Large-N_c nuclear potential puzzle

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An analysis of the baryon-baryon potential from the point of view of large-N_c QCD is performed. A comparison is made between the N_c-scaling behavior directly obtained from an analysis at the quark-gluon level to the N_c scaling of the potential for a generic hadronic field theory in which it arises via meson exchanges and for which the parameters of the theory are given by their canonical large-N_c scaling behavior. The purpose of this comparison is to use large-N_c consistency to test the widespread view that the interaction between nuclei arises from QCD through the exchange of mesons. Although at the one- and two-meson exchange level the scaling rules for the potential derived from the hadronic theory matches the quark-gluon level prediction, at the three- and higher-meson exchange level a generic hadronic theory yields a potential which scales with N_c faster than that of the quark-gluon theory.

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I. HADRONIC DYNAMICS AND LARGE-N_c COUNTING

The absence of a calculational scheme for properties and low-energy interactions of hadrons from the first principles of the underlying microscopic theory of color dynamics calls for the treatment of this domain by means of effective theories with degrees of freedom other than quarks and gluons. Traditionally nuclear physicists have envisioned nucleon-nucleon interactions as emerging from the exchange of mesons. When QCD was established as the theory of strong interactions three decades ago, the meson-exchange picture was viewed as arising from it: QCD gives rise to effective hadronic degrees of freedom and the interaction of these could then account for nuclear forces. Obviously, these effective degrees of freedom can only describe the low-lying modes of the theory where the underlying quark and gluon degrees of freedom are not easy to disentangle. Therefore, one faces the long-standing problem of the correspondence between hadron and color dynamics. The problem we wish to address is the extent to which QCD justifies the traditional meson-exchange picture of nuclear forces.

The large-N_c approximation provides a possible framework to investigate this issue since there are a number of important simplifications of QCD in this regime [1,2]. Of course, it is by no means obvious that one can directly deduce specific phenomenological consequences for the real world from the large-N_c perspective. Recall, for example, that the deuteron binding energy ε_B is of order N_c while the delta-nucleon mass splitting δM is of order $1/N_c$. Thus, if one were in a large-N_c world, one would expect a hierarchy of scales with $\varepsilon_B \sim \mathcal{O}(N_c) \gg \delta M \sim \mathcal{O}(1/N_c)$. In the real world, however, one finds these scales indeed widely separated but with the opposite order: $\varepsilon_B \ll \delta M$. However, the argument that a QCD description of low-energy nucleon-nucleon interactions should be describable in a meson-exchange picture does not appear to depend in any explicit way on the fact that $N_c=3$ in the physical world. Thus, if the argument is valid, it ought to apply equally well in a fictitious multicolor world and one would then expect quantities deduced from the meson-exchange picture to scale with N_c in the same way as the same quantities deduced from an analysis conducted directly at the quark-gluon level. The use of large-N_c scaling rules to test ideas from nuclear physics is not new. It was argued more than a decade ago that nucleon loop contributions based on pointlike nucleon-meson couplings, as was conventionally calculated in various quantum hadrodynamical models [3], did not scale with N_c in a manner consistent with large-N_c QCD and hence presumably did not capture the underlying QCD dynamics [4].

The physical spectrum of QCD consists of colorless hadronic states — baryons and mesons. As discussed by 't Hooft [1] and Witten [2], large-N_c QCD gives definite predictions for the scaling of their characteristics with N_c. For example, the baryon and meson masses are of order N_c and unity, respectively, and the single-meson-baryon coupling g_{1m} is of order $\sqrt{N_c}$ while meson-baryon scattering amplitudes are of order unity. More generally, reasoning along the lines suggested by Witten [2] implies that the coupling of N mesons to a baryon scales at most as $g_{Nm} \sim N_c^{1-N/2}$.

Apart from these generic counting rules there are additional constraints coming from the spin-flavor structure of the interaction. This can be seen by imposing the consistency of two single meson-baryon interactions vertices $\boldsymbol{V}$ (nominally of order N_c in total) with the meson-baryon scattering which unitarity restricts to be of order unity. The cancellations required for this to come about imply a contracted SU(4) symmetry for two-flavor QCD [5–7]; for a review, see Refs. [8,9]. This implies that baryons form towers of nearly degenerate states with $I=J$ and with splittings of order $1/N_c$. The contracted SU(4) relations hold for states in this tower. The commutation algebra of the spin $\boldsymbol{J}$, isospin $\boldsymbol{I}$, and spin-isospin $\boldsymbol{X}$ vertices is given schematically by

$$[\boldsymbol{I},\boldsymbol{I}] \sim \boldsymbol{I}, \quad [\boldsymbol{J},\boldsymbol{J}] \sim \boldsymbol{J}, \quad [\boldsymbol{I},\boldsymbol{X}] \sim \boldsymbol{X},$$

$$[\boldsymbol{J},\boldsymbol{X}] \sim \boldsymbol{X}, \quad [\boldsymbol{X},\boldsymbol{X}] \sim 1/N_c^2. \tag{1}$$

The last relation implies a suppression when commutators arise. This occurs in treatments of the tree-level baryon-meson scattering: for the N-meson-baryon scattering amplitude $\mathcal{A}_N$ the destructive interference [10–12] leads to the

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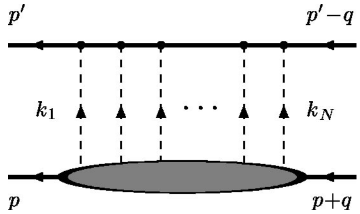

FIG. 1. A generic exchange diagram for the baryon-baryon scattering.

appearance of multiple commutators of meson vertices leading to consistency with the large-N_c counting rules predicting $\mathcal{A}_N \sim N_c^{1-N/2}$.

