Imaginary parts and infrared divergences of two-loop vector boson self-energies in thermal QCD

A. Majumder and C. Gale
Physics Department, McGill University, 3600 University Street, Montréal, Quebec, Canada H3A 2T8
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We calculate the imaginary part of the retarded two-loop self-energy of a static vector boson in a plasma of quarks and gluons at a temperature T, using the imaginary time formalism. We recombine the various cuts of the self-energy to generate physical processes. We demonstrate how cuts containing loops may be reinterpreted in terms of interference between the $O(\alpha)$ tree diagrams and the Born term along with spectators from the medium. We apply our results to the rate of dilepton production in the limit of dilepton invariant mass $E \gg T$. We find that all infrared and collinear singularities cancel in the final result obtained in this limit.

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I. INTRODUCTION

The imaginary parts of retarded self-energies represent extremely important quantities in thermal field theory. They provide information about various quantities of physical interest in the medium. Primary among these are the decay and formation rates of particles [1]. Boson self-energies provide information about quantities like Z decay rates [2] and production rates of dileptons and real photons [3] from a quark-gluon plasma (QGP). The spectrum of lepton pairs (i.e., e^+e^-, $\mu^+\mu^-$) and real photons emanating from such a plasma has been considered as a promising signature of QGP formation [4,5]. This owes to the fact that the photons or dileptons suffer essentially no final state interaction.

Some years ago the contribution to the rate of dileptons produced at rest in the plasma at first order in the strong coupling constant was evaluated [6]. This included reactions like three particle fusion ($q\bar{q}g \to \gamma^*$), Compton scattering ($qg \to q\gamma^*$ or $\bar{q}g \to \bar{q}\gamma^*$), pair annihilation ($q\bar{q} \to g\gamma^*$), Born term with vertex correction, and Born term with quark or antiquark self-energy correction. This calculation was performed in the real-time formalism, both in a Feynman diagram approach in thermofield dynamics, and by taking the imaginary part of the two-loop photon self-energy. In the case of massless QCD, each of the contributions mentioned above contain infrared or collinear singularities. These were regulated at intermediate stages of the calculation by giving masses to the quarks and gluons. The combined rate from all these processes was then found to be free of all divergences in the limit of vanishing masses. This calculation was also performed simultaneously by another group [7], who dimensionally regularized the singularities at intermediate stages of the calculation. The end result remained the same: when all the different processes were summed, the divergences canceled and dilepton rate at next-to-leading order remained finite.

Recent calculations employing a multiple scattering expansion, however, have found a remnant collinear divergence [2,8]. This result has been commented upon [9], and the issue of divergences remained unresolved [10]. In the wake of this strife, we revisit this problem in a systematic calculation. Also, to the best of our knowledge, a complete calculation of the imaginary part of a heavy vector boson retarded self-energy in the imaginary time formalism has yet to be performed. This is the subject of this paper. The scalar boson self-energy was examined recently [11]. There are various advantages to such a calculation: the basic Feynman rules are easily generalized from zero temperature, there is no doubling of degrees of freedom and no matrix structure of propagators, multiple poles that lead to ill-defined products of delta functions in the real-time formalism are easily and naturally handled both in the Matsubara sums and in the analytic continuation. The purpose of this calculation is thus manifold. The first goal is to enumerate and interpret the various physical contributions contained in the imaginary part of the two-loop self-energies. In doing this, it shall then be shown that cuts containing loops may be reexpressed as interference between tree diagrams and the Born term with a thermal medium spectator. Importantly, we also demonstrate how double poles may be simply and elegantly dealt with, in the Matsubara sum and in the analytic continuation to real energies. We finally concentrate on the eventual collinear and infrared divergences in the ensuing rates. In this study, we focus on the singularity structure in the region of phase space investigated by the authors of Refs. [2,8]. Even though we explicitly calculate the self-energies of static virtual photons, the results may be easily applied to other vector bosons in-medium, with the exception of the gluon, which admits other self-energies in a QGP.

The various sections are organized as follows. In Sec. II we begin by evaluating one of the self-energy diagrams of a static photon with an imaginary energy at two loops (the impatient reader may skip ahead to Sec. VI where the various cuts of the self-energy are recombined to provide physical interpretations of the various terms obtained, following which the infrared behavior of heavy photon production will be discussed). In Sec. III we analytically continue the photon energy to real values and obtain the imaginary part of the corresponding retarded self-energy. In Sec. IV we evaluate the other self-energy topology. In Sec. V we analytically continue this self-energy to real values of photon energy and find the retarded imaginary self-energy. In Sec. VI we combine the treelike cuts and reinterpret them as physical processes with thermal distributions on the phase space factors. In Sec. VII we attempt to interpret cuts containing loops in terms of the recently proposed spectator interpretation [12]. In Sec. VIII we take the limit of heavy photon production ($E \gg T$)

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and evaluate the various contributions. In Sec. IX we combine all cuts, demonstrate the cancellation of the collinear and infrared divergences, and present our results. We present our conclusions and brief discussions in Sec. X. Two appendixes follow. In the interest of quantitative accuracy and repeatability, we have presented many calculational details: the issue being addressed here is technical and thus demands a rigorous treatment.

II. THE SELF-ENERGY: TOPOLOGY I

We evaluate the photon self-energy with a gluon running across as shown in Fig. 1. To begin with, we derive the expression for the effective quark photon vertex corrected by a gluon running across, i.e.,

$$ie\Gamma^{\mu}=ie\gamma^{\mu}+ie\,\delta\Gamma^{\mu},$$

where e may be taken to be the electric charge of the quark. In standard notation, the expression for the effective vertex in Feynman gauge may be written down as

$$ie\Gamma^{\mu}=\frac{i}{\beta}\sum_{q^0}\int\frac{d^3q}{(2\pi)^3}\frac{-ig_{\rho\sigma}\delta^{ab}}{q^2}(it^a_{i,j}g\gamma^{\rho})\times\frac{i(\not k-\not q-\not p)}{(k-q-p)^2}(ie\gamma^{\mu})\frac{i(\not k-\not q)}{(k-q)^2}(it^b_{j,k}g\gamma^{\sigma}). \quad (2.1)$$

The Matsubara sum in the effective vertex may be simply evaluated using the method of Pisarski [13]. In our notation (see Appendix A, see also [14]), this is given in the static limit $(\vec{p}=0)$ as

$$\delta\Gamma^{\mu}=\frac{g^2C_{ik}}{4}\int\frac{d^3q}{(2\pi)^3}\sum_{s_1,s_2,s_3}\frac{(\not k-\not q)_{s_2}\gamma^{\mu}(\not k-\not q)_{s_3}}{qE_{q-k}E_{q-k}(p^0-[s_2-s_3]E_{q-k})}\times\left\{-s_3\frac{(s_1+s_2)/2-s_1\tilde{n}(E_{q-k})+s_2n(q)}{k^0-s_1q-s_2E_{q-k}}+s_2\frac{(s_1+s_3)/2-s_1\tilde{n}(E_{q-k})+s_3n(q)}{k^0-p^0-s_1q-s_3E_{q-k}}\right\}, \quad (2.2)$$

where s_1,s_2,s_3 are sign factors, which are summed over the values of ±1. We may now use the above result to write the full self-energy of the photon in the static limit as

$$i\Pi^{\mu}_{\mu}=\frac{i}{\beta}\sum_{k^0}\int\frac{d^3k}{(2\pi)^3}(-1)\times\mathrm{Tr}\sum_{ss_4}\left[e\gamma_{\mu}\delta_{ki}\frac{\gamma^{\beta}s_4\hat{k}_{\beta,s_4}}{2(k^0-p^0-s_4k)}e\,\delta\Gamma^{\mu}_{i,k}\frac{\gamma^{\alpha}s\hat{k}_{\alpha,s}}{2(k^0-sk)}\right], \quad (2.3)$$

where $\hat{k}_s$ stands for the four component quantity

$$\left\{s,\frac{k_x}{k},\frac{k_y}{k},\frac{k_z}{k}\right\}=\{s,0,0,1\}.$$

We note that the effective vertex may be written as

$$\delta\Gamma^{\mu}=\gamma^{\rho}\gamma^{\mu}\gamma^{\sigma}\delta\Gamma_{\rho\sigma}$$

to highlight the structure of γ matrices contained within it. The trace of the γ matrices is straightforward. This gives the full self-energy as

$$\Pi^{\mu}_{\mu}=\frac{-4}{\beta}\sum_{k^0}\int\frac{d^3k}{(2\pi)^3}\sum_{ss_4}e^2\left[\frac{s\hat{k}_{\alpha,s}}{k^0-sk}\delta\Gamma^{\beta\alpha}\frac{s_4\hat{k}_{\beta,s_4}}{k^0-p^0-s_4k}\right]. \quad (2.4)$$

For convenience we change $s_2\to-s_3$ and $s_3\to-s_2$. To evaluate the Matsubara sum we follow the method of Ref. [15]. This method converts the Matsubara sum into a contour integration in the complex plain of k^0, i.e.,

$$\Pi^{\mu}_{\mu}=\frac{-4e^2g^2}{2\pi i}\int_{-i\infty+\epsilon}^{i\infty+\epsilon}dk^0\int\frac{d^3kd^3q}{(2\pi)^6}[1/2-\tilde{n}(k^0)]\frac{s\hat{k}_{\alpha,s}}{s_5k^0-sk}\left[\frac{\widehat{(q-k)}^{\alpha}_{s_2}\widehat{(q-k)}^{\beta}_{s_3}}{q(p^0-[s_2-s_3]E_{q-k})}\times\left\{s_2\frac{(s_1-s_3)/2-s_1\tilde{n}(E_{q-k})-s_3n(q)}{s_5k^0-s_1q+s_3E_{q-k}}+s_3\frac{(s_1-s_2)/2-s_1\tilde{n}(E_{q-k})-s_2n(q)}{p^0-s_5k^0+s_1q-s_2E_{q-k}}\right\}\right]\frac{s_4\hat{k}_{\beta,s_4}}{s_5k^0-p^0-s_4k}, \quad (2.5)$$

where $\widehat{(q-k)}_s$ stands for the four component quantity

$$\left\{s,\frac{q_x-k_x}{|q-k|},\frac{q_y-k_y}{|q-k|},\frac{q_z-k_z}{|q-k|}\right\}=\left\{s,\frac{q\sin\theta\cos\phi}{\sqrt{k^2+q^2-2kq\cos\theta}},\frac{q\sin\theta\sin\phi}{\sqrt{k^2+q^2-2kq\cos\theta}},\frac{q\cos\theta-k}{\sqrt{k^2+q^2-2kq\cos\theta}}\right\}.$$

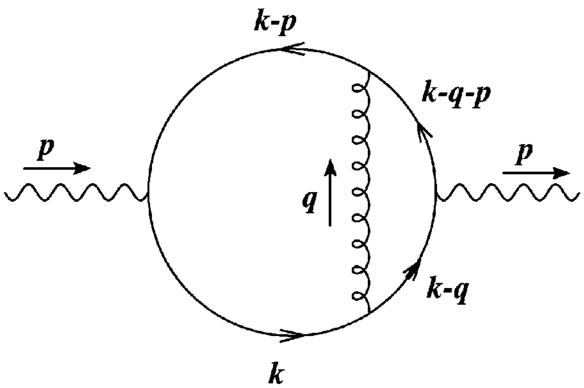

FIG. 1. The first topology for the self-energy.

The k^0 integration is from $-\infty\to\infty$ on the positive side of the real axis. We may thus close the contour on the positive side. Note that the function is vanishing as $k^0\to\infty$. The result of this integration will simply be the sum of the residues at the corresponding poles. Looking at the above expression we note that the pole structure is different depending on whether the term being considered is the first one or the second one in the curly brackets. We note the following poles:

(i) First order pole at $k^0=k$, requires $s_5=s$ (in both terms)

(ii) First order pole at $k^0=k+s_5p^0$, requires $s_5=s_4$ (in both terms)

(iii) First order pole at $k^0=s_5s_1q-s_5s_3E_{q-k}$, requires $s_5s_1q-s_5s_3E_{q-k}>0$ (only in the first term)

(iv) First order pole at $k^0=s_5p^0-s_5s_2E_{q-k}+s_5s_1q$ (only in the second term).

In the following, each of the poles are evaluated in a separate subsection and then summed up.

A. First order pole at $k^0=k$

This is the pole of the first outer propagator (i.e., not a propagator in the effective vertex), it is a pole for the entire self-energy expression. It has the obvious residue of

$$\Pi^\mu_\mu(A)=4e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}[1/2-\tilde{n}(k)]\frac{s\hat{k}_{\alpha,s}}{s}\times\left[\frac{\widehat{(q-k)}^{\alpha}_{s_2}\widehat{(q-k)}^{\beta}_{s_3}}{q(p^0-[s_2-s_3]E_{q-k})}\times\left\{s_2\frac{(s_1-s_3)/2-s_1\tilde{n}(E_{q-k})-s_3n(q)}{sk-s_1q+s_3E_{q-k}}+s_3\frac{(s_1-s_2)/2-s_1\tilde{n}(E_{q-k})-s_2n(q)}{p^0-sk+s_1q-s_2E_{q-k}}\right\}\right]\times\frac{s_4\hat{k}_{\beta,s_4}}{sk-p^0-s_4k}. \tag{2.6}$$

Note that there is an extra negative sign in the residue as the contour is being taken in the clockwise sense.

