

**Chiral  $NN$  model and  $A_y$  puzzle**

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We analyze the results by chiral  $NN$  models for the two-nucleon system and calculate the predictions for the nucleon vector analyzing power of elastic nucleon-deuteron ( $Nd$ ) scattering,  $A_y$ , by these models. Our conclusion is that a *quantitative* chiral two-nucleon potential does not resolve the  $Nd A_y$  puzzle (when only two-body forces are included).

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The term  $A_y$  puzzle refers to the inability to explain the nucleon vector analyzing power  $A_y$  in elastic nucleon-deuteron ( $Nd$ ) scattering below 30-MeV laboratory energy for the incident nucleon by means of three-body calculations in which only two-nucleon forces are applied. The problem showed up as soon as it was possible to conduct three-body continuum calculations *with realistic  $NN$  potentials*. The first such calculation was performed by Stolk and Tjon [1] in 1978 using the Reid soft-core potential [2], and the first calculations with (a separable representation of) the Paris potential [3] were conducted by the Graz-Osaka group in 1987 [4]; in both cases, the  $A_y$  predictions showed the characteristic problem. Finally, the “puzzle” became proverbial when rigorous three-nucleon continuum Faddeev calculations using realistic forces were started on a large scale [5]. Over the years, many measurements and calculations of  $Nd A_y$  were performed (including the  $pd$  reaction that involves the Coulomb force [6]) which all confirmed that the problem was real (see Ref. [7] for a review): For energies below 20 MeV,  $A_y$  is predicted about 30% too small in the angular region around a  $120^\circ$  center-of-mass angle where the maximum occurs.

There have been many attempts to solve the problem. Already in the very early stages of three-body continuum calculations, when only schematic  $NN$  potentials were applied, it was noted that the  $Nd A_y$  predictions depend very sensitively on the strength of the input  $NN$  potential in the triplet  $P$  waves [8,9]—a sensitivity that was confirmed in later calculations using realistic forces [10]. Based upon this experience, Witała and Glöckle [11] showed in 1991 that small changes in those  $^3P$ -wave potentials could remove the discrepancy. This finding gave rise to systematic investigations of the question of whether the small variations of the low-energy phase shifts of, particularly, those triplet  $P$  waves necessary to explain the  $Nd A_y$ , are consistent with the  $NN$  data base. While Tornow and co-workers [12] suggested that the low-energy  $NN$  data may leave some latitude in the  $NN$

$^3P$  waves that could improve the predictions for  $Nd A_y$ , Hüber and Friar [13] found that it is not possible with reasonable changes in the  $NN$  potential to increase the  $Nd A_y$  and at the same time to keep the two-body observables unchanged.

Another important observation has been that conventional three-nucleon forces (when added to a realistic two-nucleon potential) change the predictions for  $Nd A_y$  only slightly and do not improve them [14,6]. Therefore, the general perception in the community has shifted toward the belief that the  $A_y$  puzzle is the “smoking gun” for new types of three-nucleon forces [15–18] or new physics [19].

However, very recently, there has been an apparent indication that the above conclusion and belief may be premature. It was reported [20] that with a two-nucleon force of another type, namely, one that is based upon chiral effective field theory, the  $A_y$  puzzle is resolved.

In recent years, effective field theory methods have become increasingly popular in nuclear physics. The reason for this development is the need to link conventional nuclear physics methods one way or the other to the underlying theory of strong interactions: QCD. After quark cluster models had only a limited success, it was recognized that the symmetries of QCD are more important than the high-energy degrees of freedom of QCD (quarks and gluons). The effective field theory concept distinguishes between different energy scales and assigns appropriate degrees of freedom for each scale while observing the overall symmetries. For traditional nuclear physics with energies below 1 GeV, the right degrees of freedom are nucleons and pions interacting via a force that is controlled by (broken) chiral symmetry.

