(p_1,p_2) \approx \mathcal{D}(p,p) = D(p)$ and, according to Eq. (29),

$$\langle \cos(qx_{12}) \rangle' \approx \frac{[1+D(p)]^2}{[1+D(p+q/2)][1+D(p-q/2)]} \times \langle \cos(qx_{12}) \rangle'_{ch}. \tag{39}$$

One can see that $\langle \cos(qx_{12}) \rangle' \approx \langle \cos(qx_{12}) \rangle'_{ch}$ at small $|\mathbf{q}|$ or, in the case of a small coherent contribution $D(p) \ll 1$. Note that in the opposite case, $D(p) \gg 1$, a decrease of the correlation function towards unity with the increasing $\mathbf{q}^2$ is conditioned by the chaotic component $\langle \cos(qx_{12}) \rangle'_{ch}$ starting at $\mathbf{q}^2 \sim R^{-2} \ln D^2(\mathbf{0}) - 4\mathbf{p}^2$. At smaller $\mathbf{q}^2$ values, the behavior of the correlation function is essentially flatter due to the q dependence of the denominator in Eq. (39). For the extreme case of a pure coherent radiation, $D(p) \to \infty$ $[G(p) \to 1]$, the function $\langle \cos(qx_{12}) \rangle'$ tends to unity at all q irrespective of the assumption $R_{coh} \approx R_{ch}$:

$$\langle \cos(qx_{12}) \rangle' \to \frac{\int d^3\sigma_\mu(x_1) d^3\sigma_\nu(x_2) p^\mu p^\nu f_{coh}(x_1,p) f_{coh}(x_2,p) \cos(qx_{12})}{\int d^3\sigma_\mu(x_1) d^3\sigma_\nu(x_2) p_1^\mu p_2^\nu f_{coh}(x_1,p_1) f_{coh}(x_2,p_2)} = 1. \tag{40}$$

The last equality in Eq. (40) follows from the definition (14) of the coherent Wigner function, both the nominator and denominator in Eq. (40) being equal to $|d(p_1)d(p_2)|^2$. Experimentally, the approach to such an extreme regime can display itself as a tendency of the intercepts of the QS correlation functions to the values defined by Eqs. (32) at $G(p) \to 1$, and as a flattering of the QS correlation functions within a growing $\mathbf{q}$ interval. The latter mimics a decrease of interferometry radii; of course, it does not mean that the real size of the system tends to zero.

The effect of coherent radiation on pion spectra and $\pi^+\pi^+$ and $\pi^+\pi^-$ correlation functions is demonstrated in Figs. 1–3 for different ratios $D_{tot} = D(\mathbf{0})(R_T/R_{coh})^3$ of the total numbers of coherent and chaotic pions. The plots correspond to simple Gaussian Wigner functions (35) and (36) with $R_T \equiv (4mT)^{-1/2} \approx 0.72$ fm ($T = 0.135$ GeV) and $R_{coh} = R_{ch} = 5$ fm. Under the assumption of a common source of coherent and chaotic pions in ultrarelativistic heavy ion collisions, characterized by a typical radius $R \sim 5-10$ fm, the coherent component in the spectra is concentrated in a rather small momentum region of a characteristic width $(2R)^{-1} \sim 20-10$ MeV/c (see Fig. 1).

III. CORRELATION FUNCTIONS AFFECTED BY FINAL STATE INTERACTION AND COHERENCE

In ultrarelativistic $A+A$ collisions, free hadrons appear mainly at the late stage of the evolution after the system expands and reaches the thermal freeze-out. After the hydrodynamic tube decays and produces final particles and resonances, particles still appear from resonance decays. Thus, more than half of pions produced in high energy heavy ion collisions is of the resonance origin. As a consequence, the pion spectra and correlations are influenced by resonance production and decay spectra, as well as by resonance lifetimes. Particularly, the pions from the decays of long-lived resonances do not contribute to QS and FSI correlations and thus suppress the correlation function $C^{ij}(p,q)$; we will consider this suppression in the next section.

However, even after the thermal (hydrodynamic) system

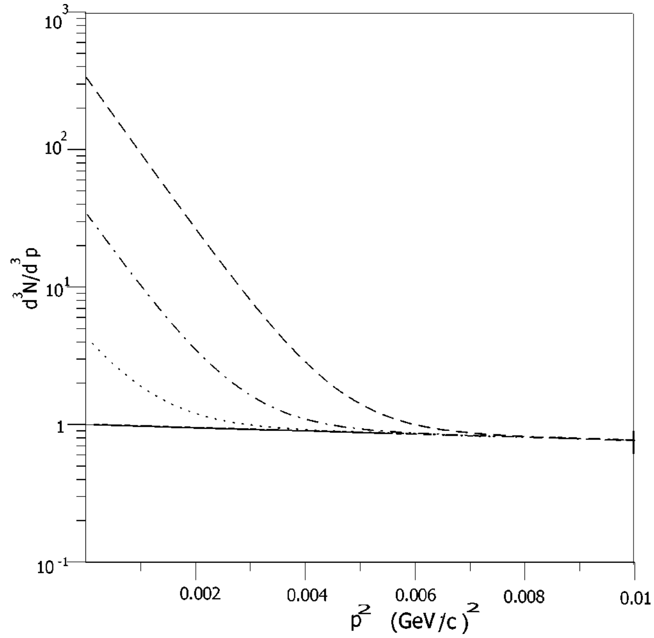

FIG. 1. The single-pion momentum spectra $d^3N/d^3\mathbf{p}$ calculated for different ratios D_{tot} of the total numbers of coherent and chaotic pions, assuming the Gaussian parametrization of the Wigner densities in Eqs. (35) and (36) with $R_T \equiv (4mT)^{-1/2} \approx 0.72$ fm ($T = 0.135$ GeV) and $R_{coh} = R_{ch} = 5$ fm. The solid, dotted, dash-dotted, and dashed curves correspond to $D_{tot} = 0$, 0.01, 0.1, and 1, respectively. The overall normalization is arbitrary.

and short-lived resonances decay, the particles in nearby phase-space points continue to interact. Due to a large effective emission volume in heavy ion collisions, the particle interaction in the final state is usually dominated by the long-range Coulomb forces. To calculate the FSI effect on two-