B. First order pole at $k^0=k+s_5p^0$

This is the pole of the second outer propagator, it is a pole for the entire self-energy expression. It gives the residue

$$\Pi^\mu_\mu(B)=4e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}[1/2-\tilde{n}(k)]\times\frac{s\hat{k}_{\alpha,s}}{p^0+s_4k-sk}\left[\frac{\widehat{(q-k)}^{\alpha}_{s_2}\widehat{(q-k)}^{\beta}_{s_3}}{q(p^0-[s_2-s_3]E_{q-k})}\times\left\{s_2\frac{(s_1-s_3)/2-s_1\tilde{n}(E_{q-k})-s_3n(q)}{p^0+s_4k-s_1q+s_3E_{q-k}}+s_3\frac{(s_1-s_2)/2-s_1\tilde{n}(E_{q-k})-s_2n(q)}{-s_4k+s_1q-s_2E_{q-k}}\right\}\right]\frac{s_4\hat{k}_{\beta,s_4}}{s_4}. \tag{2.7}$$

Note that the p^0 in the distribution function has been dropped. This may be done as $e^{p^0\beta}=1$ (p^0 is a discrete even frequency), and also, as we are eventually going to analytically continue the self-energy to complex values of p^0. The correct analytic continuation is given by that function which has no nonanalytic behavior off the real axis [16]. One may easily check that the above function with a p^0 in the distribution function will have poles at $p^0=-k+i2(n+1)\pi T$. Note that in this pole we may switch

$$s\to-s_4,\quad s_4\to-s,\quad s_2\to-s_3,\quad s_3\to-s_2, s_1\to-s_1,$$

and noting that $\hat{k}_{-s}\widehat{(q-k)}_{-s_2}=\hat{k}_s\widehat{(q-k)}_{s_2}$, we find

$$\Pi^\mu_\mu(B)=\Pi^\mu_\mu(A).$$

C. First order pole at $k^0=s_5s_1q-s_5s_3E_{q-k}$

In the expression for the effective vertex [Eq. (2.2) or Eq. (2.5)], we note the presence of two terms inside the curly brackets with different pole structures. This is a pole of the first term, and is realized only if $s_5s_1q-s_5s_3E_{q-k}>0$. Thus the residue is

$$\Pi^\mu_\mu(C)=4e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}[1/2-\tilde{n}(s_5s_1q-s_5s_3E_{q-k})]\times\frac{s\hat{k}_{\alpha,s}}{s_1q-s_3E_{q-k}-sk}\left[\frac{\widehat{(q-k)}^{\alpha}_{s_2}\widehat{(q-k)}^{\beta}_{s_3}}{q(p^0-[s_2-s_3]E_{q-k})}\times\left\{s_2\frac{(s_1-s_3)/2-s_1\tilde{n}(E_{q-k})-s_3n(q)}{s_5}\right\}\right]\times\frac{s_4\hat{k}_{\beta,s_4}\theta(s_5s_1q-s_5s_3E_{q-k})}{s_1q-s_3E_{q-k}-p^0-s_4k}. \tag{2.8}$$

D. First order pole at $k^0=s_5p^0+s_5s_1q-s_5s_2E_{q-k}$

This is the pole of the second term in the curly bracket mentioned in the preceding section. It is realized only if $s_5s_1q-s_5s_2E_{q-k}>0$. The residue is

$$\Pi^{\mu}_{\mu}(D)=4e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}[1/2-\tilde{n}(s_5s_1q-s_5s_2E_{q-k})]\times\frac{s\hat{k}_{\alpha,s}}{p^0+s_1q-s_2E_{q-k}-sk}\times\left[\frac{\widehat{(q-k)}^{\alpha}_{s_2}\widehat{(q-k)}^{\beta}_{s_3}}{q(p^0-[s_2-s_3]E_{q-k})}\times\left\{s_3\frac{(s_1-s_2)/2-s_1\tilde{n}(E_{q-k})-s_2n(q)}{-s_5}\right\}\right]\times\frac{s_4\hat{k}_{\beta,s_4}\theta(s_5s_1q-s_5s_2E_{q-k})}{s_1q-s_2E_{q-k}-s_4k}. \tag{2.9}$$

Note that in this pole we may switch

$$s_2\to-s_3,\quad s_3\to-s_2,\quad s_1\to-s_1,\quad s_5\to-s_5,$$

$$s\to-s_4,\quad s_4\to-s.$$

With this operation, we find

$$\Pi^{\mu}_{\mu}(D)=\Pi^{\mu}_{\mu}(C).$$

Thus the full photon self-energy to second order in the coupling constant for the diagram of Fig. 1 is given by summing up the results of the preceding four subsections, i.e.,

$$\Pi^{\mu}_{\mu}=2\Pi^{\mu}_{\mu}(A)+2\Pi^{\mu}_{\mu}(C).$$

III. IMAGINARY PART OF THE FIRST SELF-ENERGY TOPOLOGY

We now proceed with evaluating the discontinuity in the first self-energy, as p^0 is analytically continued towards the positive real axis from above, i.e., $p^0\to E+i\epsilon$. Analytically continuing p^0 will give us the retarded self-energy of the photon in real time in terms of a real continuous energy $p^0=E$. The expressions to be continued are $\Pi^{\mu}_{\mu}(A)$ and $\Pi^{\mu}_{\mu}(D)$. The presence of the theta function in $\Pi^{\mu}_{\mu}(D)$ complicates the pole structure that one would obtain during the analytic continuation of this expression. Using standard techniques (outlined in Sec. IV D), we decompose the theta function to obtain

$$\Pi^{\mu}_{\mu}(D)=-4e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}\frac{ss_4s_3}{q}\left\{\frac{[\hat{k}_s\cdot\widehat{(q-k)}_{-s_5}][\hat{k}_{s_4}\cdot\widehat{(q-k)}_{s_3}][1-\tilde{n}(E_{q-k})+n(q)][1/2-\tilde{n}(E_{q-k}+q)]}{[p^0-(sk-s_5E_{q-k}-s_5q)][p^0-(-s_5-s_3)E_{q-k}][s_5(E_{q-k}+q)-s_4k]}-\frac{[\hat{k}_s\cdot\widehat{(q-k)}_{s_5}][\hat{k}_{s_4}\cdot\widehat{(q-k)}_{s_3}][\tilde{n}(E_{q-k})+n(q)][1/2-\tilde{n}(q-E_{q-k})]}{[p^0-(sk+s_5E_{q-k}-s_5q)][p^0-(s_5-s_3)E_{q-k}][s_5(q-E_{q-k})-s_4k]}\right\}. \tag{3.1}$$

Analyzing the expressions for $\Pi^{\mu}_{\mu}(A)$ and $\Pi^{\mu}_{\mu}(D)$, we note the following discontinuities.

(a) Poles of type $p^0=2k$.

(i) First order pole in $\Pi^{\mu}_{\mu}(A)$ at $p^0=2k$, requires $s=-s_4=1$ [in both terms that make up $\Pi^{\mu}_{\mu}(A)$].

(b) Poles of type $p^0=2E_{q-k}$.

(ii) First order pole in $\Pi^{\mu}_{\mu}(A)$ at $p^0=2E_{q-k}$, requires $s_2=-s_3=1$ [in both terms that make up $\Pi^{\mu}_{\mu}(A)$].

(iii) First order pole in $\Pi^{\mu}_{\mu}(D)$ at $p^0=2E_{q-k}$, requires $s_5=s_3=-1$ [only in the first term of $\Pi^{\mu}_{\mu}(D)$].

(iv) First order pole in $\Pi^{\mu}_{\mu}(D)$ at $p^0=2E_{q-k}$, requires $s_5=-s_3=1$ [only in the second term of $\Pi^{\mu}_{\mu}(D)$].

(c) Poles of type $p^0=sk+s_1q+s_2E_{k-q}$.

(v) First order pole in $\Pi^{\mu}_{\mu}(A)$ at $p^0=sk-s_1q+s_2E_{q-k}$, requires $sk-s_1q+s_2E_{q-k}>0$ [only in the second term of $\Pi^{\mu}_{\mu}(A)$].

(vi) First order pole in $\Pi^{\mu}_{\mu}(B)$ at $p^0=sk-s_5q-s_5E_{q-k}$, requires $sk-s_5q-s_5E_{q-k}>0$ [only in the first term of $\Pi^{\mu}_{\mu}(B)$].

(vii) First order pole in $\Pi^{\mu}_{\mu}(B)$ at $p^0=sk-s_5q+s_5E_{q-k}$, requires $sk-s_5q+s_5E_{q-k}>0$ [only in the second term of $\Pi^{\mu}_{\mu}(B)$].

We may write down the expression for $2\Pi^{\mu}_{\mu}(A)$ highlighting its real and imaginary parts as $p^0\to E+i\epsilon$ as

$$2\Pi^{\mu}_{\mu}(A)=-8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}\frac{s_4[\hat{k}_s\cdot\widehat{(q-k)}_{s_2}][\hat{k}_{s_4}\cdot\widehat{(q-k)}_{s_3}]}{E_{q+k}}\left[\mathrm{P}\left(\frac{1}{E-(s_2-s_3)E_{q-k}}\right)-i\pi\delta(E-[s_2-s_3]E_{q-k})\right]\times\left\{s_2\frac{(s_1-s_3)/2-s_1\tilde{n}(E_{q-k})-s_3n(q)}{sk-s_1q+s_3E_{q-k}}+s_3[(s_1-s_2)/2-s_1\tilde{n}(E_{q-k})-s_2n(q)]\left[\mathrm{P}\left(\frac{1}{E-sk+s_1q-s_2E_{q-k}}\right)-i\pi\delta(E-sk+s_1q-s_2E_{q-k})\right]\right\}[1/2-\tilde{n}(k)]\left[\mathrm{P}\left(\frac{1}{E-(s-s_4)k}\right)-i\pi\delta(E-[s-s_4]k)\right]. \tag{3.2}$$

We now write down the various discontinuities as enumerated above [note that we are now looking at $2\Pi^{\mu}_{\mu}(A)$ and $2\Pi^{\mu}_{\mu}(D)$, so the overall factors have doubled],

$$\mathrm{disc}[2\Pi^{\mu}_{\mu}(A)]_a=(+2\pi i)8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}\frac{-2[\hat{k}_+\cdot\widehat{(q-k)}_{s_2}][\hat{k}_-\cdot\widehat{(q-k)}_{s_3}]}{q(E-[s_2-s_3]E_{q-k})}s_2\frac{(s_1-s_3)/2-s_1\tilde{n}(E_{q-k})-s_3n(q)}{k-s_1q+s_3E_{q-k}}\times[1/2-\tilde{n}(k)]\delta(E-2k). \tag{3.3}$$

$$\mathrm{disc}[2\Pi^{\mu}_{\mu}(A)]_b=(+2\pi i)8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}\frac{s_4[\hat{k}_s\cdot\widehat{(q-k)}_+][\hat{k}_{s_4}\cdot\widehat{(q-k)}_-]}{q}\left\{\frac{[1-2\tilde{n}(E_{q-k})]2q}{(sk-E_{q-k})^2-q^2}\right\}\frac{[1/2-\tilde{n}(k)]}{E-(s-s_4)k}\delta(E-2E_{k-q}). \tag{3.4}$$

$$\mathrm{disc}[2\Pi^{\mu}_{\mu}(D)]_b=(+2\pi i)8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}\frac{1}{q}\left\{\frac{[\hat{k}_s\cdot\widehat{(q-k)}_+][\hat{k}_s\cdot\widehat{(q-k)}_-]}{E^2_{q-k}-(sk+q)^2}+\frac{[\hat{k}_s\cdot\widehat{(q-k)}_+][\hat{k}_s\cdot\widehat{(q-k)}_+]}{q^2-(E_{q-k}-sk)^2}\right\}\times\{[1-\tilde{n}(E_{q-k})+n(q)][1/2-\tilde{n}(E_{q-k}+q)]-[\tilde{n}(E_{q-k})+n(q)][1/2-\tilde{n}(q-E_{q-k})]\}\delta(E-2E_{q-k}). \tag{3.5}$$

We now proceed to the evaluations of the discontinuities of the type $p^0=sk+s_2E_{q-k}+s_1q$. We first change $s_1\to -s_1$ in $2\Pi^{\mu}_{\mu}(A)$ and $s_5\to -s_5$ in the first part of $2\Pi^{\mu}_{\mu}(D)$. Hence the discontinuity occurs in $2\Pi^{\mu}_{\mu}(A)$ at $p^0=sk+s_2E_{q-k}+s_1q$ only when $sk+s_2E_{q-k}+s_1q>0$. This may happen in only one of four instances: $s=+,s_2=+,s_1=+$; $s=-,s_2=+,s_1=+$; $s=+,s_2=-,s_1=+$; $s=+,s_2=+,s_1=-$. In $2\Pi^{\mu}_{\mu}(D)$ the discontinuity occurs in the first term when $s=+,s_5=+$ or when $s=-,s_5=+$ and in the second term when $s=+,s_5=+$ or when $s=+,s_5=-$.