In the present study we address the issue of the consistency of large-N_c QCD with the conventional meson-exchange picture used to describe nuclear potentials. If the latter adequately describes the real world, it must possess the same multicolor asymptotics as deduced from QCD for loop amplitudes. We consider the potential used for the baryon-baryon scattering shown Fig. 1. The problem at the quark level was discussed by Kaplan and Savage [13] and by Kaplan and Manohar [14]; collectively, we refer to their analysis as KSM. The basic strategy used by KSM was based on Witten's Hartree picture where the interaction is identified as being due to quark-line-connected diagrams. KSM then equate the nucleon-nucleon potential to the sum of the quark-line-connected Feynman graphs which involve exchanges between two groups of N_c quark lines which represent baryons. This is then analyzed using the contracted SU(4) symmetry. The principal results of this analysis are that the strength of the spin-isospin structures of the nucleon-nucleon potential scale as follows:

$$\mathcal{V}_{I=J} \sim N_c, \quad \mathcal{V}_{I\neq J} \sim N_c^{-1}, \tag{2}$$

where the subscript indicates the quantum numbers of the exchange in the t channel. It is straightforward to see from the large-N_c scaling rules of meson-baryon couplings that a one-meson-exchange potential will satisfy Eq. (2). It is not immediately obvious, however, that multimeson-exchange potentials will obey this rule since superficially they are clearly larger than allowed by Eq. (2). For example, at the two-meson-exchange level, both the retardation effects from box graphs and the contributions from cross-box graphs enter $\mathcal{V}_{I=J}$ and $\mathcal{V}_{I\neq J}$ at order N_c^2. However, as shown in a detailed calculation in Ref. [15] cancellations between these two yield potentials compatible with Eq. (2). The contracted SU(4) structure played an essential role in achieving this goal. Note that to get consistency with $\mathcal{V}_{I\neq J}$ cancellations up to order N_c^{-3} are needed; in fact they occurred up to order N_c^{-4}.

The results of Ref. [15] raise the hope that similar cancellations might be expected for all multimeson exchanges. If these were true, it would show consistency at large N_c between the scaling of the potential at the quark-gluon and hadronic level and would help to justify meson-exchange-based potential models as arising from QCD. However, as will be shown in this paper, the type of cancellations seen for the two-meson exchange do not occur for general multimeson exchanges. Thus, potentials derived from generic hadronic theories calculated at a fixed number of meson exchanges do not give rise to potentials that respect the KSM scaling rules of Eq. (2). This result is puzzling in view of the general expectation that the physics of QCD at low energies can be described in terms of hadronic degrees of freedom. We will refer to this as the "large-N_c nuclear potential puzzle." There is a second puzzling aspect of this problem. In studying exchanges of quark-antiquark pairs the role of "static" pairs, i.e., pairs whose energy transfer is small (of order $1/N_c$), is special: the leading-order contribution of multipair exchanges to the potential requires all pairs to be static. If there were a one-to-one mapping between classes of quark-gluon diagrams with hadronic ones, this would correspond to static meson exchanges. In fact, however, we will see that the "dangerous" contributions at the hadronic level — the ones which contradict the KSM rules of Eq. (2) — all come from nonstatic meson exchanges.

The calculations of multimeson exchanges can get quite complicated. Accordingly, it is useful to develop tools which greatly simplify the analysis. The non-Abelian generalization of the eikonal formula [12] which can be used to compute the sum of certain multimeson amplitudes will be quite handy in this context. In the following section we will review this formalism. In Sec. III we show how sums of various multimeson diagrams lead to contributions which are incompatible with the KSM rules. Finally, we conclude with a discussion of possible resolutions of this apparent paradox.

II. NON-ABELIAN EIKONAL FORMULA

Following Ref. [12] we consider a baryon which emits or absorbs a number of virtual mesons. Since the baryon is extremely heavy at $N_c = \infty$, it can be treated as almost static. It will be a nonrelativistic particle if its three-momentum $\boldsymbol{p}$ is of $\mathcal{O}(N_c^0)$; the four-momentum is then $(M + \boldsymbol{p}^2/(2M), \boldsymbol{p})$. In this kinematic regime the heavy baryon propagator is approximated by the product of the conventional eikonal propagator and a projection matrix (which will be omitted later) on the large components of the nucleon bispinor:

$$\frac{1}{\not{p} + \not{k} - M' + i\epsilon} \to \frac{1}{\omega + i\epsilon} \frac{\mathbb{1} + \gamma_0}{2}. \tag{3}$$

Taken literally, this expression which neglects the nucleon recoil is only valid for meson energies ω, $k = (\omega, \boldsymbol{k})$, of order $\mathcal{O}(N_c^0)$. If ω is of order N_c^{-1}, then the denominator of the propagator (3) is modified to include the recoil effect as follows: $\omega \to \omega + \delta M + [\boldsymbol{p}^2 - (\boldsymbol{p} + \boldsymbol{k})^2]/(2M)$, with the mass difference of the "degenerate" states in the baryon tower, $\delta M = M - M'$. Note that δM is $\mathcal{O}(N_c^{-1})$ [17–19]. For simplicity we generally omit this modification in our expressions. This approximation has to be kept in mind since it leads to singularities in the integrand of certain loop amplitudes. However, they are easily identifiable and cured by the simple expedient of reintroducing the recoil correction.

There is an important kinematical constraint in this regime. Since the initial and final nucleons are on shell and have three-momenta of order N_c^0, the kinetic energies of the initial and final states are thus of order N_c^{-1} (due to the fact that $M \sim N_c$) and thus the total exchanged energy is also of order N_c^{-1}.

The blob on the lower baryon line in Fig. 1 represents the tree amplitude for production of N mesons $\mathcal{A}_N$; it includes all possible permutation of single-meson emissions from the baryon line and this contribution reads

$$\mathcal{A}_N \equiv \sum_{\sigma \in \{1,\ldots,N\}} \mathcal{A}[\sigma_1 \cdots \sigma_N], \tag{4}$$

where

$$\mathcal{A}[\sigma_1 \cdots \sigma_N] = \boldsymbol{V}_{\sigma_1} \boldsymbol{V}_{\sigma_2} \cdots \boldsymbol{V}_{\sigma_N} a[\sigma_1 \cdots \sigma_N], \tag{5}$$

with one of the matrices $\boldsymbol{V} = \{\boldsymbol{1}, \boldsymbol{I}, \boldsymbol{J}, \boldsymbol{X}\}$ associated with the meson-baryon interaction vertex, and

$$a[\sigma_1 \cdots \sigma_N] \equiv -2\pi i \delta\left(\sum_{j=1}^{N} \omega_j\right) \prod_{j=1}^{N-1} \frac{1}{\sum_{k=1}^{j} \omega_{\sigma_k} + i\epsilon}. \tag{6}$$

Obviously, for a single element n (and neglecting recoil as discussed above), $a[n] = -2\pi i \delta(\omega_n)$.