The derivation of the nuclear force from chiral effective field theory was initiated by Weinberg [21] and pioneered by Ordóñez and van Kolck [22] and van Kolck and co-workers [23,24]. Subsequently, many researchers became interested in the subject [25–36]. As a result, efficient methods for deriving the nuclear force from chiral Lagrangians emerged and the quantitative nature of the chiral  $NN$  potential improved. This trend shows up, in particular, in the excellent work by Epelbaum *et al.* [30] where the chiral  $NN$  force was constructed using a unitary transformation and applying systematic power counting in next-to-leading order (NLO) and next-to-NLO; and it is this potential in NLO that was applied

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TABLE I.  $^1S_0$   $np$  and  $^3S_1$  phase shifts (in degrees).

$T_{\text{lab}}$ (MeV)	PWA93 <sup>a</sup>	NLO <sup>b</sup>	Idaho <sup>c</sup>	CD-Bonn <sup>d</sup>
$^1S_0$				
1	62.068(30)	62.05	62.03	62.09
5	63.63(8)	63.85	63.52	63.67
10	59.96(11)	60.26	59.80	60.01
25	50.90(19)	50.92	50.66	50.93
50	40.54(28)	39.29	40.27	40.45
100	26.78(38)	21.68	26.87	26.38
150	16.94(41)	7.26	17.56	16.32
200	8.94(39)	-5.53	9.58	8.31
$^3S_1$				
1	147.747(10)	147.70	147.76	147.75
5	118.178(21)	118.29	118.20	118.18
10	102.611(35)	102.84	102.64	102.62
25	80.63(7)	80.69	80.67	80.63
50	62.77(10)	61.74	62.80	62.73
100	43.23(14)	38.65	43.13	43.06
150	30.72(14)	21.67	30.41	30.47
200	21.22(15)	7.20	20.90	20.95

<sup>a</sup>Nijmegen multienergy  $np$  analysis [37]. Numbers in parentheses give the uncertainties in the last digits.

<sup>b</sup>Chiral next-to-leading order (NLO) potential by Epelbaum *et al.* [30] using a Gaussian cutoff with cutoff mass  $\Lambda=600$  MeV.

<sup>c</sup>Chiral  $NN$  potential by Entem and Machleidt [40] (“Idaho-B” is used).

<sup>d</sup>Reference [39].

in Ref. [20], resulting—seemingly—in a resolution of the long-standing  $A_y$  puzzle. It is the purpose of this note to critically investigate the predictions by the chiral  $NN$  model and the implications for the  $Nd$   $A_y$  puzzle.

We start our investigation by taking a close look at important phase shifts of two-nucleon scattering. In Table I, we list  $S$ -wave phase shifts and, in Table II, we show  $^3P$ -wave phase shifts for energies between 1 and 200 MeV. Since the charge dependence of the  $NN$  interaction is not a crucial factor in the  $A_y$  puzzle [12], and since the present chiral  $NN$  potentials are all charge independent and adjusted to the neutron-proton ( $np$ ) data, we consider  $np$  phase shifts and  $np$  data.

It is of interest to compare the phase shifts produced by the chiral NLO model by Epelbaum *et al.* [30] (which is the chiral potential applied in Ref. [20]) with the empirical ones from the Nijmegen multienergy  $np$  analysis [37] (PWA93) and the predictions by one representative of the family of the high-precision potentials constructed in the 1990s (CD-Bonn [38,39]). In  $S$  waves (Table I), there is, generally, good agreement up to 50 MeV. Above 50 MeV, differences between NLO and PWA93 show up and increase with energy. However, since the  $S$  waves are not very important for  $Nd$   $A_y$ , this may not have much impact on the predictions.

We turn now to the triplet- $P$  waves (Table II) which are crucial for  $Nd$   $A_y$ , and focus, first, on the energy range below 30 MeV. The NLO  $^3P_0$  phase shifts are about 2% larger than the PWA93 shifts, which is not significant. For  $^3P_1$ , the

TABLE II. Triplet- $P$   $np$  phase shifts (in degrees). For notation, see Table I.