Thus the discontinuity in $2\Pi^{\mu}_{\mu}(A)$ occurs in four parts,

$$\begin{aligned}\mathrm{disc}[2\Pi^{\mu}_{\mu}(A)]_c=(+2\pi i)8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6q}[1/2-\tilde{n}(k)]\Bigg\{&\frac{s_3s_4[\hat{k}_+\cdot\widehat{(q-k)}_+][\hat{k}_{s_4}\cdot\widehat{(q-k)}_{s_3}][-1+\tilde{n}(E_{q-k})-n(q)]}{[k+q+s_3E_{q-k}][q+E_{q-k}+s_4k]}\delta(E-k-E_{q-k}-q)\\&+\frac{s_3s_4[\hat{k}_-\cdot\widehat{(q-k)}_+][\hat{k}_{s_4}\cdot\widehat{(q-k)}_{s_3}][-1+\tilde{n}(E_{q-k})-n(q)]}{[-k+q+s_3E_{q-k}][q+E_{q-k}+s_4k]}\delta(E+k-E_{q-k}-q)\\&+\frac{s_3s_4[\hat{k}_+\cdot\widehat{(q-k)}_-][\hat{k}_{s_4}\cdot\widehat{(q-k)}_{s_3}][\tilde{n}(E_{q-k})+n(q)]}{[k+q+s_3E_{q-k}][-E_{q-k}+q+s_4k]}\delta(E-k+E_{q-k}-q)\\&+\frac{s_3s_4[\hat{k}_+\cdot\widehat{(q-k)}_+][\hat{k}_{s_4}\cdot\widehat{(q-k)}_{s_3}][-\tilde{n}(E_{q-k})-n(q)]}{[k-q+s_3E_{q-k}][E_{q-k}-q+s_4k]}\delta(E-k-E_{q-k}+q)\Bigg\}.\end{aligned} \tag{3.6}$$

The discontinuity in $2\Pi^{\mu}_{\mu}(D)$ also occurs in four parts,

$$\begin{aligned}\mathrm{disc}[2\Pi^{\mu}_{\mu}(D)]_c=(-2\pi i)8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6q}\Bigg[&[1/2-\tilde{n}(E_{q-k}+q)]\\&\times\Bigg\{\frac{s_3s_4[\hat{k}_+\cdot\widehat{(q-k)}_+][\hat{k}_{s_4}\cdot\widehat{(q-k)}_{s_3}][1-\tilde{n}(E_{q-k})+n(q)]}{[k+q+s_3E_{q-k}][q+E_{q-k}+s_4k]}\delta(E-k-E_{q-k}-q)\\&-\frac{s_3s_4[\hat{k}_-\cdot\widehat{(q-k)}_+][\hat{k}_{s_4}\cdot\widehat{(q-k)}_{s_3}][1-\tilde{n}(E_{q-k})+n(q)]}{[-k+q+s_3E_{q-k}][q+E_{q-k}+s_4k]}\delta(E+k-E_{q-k}-q)\Bigg\}\\&+[1/2-\tilde{n}(q-E_{q-k})]\Bigg\{-\frac{s_3s_4[\hat{k}_+\cdot\widehat{(q-k)}_-][\hat{k}_{s_4}\cdot\widehat{(q-k)}_{s_3}][\tilde{n}(E_{q-k})+n(q)]}{[k+q+s_3E_{q-k}][-E_{q-k}+q+s_4k]}\delta(E-k+E_{q-k}-q)\\&-\frac{s_3s_4[\hat{k}_+\cdot\widehat{(q-k)}_+][\hat{k}_{s_4}\cdot\widehat{(q-k)}_{s_3}][\tilde{n}(E_{q-k})+n(q)]}{[k-q+s_3E_{q-k}][E_{q-k}-q+s_4k]}\delta(E-k-E_{q-k}+q)\Bigg\}\Bigg].\end{aligned} \tag{3.7}$$

IV. THE SELF-ENERGY: TOPOLOGY II

We begin by evaluating the photon self-energy with one quark line containing a gluon loop. The quark self-energy may be written as

$$-i\Sigma(k)_{i,k}=(ie)^2\frac{i}{\beta}\sum_{q^0}\int\frac{d^3q}{(2\pi)^3}t^a_{i,j}\gamma^\mu\frac{\not{q}}{q^2}t^b_{j,i}\gamma_\mu\frac{-i\delta^{a,b}}{(k-q)^2}. \tag{4.1}$$

Using the identity $\gamma^\mu\not{q}\gamma_\mu=-2\not{q}$, q being the momentum of the gluon, the Matsubara sum in the quark self-energy is evaluated using the method of Pisarski [13]. It is given in our notation (Appendix A) as

$$\Sigma(k)_{i,k}=\frac{g^2t^a_{i,j}t^b_{j,k}\delta^{a,b}}{2}\int\frac{d^3q}{(2\pi)^3}\sum_{s_1s_2}\frac{(\widehat{\not{k}-\not{q}})_{s_2}}{q}$$
$$\times\frac{(s_1+s_2)/2-s_1\tilde{n}_1(E_{k-q})+s_2n(q)}{k^0-s_2E_{k-q}-s_1q}$$
$$=\gamma^\beta\Sigma_{i,k\beta}, \tag{4.2}$$

where the self-energy has been written in the final form to highlight its matrix structure. We may use this to write the full self-energy of the photon in the static limit as

$$\Pi^\mu_\mu=\frac{1}{\beta}\sum_{k^0}\int\frac{d^3k}{(2\pi)^3}(-1)\mathrm{Tr}\sum_{s_1s_2s_3s_4s_5}\left[e\gamma_\mu\frac{\gamma^\alpha s_3\hat{k}_{\alpha,s_3}\delta_{j,i}}{2(k^0-s_3k)}\right.$$
$$\left.\times\gamma^\beta\Sigma_{i,k\beta}(k)\frac{\gamma^\gamma s_4\hat{k}_{\gamma,s_4}\delta_{k,l}}{2(k^0-s_4k)}e\gamma^\mu\frac{\gamma^\delta s_5\hat{k}_{\delta,s_5}\delta_{l,j}}{2(k^0-p^0-s_5k)}\right]. \tag{4.3}$$

We choose the z direction to be defined by the direction of k. Note that $\hat{k}_{s_4}\cdot\hat{k}_{s_5}=-2\delta_{s_4,-s_5}$. Note also that the s_4 and s_3 dependence of the photon self-energy is identical. This allows us to write down the self-energy as

$$\Pi^\mu_\mu=\frac{-1}{\beta}\sum_{k^0}\int\frac{d^3k}{(2\pi)^3}2e^2$$
$$\times\left[\frac{2(-s_3)\Sigma_{jj}\cdot\hat{k}_{s_3}}{(k^0-s_3k)(k^0-s_4k)(k^0-p^0+s_4k)}\right.$$
$$\left.-\frac{2(-s_5)\Sigma_{jj}\cdot\hat{k}_{s_5}}{(k^0-p^0-s_5k)[(k^0)^2-k^2]}\right], \tag{4.4}$$

where the summation is implied over all the sign variables present, i.e., s_1,s_2,s_3,s_4,s_5. Note that the double pole is only present in the first term.

We now need to evaluate the Matsubara sum over k^0. For this we follow the method of Ref. [15]. This method converts the Matsubara sum into a contour integration in the complex plain of k^0. The color factor from the quark self-energy combined with that from the rest of the diagram becomes $\mathrm{Tr}[t^a,t^b]\delta^{ab}=4$. Using this we obtain the self-energy of the photon as

$$\Pi^\mu_\mu=\frac{8e^2g^2}{2\pi i}\int_{-i\infty+\epsilon}^{i\infty+\epsilon}dk^0\int\frac{d^3kd^3q}{(2\pi)^6}$$
$$\times\frac{[1/2-\tilde{n}(k^0)][(s_1+s_2)/2-s_1\tilde{n}(E_{k-q})+s_2n(q)]}{q[sk^0-s_2E_{k-q}-s_1q]}$$
$$\times\left\{\frac{s_3\widehat{(k-q)}_{s_2}\cdot\hat{k}_{s_3}}{[sk^0-s_3k][sk^0-s_4k][sk^0-p^0+s_4k]}\right.$$
$$\left.-\frac{s_5\widehat{(k-q)}_{s_2}\cdot\hat{k}_{s_5}}{[sk^0-p^0-s_5k][(k^0)^2-k^2]}\right\}. \tag{4.5}$$

The k^0 integration is from $-\infty\to\infty$ on the positive side of the real axis. We may thus close the contour on the positive side. Note that the function is vanishing as $k^0\to\infty$. The result of this integration will simply be the sum of the residues at the corresponding poles. Looking at the above expression we note the following poles:

(i) Second order pole at $k^0=k$, requires $s_3=s_4=s$ (only in the first term).
(ii) First order pole at $k^0=k$, no requirement (only in the second term), requires $s_3=-s_4=s$ or $-s_3=s_4=s$ (only in the first term).
(iii) First order pole at $k^0=k+sp^0$, requires $s_4=-s$ (only in first term), requires $s_5=s$ (only in the second term).
(iv) First order pole at $k^0=ss_2E_{k-q}+ss_1q$, requires $ss_2E_{k-q}+ss_1q>0$.

In the following each of these poles will be evaluated in a separate subsection and then summed up.

A. Second order pole at $k^0=k$

We begin by evaluating the second order pole. The origin of this pole can be traced back to the two propagators that may go on-shell simultaneously. In the real-time formalism this leads to the ill-defined square of the Dirac delta function. In imaginary time, however, this pole is easily dealt with: the residue of a function $f(k^0)$ at a second order pole at $k^0=k$ is simply given as $(d/dk^0)(k^0-k)^2f(k^0)|_{k^0=k}$. Using this we get the residue of Π at this pole as

$$\Pi^\mu_\mu(A)=8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}$$
$$\times\frac{[(s_1+s_2)/2-s_1\tilde{n}(E_{k-q})+s_2n(q)]}{q}\widehat{(k-q)}_{s_2}\cdot\hat{k}_s$$
$$\times\left\{\frac{s[1/2-\tilde{n}(k)]'}{[sk-s_2E_{k-q}-s_1q][p^0-2sk]}\right.$$
$$-\frac{1/2-\tilde{n}(k)}{[sk-s_2E_{k-q}-s_1q]^2[p^0-2sk]}$$
$$\left.+\frac{1/2-\tilde{n}(k)}{[sk-s_2E_{k-q}-s_1q][p^0-2sk]^2}\right\}, \tag{4.6}$$

where the prime denotes derivation only with respect to k. Note that as in the case of the first self-energy topology there is an extra negative sign in the residue as the contour is closed in the clockwise sense.

B. First order pole at $k^0=k$

This obvious residue may be easily evaluated using the methods outlined in the preceding section,

$$\Pi^{\mu}_{\mu}(B)=-8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}\times\frac{[(s_1+s_2)/2-s_1\tilde{n}(E_{k-q})+s_2n(q)][1/2-\tilde{n}(k)]}{q[sk-s_2E_{k-q}-s_1q]}\times\left\{\frac{-s\widehat{(k-q)}_{s_2}\cdot\hat{k}_s}{2kp^0}+\frac{s\widehat{(k-q)}_{s_2}\cdot\hat{k}_{-s}}{2k[p^0-2sk]}+\frac{s_5\widehat{(k-q)}_{s_2}\cdot\hat{k}_{s_5}}{2k[p^0-(s-s_5)k]}\right\}. \tag{4.7}$$

In the above, we sum over the two possibilities of $s_5=\pm s$ to get the factor in the bracket as

$$\left\{\frac{-s\widehat{(k-q)}_{s_2}\cdot\hat{k}_s}{2kp^0}+\frac{s\widehat{(k-q)}_{s_2}\cdot\hat{k}_{-s}}{2k[p^0-2sk]}+\frac{s\widehat{(k-q)}_{s_2}\cdot\hat{k}_s}{2kp^0}+\frac{-s\widehat{(k-q)}_{s_2}\cdot\hat{k}_{-s}}{2k[p^0-2sk]}\right\}=0.$$

Hence, $\Pi^{\mu}_{\mu}(B)=0$.