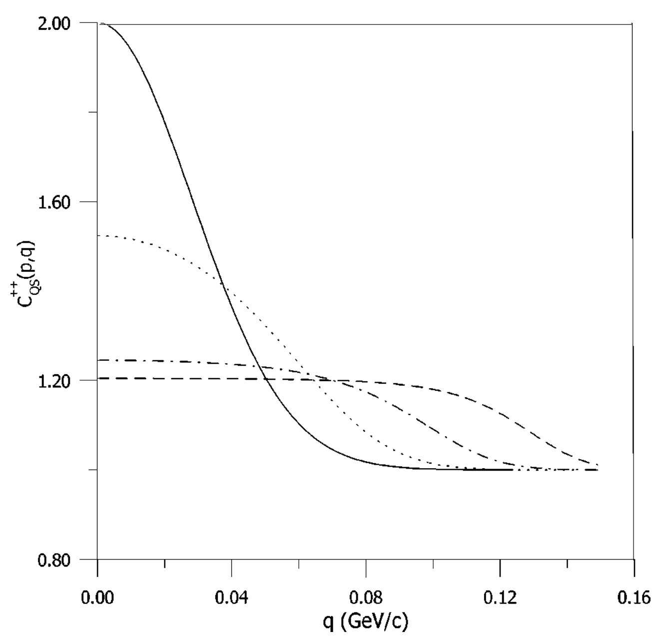

FIG. 2. The pure QS correlation functions $C_{QS}(p,q)$ calculated for $\pi^+\pi^+$ pairs at $\mathbf{p}=0$ GeV/c for the same conditions as in Fig. 1.

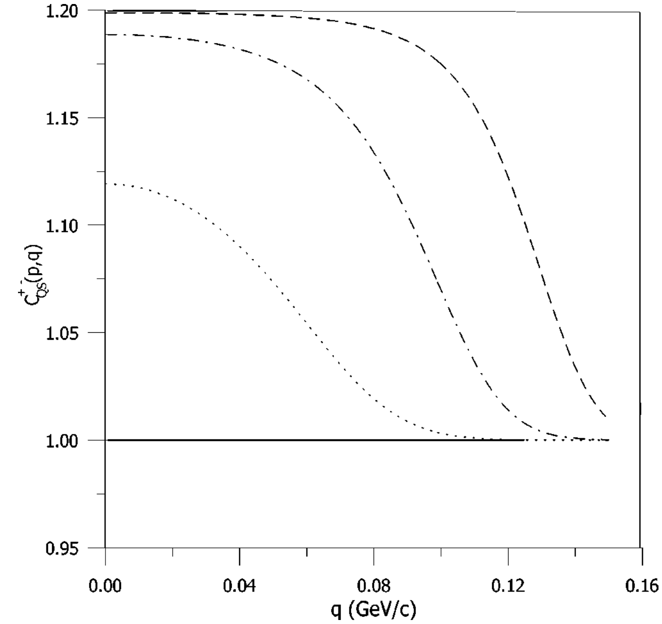

FIG. 3. The pure QS correlation functions $C_{QS}(p,q)$ calculated for $\pi^+\pi^-$ pairs at $\mathbf{p}=0$ GeV/c for the same conditions as in Fig. 1.

particle spectra, we will assume sufficiently small phase-space density of the produced particles and use the FSI theory in the two-body approximation [8,33,34] for pions, neglecting the FSI of resonances.

The single-pion spectrum in Eq. (1) then remains unchanged while the two-pion one (for pairs containing no pions from long-lived sources) takes the form

$$\omega_{\mathbf{p}_1}\omega_{\mathbf{p}_2}\frac{d^6N_{ij}}{d^3\mathbf{p}_1 d^3\mathbf{p}_2} \doteq \int d^4k_1 d^4k_2 d^4k_1' d^4k_2' \times \langle a_i^\dagger(k_1) a_j^\dagger(k_2) a_i(k_1') a_j(k_2')\rangle \times \Phi^{(-)ij}_{p_1p_2}(k_1,k_2)\Phi^{(-)ij*}_{p_1p_2}(k_1',k_2'), \quad (41)$$

where the nonsymmetrized Bethe-Salpeter amplitude $\Phi^{(-)ij}_{p_1p_2}(k_1,k_2) \equiv \Phi^{(+)ij*}_{p_1p_2}(k_1,k_2)$ in a four-momentum representation is expressed through the propagators of particles i and j and their scattering amplitude $\mathcal{F}_{ij}$ analytically continued to the unphysical region [33,34],[9]

$$\Phi^{(-)ij}_{p_1p_2}(k_1,k_2) = \delta^4(k_1-p_1)\delta^4(k_2-p_2) + \delta^4(k_1+k_2-p_1-p_2)\frac{i\sqrt{p^2}}{\pi^3} \times \frac{\mathcal{F}^*_{ij}(k_1,k_2;p_1,p_2)}{(k_1^2-m^2-i0)(k_2^2-m^2-i0)}. \quad (42)$$

[9]It is important that the relation between the production amplitude and the operator product average, as given in Eq. (1), is valid also at the off mass shell.

The averaging in Eq. (41) is performed with the help of the statistical operator ρ without FSI: $\langle \dots \rangle = Sp(\dots \rho)$. Introducing the Bethe-Salpeter amplitudes $\Psi^{(-)ij}_{p_1p_2}(x_1,x_2)$ in space-time representation,

$$\Phi^{(-)ij}_{p_1p_2}(k_1,k_2) = (2\pi)^{-8}\int d^4x_1 d^4x_2 e^{ik_1x_1+ik_2x_2}\Psi^{(-)ij}_{p_1p_2}(x_1,x_2), \quad (43)$$

one can rewrite Eq. (41) as

$$\omega_{\mathbf{p}_1}\omega_{\mathbf{p}_2}\frac{d^6N_{ij}}{d^3\mathbf{p}_1 d^3\mathbf{p}_2} \doteq \int d^4x_1 d^4x_2 d^4x_1' d^4x_2' \times \rho^{ij}(x_1,x_2;x_1',x_2')\Psi^{(-)ij}_{p_1p_2}(x_1,x_2) \times \Psi^{(-)ij*}_{p_1p_2}(x_1',x_2'), \quad (44)$$

where the space-time density matrix ρ^{ij} is just the Fourier transform of the four-operator average in Eq. (41):[10]

$$\rho^{ij}(x_1,x_2;x_1',x_2') = (2\pi)^{-16}\int d^4k_1 d^4k_2 d^4k_1' d^4k_2' \times e^{ik_1x_1+ik_2x_2}e^{-ik_1'x_1'-ik_2'x_2'} \times \langle a_i^\dagger(k_1)a_j^\dagger(k_2)a_i(k_1')a_j(k_2')\rangle. \quad (45)$$