As a result of the destructive Bose-Einstein interference, $\mathcal{A}_N$ can be expressed by a multiple-commutator formula for the sum of non-Abelian eikonal amplitudes discussed in Ref. [11],

$$\mathcal{A}_N = \sum_{\sigma \in \{1,\ldots,N\}} \tilde{\mathcal{A}}[\sigma_1 \cdots \sigma_N]. \tag{7}$$

The constructive definition of the multicommutator amplitude $\tilde{\mathcal{A}}$ goes as follows. For a given ordering of lines, if the rightmost element of the given permutation is smaller than any other element to its left, then there is only one partition and it is the whole tree. Otherwise, go to the first element whose number is smaller and draw a partition just to the right of that element. Next, start to the left of this partition and repeat the procedure again. Through the cut separating the partitions, the amplitude factorizes:

$$\begin{aligned}\tilde{\mathcal{A}}&[\sigma_1 \cdots \sigma_2 | \sigma_3 \cdots \sigma_4 | \cdots] \\ &= \tilde{\mathcal{A}}[\sigma_1 \cdots \sigma_2] \tilde{\mathcal{A}}[\sigma_3 \cdots \sigma_4] \tilde{\mathcal{A}}[\cdots],\end{aligned} \tag{8}$$

where $\sigma_2 < \sigma_4 < \cdots$. The amplitudes without partitions are given by the commutator formula with the innermost commutator formed by the last two elements of the tree,

$$\begin{aligned}\tilde{\mathcal{A}}&[\sigma_1 \cdots \sigma_{n-1} \sigma_n] \\ &= [\boldsymbol{V}_{\sigma_1}, \cdots, [\boldsymbol{V}_{\sigma_{n-1}}, \boldsymbol{V}_{\sigma_n}] \cdots] a[\sigma_1 \cdots \sigma_{n-1} \sigma_n].\end{aligned} \tag{9}$$

The first two nontrivial examples read

$$\mathcal{A}_2 = \tilde{\mathcal{A}}[1|2] + \tilde{\mathcal{A}}[21],$$

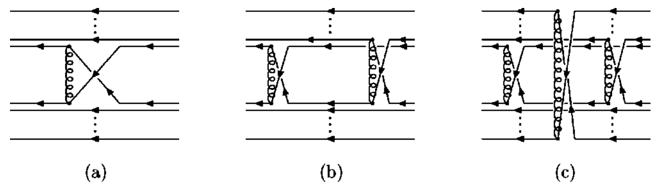

FIG. 2. Quark diagrams which can be interpreted as meson exchanges.

$$\begin{aligned}\mathcal{A}_3 = {}& \tilde{\mathcal{A}}[1|2|3] + \tilde{\mathcal{A}}[21|3] + \tilde{\mathcal{A}}[1|32] + \tilde{\mathcal{A}}[231]bf + \tilde{\mathcal{A}}[31|2] \\ &+ \tilde{\mathcal{A}}[321].\end{aligned} \tag{10}$$

The proof is straightforward by the Fourier transformation of both sides of the equality.

The generalizations are obvious. The constructive formula reads

$$\begin{aligned}\mathcal{A}_N = \sum_{\sigma = \sigma^{(1)} + \cdots + \sigma^{(j)}} & \left(\prod_{k=1}^{j} a[\{\sigma^{(k)}\}]\right) \\ & \times \boldsymbol{V}[\{\sigma^{(1)}\}] \cdots \boldsymbol{V}[\{\sigma^{(j)}\}],\end{aligned} \tag{11}$$

with a number of partitions j of a given permutation σ into sets $\sigma^{(k)}$ with elements $\{\sigma^{(k)}\} = \sigma_1^{(k)}, \ldots, \sigma_{n_k}^{(k)}$ constructed according to the definition given above. To make these notations clear and in order to establish a contact with the previous discussion, let us mention that there is a term in Eq. (11) of the form (8) where, for instance, the elements $\{\sigma^{(1)}\} = \sigma_1^{(1)}, \ldots, \sigma_{n_1}^{(1)}$ of the set $\sigma^{(1)}$ are identified as follows: $\sigma_1^{(1)} \equiv \sigma_1, \ldots, \sigma_{n_1}^{(1)} \equiv \sigma_2$; the elements $\{\sigma^{(2)}\} = \sigma_1^{(2)}, \ldots, \sigma_{n_2}^{(2)}$ of $\sigma^{(2)}$ are related as $\sigma_1^{(2)} \equiv \sigma_3, \ldots, \sigma_{n_2}^{(2)} \equiv \sigma_4$, etc. For a single element in a set the vertex function introduced in Eq. (11), $\boldsymbol{V}[n]$ coincides with the single vertex $\boldsymbol{V}_n$. For more than one element it is given by a multiple commutator

$$\boldsymbol{V}[\{\sigma^{(k)}\}] = [\boldsymbol{V}_{\sigma_1^{(k)}}, \ldots, [\boldsymbol{V}_{\sigma_{n_k-1}^{(k)}}, \boldsymbol{V}_{\sigma_{n_k}^{(k)}}] \cdots].$$

The energy-dependent functions a of a given set are defined in Eq. (6). For the Abelian case, i.e., when $\boldsymbol{V} \to \boldsymbol{1}$, all commutators vanish and one gets the well-known eikonal formula.

III. BARYON-BARYON SCATTERING

The baryon-baryon interaction at large N_c comes from the exchange of pairs of quark constituents. A quark from one of the hadrons switches places with any quark in the other baryon and exchanges a gluon to neutralize the color charge. The quark exchange, in Fig. 2(a), may naturally be reinterpreted at the hadronic level as a meson exchange,

$$\mathcal{V}(\boldsymbol{q}) = g_{1m}^2 D(-\boldsymbol{q}^2), \tag{12}$$

$$\text{with } D(q^2) = \frac{1}{q^2 - m^2 + i\epsilon},$$

where for simplicity we have chosen the example of the scalar-isoscalar channel. Clearly, this correspondence between the meson and quark exchanges is not valid on a diagram-by-diagram basis. Presumably the sum of all quark exchange diagrams including an arbitrary number of gluons connecting the exchanged quarks (which if planar are leading order in N_c) gets mapped onto the sum of all one-meson-exchange graphs for mesons with these quantum numbers.