C. First order pole at $k^0=k+sp^0$

This gives the residue

$$\Pi^{\mu}_{\mu}(C)=-8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}\times\frac{[(s_1+s_2)/2-s_1\tilde{n}(E_{k-q})+s_2n(q)][1/2-\tilde{n}(k)]}{q[p^0-(s_2E_{k-q}+s_1q-sk)][p^0+2sk]}\times\left\{\frac{s_3\widehat{(k-q)}_{s_2}\cdot\hat{k}_{s_3}}{s[p^0-(s_3-s)k]}-\frac{\widehat{(k-q)}_{s_2}\cdot\hat{k}_s}{p^0}\right\}. \tag{4.8}$$

Switching $s\to-s$ and summing over $s_3=\pm s$ we get

$$\Pi^{\mu}_{\mu}(C)=8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}\times\frac{[(s_1+s_2)/2-s_1\tilde{n}(E_{k-q})+s_2n(q)][1/2-\tilde{n}(k)]}{q[p^0-(s_2E_{k-q}+s_1q+sk)][p^0-2sk]}\times\left\{\frac{\widehat{(k-q)}_{s_2}\cdot\hat{k}_s}{(p^0-2sk)}\right\}. \tag{4.9}$$

D. First order pole at $k^0=ss_2E_{k-q}+ss_1q$

This pole is realized only if $ss_2E_{k-q}+ss_1q>0$. This condition may be enforced with the following set of delta and theta functions:

$$\delta_{s,s_2}\delta_{s,s_1}+\delta_{s,s_2}\delta_{s,-s_1}\Theta(E_{k-q}-q)+\delta_{s,-s_2}\delta_{s,s_1}\Theta(q-E_{k-q}). \tag{4.10}$$

We start with the second and third terms,

$$\Pi^{\mu}_{\mu}(D,2)=8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}\frac{[1/2-\tilde{n}(E_{k-q}-q)][\tilde{n}(E_{k-q})+n(q)]}{q}\times\left\{\frac{s_3\widehat{(k-q)}_s\cdot\hat{k}_{s_3}}{[sE_{k-q}-sq-s_3k][sE_{k-q}-sq-s_4k][p^0-(s_4k+sE_{k-q}-sq)]}-\frac{s_5\widehat{(k-q)}_s\cdot\hat{k}_{s_5}}{[p^0-(sE_{k-q}-sq-s_5k)][(E_{k-q}-q)^2-k^2]}\right\}\theta(E_{k-q}-q). \tag{4.11}$$

Similarly we find for the third term

$$\Pi^{\mu}_{\mu}(D,3)=8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}\frac{[1/2-\tilde{n}(q-E_{k-q})][-\tilde{n}(E_{k-q})-n(q)]}{q}\times\left\{\frac{s_3\widehat{(k-q)}_{-s}\cdot\hat{k}_{s_3}}{[-sE_{k-q}+sq-s_3k][-sE_{k-q}+sq-s_4k][p^0-(s_4k-sE_{k-q}+sq)]}-\frac{s_5\widehat{(k-q)}_{-s}\cdot\hat{k}_{s_5}}{[p^0-(-sE_{k-q}+sq-s_5k)][(E_{k-q}-q)^2-k^2]}\right\}\theta(q-E_{k-q}). \tag{4.12}$$

Now, switching $s\to -s$ in the third term and noting that $1/2-\tilde{n}(q-E_{k-q})=-1/2+\tilde{n}(E_{k-q}-q)$, we observe that the second and third terms can be combined to give

$$\Pi_\mu^\mu(D,2+3)=8e^2g^2\int \frac{d^3k d^3q}{(2\pi)^6}\frac{[1/2-\tilde{n}(E_{k-q}-q)][\tilde{n}(E_{k-q})+n(q)]}{q}\times\Bigg\{\frac{s_3\widehat{(k-q)}_s\cdot\hat{k}_{s_3}}{[sE_{k-q}-sq-s_3k][sE_{k-q}-sq-s_4k][p^0-(s_4k+sE_{k-q}-sq)]}-\frac{s_5\widehat{(k-q)}_s\cdot\hat{k}_{s_5}}{[p^0-(sE_{k-q}-sq-s_5k)][(E_{k-q}-q)^2-k^2]}\Bigg\}. \tag{4.13}$$

Note the absence of the theta functions in the above equation. Now we may also write down the residue from the first set of delta functions in Eq. (4.10) as

$$\Pi_\mu^\mu(D,1)=8e^2g^2\int \frac{d^3k d^3q}{(2\pi)^6}\frac{[1/2-\tilde{n}(E_{k-q}+q)][1-\tilde{n}(E_{k-q})+n(q)]}{q}\times\Bigg\{\frac{s_3\widehat{(k-q)}_s\cdot\hat{k}_{s_3}}{[sE_{k-q}+sq-s_3k][sE_{k-q}+sq-s_4k][p^0-(s_4k+sE_{k-q}+sq)]}-\frac{s_5\widehat{(k-q)}_s\cdot\hat{k}_{s_5}}{[p^0-(sE_{k-q}+sq-s_5k)][(E_{k-q}+q)^2-k^2]}\Bigg\}. \tag{4.14}$$

The total expression obtained by summing up the results from the preceding four subsections will give us the full self-energy of the photon to second order in the coupling constant for the diagram of Fig. 2, i.e.,

$$\Pi_\mu^\mu=\Pi_\mu^\mu(A)+\Pi_\mu^\mu(B)+\Pi_\mu^\mu(C)+\Pi_\mu^\mu(D,1)+\Pi_\mu^\mu(D,2+3).$$

V. IMAGINARY PART OF THE SECOND SELF-ENERGY TOPOLOGY

We now proceed with evaluating the discontinuity in the second self-energy as p^0 is analytically continued to a positive real value, i.e., $p^0\to E+i\epsilon$. Analyzing the expressions derived in the above sections we note the following discontinuities.

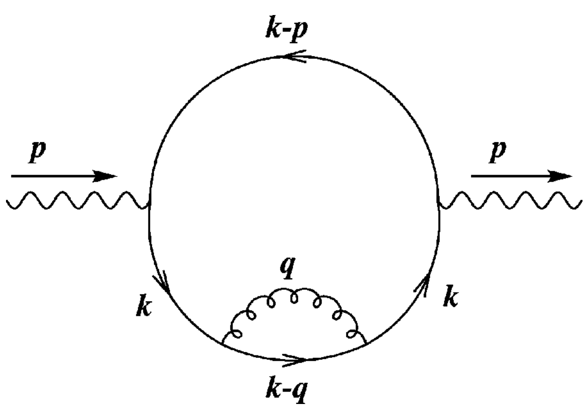

FIG. 2. The second topology for the self-energy.

(a) Poles of type $p^0=2k$.

(i) First order pole in $\Pi_\mu^\mu(A)$ at $p^0=2k$, requires $s=1$ (only in the first and second terms).

(ii) Second order pole in $\Pi_\mu^\mu(A)$ at $p^0=2k$. This occurs in the third term in the bracket and requires $s=1$.

(iii) Second order pole in $\Pi_\mu^\mu(C)$ at $p^0=2k$, requires $s=1$ and $s_3=1$ (only in the first term).

(b) Poles of type $p^0=sk+s_1q+s_2E_{k-q}$.

(iv) First order pole in $\Pi_\mu^\mu(C)$ at $p^0=sk+s_1q+s_2E_{k-q}$, requires $s=s_1=s_2=1$, $-s=s_1=s_2=1$, $s=-s_1=s_2=1$, or $s=s_1=-s_2=1$ (in both terms).

(v) First order pole in $\Pi_\mu^\mu(D,2+3)$ at $p^0=s_4k+sE_{k-q}-sq$, requires $s_4=s=1$ or $s_4=-s=1$ (only in the first term); at $p^0=-s_5k+sE_{k-q}-sq$, requires $-s_5=s=1$ or $s_5=s=-1$ (only in the second term).

(vi) First order pole in $\Pi_\mu^\mu(D,1)$ at $p^0=s_4k+sE_{k-q}+sq$, requires $s_4=s=1$ or $-s_4=s=1$ (only in the first term); at $p^0=-s_5k+sE_{k-q}+sq$, requires $-s_5=s=1$ or $s_5=s=1$ (only in the second term).

The discontinuity across a second order pole is derived in Appendix B. We now write down the various discontinuities as enumerated above,

$$\mathrm{disc}[\Pi^{\mu}_{\mu}(A)]_a$$
$$=(-2\pi i)8e^2g^2\int\frac{dk\,d\theta\,d\phi\,\sin\theta\,d^3q}{(2\pi)^6q}\,\delta(E-2k)$$
$$\times\Bigg\{\frac{k^2\mathcal{NS}[1/2-\tilde{n}(k)]'}{[k-s_2E_{k-q}-s_1q]}-\frac{k^2\mathcal{NS}[1/2-\tilde{n}(k)]}{[k-s_2E_{k-q}-s_1q]^2}$$
$$-\frac{1}{2}\frac{2k\mathcal{NS}[1/2-\tilde{n}(k)]}{[k-s_2E_{k-q}-s_1q]}-\frac{1}{2}\frac{k^2(\mathcal{NS})'[1/2-\tilde{n}(k)]}{[k-s_2E_{k-q}-s_1q]}$$
$$-\frac{1}{2}\frac{k^2\mathcal{NS}[1/2-\tilde{n}(k)]'}{[k-s_2E_{k-q}-s_1q]}$$
$$-\frac{1}{2}\frac{k^2\mathcal{NS}[1/2-\tilde{n}(k)][-1+s_2E'_{k-q}]}{[k-s_2E_{k-q}-s_1q]^2}\Bigg\},\qquad(5.1)$$

where the prime denotes derivation only with respect to k. The symbol $\mathcal{N}$ stands for the factor $[(s_1+s_2)/2-s_1\tilde{n}(E_{k-q})+s_2n(q)]$, while the factor $\widehat{(k-q)}_{s_2}\cdot\hat{k}_+$ is represented by the symbol $\mathcal{S}$,

$$\mathrm{disc}[\Pi^{\mu}_{\mu}(C)]_a$$
$$=(2\pi i)8e^2g^2\int\frac{dk\,d\theta\,d\phi\,\sin\theta\,d^3q}{(2\pi)^6}\,\delta(E-2k)$$
$$\times\Bigg\{\frac{1}{2}\frac{2k\mathcal{NS}[1/2-\tilde{n}(k)]}{[E-s_2E_{k-q}-s_1q-k]}+\frac{1}{2}$$
$$\times\frac{k^2(\mathcal{NS})'[1/2-\tilde{n}(k)]}{[E-s_2E_{k-q}-s_1q-k]}+\frac{1}{2}\frac{k^2\mathcal{NS}[1/2-\tilde{n}(k)]'}{[E-s_2E_{k-q}-s_1q-k]}$$
$$+\frac{1}{2}\frac{k^2\mathcal{NS}[1/2-\tilde{n}(k)][1+s_2E'_{k-q}]}{[E-s_2E_{k-q}-s_1q-k]^2}\Bigg\}.\qquad(5.2)$$

The two terms above are the result of the discontinuities at $p^0\to E=2k$. In the following we shall enumerate those terms that result as we take the discontinuities at $p^0\to E=sk+s_1q+s_2E_{k-q}$:

$$\mathrm{disc}[\Pi^{\mu}_{\mu}(C)]_c$$
$$=(-2\pi i)8e^2g^2\int\frac{d^3k\,d^3q}{(2\pi)^6}$$
$$\times\frac{[(s_1+s_2)/2-s_1\tilde{n}(E_{k-q})+s_2n(q)][1/2-\tilde{n}(k)]}{q[s_2E_{k-q}+s_1q-sk]}$$
$$\times\Bigg\{\frac{s_3\widehat{(k-q)}_{s_2}\cdot\hat{k}_{s_3}}{s[s_2E_{k-q}+s_1q-s_3k]}-\frac{\widehat{(k-q)}_{s_2}\cdot\hat{k}_{-s}}{s_2E_{k-q}+s_1q+sk}\Bigg\}$$
$$\times\delta(E-sk-s_1q-s_2E_{k-q})[\delta_{s,+}\delta_{s_1,+}\delta_{s_2,+}$$
$$+\delta_{s,-}\delta_{s_1,+}\delta_{s_2,+}+\delta_{s,+}\delta_{s_1,-}\delta_{s_2,+}$$
$$+\delta_{s,+}\delta_{s_1,+}\delta_{s_2,-}].\qquad(5.3)$$

Recall that even though not explicitly mentioned, there is an implied summation over all sign factors. We may now perform the sum over $s_3=\pm s$ to get

$$\mathrm{disc}[\Pi^{\mu}_{\mu}(C)]_c$$
$$=(-2\pi i)8e^2g^2\int\frac{d^3k\,d^3q}{(2\pi)^6}$$
$$\times\frac{[(s_1+s_2)/2-s_1\tilde{n}(E_{k-q})+s_2n(q)][1/2-\tilde{n}(k)]}{q[s_2E_{k-q}+s_1q-sk]^2}$$
$$\times\widehat{(k-q)}_{s_2}\cdot\hat{k}_s\,\delta(E-sk-s_1q-s_2E_{k-q})$$
$$\times[\delta_{s,+}\delta_{s_1,+}\delta_{s_2,+}+\delta_{s,-}\delta_{s_1,+}\delta_{s_2,+}$$
$$+\delta_{s,+}\delta_{s_1,-}\delta_{s_2,+}+\delta_{s,+}\delta_{s_1,+}\delta_{s_2,-}].\qquad(5.4)$$

Finally, the discontinuity in parts D of the second self-energy is given as

$$\mathrm{disc}[\Pi^{\mu}_{\mu}(D,2+3)]_c$$
$$=(-2\pi i)8e^2g^2\int\frac{d^3k\,d^3q}{(2\pi)^6}$$
$$\times\frac{[1/2-\tilde{n}(E_{k-q}-q)][\tilde{n}(E_{k-q})+n(q)]}{q}$$
$$\times\Bigg\{\frac{s_3\widehat{(k-q)}_s\cdot\hat{k}_{s_3}}{[sE_{k-q}-sq-s_3k][sE_{k-q}-sq-s_4k]}$$
$$\times[\delta_{s_4,+}\delta_{s,+}\delta(E-k-E_{k-q}+q)$$
$$+\delta_{s_4,+}\delta_{s,-}\delta(E-k+E_{k-q}-q)]$$
$$-\frac{s_5\widehat{(k-q)}_s\cdot\hat{k}_{s_5}}{[(E_{k-q}-q)^2-k^2]}[\delta_{s_5,-}\delta_{s,+}\delta(E-k-E_{k-q}+q)$$
$$+\delta_{s_5,-}\delta_{s,-}\delta(E-k+E_{k-q}-q)]\Bigg\}.\qquad(5.5)$$