$T_{\text{lab}}$ (MeV)	PWA93 <sup>a</sup>	NLO	Idaho	Modified <sup>b</sup>	CD-Bonn
$^3P_0$					
1	0.18	0.19	0.18	0.18	0.18
5	1.63(1)	1.67	1.64	1.61	1.61
10	3.65(2)	3.72	3.69	3.62	3.62
25	8.13(5)	8.22	8.19	7.98	8.10
50	10.70(9)	10.84	10.63	10.28	10.74
100	8.46(11)	8.31	8.17	7.91	8.57
150	3.69(14)	2.52	3.60	3.66	3.72
200	-1.44(17)	-4.10	-1.21	-0.93	-1.55
$^3P_1$					
1	-0.11	-0.12	-0.11	-0.11	-0.11
5	-0.94	-0.99	-0.93	-0.94	-0.93
10	-2.06	-2.16	-2.04	-2.09	-2.04
25	-4.88(1)	-5.03	-4.82	-4.98	-4.81
50	-8.25(2)	-8.32	-8.22	-8.57	-8.18
100	-13.24(3)	-12.66	-13.36	-13.86	-13.23
150	-17.46(5)	-15.94	-17.59	-17.85	-17.51
200	-21.30(7)	-18.86	-21.28	-21.18	-21.38
$^3P_2$					
1	0.02	0.02	0.02	0.02	0.02
5	0.25	0.24	0.26	0.28	0.26
10	0.71	0.70	0.72	0.78	0.72
25	2.56(1)	2.89	2.58	2.78	2.60
50	5.89(2)	8.29	5.86	6.29	5.93
100	10.94(3)	22.61	10.77	11.24	11.01
150	13.84(4)	35.98	13.72	13.82	13.98
200	15.46(5)	44.31	15.58	15.33	15.66

<sup>a</sup>Uncertainties smaller than  $0.005^\circ$  are not shown.

<sup>b</sup>Modified version of the Idaho potential with enhanced spin-orbit force at low energies.

differences are more drastic: the NLO value at 10 MeV is about 5% smaller and the one at 25 MeV is 3% smaller than the PWA93 value. Finally, the NLO prediction for  $^3P_2$  at 25 MeV is enhanced by 13%.

To understand how variations of the  $^3P$  phase shifts may effect observables, we consider the spin-orbit phase-shift combination

$$\Delta_{LS} = \frac{1}{12}(-2\delta_{3P_0} - 3\delta_{3P_1} + 5\delta_{3P_2}), \quad (1)$$

which is a measure for the strength of the spin-orbit force. Results are shown in Table III. At 10 MeV, one obtains  $\Delta_{LS}^{\text{PWA93}} = 0.203^\circ$  and  $\Delta_{LS}^{\text{NLO}} = 0.212^\circ$  for PWA93 and NLO, respectively, implying that the NLO value is larger by 4.4% as compared to the PWA93 value. At 25 MeV, the corresponding figures are  $\Delta_{LS}^{\text{PWA93}} = 0.93^\circ$  and  $\Delta_{LS}^{\text{NLO}} = 1.09^\circ$ , implying that the NLO model is 17% larger. These numbers show that the spin-orbit force of the NLO model is enhanced as compared to the Nijmegen analysis, and similar enhancements are obtained when comparing to any of the high-precision potentials, like the CD-Bonn (cf. Table III) poten-

TABLE III. Spin-orbit phase shift combination  $\Delta_{LS}$  (in degrees), [Eq. (1)] at 10, 25, and 50 MeV for various models explained in Tables I and II.

$T_{\text{lab}}$ (MeV)	PWA93	NLO	Idaho	Modified	CD-Bonn
10	0.203	0.212	0.195	0.244	0.207
25	0.93	1.09	0.92	1.07	0.94
50	2.73	3.73	2.73	3.05	2.73

tial. It is well known that the two-nucleon spin-orbit force is magnified in the  $Nd$  system. Therefore, a moderate enhancement of the  $NN$   $LS$  potential leads to a substantial enlargement of the  $Nd$   $A_y$ . Thus the enhanced NLO spin-orbit force as reflected in the larger values for  $\Delta_{LS}^{\text{NLO}}$  could very well be the explanation of the large  $Nd$   $A_y$  predictions by NLO reported in Ref. [20].