In analyzing the contributions to the potential one must recall that the full amplitude is obtained from the potential by an iteration. Thus, nonrelativistic amplitudes satisfy the Lippmann-Schwinger equation

$$\mathcal{T}(\boldsymbol{p},\boldsymbol{p}+\boldsymbol{q}) = -\mathcal{V}(\boldsymbol{q}) + \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\mathcal{V}(\boldsymbol{k})\mathcal{G}(\boldsymbol{k})\mathcal{T}(\boldsymbol{p}+\boldsymbol{k},\boldsymbol{p}+\boldsymbol{q}), \tag{13}$$

with the potential $\mathcal{V}$ and the two-baryon propagator

$$\mathcal{G}(\boldsymbol{k}) \equiv \frac{1}{\dfrac{\boldsymbol{p}^2}{M} - \dfrac{(\boldsymbol{k}+\boldsymbol{p})^2}{M} + i\epsilon} = \int \frac{d\omega}{2\pi i}\frac{1}{\left(\omega - \dfrac{\boldsymbol{p}^2}{2M} + \dfrac{(\boldsymbol{k}+\boldsymbol{p})^2}{2M} - i\epsilon\right)\left(\omega + \dfrac{\boldsymbol{p}^2}{2M} - \dfrac{(\boldsymbol{k}+\boldsymbol{p})^2}{2M} + i\epsilon\right)}. \tag{14}$$

Here we defined $\mathcal{G}$ as the conventional nonrelativistic Green function for a two-baryon system of the reduced mass $M/2$ and in the last form we reexpressed it in terms of the single-particle propagators neglecting δM. The kinematics is chosen according to Fig. 1 where $p=(E,\boldsymbol{p})$ and $p'=(E,-\boldsymbol{p})$ with nonrelativistic expansion $E\approx M+\boldsymbol{p}^2/(2M)$. The momentum transferred $q=p-p'=(q_0,\boldsymbol{q})$ has a small time component $q_0\sim\mathcal{O}(1/N_c)$ since it is inversely proportional to the baryon mass.

When assessing the contribution of some hadronic-level Feynman diagram to the potential, it is essential to note that the Feynman diagrams sum to give the full amplitude and not just the potential. Accordingly, to extract the contributions to the potential, one must remove all contributions which correspond to iterates of the potential. Fortunately, they are easily identifiable. From the nonrelativistic form (14) it is clear that as $N_c\to\infty$, $\mathcal{G}$ diverges since $M\sim N_c$. Thus, in the large-N_c limit, these potential iterates are associated with infrared singularities and these are the only infrared singularities in the problem. Thus, when one encounters them in the integrals one may make the substitution

$$\int d\omega \frac{\delta(\omega)}{\omega+i\epsilon} \to \mathcal{G}, \tag{15}$$

where one inputs $\mathcal{G}$ with appropriate kinematics. More significantly, having identified these terms as arising from a potential iterate, one can remove them from the amplitude to extract an irreducible contribution to the potential only.

The baryon-baryon interaction at the QCD level is generated by various complicated quark-exchange processes sampled in Fig. 2. One expects that these are connected diagrams which contribute to the potential while the quark-line disconnected amplitudes, such as Fig. 2(c), are associated with iterates of the potential which arise in the Lippmann-Schwinger equation. On the hadronic side, the sum of the quark-level diagrams can presumably be interpreted as meson exchanges. A few examples of these can be found in Fig. 3. As we will demonstrate momentarily, at multiple-meson exchanges the correspondence between quark- and hadron-level diagrams is lost.

For the sake of concreteness, we illustrate the issues by considering bosons with scalar-isoscalar quantum numbers. For other channels, the presence of nontrivial spin-isospin indices introduces significant complications due to the noncommutativity of vertices but does not affect the main line of reasoning. We will return to the effects of noncommutativity when we consider ladder and crossed-ladder diagrams. We also will neglect all effects of momentum dependence in the meson-baryon couplings. Again it easy to see that they do not alter our conclusions.

Let us analyze what kinds of quark configurations give contributions to the potential. The one quark-pair exchange in Fig. 2(a) is translated into the one-meson exchange of Fig. 3(a). At the quark level, there is a N_c^2 combinatorial factor from the number of possibilities to join the quark lines in both baryons and there is a $1/(\sqrt{N_c})^2$ from the two gluon couplings. This results in an overall scaling as N_c which is compatible with Eq. (2). On the hadronic side the meson-baryon coupling behaves as $\sqrt{N_c}$ and the one-meson-exchange diagram thus also scales as N_c. Note that the energy transfer scales as $1/N_c$ and vanishes for $N_c\to\infty$. Thus, both the exchanged meson and the exchanged quark pair are static with this kinematics.

The two-quark-pair-exchange diagram in Fig. 2(b) is of order N_c: a combinatoric factor of N_c^3 and a factor of $1/(\sqrt{N_c})^4$ from the coupling constants. Note that this diagram has both pairs of quarks coupled to the same quark line in one of the baryons and thus corresponds to the diagram with

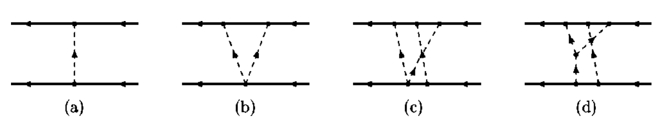

FIG. 3. Typical meson-exchange diagrams contributing to the baryon-baryon scattering amplitude. They are expected to correspond to the quark-pair exchange graphs in Fig. 2. The last two graphs (c) and (d) violate the KSM large-N_c counting rule.