We may now sum over $s_3=\pm1$ to get

$$\mathrm{disc}[\Pi^{\mu}_{\mu}(D,2+3)]_c$$
$$=(-2\pi i)8e^2g^2\int\frac{d^3k\,d^3q}{(2\pi)^6}$$
$$\times\frac{[1/2-\tilde{n}(E_{k-q}-q)][\tilde{n}(E_{k-q})+n(q)]}{q}$$
$$\times\Bigg\{\frac{\widehat{(k-q)}_-\cdot\hat{k}_+}{[k+E_{k-q}-q]^2}\,\delta(E-k+E_{k-q}-q)$$
$$+\frac{\widehat{(k-q)}_+\cdot\hat{k}_+}{[k+q-E_{k-q}]^2}\,\delta(E-k-E_{k-q}+q)\Bigg\}.\qquad(5.6)$$

$$\mathrm{disc}[\Pi^{\mu}_{\mu}(D,1)]_c=(-2\pi i)8e^2g^2\int\frac{d^3kd^3q[1/2-\tilde{n}(E_{k-q}+q)][1-\tilde{n}(E_{k-q})+n(q)]}{(2\pi)^6\qquad q}$$
$$\times\left\{\frac{s_3\widehat{(k-q)}_s\cdot\hat{k}_{s_3}}{[sE_{k-q}+sq-s_3k][sE_{k-q}+sq-s_4k]}[\delta_{s_4,+}\delta_{s,+}\delta(E-k-E_{k-q}-q)+\delta_{s_4,-}\delta_{s,+}\delta(E+k-E_{k-q}-q)]\right.$$
$$\left.-\frac{s_5\widehat{(k-q)}_s\cdot\hat{k}_{s_5}}{[(E_{k-q}+q)^2-k^2]}[\delta_{s_5,-}\delta_{s,+}\delta(E-k-E_{k-q}-q)+\delta_{s_5,+}\delta_{s,+}\delta(E+k-E_{k-q}-q)]\right\}. \tag{5.7}$$

We may sum over $s_3=\pm1$ to obtain

$$\mathrm{disc}[\Pi^{\mu}_{\mu}(D,1)]_c=(-2\pi i)8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6}\times\frac{[1/2-\tilde{n}(E_{k-q}+q)][1-\tilde{n}(E_{k-q})+n(q)]}{q}$$
$$\times\left\{\frac{\widehat{(k-q)}_+\cdot\hat{k}_+}{[E_{k-q}+q-k]^2}\delta(E-k-E_{k-q}-q)-\frac{\widehat{(k-q)}_+\cdot\hat{k}_-}{[E_{k-q}+q+k]^2}\delta(E+k-E_{k-q}-q)\right\}. \tag{5.8}$$

VI. PHYSICAL INTERPRETATION: TREELIKE CUTS

We now begin the process of combining terms from the discontinuities of the two self-energies to obtain the square of amplitudes of physical processes. Essentially we shall follow the method outlined by Weldon [1]. Our method is essentially a three step process: (i) Collect together terms that have the same energy conserving delta functions, (ii) reorganize the thermal distribution functions to express them as a difference of the thermal weights for particle emission and absorption. (iii) reorganize the remaining momentum dependent part as the square of the amplitude of the process hinted at by the previous two steps.

For easy identification we indicate the contribution from the first self-energy topology by Π^1 and from the second topology by Π^2. We begin with the discontinuities where no loops are left in the final result. These are the discontinuities given by Eqs. (3.6) and (3.7) for the first self-energy topology and Eqs. (5.4), (5.6), and (5.8) for the second self-energy topology. These discontinuities will result in physical amplitudes for three kinds of processes: photon decay, Compton scattering, and pair creation.

A. Photon decay and formation

We begin by analyzing the terms that containing the delta function $\delta(E-k-q-E_{k-q})$. The contributions to this from Π^1 are

$$\mathrm{disc}[\Pi^{1\mu}_{\mu}(A)](E-k-E_{q-k}-q)=(+2\pi i)8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6q}[1/2-\tilde{n}(k)]\delta(E-k-E_{q-k}-q)$$
$$\times\frac{s_3s_4[\hat{k}_+\cdot\widehat{(q-k)}_+][\hat{k}_{s_4}\cdot\widehat{(q-k)}_{s_3}][-1+\tilde{n}(E_{q-k})-n(q)]}{[k+q+s_3E_{q-k}][q+E_{q-k}+s_4k]} \tag{6.1}$$

and

$$\mathrm{disc}[\Pi^{1\mu}_{\mu}(D)](E-k-E_{q-k}-q)=(-2\pi i)8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6q}[1/2-\tilde{n}(E_{q-k}+q)]\delta(E-k-E_{q-k}-q)$$
$$\times\frac{s_3s_4[\hat{k}_+\cdot\widehat{(q-k)}_+][\hat{k}_{s_4}\cdot\widehat{(q-k)}_{s_3}][1-\tilde{n}(E_{q-k})+n(q)]}{[k+q+s_3E_{q-k}][q+E_{q-k}+s_4k]}. \tag{6.2}$$

Note that

$$\tilde{n}(E_{q-k}+q)[1-\tilde{n}(E_{q-k})+n(q)]=\tilde{n}(E_{q-k})n(q).$$

Using the above identity we may combine the two terms and rewrite the distribution functions to give

$$\begin{aligned}
&\mathrm{disc}[\Pi^{1\mu}_{\mu}](E-k-E_{q-k}-q)\\
&=(-2\pi i)8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6q}\{[1-\tilde{n}(k)][1+n(q)]\\
&\times[1-\tilde{n}(E_{q-k})]-\tilde{n}(k)n(q)\tilde{n}(E_{q-k})\}\\
&\times\frac{s_3s_4[\hat{k}_+\cdot\widehat{(q-k)}_+][\hat{k}_{s_4}\cdot\widehat{(q-k)}_{s_3}]}{[k+q+s_3E_{q-k}][q+E_{q-k}+s_4k]}\\
&\times\delta(E-k-E_{q-k}-q). \qquad (6.3)
\end{aligned}$$

We may combine the coefficients of the same delta function from the second self-energy to get

$$\begin{aligned}
&\mathrm{disc}[\Pi^{2\mu}_{\mu}](E-k-q-E_{k-q})\\
&=2\times(-2\pi i)8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6q}\{[1-\tilde{n}(k)][1+n(q)]\\
&\times[1-\tilde{n}(E_{q-k})]-\tilde{n}(k)n(q)\tilde{n}(E_{q-k})\}\\
&\times\frac{\widehat{(k-q)}_+\cdot\hat{k}_+}{[E_{k-q}+q-k]^2}\,\delta(E-k-E_{k-q}-q). \qquad (6.4)
\end{aligned}$$

The overall factor of 2 is the ratio of the symmetry factor of this diagram to the denominator obtained from perturbation theory. Note that we obtain the same form of the distribution functions, this indicates the generic structure of heavy photon decay and reformation. In the distribution function factor, terms like $1+n(q)$ indicate Bose-Einstein enhancement in the emission of a gluon. The 1 is from spontaneous emission and $n(q)$ represents stimulated emission of a boson into a thermal bath. Terms like $1-\tilde{n}(k)$ represent the "Pauli blocked" emission of a quark of momentum k into the thermal bath. The product of the three factors $[1-\tilde{n}(k)][1+n(q)][1-\tilde{n}(E_{q-k})]$ (along with the phase space integral and delta function) can thus be interpreted as the statistical factor associated with a heavy photon, outside a thermal bath, decaying by emitting a quark, an antiquark, and a gluon into a thermal bath. Subtracted from this is the factor $\tilde{n}(k)n(q)\tilde{n}(E_{q-k})$; this represents the formation of a heavy photon from a quark, an antiquark, and a gluon, all three emitted from the thermal bath (Fig. 3). The photon subsequently escapes from the bath without further interaction.

To convert the above expressions into cross sections for heavy photon decay and reformation, we start by first defining a new four-vector $\mathbf{w}=(w,\vec{w})$, such that $w=E-k-q$ (in order to avoid confusion we introduce the notation of four-vectors as boldface characters). This relation is indicated by the one-dimensional delta function. To obtain the probability

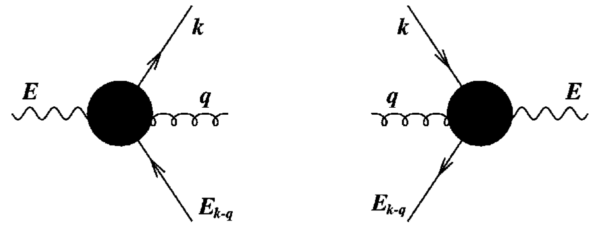

FIG. 3. Heavy-photon decay and formation.

of photon decay, we need to generalize the delta function to a four-delta function. We thus need to generalize the definition of $\mathbf{w}$,

$$\mathbf{w}=\mathbf{p}-\mathbf{k}-\mathbf{q},$$

where $\mathbf{p}=(E,0,0,0)$ is the mass of the off-shell photon. As denoted by Fig. 3, $\mathbf{k}$, $\mathbf{q}$, and $\mathbf{w}$ are all on-shell. The above relation also implies $\vec{w}=-\vec{k}-\vec{q}$. We may set $\mathbf{k}^0=k=|\vec{k}|$ and $\mathbf{q}^0=q=|\vec{q}|$. Now, requiring that $\mathbf{w}$ be on-shell imposes the condition that

$$(E-k-q)^2=k^2+q^2+2kq\cos\theta$$

$$\Rightarrow E(E-2k-2q)=-\mathbf{k}\cdot\mathbf{q},$$

where θ is the angle between the three-vectors $\vec{q}$ and $\vec{k}$. Using the above relations we many now rewrite the discontinuity obtained from Π^2. In the numerator of the integrand we notice the factor $\widehat{(k-q)}_+\cdot\hat{k}_+$, this may be changed appropriately by setting $\vec{k}\leftrightarrow-\vec{k}$ in the integrand. Noting that $\widehat{(-k)}_s=-\hat{k}_{-s}$ we get the above factor as $-\hat{w}_+\cdot\hat{k}_-$. Introducing the standard denominators $2k$,$2w$, and factors of 2π we obtain Eq. (6.4) as

$$\begin{aligned}
&\mathrm{disc}[\Pi^{2\mu}_{\mu}](E-k-q-E_{k-q})\\
&=-2i\times8e^2g^2\int\frac{d^3kd^3qd^3w}{(2\pi)^9 2q2k2w}8(2\pi)^4\\
&\times\delta^4(\mathbf{p}-\mathbf{k}-\mathbf{q}-\mathbf{w})\{[1-\tilde{n}(k)][1+n(q)]\\
&\times[1-\tilde{n}(w)]-\tilde{n}(k)n(q)\tilde{n}(w)\}\left[-\frac{\mathbf{w}_+\cdot\mathbf{k}_-}{[w+q-k]^2}\right]. \qquad (6.5)
\end{aligned}$$

We now split the above integrand into two parts and in one of them switch $w\leftrightarrow k$. Note that $-\mathbf{w}_+\cdot\mathbf{k}_-=wk+\vec{w}\cdot\vec{k}=(E-2k)(E-2w)/2$ and we finally get the above discontinuity as

$$\begin{aligned}
&\mathrm{disc}[\Pi^{2\mu}_{\mu}](E-k-q-E_{k-q})\\
&=-i\int\frac{d^3kd^3qd^3w}{(2\pi)^9 2q2k2w}(2\pi)^4\delta^4(\mathbf{p}-\mathbf{k}-\mathbf{q}-\mathbf{w})\\
&\times\{[1-\tilde{n}(k)][1+n(q)][1-\tilde{n}(w)]\\
&-\tilde{n}(k)n(q)\tilde{n}(w)\}32e^2g^2\left[\frac{E-2k}{E-2w}+\frac{E-2w}{E-2k}\right]. \qquad (6.6)
\end{aligned}$$

We now perform the same procedure on the corresponding discontinuity from Π^1 to get

$$\begin{aligned}\text{disc}[\Pi^{1\mu}_{\mu}](E-k-q-E_{k-q}) &= -i\times 8e^2g^2\int \frac{d^3kd^3qd^3w}{(2\pi)^9 2q2k2w}8(2\pi)^4\\ &\times \delta^4(\mathbf{p}-\mathbf{k}-\mathbf{q}-\mathbf{w})\{[1-\tilde{n}(k)][1+n(q)]\\ &\times[1-\tilde{n}(w)]-\tilde{n}(k)n(q)\tilde{n}(w)\}\\ &\times\frac{s_3s_4[\mathbf{k}_+\cdot\mathbf{w}_+][\hat{k}_{s_4}\cdot\hat{w}_{s_3}]}{[k+q+s_3w][q+w+s_4k]}. \end{aligned} \tag{6.7}$$