Separating the phase factor due to free motion of the two-particle center-of-mass system (c.m.s.):

$$\Psi^{(-)ij}_{p_1p_2}(x_1,x_2) = e^{-iPX_{12}}\psi^{(-)ij}_q(x_{12}),$$

$$X_{12} = \tfrac{1}{2}(x_1+x_2), \quad x_{12} = x_1 - x_2, \quad P \equiv 2p = p_1 + p_2, \quad (46)$$

and integrating over the pair c.m.s. four coordinate X_{12} and X_{12}' in Eq. (44), one can express the two-particle spectrum through the reduced space-time density matrix $\rho_P^{ij}(x_{12};x_{12}')$, the latter depending on the pair total four-momentum P and the relative four coordinates of the emission points only:

$$\omega_{\mathbf{p}_1}\omega_{\mathbf{p}_2}\frac{d^6N_{ij}}{d^3\mathbf{p}_1 d^3\mathbf{p}_2} \doteq \int d^4x_{12} d^4x_{12}' \rho_P^{ij}(x_{12};x_{12}')\psi^{(-)ij}_q(x_{12})\psi^{(-)ij*}_q(x_{12}'), \quad (47)$$

$$\rho_P^{ij}(x_{12};x_{12}') = (2\pi)^{-8}\int d^4k_1 d^4k_1' e^{i(k_1-p)x_{12}}e^{-i(k_1'-p)x_{12}'} \times \langle a_i^\dagger(k_1)a_j^\dagger(P-k_1)a_i(k_1')a_j(P-k_1')\rangle. \quad (48)$$

Note that in the two-particle c.m.s., where $P=\{m_{12},0,0,0\}$, $q=\{0,2\mathrm{k}^*\}$, $x_{12}=\{t^*,\mathrm{r}^*\}$, the reduced Bethe-Salpeter amplitude $\psi^{(-)ij*}_q(x_{12}) = \psi^{(+)ij}_q(x_{12})$ at $t^* = t_1^* - t_2^* = 0$ coincides with a stationary solution $\psi_{-\mathrm{k}^*}(\mathbf{r}^*)$ of the scattering problem having at large distances r^* the asymptotic form of a superposition of of the plane and outgoing spherical waves (the minus sign of the vector $\mathbf{k}^*$ corresponds to the reverse in time direction of the emission process). This amplitude can be substituted by this solution (*equal time* approximation) on condition [34] $|t^*| \ll m r^{*2}$ which is usually satisfied for particle production in heavy ion collisions.

Since the resonances have finite lifetimes, their decay products are created in an essentially four-dimensional space-time region. At the post-thermal freeze-out stage, the resonances are usually described by semiclassical techniques; they are considered as unstable particles moving along classical trajectories and decaying according to the exponential law [35] (see, however, Refs. [33,36,37]). This approximation neglects a small correlation effect in pairs of unlike pions appearing due to QS correlations of identical resonances. The resonances are assumed to be described according to the Gibbs density matrix prior to the thermal freeze-out; this guarantees the chaoticity of the decay pions.[11] Therefore, the pions from resonance decays do not destroy the structure of the decomposition of the operator averages in Eqs. (6) and (7) into irreducible parts based on the thermal Wick theorem.

After the production, the pions in nearby phase-space points, chaotic as well as coherent ones, undergo a long-time scale interaction in the final state. According to Eqs. (44) or (47), the intensity of FSI interaction is conditioned by the two-particle Bethe-Salpeter amplitudes $\Psi_{p_1p_2}(x_1,x_2)$ or $\psi_q(x_{12})$ and the corresponding two-particle space-time density matrices $\rho(x_1,x_2;x_1',x_2')$ or $\rho_P(x_{12};x_{12}')$. Clearly, in the case of absent FSI, the two-pion spectrum merely reduces to the Fourier transform of the space-time density matrix. It can be represented as an integral over the mean four coordinates $\overline{x} = (x + x')/2$ of a combination of bilinear products of single-particle chaotic and coherent emission functions $g_{ch}(\overline{x},p)$ and $g_{coh}(\overline{x},p)$, respectively defined in Eqs. (50) and (51) below.

The emission function $g(\overline{x},p)$ is closely related with the Wigner phase-space density $f(x,p)$ at asymptotic times $t \geqslant t_{out}$. Let us denote by $\overline{x} \equiv \{\overline{t}, \mathbf{x} - (\mathbf{p}/p_0)(t - \overline{t})\}$ the space-time point, starting from which a free particle moving with velocity p/p_0 reaches a point x; the portion of such particles

[10]For identical particles, it differs from the space-time density matrix of Ref. [33], where the effect of QS enters through the symmetrization of the Bethe-Salpeter amplitudes, while here it is through the Wigner decomposition of the four-operator average in Eq. (52) below.

[11]Note that the chaotization of decay pions partially happens irrespective of the form of the density matrix if pions were emitted by a large number of many different sorts of resonances.

is $g(\overline{x},p)$. Collecting all the contributions (starting in our case from the thermal freeze-out time t_f), we have

$$p_0 f(x,p)=\int d^4\overline{x}\,\delta^3[\overline{\mathbf{x}}-\mathbf{x}+(\mathbf{p}/p_0)(t-\overline{t})]g(\overline{x},p), \tag{49}$$

where $g(\overline{x},p)=p_0\delta(\overline{t}-t_f)f(\overline{x},p)+s(\overline{x},p)$ and $s(\overline{x},p)\sim\theta(t-\overline{t})\theta(\overline{t}-t_f)$ is the density of pion emission at the post-thermal stage, $t>t_f$. Therefore we can rewrite the irreducible (thermal) part of the two-operator average through the chaotic emission function as

$$\langle a_i^\dagger(p_1)a_i(p_2)\rangle_{ch}=\int_{\sigma_{out}} d\sigma_\mu p^\mu e^{-iqx}f_{ch}(x,p)$$

$$=\int d^4\overline{x}e^{-iq\overline{x}}g_{ch}(\overline{x},p),$$

$$p\equiv P/2=(p_1+p_2)/2,\quad q=p_1-p_2, \tag{50}$$

where we have used the equality $qx=q\overline{x}$ following from the relation $qp\equiv q_0p_0-\mathbf{qp}=0$. Similarly, for the coherent component of the two-operator average at fixed $\mathbf{e}$, we get

$$\langle a_i^\dagger(p_1)\rangle_{\mathbf{e}}\langle a_i(p_2)\rangle_{\mathbf{e}}=|e_i|^2d^*(p_1)d(p_2)$$

$$=|e_i|^2\int_{\sigma_{out}} d\sigma_\mu p^\mu e^{-iqx}f_{coh}(x,p)$$

$$=|e_i|^2\int d^4\overline{x}e^{-iq\overline{x}}g_{coh}(\overline{x},p). \tag{51}$$

The results of Sec. II can thus be rewritten in terms of the emission functions in accordance with a formal substitution $\int_{\sigma_{out}}d\sigma_\mu p^\mu f(x,p)\rightarrow\int d^4x g(x,p)$.