a two-meson-baryon (seagull) vertex, Fig. 3(b). The reason for both pairs to couple to the same line in a baryon is simple. If they do not, then the graph would be quark-line disconnected and thus, by hypothesis, not associated with the irreducible part of the potential. By the N_c counting for this disconnected quark graph to agree with the seagull diagram, it would require an extra gluon exchange to yield the same total large-N_c behavior. On the quark level one again realizes that the leading-order contribution of this type comes from the exchange of static pairs. In this case the requirement goes beyond simple kinematics. Kinematically the energies of the two exchanges must be equal and opposite (up to $1/N_c$ corrections) but need not be static. However, the two affected quark lines in the upper baryon in Fig. 2(b) do not communicate by a gluon and, therefore, if each quark exchange carries an energy, then the energy of the final-state quarks would not correspond to the single-particle energies in the Hartree ground state. On the other hand, if they do exchange a gluon, there is an extra $1/N_c$ suppression without an offsetting combinatoric gain, so the term is not of leading order in the $1/N_c$ expansion. On the hadronic level the static nature of the exchange can be seen to arises as follows from the Feynman diagram of Fig. 3(b):

$$\begin{aligned} i\mathcal{T} &= g_{1m}^2 g_{2m} \int \prod_{j=1}^{2} \frac{d^3\boldsymbol{k}_j}{(2\pi)^3} (2\pi)^3 \delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{q}) \\ &\quad \times \int \prod_{j=1}^{2} \frac{d\omega_j}{2\pi} (2\pi) \frac{\delta(\omega_1+\omega_2)}{\omega_1 - i\epsilon} D(k_1^2) D(k_2^2) \\ &= i\pi g_{1m}^2 g_{2m} \int \prod_{j=1}^{2} \frac{d^3\boldsymbol{k}_j}{(2\pi)^3} (2\pi)^3 \delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{q}) \\ &\quad \times \int \prod_{j=1}^{2} \frac{d\omega_j}{2\pi} (2\pi) \delta(\omega_1) \delta(\omega_2) D(-\boldsymbol{k}_1^2) D(-\boldsymbol{k}_2^2), \end{aligned} \tag{16}$$

where $g_{1m} \sim \sqrt{N_c}$ is the single meson-nucleon coupling constant, $g_{2m} \sim N_c^0$ is the "seagull" coupling, and $D(-\boldsymbol{k}^2) = (-\boldsymbol{k}^2 - m^2 + i\epsilon)^{-1}$ is the meson propagator for a meson of mass m, three-momentum $\boldsymbol{k}$, and zero energy. The second equality follows from the fact that the meson propagators are even functions of the meson energies ω_j so that one immediately finds that the principal value part of the baryon propagator cancels under integration and only the δ-function piece survives. Thus only static-exchange mesons contribute.

In addition to the seagull-type two-meson exchange there are also box and crossed-box contributions to the potential. As discussed at length in Ref. [15], the retardation effects in the box and crossed-box graphs separately violate the counting rules of Eq. (2). Moreover, the contributions from these graphs come entirely from nonstatic meson exchanges. However, these terms when summed cancel one another, yielding the total result consistent with Eq. (2) and without contributions from nonstatic mesons. At the level of the amplitude at the two-meson-exchange level one also has an iterate of the static one-meson exchanges. This term presumably is associated with the non-quark-line-connected part of the two-quark-pair-exchange graphs at the quark-gluon level.

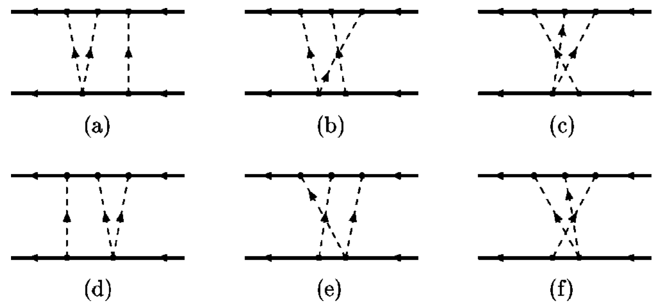

FIG. 4. Three-meson-exchange diagrams with one seagull vertex.

A. Problem at three-meson-exchange level

The three-quark-pair exchange of the type displayed in Fig. 2(c) is not quark-line connected. Rather two of the exchanges are connected while the third one is disconnected. By hypothesis, then, this graph is not to be associated with the potential but presumably with a potential iterate. Using the analysis similar to what was done previously in this paper it is easy to see that all of the exchanged quark pairs must be static at leading order in the $1/N_c$ expansion (which for this class of graph is formally of order N_c^2 — larger than the allowed scaling of the potential — due to the disconnected nature of the graph).

At the hadronic level this graph should be associated with a seagull exchange as in Fig. 3(b) dressed with an extra single-meson exchange, Fig. 3(a). This generates the six diagrams displayed in Fig. 4. The generalized eikonal formula of Sec. II can be applied to describe the lower line in Fig. 4 in a straightforward manner (with the "seagull" vertex taking the place of a single-particle one).[1] Because the vertices commute, only the δ function contributes for the propagators in appropriate pairs of graphs. This implies a cancellation of the fully nonstatic parts of diagrams Fig. 4(a) with 4(c) and Fig. 4(d) with 4(f), where "fully nonstatic" implies that all the mesons exchanged are nonstatic. These cancellations are reminiscent of the ones of the retardation effects in box graphs against crossed-box graphs discussed in Ref. [15]. The δ-function contributions to Figs. 4(a), 4(c) and Fig. 4(d), 4(f) give combinations of propagators of the form seen in Eq. (15). Obviously, they are iterates of the lowest-order Lippmann-Schwinger kernel and do not affect the potential itself. Finally, there is a δ-function contribution to Figs. 4(b), 4(e). This term is not an iterate of the potential but rather an

[1]We use the eikonal formula on the lower line since it is simpler. When applied to the upper line, the potential iterate contributions are not identifiable as easily.

irreducible contribution to it given by

$$\delta\mathcal{V}=-g_{1m}^4 g_{2m}\int\prod_{j=1}^{3}\frac{d^3\boldsymbol{k}_j}{(2\pi)^3}(2\pi)^3\delta^{(3)}\left(\sum_{j=1}^{3}\boldsymbol{k}_j-\boldsymbol{q}\right)\times\int\prod_{j=1}^{3}\frac{d\omega_j}{2\pi}(2\pi)\delta\left(\sum_{j=1}^{3}\omega_j\right)\left(\prod_{j=1}^{3}D(k_j^2)\right)\times\frac{1}{\omega_1-i\epsilon}\frac{1}{\omega_1+\omega_2-i\epsilon}(-2\pi i)\delta(\omega_2). \tag{17}$$