The part of the integrand besides the distribution function part (depends on the angle between $\vec{k}$ and $\vec{q}$, and will be denoted as the matrix part) may be expanded by summing over s_3, s_4 as

$$\frac{\mathbf{k}\cdot\mathbf{w}}{kw}\left[\frac{\mathbf{k}_+\cdot\mathbf{w}_+}{(k+q+w)^2}+\frac{\mathbf{k}_+\cdot\mathbf{w}_-}{(k+q)^2-w^2}+\frac{\mathbf{k}_-\cdot\mathbf{w}_+}{(q+w)^2-k^2}+\frac{\mathbf{k}_-\cdot\mathbf{w}_-}{q^2-(k-w)^2}\right].$$

Using the relations

$$\mathbf{k}_+\cdot\mathbf{w}_-=\mathbf{k}_-\cdot\mathbf{w}_+=-(1/2)[E-2k][E-2w],$$

$$\mathbf{k}_+\cdot\mathbf{w}_+=\mathbf{k}_-\cdot\mathbf{w}_-=(E/2)[E-2q],$$

and the relation imposed by the delta function (i.e., $E=k+q+w$) we can simplify the matrix part to give

$$\frac{E(E-2q)}{[E-2w][E-2k]},$$

substituting the above into the expression for Π^1 and then combining the results from Π^1 and Π^2 we get

$$\begin{aligned}\text{disc}[\Pi^{\mu}_{\mu}](E-k-q-E_{k-q}) &= -i\int \frac{d^3kd^3qd^3w}{(2\pi)^9 2q2k2w}(2\pi)^4\delta^4(\mathbf{p}-\mathbf{k}-\mathbf{q}-\mathbf{w})\\ &\times\{[1-\tilde{n}(k)][1+n(q)][1-\tilde{n}(w)]\\ &-\tilde{n}(k)n(q)\tilde{n}(w)\}32e^2g^2\\ &\times\left[\frac{E-2k}{E-2w}+\frac{E-2w}{E-2k}+2\frac{E(E-2q)}{[E-2w][E-2k]}\right]. \end{aligned} \tag{6.8}$$

Photon decay into a quark, an antiquark, and a gluon at first order in the electromagnetic and strong coupling constant can occur by two types of Feynman diagrams [17] as shown in Fig. 4. The matrix element for the first diagram may be written as $\mathcal{M}_1=\mathcal{M}_1^{\mu}\epsilon_{\mu}(\mathbf{p})$, where

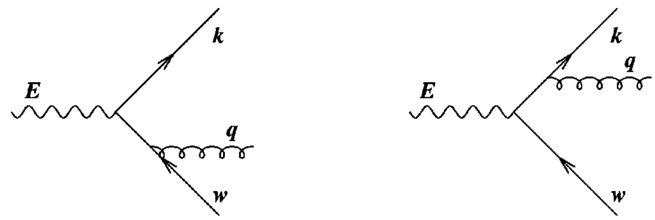

FIG. 4. Heavy-photon decay at first order in α and α_s.

$$\mathcal{M}_1^{\mu}=\overline{u}(k)ie\gamma^{\mu}\frac{i(\not{k}-\not{p})}{(k-p)^2}ig\gamma^{\rho}\epsilon^*_{\rho}(q)v(w),$$

and for the second diagram as

$$\mathcal{M}_2^{\mu}=\overline{u}(k)ig\gamma^{\rho}\epsilon^*_{\rho}(q)\frac{i(\not{p}-\not{w})}{(p-w)^2}ie\gamma^{\mu}v(w).$$

Taking the product $\mathcal{M}_1^{*\mu}\mathcal{M}_{1\mu}$ and summing over the spins and colors of the quark, the antiquark, and the gluon gives

$$\mathcal{M}_1^{*\mu}\mathcal{M}_{1\mu}=-32e^2g^2\frac{E-2w}{E-2k}. \tag{6.9}$$

Similarly

$$\mathcal{M}_2^{*\mu}\mathcal{M}_{2\mu}=-32e^2g^2\frac{E-2k}{E-2w}. \tag{6.10}$$

Notice that as the three three-vectors $\vec{k},\vec{q},\vec{w}$ form a triangle, $E-2k=w+q-k$ is always positive. By the same argument $E-2w$ and $E-2q$ are also positive. We thus note that $\mathcal{M}_1^{*\mu}\mathcal{M}_{1\mu}$ and $\mathcal{M}_2^{*\mu}\mathcal{M}_{2\mu}$ are negative. This is to be expected, as the square of the full matrix element $|\mathcal{M}|^2$ is positive, where from the sum over the photon's spin we get

$$|\mathcal{M}|^2=-g_{\mu,\nu}\mathcal{M}^{*\mu}\mathcal{M}^{\nu}=-\mathcal{M}^{*\mu}\mathcal{M}_{\mu}.$$

The cross term is

$$\mathcal{M}_2^{*\mu}\mathcal{M}_{1\mu}=-32e^2g^2\frac{2E(E-2q)}{[E-2w][E-2k]}. \tag{6.11}$$

Comparing the above three equations with the result from the loop calculation [Eqs. (6.8) and (6.6)] gives us the relations

$$\begin{aligned}\text{disc}[\Pi^{2\mu}_{\mu}](E-k-q-E_{k-q}) &= i\int \frac{d^3kd^3qd^3w}{(2\pi)^9 2q2k2w}(2\pi)^4\delta^4(\mathbf{p}-\mathbf{k}-\mathbf{q}-\mathbf{w})\{[1-\tilde{n}(k)]\\ &\times[1+n(q)][1-\tilde{n}(w)]-\tilde{n}(k)n(q)\tilde{n}(w)\}\\ &\times[\mathcal{M}_1^{*\mu}\mathcal{M}_{1\mu}+\mathcal{M}_2^{*\mu}\mathcal{M}_{2\mu}], \end{aligned} \tag{6.12}$$

$$\text{disc}[\Pi^{1\mu}_{\mu}](E-k-q-E_{k-q})$$
$$=i\int\frac{d^3kd^3qd^3w}{(2\pi)^9 2q2k2w}(2\pi)^4\delta^4(\mathbf{p}-\mathbf{k}-\mathbf{q}-\mathbf{w})$$
$$\times\{[1-\tilde{n}(k)][1+n(q)][1-\tilde{n}(w)]-\tilde{n}(k)n(q)\tilde{n}(w)\}$$
$$\times[\mathcal{M}_2^{*\mu}\mathcal{M}_{1\mu}+\mathcal{M}_1^{*\mu}\mathcal{M}_{2\mu}], \quad (6.13)$$

and hence we get the relation written down by Weldon [1],

$$\text{disc}[\Pi^{\mu}_{\mu}](E-k-q-E_{k-q})$$
$$=i\int\frac{d^3kd^3qd^3w}{(2\pi)^9 2q2k2w}(2\pi)^4\delta^4(\mathbf{p}-\mathbf{k}-\mathbf{q}-\mathbf{w})$$
$$\times\{[1-\tilde{n}(k)][1+n(q)][1-\tilde{n}(w)]-\tilde{n}(k)n(q)\tilde{n}(w)\}$$
$$\times[\mathcal{M}^{*\mu}\mathcal{M}_{\mu}], \quad (6.14)$$

where $\mathcal{M}=\mathcal{M}^{\mu}\epsilon_{\mu}(p)=\mathcal{M}_1+\mathcal{M}_2$ is the full matrix element of heavy photon decay.

B. Compton scattering

The analysis for Compton scattering is slightly more tricky. Note that there are two sets of terms from Eqs. (3.6) and (3.7) and Eqs. (5.4), (5.6), and (5.8) that may lead to Compton scattering. One appears with the delta function $\delta(E+k-q-E_{k-q})$ and the other with the delta function $\delta(E+E_{k-q}-k-q)$. The delta functions can be converted into one another simply by replacing $\vec{k}\to\vec{k}+\vec{q}$, followed by $\vec{q}\to-\vec{q}$. One notes on performing this operation that the rest of the integrand looks rather different. This happens as there are four topologically distinct diagrams that may fall under the category of Compton scattering (it is well known that for a given in-state there are two diagrams that lead to Compton scattering; there are four here as we sum over the possibilities of the incoming fermion being a quark or an antiquark). Let us consider the contribution from $\Pi^{1\mu}_{\mu}$,

$$\text{disc}[\Pi^{1\mu}_{\mu}](E+k-E_{q-k}-q)$$
$$=(2\pi i)8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6 q}\{\tilde{n}(k)[1-\tilde{n}(E_{q-k})]$$
$$\times[1+n(q)]-[1-\tilde{n}(k)]\tilde{n}(E_{q-k})n(q)\}$$
$$\times\frac{s_3s_4[\hat{k}_-\cdot\widehat{(q-k)}_+][\hat{k}_{s_4}\cdot\widehat{(q-k)}_{s_3}]}{[-k+q+s_3E_{q-k}][q+E_{q-k}+s_4k]}$$
$$\times\delta(E+k-E_{q-k}-q). \quad (6.15)$$

For the contribution from $\Pi^{2\mu}_{\mu}$, recall that we have an overall factor of 2 in each of the results of Eqs. (5.4), (5.6), and (5.8) coming from the overall symmetry factor of $\Pi^{2\mu}_{\mu}$ being double that of $\Pi^{1\mu}_{\mu}$. We take half of the contribution from the $\delta(E+k-E_{k-q}-q)$ term and half from the $\delta(E+E_{k-q}-k-q)$ term, and in the second contribution change $\vec{k}\to\vec{k}+\vec{q}$, followed by $\vec{q}\to-\vec{q}$. This gives the total contribution from $\Pi^{2\mu}_{\mu}$ as

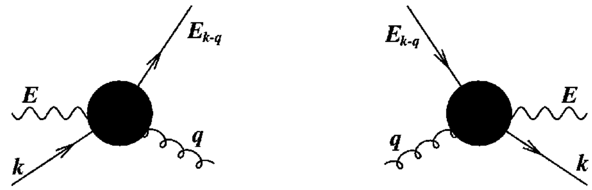

FIG. 5. Quark Compton scattering.

$$\text{disc}[\Pi^{2\mu}_{\mu}](E+k-E_{q-k}-q)$$
$$=(2\pi i)8e^2g^2\int\frac{d^3kd^3q}{(2\pi)^6 q}\{\tilde{n}(k)[1-\tilde{n}(E_{q-k})]$$
$$\times[1+n(q)]-[1-\tilde{n}(k)]\tilde{n}(E_{q-k})n(q)\}$$
$$\times\left[\frac{\hat{k}_-\cdot\widehat{(q-k)}_+}{[k+q+E_{q-k}]^2}+\frac{\hat{k}_-\cdot\widehat{(q-k)}_+}{[k-q+E_{q-k}]^2}\right]$$
$$\times\delta(E+k-E_{q-k}-q). \quad (6.16)$$

Notice that the combinations of distribution functions appearing in the curly brackets are identical to Eq. (6.15). The product of the three factors $\tilde{n}(k)[1-\tilde{n}(E_{q-k})][1+n(q)]$ has the interpretation of an incoming quark (or an antiquark) from the medium fusing with a photon coming in from outside the bath, resulting in a gluon and a quark (or an antiquark) going into the medium. Subtracted from this is the product $[1-\tilde{n}(k)]\tilde{n}(E_{q-k})n(q)$, which has the interpretation of an incoming quark (antiquark) from the medium fusing with an incoming gluon from the medium, resulting in a quark (antiquark) going back into the medium, and a virtual photon that leaves the medium (Fig. 5).