To express the four-operator average in Eq. (48) through the emission functions, we can exploit the decomposition similar to that in Eq. (6):

$$\begin{aligned}&\langle a_i^\dagger(k_1)a_j^\dagger(P-k_1)a_i(k_1')a_j(P-k_1')\rangle_{\mathbf{e}}\\&=\langle a_i^\dagger(k_1)a_i(k_1')\rangle_{\mathbf{e}}\langle a_j^\dagger(P-k_1)a_j(P-k_1')\rangle_{\mathbf{e}}\\&\quad+\delta_{ij}[\langle a_i^\dagger(k_1)a_i(P-k_1')\rangle_{\mathbf{e}}\langle a_i^\dagger(P-k_1)a_i(k_1')\rangle_{\mathbf{e}}\\&\quad-\langle a_i^\dagger(k_1)\rangle_{\mathbf{e}}\langle a_i^\dagger(P-k_1)\rangle_{\mathbf{e}}\langle a_i(k_1')\rangle_{\mathbf{e}}\langle a_i(P-k_1')\rangle_{\mathbf{e}}].\end{aligned} \tag{52}$$

Using Eqs. (50) and (51) for the two-operator averages in Eq. (52), we get

$$\begin{aligned}&\langle a_i^\dagger(k_1)a_j^\dagger(P-k_1)a_i(k_1')a_j(P-k_1')\rangle_{\mathbf{e}}\\&=\int d^4\overline{x}_1d^4\overline{x}_2(e^{-i(k_1-k_1')\cdot\overline{x}_{12}}g_{\mathbf{e},i}[\overline{x}_1,\tfrac{1}{2}(k_1+k_1')]\\&\quad\times g_{\mathbf{e},j}[\overline{x}_2,P-\tfrac{1}{2}(k_1+k_1')]+\delta_{ij}e^{-i(k_1+k_1'-P)\cdot\overline{x}_{12}}\\&\quad\times\{g_{\mathbf{e},i}[\overline{x}_1,p+\tfrac{1}{2}(k_1-k_1')]\\&\quad\times g_{\mathbf{e},i}[\overline{x}_2,p-\tfrac{1}{2}(k_1-k_1')]-|e_i|^4g_{coh}[\overline{x}_1,p\\&\quad+\tfrac{1}{2}(k_1-k_1')]g_{coh}[\overline{x}_2,p-\tfrac{1}{2}(k_1-k_1')]\}),\end{aligned} \tag{53}$$

where $\overline{x}_{12}=\overline{x}_1-\overline{x}_2$ and

$$g_{\mathbf{e},i}(\overline{x},k)=g_{ch}(\overline{x},k)+|e_i|^2g_{coh}(\overline{x},k). \tag{54}$$

After the averaging over the orientation of the isospin vector $\mathbf{e}$, we get

$$\begin{aligned}\langle a_i^\dagger(k_1)a_j^\dagger(P-k_1)a_i(k_1')a_j(P-k_1')\rangle=\int d^4\overline{x}_1d^4\overline{x}_2\cdot\Big[&e^{-i(k_1-k_1')\cdot\overline{x}_{12}}\{g[\overline{x}_1,\tfrac{1}{2}(k_1+k_1')]g[\overline{x}_2,P-\tfrac{1}{2}(k_1+k_1')]\\&+(\langle|e_ie_j|^2\rangle-\tfrac{1}{9})g_{coh}[\overline{x}_1,\tfrac{1}{2}(k_1+k_1')]g_{coh}[\overline{x}_2,P-\tfrac{1}{2}(k_1+k_1')]\}\\&+\delta_{ij}e^{-i(k_1+k_1'-P)\cdot\overline{x}_{12}}\Big(g[\overline{x}_1,p+\tfrac{1}{2}(k_1-k_1')]g[\overline{x}_2,p-\tfrac{1}{2}(k_1-k_1')]\\&-\frac{1}{9}g_{coh}[\overline{x}_1,p+\tfrac{1}{2}(k_1-k_1')]g_{coh}[\overline{x}_2,p-\tfrac{1}{2}(k_1-k_1')]\Big)\Big],\end{aligned} \tag{55}$$

where

$$g(\overline{x},k)=g_{ch}(\overline{x},k)+\tfrac{1}{3}g_{coh}(\overline{x},k). \tag{56}$$

Inserting expression (55) for the four-operator average into Eq. (48) and, integrating in the first and second term over (k_1-k_1') and $(k_1+k_1'-P)$, respectively, one can rewrite the reduced space-time density matrix as

$$\rho_P^{ij}(x_{12};x'_{12})=(2\pi)^{-4}\int d^4\overline{x}_1 d^4\overline{x}_2 d^4\kappa\cdot\{e^{i\kappa\cdot(x_{12}-x'_{12})}\delta^4(\tfrac{1}{2}(x_{12}+x'_{12})-\overline{x}_{12})[g(\overline{x}_1,p+\kappa)g(\overline{x}_2,p-\kappa)$$
$$+(\langle|e_ie_j|^2\rangle-\tfrac{1}{9})g_{coh}(\overline{x}_1,p+\kappa)g_{coh}(\overline{x}_2,p-\kappa)]+\delta_{ij}e^{i\kappa\cdot(x_{12}+x'_{12})}\delta^4(\tfrac{1}{2}(x_{12}-x'_{12})-\overline{x}_{12})$$
$$\times[g(\overline{x}_1,p+\kappa)g(\overline{x}_2,p-\kappa).-\tfrac{1}{9}g_{coh}(\overline{x}_1,p+\kappa)g_{coh}(\overline{x}_2,p-\kappa)]\}. \quad (57)$$