Note that there are two δ functions in energy but three energies. Thus, they do not force all three mesons to be static. Rather, the single exchanged meson is static, while the two mesons connected to the seagull vertex have equal and opposite (but generally nonzero) energy transfer. Performing the energy integrations yields

$$\delta\mathcal{V}=-g_{1m}^4 g_{2m}\int\prod_{j=1}^{3}\frac{d^3\boldsymbol{k}_j}{(2\pi)^3}(2\pi)^3\delta^{(3)}\left(\sum_{j=1}^{3}\boldsymbol{k}_j-\boldsymbol{q}\right)\times D(-\boldsymbol{k}_2^2)\mathcal{R}_2(\varepsilon_1,\varepsilon_3). \tag{18}$$

For simplicity, have introduced a general function $\mathcal{R}_n$ which shows commonly in expressions for multimeson contributions with two nonstatic mesons,

$$\mathcal{R}_n(\varepsilon_j,\varepsilon_k)\equiv\int\frac{d\omega}{2\pi i}\frac{1}{(\omega-i\epsilon)^n}D(\omega^2-\boldsymbol{k}_j^2)D(\omega^2-\boldsymbol{k}_k^2)=\frac{1}{2(\varepsilon_j\varepsilon_k)^{n+1}}\frac{\varepsilon_j^{n+1}-\varepsilon_k^{n+1}}{\varepsilon_j^2-\varepsilon_k^2}, \tag{19}$$

with $\varepsilon_j\equiv\sqrt{\boldsymbol{k}_j^2+m^2}$.

A couple of comments are in order at this point. The first is that $\delta\mathcal{V}$ as given in Eq. (18) scales as N_c^2. This is easily seen since $g_{1m}\sim\sqrt{N_c}$ while $g_{2m}\sim N_c^0$ and both $D(-\boldsymbol{k}_2^2)$ and $\mathcal{R}_n(\varepsilon_j,\varepsilon_k)$ are independent of N_c. The second is that this contribution to $\delta\mathcal{V}$ does not vanish. Both $D(-\boldsymbol{k}_2^2)$ and $\mathcal{R}_n(\varepsilon_j,\varepsilon_k)$ are positive definite, so no cancellations are possible in the integral. This is the heart of our puzzle. The three-meson-exchange contribution to the potential is of order N_c^2. This is incompatible with Eq. (2) which was derived at the quark level. Moreover, the contribution comes entirely from nonstatic meson contributions. At the three-quark-pair-exchange level the amplitude only gets contributions from nonstatic pairs at order N_c (even including quark-line-disconnected pieces).

In the explicit computation done above, pointlike meson-nucleon couplings were used. It is clear that had momentum-dependent vertices been included the functional form of the integral used to derive the potential would be altered but the N_c counting would not. Similarly, it is clear that this problem is not restricted to effects from seagull graphs. It is trivial to see that order N_c^2 contributions will arise in other topologies with multimeson couplings such as exemplified in Fig. 3(d). Note, though, that this diagram corresponds to the same quark-level topology in Fig. 2(c), hinting at a possible conspiracy of the seagull and three-meson vertex contributions.

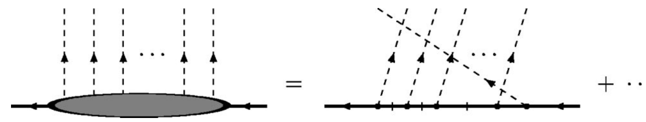

FIG. 5. A crossed-ladder diagram (on the right-hand side of the equality) which generates a superleading contribution to the baryon potential. Dashes on the baryon line denote the on-shell condition (the cuts) of the corresponding propagators.

B. Ladders and crossed ladders

A similar problem shows up in the contributions form ladder and crossed-ladder diagrams. Here, we will consider the effect of N identical mesons with nonderivative pointlike couplings to the baryons. Derivative couplings will not alter the conclusions, but as seen in Ref. [15], greatly complicate the analysis. Non-point-like couplings can also be easily included and do not alter the qualitative results either. Similarly, the restriction to identical mesons is done for simplicity; again, the conclusions will not be strongly dependent on this. Since we are interested in the possibility of effects which violate the KSM rules by having a ''super-leading'' N_c dependence, we will consider the exchanges of mesons which have couplings to baryons of order $\sqrt{N_c}$, the maximum allowable. From the analysis of the contracted SU(4) symmetry in Ref. [16] these will be a scalar-isoscalar vertex or a spin one, isospin one $\boldsymbol{X}_{ia}$ type. As will be seen below, the superleading effect depends on the nonvanishing of the commutators of the vertices. Thus, the scalar-isoscalar exchanges, which commute, will not contribute (unless other noncommuting exchanges are also present). Accordingly we will restrict our attention to mesons that couple in a nonderivative way to $\boldsymbol{X}_{ia}$. An example of such a meson is the spatial part of the A_1 (recalling that for nonrelativistic baryons, the couplings to the temporal and spatial parts of vector mesons can be separated).

Note that our result will depend on the fact that our problem is $[\boldsymbol{X}_{ia},\boldsymbol{X}_{jb}]\neq 0$. Of course, for contracted SU(4) this commutator is zero. However, one only has the contracted SU(4) symmetry for infinite N_c. For finite N_c the commutator is small, scaling as N_c^{-2} but not zero. If the commutator is multiplied by a function which grows with N_c rapidly enough, it will contribute and can indeed be associated with superleading effects.

For any given number of meson exchanges one has to sum the blob in Fig. 5 representing the emission of mesons with all orderings. For this purpose the non-Abelian generalization of the eikonal formula in Eq. (11) is indispensable.