To convert the above expressions into cross sections for Compton scattering, we again define the new four-vector $\mathbf{w}=(w,\vec{w})$, such that $w=E+k-q$. This is generalized to

$$\mathbf{w}=\mathbf{p}+\mathbf{k}-\mathbf{q},$$

as a result $\vec{w}=\vec{k}-\vec{q}$. Now, requiring that $\mathbf{w}$ be on-shell imposes the condition that

$$(E+k-q)^2=k^2+q^2-2kq\cos\theta$$
$$\Rightarrow E(E+2k-2q)=\mathbf{k}\cdot\mathbf{q}.$$

Using the above relations we many now rewrite the discontinuity obtained from Π^2. In the numerators of the integrand we notice the factor $\widehat{(k-q)}_+\cdot\hat{k}_-$, which may be written as $\hat{\mathbf{w}}_+\cdot\hat{\mathbf{k}}_-=(E+2k)(E-2w)/2$. We introduce the standard denominators $2k$,$2w$, and factors of 2π and perform a similar set of operations as for photon decay to obtain the full result for Compton scattering as

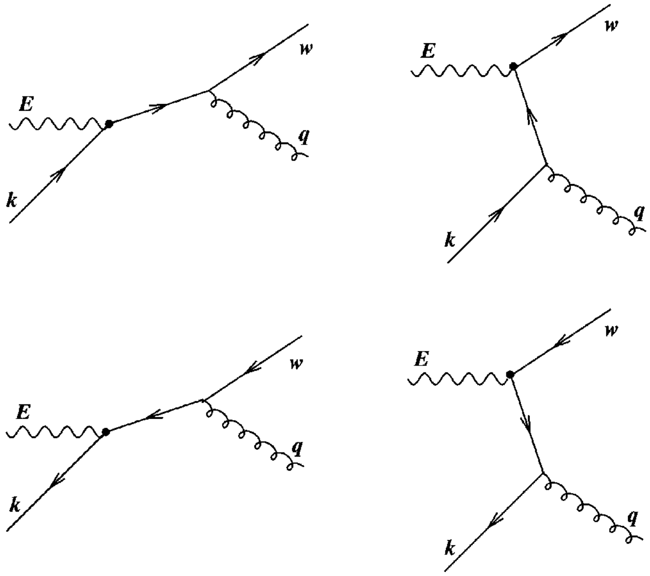

FIG. 6. Compton scattering at first order in α and α_s.

$$\mathrm{disc}[\Pi_\mu^\mu](E+k-q-E_{k-q})$$
$$=i\int \frac{d^3k d^3q d^3w}{(2\pi)^9 2q2k2w}(2\pi)^4\delta^4(\mathbf{p}+\mathbf{k}-\mathbf{q}-\mathbf{w})$$
$$\times\{\tilde{n}(k)[1+n(q)][1-\tilde{n}(w)]$$
$$-[1-\tilde{n}(k)]n(q)\tilde{n}(w)\}32e^2g^2$$
$$\times\left[\frac{E+2k}{E-2w}+\frac{E-2w}{E+2k}+2\frac{E(E-2q)}{[E-2w][E+2k]}\right]. \quad (6.17)$$

Recall that in $\Pi_\mu^{1\mu}$ there is another term, the coefficient of the delta function $\delta(E+E_{q-k}-k-q)$, which leads to Compton scattering. Also, in the Compton scattering contributions from Π^2, we only used a half of both the terms. Following almost the same method as above, one can demonstrate that the form of the contribution from these terms is almost the same as above with k and w interchanged. In it, one may interchange $\vec{w}\to\vec{k}$ to get the same contribution as Eq. (6.17), hence doubling the total contribution from Compton scattering.

Compton scattering by an incoming photon of a thermal medium of quarks and antiquarks, at first order in the electromagnetic and strong coupling constant, can occur as a result of four processes as shown in Fig. 6. The matrix element for the diagrams may be written as $\mathcal{M}_n=\mathcal{M}_n^\mu\epsilon_\mu(\mathbf{p})$, where

$$\mathcal{M}_1^\mu=\overline{u(\mathbf{w})}ig\gamma^\rho\epsilon_\rho^*(\mathbf{q})i\frac{\not{\mathbf{p}}+\not{\mathbf{k}}}{(\mathbf{p}+\mathbf{k})^2}ie\gamma^\mu\epsilon_\mu(\mathbf{p})u(\mathbf{k}),$$

$$\mathcal{M}_2^\mu=\overline{u(\mathbf{w})}ie\gamma^\mu\epsilon_\mu(\mathbf{p})\frac{\not{\mathbf{w}}-\not{\mathbf{p}}}{(\mathbf{w}-\mathbf{p})^2}ig\gamma^\rho\epsilon_\rho^*(\mathbf{q})u(\mathbf{k}).$$

The amplitude for the third and fourth diagram can be obtained from the two amplitudes above by simply changing $u\to v$. Taking the products and summing over spins and colors (remember diagrams 1 and 2 interfere with each other, and so do 3 and 4), we get

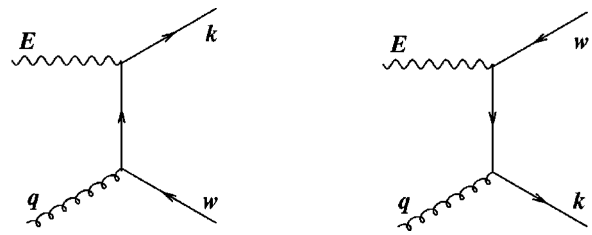

FIG. 7. Pair creation at first order in α and α_s.

$$\mathcal{M}_1^{*\mu}\mathcal{M}_{1\mu}=32e^2g^2\frac{E-2w}{E+2k}, \quad (6.18)$$

$$\mathcal{M}_2^{*\mu}\mathcal{M}_{2\mu}=32e^2g^2\frac{E+2k}{E-2w}. \quad (6.19)$$

Once again, note that $\mathcal{M}_1^{*\mu}\mathcal{M}_{1\mu}$ and $\mathcal{M}_2^{*\mu}\mathcal{M}_{2\mu}$ are negative. This is because $E-2w=q-k-w$ is always negative due to the triangle condition mentioned in the preceding section. The cross term is

$$\mathcal{M}_2^{*\mu}\mathcal{M}_{1\mu}=32e^2g^2\frac{2E(E-2q)}{[E-2w][E+2k]}. \quad (6.20)$$

Comparing the above three equations with the result from the loop calculation [Eq. (6.17)] gives us the relation

$$\mathrm{disc}[\Pi_\mu^\mu](E+k-q-E_{k-q})$$
$$=i\int \frac{d^3k d^3q d^3w}{(2\pi)^9 2q2k2w}(2\pi)^4\delta^4(\mathbf{p}+\mathbf{k}-\mathbf{q}-\mathbf{w})$$
$$\times\{\tilde{n}(k)[1+n(q)][1-\tilde{n}(w)]-[1-\tilde{n}(k)]n(q)\tilde{n}(w)\}$$
$$\times[\mathcal{M}_1^{*\mu}\mathcal{M}_{1\mu}+\mathcal{M}_2^{*\mu}\mathcal{M}_{2\mu}+2\mathcal{M}_2^{*\mu}\mathcal{M}_{1\mu}]. \quad (6.21)$$

Once again we note the interesting fact that in this gauge the mixed terms $\mathcal{M}_2^{*\mu}\mathcal{M}_{1\mu}+\mathcal{M}_1^{*\mu}\mathcal{M}_{2\mu}$ are always given by $\Pi_\mu^{1\mu}$ and the square terms $\mathcal{M}_1^{*\mu}\mathcal{M}_{1\mu}+\mathcal{M}_2^{*\mu}\mathcal{M}_{2\mu}$ are furnished by $\Pi_\mu^{2\mu}$.

C. Pair creation

The analysis for pair creation (often referred to as photon-gluon fusion) is almost identical to that done in the two preceding sections. Its contribution is furnished by the only remaining delta functions in Eqs. (3.6) and (3.7) in the first self-energy and Eqs. (5.4), (5.6), and (5.8) in the second self-energy, i.e., $\delta(E+q-k-E_{k-q})$. We simply state the results here: pair creation can occur through two types of processes (Fig. 7) and has the discontinuity in the total self-energy

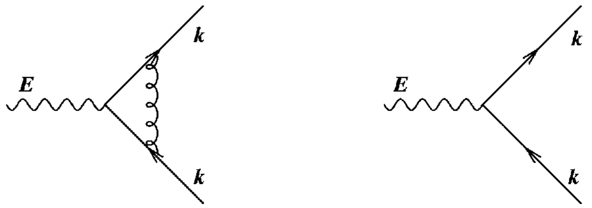

FIG. 8. Photon decay at one loop corresponding to the cut $\delta(E-2E_{k-q})$.

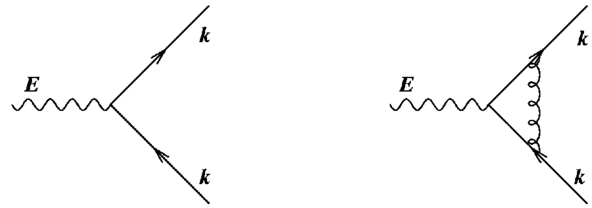

FIG. 9. Photon decay at one loop corresponding to the cut $\delta(E-k)$.

$$\begin{aligned}
&\mathrm{disc}[\Pi^{2\mu}_{\mu}](E+q-k-E_{k-q})\\
&\quad=-i\int\frac{d^3k d^3q d^3w}{(2\pi)^9 2q2k2w}(2\pi)^4\delta^4(\mathbf{p}-\mathbf{k}+\mathbf{q}-\mathbf{w})\\
&\qquad\times\{[1-\tilde{n}(k)]n(q)[1-\tilde{n}(w)]-[1+n(q)]\tilde{n}(k)\tilde{n}(w)\}\\
&\qquad\times 32e^2g^2\left[\frac{E-2k}{E-2w}+\frac{E-2w}{E-2k}+2\frac{E(E+2q)}{[E-2w][E-2k]}\right].
\end{aligned}\tag{6.22}$$

VII. PHYSICAL INTERPRETATION: LOOP-CONTAINING CUTS

We now analyze the various discontinuities of $\Pi^{1\mu}_{\mu}$ and $\Pi^{2\mu}_{\mu}$ that contain loops. We start with the discontinuities of $\Pi^{1\mu}_{\mu}$. These are given by Eqs. (3.3)–(3.5). We note that there are two terms with the delta function $\delta(E-2E_{k-q})$, these correspond to the cut of Fig. 8. There is also one term with the cut $\delta(E-2k)$, this corresponds to the cut of Fig. 9.

One may be satisfied with this interpretation of the cut diagrams and not proceed further. A recent paper [12], however, has drawn attention to the fact that one can obtain a somewhat different interpretation of these diagrams, in terms of interference between simple treelike diagrams and diagrams containing particles called "spectators." Spectators are essentially on-shell particles from the heat bath that enter with the "in" state and leave with the "out" state without having interacted with the the rest of the "participants."

We start by summing over the variable s_1 in Eq. (3.3). This immediately gives two terms, distinguished by the combination of distribution functions they carry,

$$\begin{aligned}
&\mathrm{disc}[2\Pi^{\mu}_{\mu}(A)]_{a1}\\
&\quad=(-2\pi i)8e^2g^2\\
&\qquad\times\int\frac{d^3k d^3q}{(2\pi)^6}\frac{[\hat{k}_+\cdot\widehat{(q-k)}_{s_2}][\hat{k}_-\cdot\widehat{(q-k)}_{s_3}]}{q(E-[s_2-s_3]E_{q-k})}\\
&\qquad\times s_2\frac{[1-2\tilde{n}(k)][1-2\tilde{n}(E_{q-k})]q}{(k+s_3E_{k-q})^2-q^2}\delta(E-2k)
\end{aligned}\tag{7.1}$$

and

$$\begin{aligned}
&\mathrm{disc}[2\Pi^{\mu}_{\mu}(A)]_{a2}\\
&\quad=(-2\pi i)8e^2g^2\\
&\qquad\times\int\frac{d^3k d^3q}{(2\pi)^6}\frac{[\hat{k}_+\cdot\widehat{(q-k)}_{s_2}][\hat{k}_-\cdot\widehat{(q-k)}_{s_3}]}{q(E-[s_2-s_3]E_{q-k})}\\
&\qquad\times\delta(E-2k)(-s_2)[1-2\tilde{n}(k)][1+2n(q)]\\
&\qquad\times\frac{s_3k+E_{q-k}}{(k+s_3E_{k-q})^2-q^2}.
\end{aligned}\tag{7.2}$$

In Eq. (7.1), if we replace $s_2\to -s_4$, $s_3\to -s$, followed by $\vec{k}\to\vec{q}-\vec{k}$ we get

$$\begin{aligned}
&\mathrm{disc}[2\Pi^{\mu}_{\mu}(A)]_{a1}\\
&\quad=(+2\pi i)8e^2g^2\\
&\qquad\times\int\frac{d^3k d^3q}{(2\pi)^6}\frac{[\widehat{(q-k)}_-\cdot\hat{k}_{s_4}][\widehat{(q-k)}_+\cdot\hat{k}_s]}{q(E-[s-s_4]E_{q-k})}\\
&\qquad\times s_4\frac{[1-2\tilde{n}(k)][1-2\tilde{n}(E_{q-k})]q}{(E_{q-k}-sk)^2-q^2}\delta(E-2E_{q-k}).
\end{aligned}\tag{7.3}$$

The above is exactly the same as Eq. (3.4). This is to be expected as the two cuts should, in principle, represent the same diagram up to a shift in momenta. We thus double this contribution and focus on it. It represents photon decay into two quarks with quark emission and absorption from the final state quarks. The other part from Eq. (7.2) along with Eq. (3.5) will represent photon decay with gluon emission and absorption off the external quarks.