Next, we consider the  ${}^3P$  phase shifts above 30 MeV. Here the differences between PWA93 and CD-Bonn, on the one hand, and the NLO model, on the other, are in general larger and increase with energy (cf. Table II). This trend is most dramatic in  ${}^3P_2$  where the discrepancies quickly grow into hundreds of standard deviations. The sign of these differences are such as to drastically enhance the spin-orbit force, see Table III for the value at 50 MeV. Note that the  $NN$   $t$ -matrix, on- and off-shell, is input to the three-body continuum calculations. Thus the description of the two-nucleon data at energies above 30 MeV has an impact on low-energy three-body predictions. Therefore, the unrealistically strong NLO spin-orbit force above 30 MeV may—by means of an off-shell effect—further enhance the  $Nd$   $A_y$  predictions. However, one may not have much confidence in this type of off-shell effect.

The above observations trigger the question of whether chiral models can also make more accurate predictions for  $NN$  phase shifts; and if so, what the implications for the  $Nd$   $A_y$  problem, are in such a case. The natural way is to include higher-order terms in power counting which should improve not only the quality of the  $NN$  phase shift reproduction but also extend the energy range in which it works. For that purpose, we pick up the chiral  $NN$  potential of Ref. [40] (subsequently denoted by “Idaho”) that was recently developed. In the chiral Idaho model [40], contact terms up to order 4 are included which introduces more parameters allowing for a better fit of the lower partial waves in a much wider energy range. In Tables I and II, it is clearly seen that the chiral Idaho  $NN$  potential reproduces the empirical  $np$  phase shifts of the PWA93 analysis up to 200 MeV, accurately.

We now consider the observable  $Nd$   $A_y$ , which is the focus of this paper. We have calculated the predictions for  $Nd$   $A_y$  at energies 3, 10, and 65 MeV for the incident nucleon. The results are shown in Fig. 1. In this figure, the shaded band represents the prediction by the family of high-precision potentials (using always the  $np$  version of those models), namely, CD-Bonn [39], Argonne  $V_{18}$  [42], and the Nijmegen potentials Nijm-I, Nijm-II, and Nijm93 [41]. The dashed line is the prediction by the Idaho chiral  $NN$  potential [40] and it is clearly seen that this prediction follows accurately the nar-

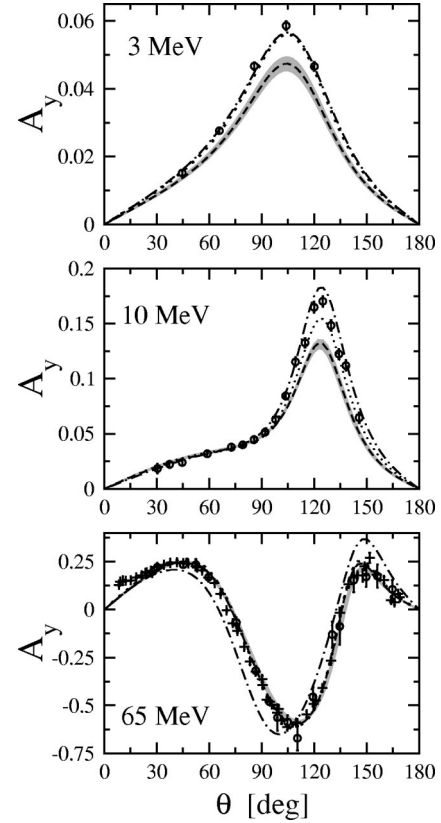


FIG. 1. Nucleon analyzing power  $A_y$  for elastic  $Nd$  scattering at 3, 10, and 65 MeV. The shaded band represents the predictions by the high-precision potentials CD-Bonn [39], Argonne  $V_{18}$  [42], and the Nijmegen potentials Nijm-I, Nijm-II, and Nijm93 [41] (always using the  $np$  versions of these potentials). The dashed line is the prediction by the Idaho chiral  $NN$  potential [40]. The dotted curve represents the result from the “modified” chiral potential (see the text) and the dash-dotted line is predicted by the NLO chiral potential by Epelbaum *et al.* [30]. Data at 3 MeV are from Ref. [45] ( $nd$ , squares), those at 10 MeV from Ref. [47] ( $nd$ , squares), and those at 65 MeV from Refs. [49] ( $nd$ , squares) and [50] ( $pd$ , crosses).

row band made up from the variations among those high-precision potentials. In conclusion, at 3 and 10 MeV, we have an  $A_y$  problem if the chiral  $NN$  potential is a *quantitative* one.