According to Eq. (47) and using the equality $\psi_q(-\overline{x}_{12})=\psi_{-q}(\overline{x}_{12})$, the two-pion spectrum then becomes

$$\omega_{\mathbf{p}_1}\omega_{\mathbf{p}_2}\frac{d^6N_{ij}}{d^3\mathbf{p}_1d^3\mathbf{p}_2}\doteq(2\pi)^{-4}\int d^4\overline{x}_1d^4\overline{x}_2d^4\kappa d^4\epsilon e^{i\kappa\cdot\epsilon}\cdot\{[g(\overline{x}_1,p+\kappa)g(\overline{x}_2,p-\kappa)$$
$$+(\langle|e_ie_j|^2\rangle-\tfrac{1}{9})g_{coh}(\overline{x}_1,p+\kappa)g_{coh}(\overline{x}_2,p-\kappa)]\psi_q^{(-)ij}(\overline{x}_{12}+\tfrac{1}{2}\epsilon)\psi_q^{(-)ij*}(\overline{x}_{12}-\tfrac{1}{2}\epsilon)$$
$$+\delta_{ij}[g(\overline{x}_1,p+\kappa)g(\overline{x}_2,p-\kappa)-\tfrac{1}{9}g_{coh}(\overline{x}_1,p+\kappa)g_{coh}(\overline{x}_2,p-\kappa)]\psi_q^{(-)ij}(\overline{x}_{12}+\tfrac{1}{2}\epsilon)\psi_{-q}^{(-)ij*}(\overline{x}_{12}-\tfrac{1}{2}\epsilon)\}$$
$$=(2\pi)^{-4}\int d^4\overline{x}_1d^4\overline{x}_2d^4\kappa d^4\epsilon,e^{i\kappa\cdot\epsilon}\cdot\{[g_{ch}(\overline{x}_1,p+\kappa)g_{ch}(\overline{x}_2,p-\kappa)+\tfrac{1}{3}(g_{ch}(\overline{x}_1,p+\kappa)$$
$$\times g_{coh}(\overline{x}_2,p-\kappa)+g_{coh}(\overline{x}_1,p+\kappa)g_{ch}(\overline{x}_2,p-\kappa))]\cdot[\psi_q^{(-)ij}(\overline{x}_{12}+\tfrac{1}{2}\epsilon)\psi_q^{(-)ij*}(\overline{x}_{12}-\tfrac{1}{2}\epsilon)$$
$$+\delta_{ij}\psi_q^{(-)ij}(\overline{x}_{12}+\tfrac{1}{2}\epsilon)\psi_{-q}^{(-)ij*}(\overline{x}_{12}-\tfrac{1}{2}\epsilon)]$$
$$+\langle|e_ie_j|^2\rangle g_{coh}(\overline{x}_1,p+\kappa)g_{coh}(\overline{x}_2,p-\kappa)\psi_q^{(-)ij}(\overline{x}_{12}+\tfrac{1}{2}\epsilon)\psi_q^{(-)ij*}(\overline{x}_{12}-\tfrac{1}{2}\epsilon)\}. \quad (58)$$

If the FSI were absent, i.e., $\psi_q^{(-)ij}(\overline{x}_{12})=\exp(-iq\cdot\overline{x}_{12}/2)$, one would get

$$\omega_{\mathbf{p}_1}\omega_{\mathbf{p}_2}\frac{d^6N_{ij}}{d^3\mathbf{p}_1d^3\mathbf{p}_2}$$
$$\doteq\int d^4\overline{x}_1d^4\overline{x}_2\{g(\overline{x}_1,p_1)g(\overline{x}_2,p_2)+(\langle|e_ie_j|^2\rangle-\tfrac{1}{9})g_{coh}(\overline{x}_1,p_1)g_{coh}(\overline{x}_2,p_2)$$
$$+\delta_{ij}[g(\overline{x}_1,p)g(\overline{x}_2,p)-\tfrac{1}{9}g_{coh}(\overline{x}_1,p)g_{coh}(\overline{x}_2,p)]\cos(q\overline{x}_{12})\}$$
$$=\int d^4\overline{x}_1d^4\overline{x}_2\{g(\overline{x}_1,p_1)g(\overline{x}_2,p_2)+[\langle|e_ie_j|^2\rangle-\tfrac{1}{9}(1+\delta_{ij})]g_{coh}(\overline{x}_1,p_1)g_{coh}(\overline{x}_2,p_2)$$
$$+\delta_{ij}g(\overline{x}_1,p)g(\overline{x}_2,p)\cos(q\overline{x}_{12})\}$$
$$=\int d^4\overline{x}_1d^4\overline{x}_2\{g_{ch}(\overline{x}_1,p_1)g_{ch}(\overline{x}_2,p_2)+\tfrac{1}{3}[g_{ch}(\overline{x}_1,p_1)g_{coh}(\overline{x}_2,p_2)+g_{coh}(\overline{x}_1,p_1)g_{ch}(\overline{x}_2,p_2)]$$
$$+\langle|e_ie_j|^2\rangle g_{coh}(\overline{x}_1,p_1)g_{coh}(\overline{x}_2,p_2)+\delta_{ij}[g_{ch}(\overline{x}_1,p)g_{ch}(\overline{x}_2,p)+\tfrac{2}{3}g_{ch}(\overline{x}_1,p)g_{coh}(\overline{x}_2,p)]\cos(q\overline{x}_{12})\} \quad (59)$$

and recover Eqs. (26) for the pure QS correlation functions.

In the case of absent coherent emission, i.e., $d=g_{coh}=0$, and on the usual assumption ($R_T^2\ll R_{ch}^2$) of sufficiently smooth four-momentum dependence of the chaotic emission function $g_{ch}(\overline{x},p)$ as compared with a sharp q dependence of the QS and FSI correlations (determined by the inverse characteristic distance between the emission points), the chaotic emission functions in Eq. (58) can be taken out of the integral over κ at small values of κ, this integral thus being close to $\delta^4(\epsilon)$. Choosing the momentum arguments in g_{ch} functions in accordance with Eq. (59) for the case of absent FSI, we get for the two-pion spectrum and the correlation function

$$\omega_{\mathbf{p}_1}\omega_{\mathbf{p}_2}\frac{d^6N_{ij}}{d^3\mathbf{p}_1d^3\mathbf{p}_2}$$
$$\approx\int d^4\overline{x}_1d^4\overline{x}_2\cdot\{g_{ch}(\overline{x}_1,p_1)g_{ch}(\overline{x}_2,p_2)|\psi_q^{(-)ij}(\overline{x}_{12})|^2$$
$$+\delta_{ij}g_{ch}(\overline{x}_1,p)g_{ch}(\overline{x}_2,p)\psi_q^{(-)ij}(\overline{x}_{12})\psi_{-q}^{(-)ij*}(\overline{x}_{12})\}, \quad (60)$$

$$C_{ch}^{ij} \approx \langle | \psi_q^{(-)ij}(\overline{x_{12}})|^2\rangle_{ch} + \delta_{ij}\langle \psi_q^{(-)ij}(\overline{x_{12}})\psi_{-q}^{(-)ij*}(\overline{x_{12}})\rangle'_{ch}, \tag{61}$$

where the average $\langle \mathcal{A}\rangle_{ch}$ and quasi-average $\langle \mathcal{A}\rangle'_{ch}$ are defined as

$$\langle \mathcal{A}\rangle_{ch} = \frac{\int d^4\overline{x_1} d^4\overline{x_2} \mathcal{A} g_{ch}(\overline{x_1},p_1) g_{ch}(\overline{x_2},p_2)}{\int d^4\overline{x_1} g_{ch}(\overline{x_1},p_1) \int d^4\overline{x_2} g_{ch}(\overline{x_2},p_2)}, \tag{62}$$

$$\langle \mathcal{A}\rangle'_{ch} = \frac{\int d^4\overline{x_1} d^4\overline{x_2} \mathcal{A} g_{ch}(\overline{x_1},p) g_{ch}(\overline{x_2},p)}{\int d^4\overline{x_1} g_{ch}(\overline{x_1},p_1) \int d^4\overline{x_2} g_{ch}(\overline{x_2},p_2)}. \tag{63}$$