The amplitude for the exchange of N point-coupled A_1 mesons is given by

$$\mathcal{T}_N=(-i)^N g_{1m}^{2N}\int\prod_{j=1}^{N}\frac{d^3\boldsymbol{k}_j}{(2\pi)^3}(2\pi)^3\delta^{(3)}\left(\sum_{j=1}^{N}\boldsymbol{k}_j-\boldsymbol{q}\right)$$
$$\times\int\prod_{j=1}^{N}\frac{d\omega_j}{2\pi}\left(\prod_{j=1}^{N}D(k_j^2)\right)$$
$$\times\boldsymbol{V}_1\frac{1}{\omega_1-i\epsilon}\boldsymbol{V}_2\frac{1}{\omega_1+\omega_2-i\epsilon}$$
$$\times\boldsymbol{V}_3\cdots\boldsymbol{V}_{N-1}\frac{1}{\sum_{j=1}^{N-1}\omega_j-i\epsilon}\boldsymbol{V}_N\otimes\mathcal{A}_N, \tag{20}$$

where $\mathcal{A}_N$ is the sum of all permutations of the positions of the mesons coupling to the lower line; see Eq. (4). The generalization of the eikonal formula in Eq. (11) gives a straightforward way to express $\mathcal{A}_N$. The notation in Eq. (20) is to be understood as follows. The vertex factors $\boldsymbol{V}_N$ represent the spin-isospin structure of the couplings. Since in this example we are considering A_1 exchange at leading order in N_c, it is simply represented by the $\boldsymbol{X}_{ia}$ of the contracted SU(4) symmetry. The tensor product is used to describe the spin-isospin structures which appear on each of two baryon lines. Note that $\mathcal{A}_N$ also contains the structures $\boldsymbol{V}_1\cdots\boldsymbol{V}_N$. There is an implicit contraction of spin and isospin indices when we write $\boldsymbol{V}_k\otimes\boldsymbol{V}_k$, so that, for this example,

$$\boldsymbol{V}_k\otimes\boldsymbol{V}_k\equiv\sum_{ia}\boldsymbol{X}_{ia}\otimes\boldsymbol{X}_{ia}.$$

This contraction comes about for obvious reasons: when an A_1 meson is exchanged the spatial i and isospin a components couple to both the upper and lower baryon lines.

The generalized eikonal formula allows us to evaluate $\mathcal{A}_N$ is a straightforward way. There is a large number of terms contained in $\mathcal{A}_N$. These can be organized by the number of commutators they contain. From the form of Eq. (11) one sees that each time a commutator is added there is one fewer δ function in energy. The term with no commutators has a total of N δ functions. Thus, these meson exchanges are static. However, it is straightforward to see that this term when combined with the meson propagators and the upper baryon line contains $N-1$ combinations of propagators which are divergent in the infrared and have the form of Eq. (15). They obviously correspond to N iterates of the one-meson-exchange potential. (This can be checked by adding back the recoil corrections and observing that Lippmann-Schwinger propagator emerges.) Thus, this term does not contribute to the potential. Next there are terms with a single commutator in $\mathcal{A}_N$. The term with the commutator $[\boldsymbol{V}_i,\boldsymbol{V}_j]$ (where i and j label the position of the meson vertex in the standard ordering, $i>j$) contains $N-1-(i-j)$ combinations of the form of Eq. (15). For $i-j<N-1$, these terms again are iterates of some lower meson-exchange potentials and do not contribute to the potential itself. However, for $i-j=N-1$, i.e., where $i=N$ and $j=1$, there are no such combinations of propagators and, thus, this does not correspond to an iterate and directly contributes to the potential. This term is unique and leads to a contribution of the form

$$\delta\mathcal{V}_N=(-1)^{N+1}g_{1m}^{2N}\int\prod_{j=1}^{N}\frac{d^3\boldsymbol{k}_j}{(2\pi)^3}(2\pi)^3\,\delta^{(3)}\left(\sum_{j=1}^{N}\boldsymbol{k}_j-\boldsymbol{q}\right)$$
$$\times\boldsymbol{V}_1\cdots\boldsymbol{V}_N\otimes[\boldsymbol{V}_N,\boldsymbol{V}_1]\boldsymbol{V}_2\cdots\boldsymbol{V}_{N-1}$$
$$\times\left(\prod_{j=2}^{N-1}D(-\boldsymbol{k}_j^2)\right)\mathcal{R}_N(\varepsilon_1,\varepsilon_N). \tag{21}$$

Note that apart from the $\boldsymbol{V}$ factors (which are multiplicative) the integrand is positive definite. Thus $\delta\mathcal{V}$ cannot vanish after integration. Recall that the commutator of two $\boldsymbol{X}$'s is of order $1/N_c^2$. Note also that there are contractions of the spin and isospin on the lower baryon line with the upper. Thus the commutator on the lower line induces a commutator on the upper line in a fashion similar to that seen in Ref. [15]. Thus one expects the commutators to give rise to an overall suppression factor of N_c^{-4}. The coupling constants g_{1m} scales as $\sqrt{N_c}$. Combining the suppression due to the commutator with the coupling constants produces $\delta\mathcal{V}_N\sim N_c^{N-4}$ in the potential. For $N>3$ this is incompatible with KSM scaling rules of Eq. (2).

One could continue in the application of the non-Abelian generalization of the eikonal formula in the evaluation of $\mathcal{A}_N$. All additional terms will have two or more commutators. Some of these terms will correspond to potential iterates, but some will be contributions to the potential. Recall, however, that commutators typically lead to suppression factors in the large-N_c expansion. For example, two single commutators will be suppressed from the single commutator by a factor of N_c^{-2} (with an additional factor N_c^{-2} induced on the upper line). Similarly a triple commutator is suppressed by a factor of N_c^{-2}. Such contributions are therefore subleading compared to the result in Eq. (21). However, the double commutator is not down by powers of N_c^{-1}. It is a simple exercise to see that such terms have a different dependence on the momentum transfer than the contribution in Eq. (21). Thus, such a contribution cannot generally cancel terms coming from a single commutator. Thus one concludes that the sum of ladder and crossed-ladder diagrams with N rungs contributes to the potential with the strength $\delta\mathcal{V}_N\sim N_c^{N-4}$.