We note that, very recently, the Bochum/Jülich group has applied their next-to-next-to-leading order (NNLO) two-nucleon potential to the  $nd$   $A_y$  problem [43]. The result is very similar to ours (with the NNLO Idaho potential), and in close agreement with the predictions from conventional  $NN$  potentials. Or, in other words, at NNLO (where the reproduction of the two-nucleon data is better than at NLO) the  $A_y$  problem is back.

The evidence presented may be perceived as a convincing proof that a *quantitative* chiral potential does not resolve the  $Nd$   $A_y$  puzzle. However, there remains one objection that can be raised. In the literature, notably in Ref. [12], one can find the suggestion that the  ${}^3P$  waves at low energy are not as well determined as claimed in the PWA93 analysis [37]. If true, then moderate variations of the  ${}^3P$  phase shifts at low energy could be consistent with the low-energy  $NN$  data. This variations could be such as to enhance the low-energy

TABLE IV.  $\chi^2/\text{datum}$  for the reproduction of the 1999  $np$  database [44] up to  $T_{\text{lab}}=210$  MeV by various models explained in Tables I and II.

Bin (MeV)	No. of data	PWA93	NLO	Idaho	Modified	CD-Bonn
0-8	81	1.05	1.05	1.03	1.32	1.06
8-17	192	0.86	1.22	0.85	1.87	0.91
17-35	292	0.82	2.15	0.81	1.27	0.81
35-75	340	1.03	10.8	1.09	1.24	1.01
75-125	239	1.00	11.6	0.99	1.06	0.97
125-183	414	1.06	95.7	1.07	1.15	1.03
183-210	141	0.96	111.7	0.97	1.18	1.01
0-210	1699	0.97	37.0	0.98	1.27	0.97

spin-orbit force and, thus, lead to an improved prediction for  $Nd A_y$ . For the purpose of seriously checking out this possibility, we have constructed a variation of the Idaho chiral potential with modified  $^3P$  phase shifts at low energy. The column “Modified” of Table II shows the  $^3P$  phase shifts, and the corresponding column of Table III reveals that for this fit the spin-orbit force is enhanced, similarly to the NLO model. However, in contrast to the NLO model, the modified model is much more realistic, since the phase shifts do not diverge to unrealistic values at higher energies.

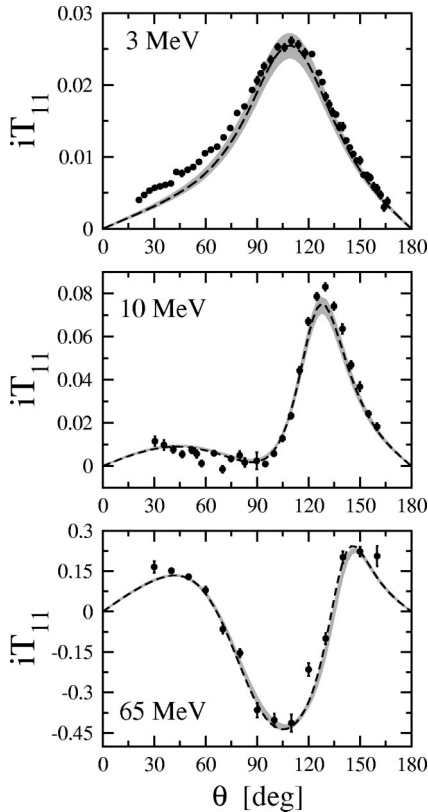


FIG. 2. Deuteron vector analyzing power  $iT_{11}$  for elastic  $Nd$  scattering. Energies, bands, and curves as in Fig. 1. The circles are  $pd$  data at 3 MeV from Ref. [46], at 10 MeV from Ref. [48], and at 65 MeV from Ref. [52].