In the case of a nonzero coherent contribution, the $\epsilon/2$ and $\overline{x_{12}}$ dispersions in the pure coherent term in Eq. (58) are the same $(2R_{coh}^2)$, contrary to usually negligible $\epsilon/2$ dispersion in the pure chaotic term $2R_T^2 \ll 2R_{ch}^2$. As for the mixed term, the $\epsilon/2$ dispersion would be negligible if only the characteristic size R_{coh} of the coherent source were sufficiently small; with the increasing R_{coh}, this dispersion may become important—for $R_{coh} \approx R_{ch}$ it amounts to about half of the $\overline{x_{12}}$ dispersion. Therefore, the ϵ dependence of the Bethe-Salpeter amplitudes should be generally retained in these terms. The important exception is the case of practical interest in heavy ion collisions, when the two charged pions are created in their c.m.s. at a distance much larger than the corresponding s-wave scattering length (of a fraction of fm) and much smaller than their Bohr radius (of 387.5 fm). The two-pion FSI interaction at small q is then dominated by the Coulomb FSI and depends only weakly on the space-time separation of the emission points. In this case,

$$\begin{aligned} C_{ch}^{ij} \approx & \langle | \psi_q^{(-)ij}(\overline{x_{12}})|^2\rangle + \delta_{ij}\langle \psi_q^{(-)ij}(\overline{x_{12}})\psi_{-q}^{(-)ij*}(\overline{x_{12}})\rangle' \\ & + (9\langle |e_i e_j|^2\rangle - 1 - \delta_{ij}) G(p_1) G(p_2) \\ & \times \langle | \psi_q^{(-)ij}(\overline{x_{12}})|^2\rangle_{coh}, \end{aligned} \tag{64}$$

where the averages are defined as in Eqs. (62) and (63) with the substitutions $g_{ch} \to g$ or $g_{ch} \to g_{coh}$, and the relative coherent contribution $G(p)$ in Eq. (24) with a formal substitution $\int_{\sigma_{out}} d\sigma_\mu p^\mu f(x,p) \to \int d^4x g(x,p)$.

IV. EXTRACTING THE COHERENT COMPONENT OF PARTICLE RADIATION

Up to now, we have ignored the contributions $d^3N_i^{(l)}/d^3\mathbf{p}$ arising in the pion spectra from the decays of long-lived (l) sources such as η, η' mesons, and also the unregistered kaons and hyperons. The pions from these sources possess no observable FSI (due to the very large relative distance of the emission points) as well as no noticeable interference effect, because the corresponding correlation width is much smaller than the relative momentum resolution $q_{\min}$ of a detector. Therefore the measured correlation functions, defined in Eq. (1), can be expressed through the correlation functions $\tilde{C}^{ij}(p,q)$ (discussed in the previous section) of all pion pairs $\pi^i\pi^j$ except for those containing pions from long-lived sources as follows [38]:

$$\begin{aligned} C^{ij}(p,q) &= n_{ij}(p_1,p_2)/n_i(p_1)n_j(p_2) \\ &= \Lambda^{ij}(p)\tilde{C}^{ij}(p,q) + 1 - \Lambda^{ij}(p), \end{aligned} \tag{65}$$

where the suppression parameter $\Lambda^{ij}(p)$ measures the fraction of pion pairs containing no pions from long-lived sources:[12]

$$\Lambda^{ij}(p) = \left(1 - \frac{d^3N_i^{(l)}/d^3\mathbf{p}}{d^3N_i/d^3\mathbf{p}}\right)\left(1 - \frac{d^3N_j^{(l)}/d^3\mathbf{p}}{d^3N_j/d^3\mathbf{p}}\right) < 1. \tag{66}$$

In the (artificial) case of an absent FSI effect, the correlation function $\tilde{C}^{ij}(p,q) = C_{QS}^{ij}(p,q)$, and the averaging in $\langle \cos(qx_{12})\rangle'$ in the QS correlation functions in Eqs. (26) should be applied only to the pion pairs containing no pions from long-lived sources. Then, assuming sufficiently good detector resolution, $q_{\min} \ll R^{-1}$, we can determine the intercepts $C^{ij}(p,0)$ calculating the correlation functions at $|q| \sim q_{\min}$:

$$C^{ij}(p,q_{\min}) = 1 + \Lambda^{ij}(p)[\delta_{ij} + (9\langle |e_i e_j|^2\rangle - 1 - \delta_{ij}) G^2(p)]. \tag{67}$$

The intercepts are lower than 2 for any system of identical pions and they are higher (lower) than 1 for $\pi^+\pi^-$ $(\pi^\pm\pi^0)$ systems.

Since the suppression parameters $\Lambda(p)$ are generally different for different pion pairs, e.g., due to different contributions of hyperon decays, it is impossible, using only apparent intercepts in Eq. (67), to separate the contributions of the coherent and long-lived sources, unless there is a known ratio of the suppression parameters $\Lambda(p)$ for identical and nonidentical pions: $\Lambda^{ii}(p)/\Lambda^{ij}(p)$. Then, for example, from the intercepts of the $\pi^+\pi^+$ and $\pi^+\pi^-$ correlation functions, one obtains the coherent fraction squared:

$$G^2(p) = \frac{\Lambda^{++}(p)}{\Lambda^{+-}(p)}\left[\frac{4}{5}\frac{\Lambda^{++}(p)}{\Lambda^{+-}(p)} + \frac{1}{5}\frac{C^{++}(p,q_{\min}) - 1}{C^{+-}(p,q_{\min}) - 1}\right]^{-1}. \tag{68}$$