IV. DISCUSSION

As discussed in the Introduction, the principal reason for undertaking the present investigation is to try to understand whether the traditional meson-exchange picture of nucleon-nucleon forces can be understood as arising from QCD. The central idea was that the general argument that a meson exchange picture ought to work was not based on the details of QCD other than confinement and hence ought to work for all N_c. Thus, the fact that the N_c counting of the nucleon-nucleon potential calculated at the quark-gluon level does not match the N_c counting based on a meson-exchange picture might be taken as a strong evidence against the latter. The conclusion that QCD is not compatible with a meson-exchange dynamics of nucleon-nucleon interactions at low momenta is quite radical. In the first place it goes against the conventional wisdom of nuclear physics. Moreover, it might be troubling from the perspective of large-N_c QCD. Heretofore in all known examples in the purely mesonic sector the large-N_c QCD counting matches what would be found in a purely hadronic theory with parameters scaling in a manner consistent with large-N_c QCD. In the baryon-number one sector problems discussed so far in the literature have a correspondence between hadronic- and quark-gluon-based descriptions. This was demonstrated so far for the tree-level Compton (multi-)meson-baryon scattering amplitudes [5–7,12] and chiral corrections to decay constants [7]. Note, however, that on the basis of the considerations advocated presently one expects a potential inconsistency in the meson-baryon Compton amplitude from the loop diagrams of the type 3(d) (obviously, with the bottom baryon line being removed). Before one accepts this radical conclusion, it is essential to explore other ways to resolve the puzzle of why the two descriptions have N_c-scaling behaviors which do not match. There are a number of possible explanations which are consistent with what we know about the system. However, all of them are unattractive in one way or another.

One general class of possibilities is that the way the hadronic-level calculation is organized is in some way defective and this hides cancellations which might bring consistency. Here we have seen that diagrams for generic exchanges of mesons of some fixed type do not cancel among themselves. It is certainly logically possible that they may cancel with some other class of graphs to preserve the scaling results of Eq. (2). One might hope that there is some way to reorganize the calculation so that the cancellations do occur. We see no simple way for this to come about. We do see two obvious scenarios where these cancellations could come about but have serious drawbacks as a resolution.

In the first scenario the cancellations would still occur for generic meson coupling constants with the large-N_c rules but would require a larger class of graphs. Since additional meson exchanges lead to increasingly superleading terms, it is conceivable that they can be resummed. Such a resummation could lead to a small result. If, for example, the series were essentially geometric — $N_c+N_c^2+N_c^3+\cdots$ — one could sum it to $1/(1-N_c)-1$ which goes to (minus) 1 in the large-N_c limit. Unfortunately, this scenario has a manifest drawback: we see no reason from the mathematical structures as to why we might expect this to happen.

A second scenario is that the cancellations will not occur for a generic hadronic theory with parameters consistent with general N_c-counting rules but depend rather on a conspiracy between the coupling constants, masses, and so on for the various mesons to yield cancellations. As a matter of principle, of course, this cannot be ruled out. Two-flavor isospin-symmetric QCD is a theory with essentially two free parameters Λ_{QCD} and m_q (defined with some scheme) and all hadronic parameters derived from QCD depend on these two in a very complicated way. Thus, there are very complex correlations between the hadronic parameters and one could imagine that these correlations conspire to enforce massive cancellations between diagrams with different topologies on the hadronic level. Since we do not know the structure of this theory, it is very difficult to conceive of how such cancellations would come about in practice and why they would hold for arbitrary momentum transfers. We note also that if this scenario were correct, it becomes hard to justify the use of the meson exchange in practice. In any practical implementation of a potential based on a meson exchange, the number of mesons included and the forms of the particular interactions are necessarily restricted. It is very implausible that the restricted form chosen would be capable of enforcing these nongeneric cancellations.

An alternative class of explanations focuses on the validity of KSM rules. Equation (2) is supposed to apply in the kinematic regime $\boldsymbol{p}\sim N_c^0$. However, it is conceivable that this regime is simply not suitable for a large-N_c expansion. It was long ago noted by Witten that scattering observables cannot have a smooth large-N_c limit in this kinematic regime [2]; thus, Refs. [13,14] focus on the potential rather than the amplitude. However, there has never been a systematic demonstration that the potential in this regime has a smooth limit. Of course, the derivation of Eq. (2) is very plausible. It is based on the Hartree picture which in turn implies that only quark-line-connected graphs contribute. It is worth noting, however, that despite its plausibility, the derivation may be flawed. Witten justified the Hartree picture for N_c quarks in their ground state where it can be shown that non-Hartree-type correlations are suppressed in the $1/N_c$ expansion. On the other hand, the potential only has meaning as an ingredient in a Schrödinger (or Lippmann-Schwinger) equation. The Schrödinger wave function implies strong correlations between the nucleons which at the quark level does not correspond to a Hartree-type wave function. Thus, the question arises of whether the Hartree- and quark-line-connected approximation can be justified. Indeed at a philosophical level one might ask whether the potential, which, after all, is not experimentally accessible in any direct way, can even be assigned an N_c scaling. At a more practical level the problem is that quark-line-disconnected pieces certainly contribute to amplitudes but by hypothesis do not contribute to the potential. Thus, for the approach to be consistent, these contribution must be associated with iterates of the potential. However, there is no general argument of which we aware of which demonstrates that the quark-line-disconnected contributions are in fact described by iterating the potential.

Explanations of the mismatch in N_c counting based on the possibility that Eq. (2) is invalid face a major hurdle. The problem is that most likely that Eq. (2) could fail is that its derivation assumes that the quark-line-disconnected parts are associated with potential iterates and that this assumption could be wrong. However, this mismatch between the N_c dependence at the quark-gluon level from the hadronic one occurs even at the amplitude level where the question of determining which contribution is an iterate does not arise. Note that although the amplitude for the three-quark-pair exchange is of order N_c^3, this contribution comes from purely quark-line-disconnected contributions and for the reason discussed above is necessarily associated with the static quark-pair exchanges. The leading contribution arising from two pairs being nonstatic and one being static is of order N_c [see

Fig. 2(c) and add a gluon connecting exchanged quarks in the top baryon line in the subgraph (b) of (c)]. However, at the hadronic level the term with a static and two nonstatic mesons, as discussed in Sec. III A, contributed to the total amplitude at order N_c^2. Thus, even at the amplitude level the contribution from the exchange of one static and two nonstatic mesons does not match the contribution from the exchange of two nonstatic quark pairs and a static pair. This mismatch cannot be ascribed to the distinction in how things are apportioned between the potential and its iterates.

In summary, the mismatch between the N_c counting of contributions to the potential between quark- and hadron-based descriptions remains puzzling. It seems likely to us that an understanding of the roles played by static and nonstatic exchanges of mesons in the hadronic picture and quark-antiquark pairs in the quark-based formalism is essential to resolve this puzzle definitively. The resolution of the problem is of real importance as it provides insights into the relationship of nuclear phenomenology to QCD.

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