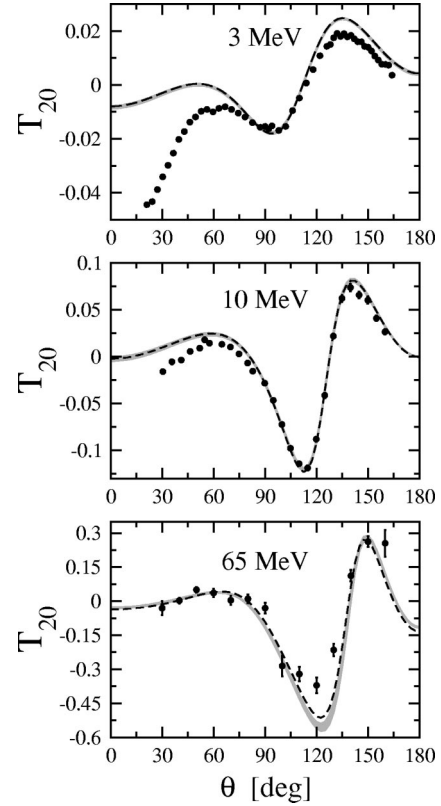


FIG. 3. Tensor analyzing power  $T_{20}$  for elastic  $Nd$  scattering. Energies, bands, and curves as in Fig. 1. The circles are  $pd$  data at 3 MeV from Ref. [46], at 10 MeV from Ref. [48], and at 65 MeV from Ref. [52].

We have calculated the  $Nd A_y$  as predicted by the modified chiral model and find, indeed, a considerable improvement (see dotted curve in Fig. 1). Is this the resolution of the  $A_y$  puzzle by a chiral  $NN$  potential? To answer this question one needs to know if the modified chiral model is a realistic and quantitative  $NN$  potential. A precise and reliable answer cannot be given by just looking at phase shifts. As stressed repeatedly by the Nijmegen group in the past, only a direct confrontation with the  $NN$  data can reveal if an  $NN$  potential is quantitative or not. For that reason we have calculated the  $\chi^2/\text{datum}$  for the fit of the world  $np$  data as represented by the 1999 database [44]; see Table IV. For a proper interpretation of the results of Table IV, it is necessary to establish a standard concerning what  $\chi^2/\text{datum}$  represents a “quantitative” reproduction of the data. This issue was debated a great deal in the 1990s, and the consensus that emerged was that only values below 1.1 are acceptable. Deliberately, we lose this standard and consider a fit with  $\chi^2/\text{datum} \leq 1.2$  as quantitative, while we will perceive higher values as not acceptable.

Applying this standard, the modified model produces unacceptable values for  $\chi^2/\text{datum}$  for all energy intervals below 75 MeV (cf. Table IV). Thus the modified chiral model is *not* a quantitative one and, consequently, it is *not* the resolution of the  $Nd A_y$  puzzle. Since it is well known that the off-shell character of the  $NN$  potential plays essentially no role in three-nucleon scattering, one can further draw the more gen-

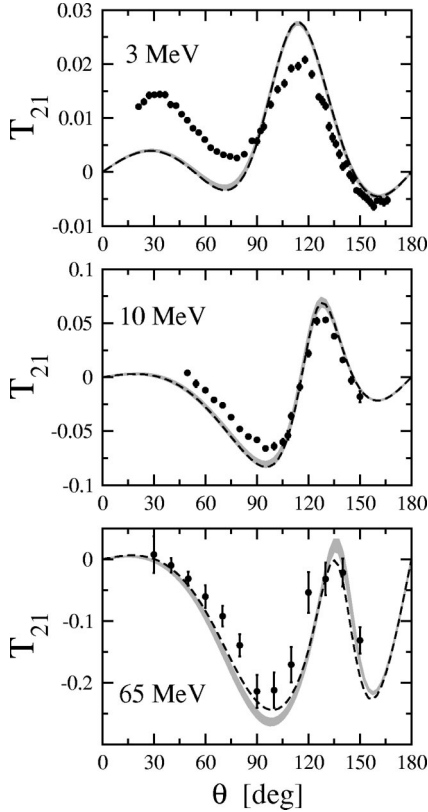


FIG. 4. Tensor analyzing power  $T_{21}$  for elastic  $Nd$  scattering. Energies, bands, and curves as in Fig. 1. The circles are  $pd$  data: at 3 MeV from Ref. [46], at 10 MeV from Ref. [48], and at 65 MeV from Ref. [52].

eral conclusion: No model that reproduces the  $NN$  data correctly can solve that  $Nd$   $A_y$  puzzle.