In fact, the knowledge of the ratio $\Lambda^{ii}(p)/\Lambda^{ij}(p)$ is not of principal importance for the extraction of the coherent fraction $G(p)$. Besides the intercepts, one can exploit also the q dependence of $C_{QS}(p,q)$ in a sufficiently wide interval to follow Eq. (26), and perform simultaneous or separate fits of the correlation functions C^{ij}, suitably parametrizing the correlator $\langle \cos(qx_{12})\rangle$ and the function $G(p \pm q/2)$. For example, one can use the usual Gaussian correlator parametrization

[12] One can include in $N_i^{(l)}$ and the corresponding suppression parameters Λ^{ij} the contribution of misidentified particles which also introduce practically no correlation.

$$\langle\cos(qx_{12})\rangle'_{ch}\simeq\exp(-q_x^2R_x^2-q_y^2R_y^2-q_z^2R_z^2) \quad (69)$$

in the longitudinally comoving system in which the pion pair is emitted transverse to the collision axis ($p_L=0$). The components of the vector $\mathbf{q}$ are chosen parallel to the collision axis (z=longitudinal), parallel to the vector $\mathbf{p}_t$ (x=outward), and perpendicular to the production plane (z,x) of the pair (y=sideward). Assuming the same radii also for the coherent emission region, and a transverse thermal law $\exp(-m_t/T)$ for the chaotic radiation with the temperature T (m_t is the pion transverse mass), we can parametrize the coherent fraction $G(p)$ similar to Eq. (38) for the nonrelativistic case with [16]

$$D(p)\simeq D(0)\exp\left[-2\left((p_x^2R_x^2+p_y^2R_y^2+p_z^2R_z^2)+\frac{m_t}{T}\right)\right], \quad (70)$$

and use Eq. (39) to calculate $\langle\cos(qx_{12})\rangle'$.

The presence of the FSI effect introduces the additional q dependence of the correlation functions and thus improves, in principle, the accuracy of the extraction of the coherent contribution $G(p)$. Consider, for example, only the effect of the Coulomb FSI and assume that the emission functions, g_{ch} and g_{coh}, are localized in the regions of characteristic sizes much smaller than the two-pion Bohr radius $|a|=387.5$ fm so that the modulus of the nonsymmetrized Coulomb wave function can be substituted by its value at zero separation. As a result the Coulomb effect factorizes in a form of the so called Gamow or Coulomb factor $A_c(ak^*)=|\psi_q^{coul}(0)|^2$ (see, e.g., Ref. [8]):

$$\tilde{C}(p,q)=A_c(ak^*)C_{QS}(p,q),$$

$$A_c(x)=(2\pi/x)/[\exp(2\pi/x)-1], \quad (71)$$

where $k^*=|\mathbf{q}^*|/2$ is the momentum of one of the two pions in their c.m.s. For the correlation functions of like ($a=|a|$) and unlike ($a=-|a|$) charged pions, we get

$$C^{\pm\pm}(p,q)=\Lambda^{\pm\pm}(p)A_c(|a|k^*)\left[1+\langle\cos(qx_{12})\rangle' - \frac{4}{5}G(p+q/2)G(p-q/2)\right]+[1-\Lambda^{\pm\pm}(p)],$$

$$C^{+-}(p,q)=\Lambda^{+-}(p)A_c(-|a|k^*)\times\left[1+\frac{1}{5}G(p+q/2)G(p-q/2)\right]+[1-\Lambda^{+-}(p)]. \quad (72)$$

Similar to the case of absent FSI, we can again use the parametrizations (69), (70), and the relation (39), and fit, simultaneously or separately, the correlation functions of like and unlike charged pions according to Eqs. (72). Moreover, the known q dependence of the Gamow factors allows to separate the coherent fraction $G(p)$ from the suppression parameter $\Lambda(p)$ in a model-independent way, without exploiting the q dependence of $\langle\cos(qx_{12})\rangle_{ch}$ and $G(p\pm q/2)$. Indeed, one can perform the fits according to Eqs. (72) in an interval of $q_{\min}<|q|\ll R^{-1}$ guaranteeing $\langle\cos(qx)\rangle'\approx 1$ and $G(p_{1,2})\approx G(p)$. The q dependence of the correlation functions is then uniquely determined by the known functions $A_c(|a|k^*)$ and $A_c(-|a|k^*)$, and the three fitted parameters $G(p)$, $\Lambda^{\pm\pm}(p)$, and $\Lambda^{+-}(p)$. Of course, such an analysis requires very good detector resolution and its good understanding.

Note that Eqs. (72) are not applicable for very small (~ 1 fm) as well as for large sources. In the former case one has to account for the strong FSI, in the latter for the finite-size Coulomb effects. For ultrarelativistic heavy ion collisions, the strong FSI effect on two-pion correlation functions is negligible for like charge pions and small (a few percent) for unlike pions. The Coulomb finite-size effects can be approximately taken into account, substituting the Gamow factor $A_c(ak^*)$ in Eq. (17) by the finite-size Coulomb factor $\tilde{A}_c(ak^*,\langle r^*\rangle/a)$ [39]. The latter represents a simple function of the arguments ak^* and $\langle r^*\rangle/a$, where $\langle r^*\rangle$ is the mean distance of the pion emission points in the pair c.m.s., corresponding to a given momentum $\mathbf{p}$. Particularly, $\tilde{A}_c\doteq A_c(ak^*)[1+2\langle r^*\rangle/a]$ at $k^*<\sim 1/\langle r^*\rangle$.

The dependence of the Coulomb factor on the unknown parameter $\langle r^*\rangle$ somewhat complicates the model-independent method for the extraction of coherent component $G(p)$ exploiting only the correlation functions in the region of very small relative momenta. Now, the simultaneous analysis of the correlation functions of like and unlike charged pions is required because their separate analysis yields the coherent contribution $G(p)$ up to a correction $\langle r^*\rangle/a$ only. As for the method based on a fit in a wide $|\mathbf{q}|$ interval, the quantity $\langle r^*\rangle$ being a unique function of the parameters characterizing the emission density actually represents no new free parameter. Particularly, for a universal anisotropic Gaussian $\mathbf{r}^*$distribution of the chaotic and coherent emission functions, the quantity $\langle r^*\rangle$ can be expressed analytically through the Gaussian interferometry radii R_y, R_z, and $R_x^*=M_t/MR_x$ (M and M_t are the two-pion effective and transverse masses, respectively) in the case of practical interest, when $R_x^*\geqslant R_y\approx R_z$ [39].