Table IV shows also the  $\chi^2/\text{datum}$  of the other models discussed in this paper. It is seen that PWA93, Idaho, and CD-Bonn models reproduce the  $np$  data below 210 MeV with the perfect  $\chi^2/\text{datum}=0.97$  and  $0.98$ . The chiral NLO potential by Epelbaum *et al.* [30] produces  $\chi^2/\text{datum}=37$  which is grossly unacceptable. In fact, only for the interval 0-8 MeV is NLO acceptable. This range of validity is so tiny that no serious implications can be drawn from any prediction by this potential.

Finally, we also like to take this opportunity to present an overview of other interesting  $Nd$  observables, which are shown in Figs. 2–8. Concerning the deuteron vector analyzing power,  $iT_{11}$ , at 3 and 10 MeV (Fig. 2), it should be noted that a seemingly drastic reduction of the discrepancy between theory and  $pd$  data seen at 3 MeV has its origin in large effects of the long-range Coulomb force acting between two protons, which is not taken into account in our calculations [6]. Taking this Coulomb force into account,  $iT_{11}$  is also underpredicted in the peak region [6]—an equally well-known problem, which is why it would be appropriate to speak more generally of the vector analyzing power puzzle in elastic  $Nd$  scattering. In almost all cases, the Idaho chiral  $NN$  potential follows the trend of the high-precision potentials. The only exceptions are  $T_{20}$  and  $T_{21}$  at 65 MeV, where the chiral potential predictions describe slightly better the

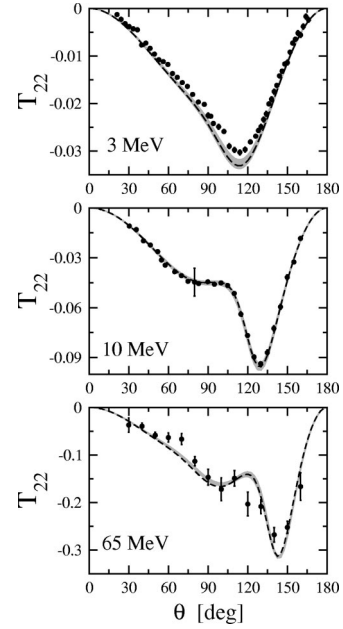


FIG. 5. Tensor analyzing power  $T_{22}$  for elastic  $Nd$  scattering. Energies, bands, and curves as in Fig. 1. The circles are  $pd$  data at 3 MeV from Ref. [46], at 10 MeV from Ref. [48], and at 65 MeV from Ref. [52].

data in the minimum region as compared to conventional potentials. But, apart from this, the quantitative chiral  $NN$  model containing contributions of higher orders in power counting does not produce any new signatures.

In summary, our main conclusions are that a *quantitative* chiral two-nucleon potential does not resolve the  $Nd$   $A_y$  puzzle on the two-body force level, and low-energy  ${}^3P_J$   $NN$

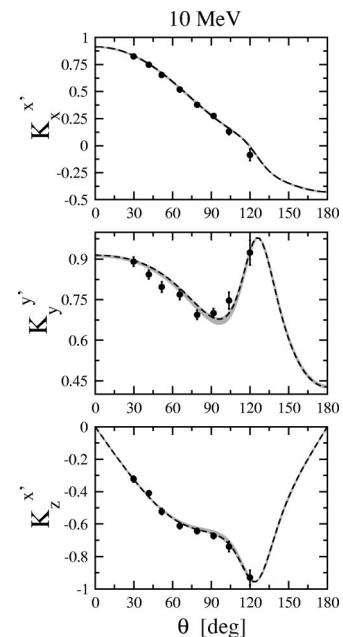


FIG. 6. Spin transfer coefficients  $K_x^{x'}$ ,  $K_y^{y'}$ , and  $K_z^{z'}$  for elastic  $Nd$  scattering at 10 MeV. Bands and curves as in Fig. 1. The circles are  $pd$  data from Ref. [48].

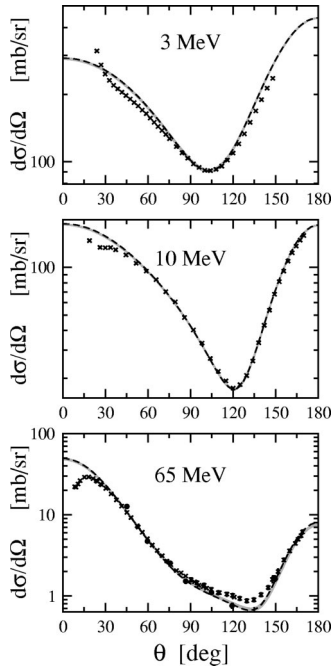


FIG. 7. Differential cross section  $d\sigma/d\Omega$  of elastic  $Nd$  scattering. Energies, bands, and curves as in Fig. 1. The crosses are  $pd$  data at 3 MeV from Ref. [51], at 10 MeV from Ref. [48], and at 65 MeV from Ref. [50]. The circles at 65 MeV are  $nd$  data from Ref. [49].

phase shifts that “solve” the  $Nd A_y$  puzzle are inconsistent with the low-energy  $NN$  data. Finally, as a consequence of the above two points, one may expect that *no quantitative two-nucleon force—no matter what the basis is, pure phenomenology, meson theory, chiral EFT, or anything—will ever solve the  $Nd A_y$  puzzle.*

An accurate  $NN$  model requires to take chiral perturbation theory ( $\chi$ PT) to order 4 in small momenta. At this order, also many three-body forces (3NF) occur. According to the basic rules of  $\chi$ PT, all two- and many-body terms must be included for a complete calculation. Conventional 3NF terms were shown to be ineffective for the  $A_y$  problem [14,6]. The advantage of  $\chi$ PT is that it provides a systematic scheme to generate all terms at a given order. As it turns out, there are several such chiral 3NF terms that were never considered in few-nucleon physics before [15–18], to our knowledge. It is natural to expect the resolution of the  $Nd A_y$  puzzle from such chiral 3NF terms which, therefore, should be at the focus of future work in the field.

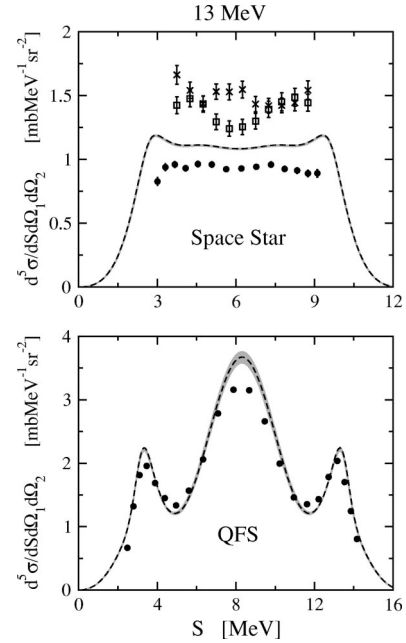


FIG. 8. Neutron-deuteron breakup cross sections for the space-star and quasifree-scattering (QFS) configurations at 13 MeV along the kinematical locus  $S$ . Bands and curves as in Fig. 1. The circles are  $pd$  data from Ref. [53]. The crosses and squares are  $nd$  data from Refs. [54] and [55], respectively.

At next-to-leading order in  $\chi$ PT, there are no 3NF contributions. So, a calculation with a NLO two-nucleon potential and no 3NF term seems to have a formal validity. However, since at NLO the  $NN$  data can only be reproduced for  $T_{\text{lab}} \leq 8$  MeV, such a calculation is doomed to be inconclusive, from the outset, and higher-order terms must be taken into account for any meaningful calculation.

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