In practice, however, the Gaussian parametrization of the relative distances between the emission points may happen to be insufficient. Particularly, it can lead to apparent inconsistencies in the treatment of QS and FSI effects because the latter is more sensitive to the tail of the distribution of the relative distances. If, for example, the r^* distribution was represented by a sum of two Gaussians with essentially different mean squared radii, the r^* "tail," determined by the larger Gaussian radius, would influence the observed correlation functions in different ways. For identical pions, the "tail" results in an additional rather narrow peak in the QS correlation function; however, this "tail" would show up only as a suppression of the correlation function if the peak were concentrated at $q\lesssim q_{\min}$ or if one measured a given projection of the correlation function (e.g., in the q_{side} direction) fixing others (q_{long} and q_{out}) in the interval exceeding the width of the narrow peak. At the same time, the r^* "tail" would influence Coulomb correlations at small $q\gtrsim q_{\min}$ since the long-distance nature of Coulomb forces leads to the observable effect conditioned by the "tail" up to $r^* \sim |a|$. In such a situation, one can no more rely on the equality between $\langle r^* \rangle_{QS}$, determined by the interferometry radii, and the characteristic size $\langle r^* \rangle_C$ determining the Coulomb FSI effect. Generally, one has to introduce also different suppression parameters $\Lambda_{QS} < \Lambda_C$ corresponding to $\langle r^* \rangle_{QS} < \langle r^* \rangle_C$. Equations (72) for the correlation functions of like and unlike charged pions, with the substitution of the Gamow factor $A_c(ak^*)$ by the finite-size Coulomb factor $\widetilde{A}_c(ak^*, \langle r^* \rangle / a)$ [39], are then modified to the form

$$\begin{aligned} C^{\pm\pm}(p,q) = {} & \Lambda_{QS}^{\pm\pm}(p)\widetilde{A}_c(|a|k^*, \langle r^* \rangle_{QS}^{\pm\pm}/|a|) \\ & \times\left[\langle \cos(qx)\rangle' - \frac{4}{5}G(p+q/2)G(p-q/2)\right] \\ & + \Lambda_C^{\pm\pm}(p)\widetilde{A}_c(|a|k^*, \langle r^* \rangle_C^{\pm\pm}/|a|) \\ & + [1-\Lambda_C^{\pm\pm}(p)], \end{aligned}$$

$$\begin{aligned} C^{+-}(p,q) = {} & \Lambda_{QS}^{+-}(p)\widetilde{A}_c(-|a|k^*, -\langle r^* \rangle_{QS}^{+-}/|a|) \\ & \times \frac{1}{5}G(p+q/2)G(p-q/2) + \Lambda_C^{+-}(p)\widetilde{A}_c \\ & \times(-|a|k^*, -\langle r^* \rangle_C^{+-}/|a|) + [1-\Lambda_C^{+-}(p)]. \end{aligned} \tag{73}$$

To simplify the analysis, one can neglect a small difference between the suppression parameters Λ_{QS} and Λ_C due to the tail of the r^* distribution and also neglect a presumably small difference between $\langle r^* \rangle^{\pm\pm}$ and $\langle r^* \rangle^{+-}$.

Note that at SPS and RHIC energies the effect of strong FSI on $\pi^+\pi^-$ correlations is still quite noticeable and, when neglected, it can lead to a suppression of a fitted $\langle r^* \rangle^{+-}$ by $\sim 50\%$. Also, due to a substantial inaccuracy of the Coulomb factor $\widetilde{A}_c(ak^*, \langle r^* \rangle / a)$ near the tailing point $k^* \sim 1/\langle r^* \rangle$, the parameters $\langle r^* \rangle^{++}$ and $\langle r^* \rangle^{+-}$ can be, respectively, overestimated and underestimated if the fitted region was not sufficiently wide. Further, in the case of different chaotic and coherent emission volumes, one has to use finite-size Coulomb factors with different $\langle r^* \rangle$ in the chaotic, coherent, and mixed terms. All these problems can be overcome exploiting the exact formulas for the two-pion wave functions (in the equal time approximation) and calculating the correlation functions according to the approximate Eq. (64). To control the systematic errors due to the smoothness assumption in Eq. (64), one can give up this assumption (at least in the pure coherent term) and check the results using instead the general expression for the two-pion spectrum in Eq. (58).

After the extraction of the fractions $G(p)$ and $\Lambda^{++}(p)$ or $\Lambda^{--}(p)$, one can obtain the coherent part of the measured single-pion spectra $\omega_{\mathbf{p}} d^3N_{\pm}/d^3\mathbf{p}$. Using Eq. (66), and substituting $d^3N/d^3\mathbf{p} \rightarrow (d^3N_{\pm}/d^3\mathbf{p} - d^3N_{\pm}^{(l)}/d^3\mathbf{p})$ in Eq. (24), one gets

$$\omega_{\mathbf{p}} \frac{d^3N_{coh}}{d^3\mathbf{p}} \equiv \frac{1}{3}|d(p)|^2 = \omega_{\mathbf{p}} \frac{d^3N_{\pm}}{d^3\mathbf{p}} G(p)\sqrt{\Lambda^{\pm\pm}(p)}. \tag{74}$$

The coherent part of the observed spectra is thus directly connected with the intensity $|d(p)|^2$ of the quasiclassical source of coherent pions.

V. CONCLUSIONS

Using the density matrix formalism, satisfying the requirements of the isospin symmetry and the superselection rule for generalized coherent states, and accounting for the final state interaction in the two-body approximation, we have developed methods allowing one to study the coherent component of pion radiation which, in heavy ion collisions, is likely conditioned by formation of a quasiclassical pion source.

These methods are based on a nontrivial modification of the effects of quantum statistics and final state interaction on two-pion correlation functions (including those of nonidentical pions) in the presence of a coherent pion radiation generated by the decay of the quasipionic ground state ("condensate"). It has been shown that the combined analysis of the correlation functions of like and unlike pions gives the possibility to discriminate between the suppression of the like-pion correlation functions conditioned by the coherent pion component and that due to the decays of long-lived sources.

The methods allowing to extract the coherent pion component from $\pi^+\pi^-$ and $\pi^\pm\pi^\pm$ correlation functions and single-pion spectra have been discussed in detail for large expanding systems produced in ultrarelativistic heavy ion collisions. For such systems, the two-pion final state interaction is dominated by the Coulomb one and plays an important role in this analysis, allowing one to determine the coherent fraction using a suitable model for the coherent and chaotic emission functions and fitting the corresponding correlation functions. For rough estimations the procedure can be substantially simplified accounting for the finite-size Coulomb effects in an approximate analytic form [39].

Finally, the coherent fractions extracted from the correlation analysis, combined with the single-pion spectra, give us the possibility to determine the spectrum of the coherent pion radiation above the thermal background and, therefore, to estimate the quasipionic condensate at the predecaying stage of the matter evolution and discriminate between possible mechanisms of coherent production in ultrarelativistic $A+A$ collisions.

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