A. Photon decay with quark emission absorption off vertex and final state

We begin by summing over the remaining sign variables s_2, s_3 in $\mathrm{disc}[2\Pi^{\mu}_{\mu}(A)]_{a1}$ to get

$$\text{disc}[2\Pi^{\mu}_{\mu}]_4 = 2\times(-2\pi i)8e^2g^2 \int \frac{d^3k d^3q}{(2\pi)^6}[1-2\tilde{n}(k)]$$
$$\times[1-2\tilde{n}(E_{q-k})]\delta(E-2k)$$
$$\times\left[\frac{[\hat{k}_+\cdot\widehat{(q-k)}_+][\hat{k}_-\cdot\widehat{(q-k)}_+]}{E[(k+E_{k-q})^2-q^2]}\right.$$
$$-\frac{[\hat{k}_+\cdot\widehat{(q-k)}_-][\hat{k}_-\cdot\widehat{(q-k)}_+]}{(E+2E_{q-k})[(k+E_{k-q})^2-q^2]}$$
$$+\frac{[\hat{k}_+\cdot\widehat{(q-k)}_+][\hat{k}_-\cdot\widehat{(q-k)}_-]}{(E-2E_{q-k})[(k-E_{k-q})^2-q^2]}$$
$$\left.-\frac{[\hat{k}_+\cdot\widehat{(q-k)}_-][\hat{k}_-\cdot\widehat{(q-k)}_-]}{E[(k-E_{k-q})^2-q^2]}\right]. \quad (7.4)$$

As in the preceding section, the distribution functions will be reorganized to allow for an interpretation in terms of thermal weights for particle emission and absorption. In the first two terms we define the new lightlike four-vector $\mathbf{w}$ such that $\vec{w}=\vec{q}-\vec{k}$. In the last two terms we define $\mathbf{w}$ such that $\vec{w}=-\vec{q}+\vec{k}$. This allows us to change the variable of integration $d^3q \rightarrow d^3w$, as k is a constant as far as the q integration is concerned. We may also redefine the distribution functions as

$$[1-2\tilde{n}(k)][1-2\tilde{n}(w)]$$
$$=\{[1-\tilde{n}(k)][1-\tilde{n}(k)]-\tilde{n}(k)\tilde{n}(k)\}$$
$$\times\{[1-\tilde{n}(w)]-[\tilde{n}(w)]\}.$$

The first set of factors in the curly brackets has the usual interpretation [1] of the thermal factors that are associated with the probability of particle emission into a heat bath or particle absorbtion from a heat bath. In this case they carry the obvious meaning of [emit fermion of energy k][emit fermion of energy $k-$][absorb fermion of energy k][absorb fermion of energy k]. The reader will note that unlike the self-energy cuts considered in [1] or those of the preceding section, the two cut diagrams that will result from this imaginary part of the self-energy will not be symmetric, in the sense that it will be the interference between a diagram with a loop and a simple tree diagram. The thermal factors discussed above will be the same for either diagram as they pertain to the quark and antiquark that emanate from the decay of the photon (or those that combine to form the photon). Both amplitudes that result from this imaginary part contain this process and thus have identical thermal factors.

The second set of thermal factors has a new interpretation. These thermal factors pertain to the particles in the remaining loop and thus are germane to only one of the two interfering amplitudes. We will demonstrate that these signal the difference of two amplitudes: that for the emission of a quark or an antiquark into the bath and its subsequent absorption from the bath, and vice versa. Thus the second set of distribution functions is to be understood as [emit fermion of four-momentum $\mathbf{w}$][absorb the same fermion of four-momentum $w-$][absorb fermion of four-momentum $\mathbf{w}$][emit the same fermion of four-momentum $\mathbf{w}$].

The process of emission of a fermion of four-momentum $\mathbf{w}$ into a bath followed by its reabsorption is formally achieved by the action of creation and annihilation operators on the bath state $|\tilde{n}_{\mathbf{w}}\rangle$, i.e.,

$$aa^{\dagger}|\tilde{n}_{\mathbf{w}}\rangle=(1-\tilde{n}_{\mathbf{w}})|\tilde{n}_{\mathbf{w}}\rangle. \quad (7.5)$$

The reverse process, i.e., the absorption of a fermion from the bath and its subsequent reemission into the bath is formally achieved by the action of annihilation and creation operators on the bath state, i.e.,

$$a^{\dagger}a|\tilde{n}_{\mathbf{w}}\rangle=(\tilde{n}_{\mathbf{w}})|\tilde{n}_{\mathbf{w}}\rangle. \quad (7.6)$$

The discontinuity of the self-energy will represent the amplitude of a particular process multiplied with the complex conjugate of another. In one of these processes the above-mentioned fermion will perform the emission and absorption procedure referred to above. In the other amplitude, as we will show shortly, it will simply enter and leave without having interacted with the rest of the particles. Due to this reason, it has been referred to previously (see [12,8]) as a spectator.

We introduce the usual denominators of $2w$,$2k$ to get

$$\text{disc}[\Pi^{\mu}_{\mu}]_4$$
$$=2\times(-2\pi i)8e^2g^2$$
$$\times\int\frac{d^3k d^3\omega}{(2\pi)^6 kwkw}\delta(E-2k)\{[1-\tilde{n}(k)]$$
$$\times[1-\tilde{n}(k)]-\tilde{n}(k)\tilde{n}(k)\}\{[1-\tilde{n}(w)]-[\tilde{n}(w)]\}$$
$$\times\left[\frac{[\mathbf{k}_+\cdot\mathbf{w}_+][\mathbf{k}_-\cdot\mathbf{w}_+]}{E[(k+w)^2-q^2]}-\frac{[\mathbf{k}_+\cdot\mathbf{w}_-][\mathbf{k}_-\cdot\mathbf{w}_+]}{(E+2w)[(k+w)^2-q^2]}\right.$$
$$\left.+\frac{[\mathbf{k}_+\cdot\mathbf{w}_-][\mathbf{k}_-\cdot\mathbf{w}_+]}{(E-2w)[(k-w)^2-q^2]}-\frac{[\mathbf{k}_+\cdot\mathbf{w}_+][\mathbf{k}_-\cdot\mathbf{w}_+]}{E[(k-w)^2-q^2]}\right]. \quad (7.7)$$

We now introduce the new four-vector $\mathbf{k}_b=(k,-\vec{k})$ and generalize the delta function to a four-delta function. We then combine the first two terms and the last two terms to write

$$\text{disc}[\Pi^{\mu}_{\mu}]_4 = i8e^2g^2\int\frac{d^3k d^3w d^3k_b}{(2\pi)^9 8kwk_b}16(2\pi)^4$$
$$\times\delta^4(\mathbf{p}+\mathbf{w}-\mathbf{k}-\mathbf{k}_b-\mathbf{w})\{[1-\tilde{n}(k)][1-\tilde{n}(k_b)]$$
$$-\tilde{n}(k)\tilde{n}(k_b)\}\{[1-\tilde{n}(w)]-[\tilde{n}(w)]\}$$
$$\times\left[\frac{[\mathbf{k}_b\cdot\mathbf{w}][\mathbf{k}_a\cdot(\mathbf{p}+\mathbf{w})]}{[(E+w)^2-w^2][(k+w)^2-q^2]}\right.$$
$$\left.+\frac{[\mathbf{k}_a\cdot\mathbf{w}][\mathbf{k}_b\cdot(\mathbf{w}-\mathbf{p})]}{[(w-E)^2-w^2][(k-w)^2-q^2]}\right]. \quad (7.8)$$

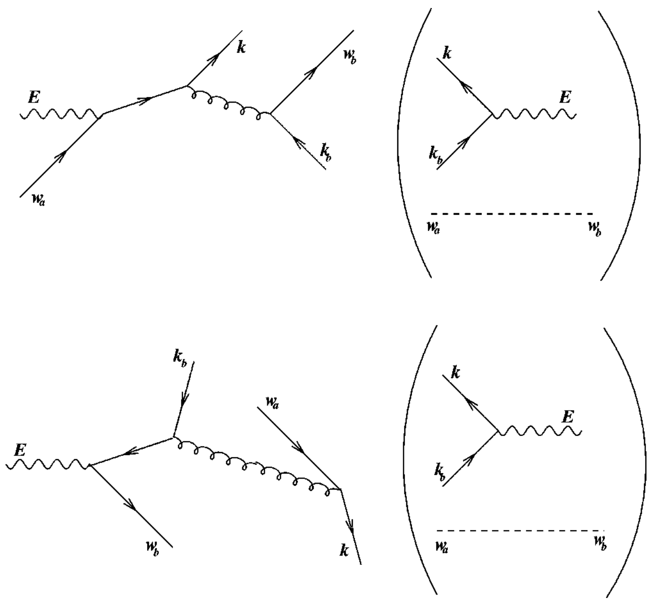

FIG. 10. Interference between diagrams of different order in α_s. The diagrams on the left indicate $2\rightarrow 3$ reactions such as $\gamma Q \rightarrow q\overline{q}Q$ (where Q indicates that the incoming and outgoing quarks are identical). The diagrams on the right indicate the complex conjugate of the Born term photon decay with a comoving quark spectator, i.e., $(\gamma\rightarrow q\overline{q})\otimes(Q\rightarrow Q)$.

The above has the interpretation of Fig. 10. This indicates the interference between two diagrams of different order in coupling constants. Let the matrix elements of the two tree-level diagrams with two propagators be denoted as $\mathcal{M}_1 = \mathcal{M}_1^{\mu}\epsilon_{\mu}(\mathbf{p})$ and $\mathcal{M}_2 = \mathcal{M}_2^{\mu}\epsilon_{\mu}(\mathbf{p})$. The matrix element of the term in brackets is simply denoted as $m^{\mu}\epsilon_{\mu}(\mathbf{p})$. Where the dotted line called the spectator is simply a product of Dirac delta functions over four-momenta and Kronecker delta functions over the spins and colors of the incoming and outgoing fermions denoted by w_a and w_b (here, for brevity we indicate all the different quantum numbers, both continuous and discreet, of the incoming and outgoing particles by a single label).

It is now simple to verify that the result obtained in Eq. (7.8) can be written as

$$\mathrm{disc}[\Pi^{\mu}_{\mu}]_4 = i\int \frac{d^3k d^3w d^3k_b}{(2\pi)^9 8kwk_b}(2\pi)^4\delta^4(\mathbf{p}+\mathbf{w}-\mathbf{k}-\mathbf{k}_b-\mathbf{w}) \times\{[1-\tilde{n}(k)][1-\tilde{n}(k_b)]-\tilde{n}(k)\tilde{n}(k_b)\}[\{1 -\tilde{n}(w)\}-\tilde{n}(w)][2m^{\mu *}\mathcal{M}_1^{\mu}+2m^{\mu *}\mathcal{M}_2^{\mu}], \tag{7.9}$$

where the Kronecker and Dirac delta functions over the fermions w_a and w_b have been used to set $w_a = w_b = w$. The factor of 2 preceding the interference matrix elements is due to the fact that a similar process may be obtained by replacing an incomming quark spectator with an antiquark spectator.

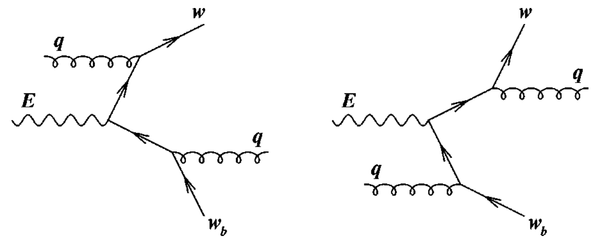

FIG. 11. Photon decay with spectator gluon (the Born term with spectator gluon is implied, see Fig. 10).

B. Photon decay with gluon emission absorption from final state quarks

This term receives contributions from $\mathrm{disc}[2\Pi^{\mu}_{\mu}(A)]_{a2}$ and $\mathrm{disc}[2\Pi^{\mu}_{\mu}(D)]_b$. The fate of this discontinuity is essentially similar to that in the preceding section and results once again in the interference of tree-level diagrams of different order. There are two sets of diagrams with two propagators here as well, the difference being that the incoming and outgoing particle with the same set of quantum numbers, or in other words the spectator, is a gluon. We once again introduce the on-shell four-vector $\mathbf{w}$, such that $\vec{w}=\vec{k}-\vec{q}$. We use this to change the variable of integration in $\mathrm{disc}[2\Pi^{\mu}_{\mu}(D)]_b$. In $\mathrm{disc}[2\Pi^{\mu}_{\mu}(A)]_{a2}$, we relabel the dummy variable $\vec{k}\rightarrow\vec{w}$. Both discontinuities give essentially the same contribution, thus the total discontinuity from such processes (Fig. 11) is given as

$$\mathrm{disc}[\Pi^{\mu}_{\mu}]_5 = i8e^2g^2\int \frac{d^3w d^3q d^3w_b}{(2\pi)^9 8kwk_b}32(2\pi)^4\delta^4(\mathbf{p}+\mathbf{q}-\mathbf{w} -\mathbf{w}_b-\mathbf{q})\{[1-\tilde{n}(w)][1-\tilde{n}(w_b)] -\tilde{n}(w)\tilde{n}(w_b)\}\{[1+n(q)]+[n(q)]\} \times\left[\frac{[\mathbf{w}_b\cdot(\mathbf{q}-\mathbf{w})][\mathbf{w}_a\cdot(\mathbf{q}-\mathbf{w}_b)]}{[(q-w)^2-k^2][(q+w)^2-k^2]}\right], \tag{7.10}$$

which is once again equal to

$$\mathrm{disc}[\Pi^{\mu}_{\mu}]_5 = i\int \frac{d^3w d^3q d^3w_b}{(2\pi)^9 8kwk_b}(2\pi)^4\delta^4(\mathbf{p}+\mathbf{q}-\mathbf{w}-\mathbf{w}_b-\mathbf{q})\} \times[1-\tilde{n}(w)][1-\tilde{n}(w_b)]-\tilde{n}(w)\tilde{n}(w_b)\} \times\{[1+n(q)]+[n(q)]\}[m^{\mu *}\mathcal{M}_1^{\mu}+m^{\mu *}\mathcal{M}_2^{\mu}]. \tag{7.11}$$

where m represents the same process as in the preceding section. The amplitudes $\mathcal{M}_1$ and $\mathcal{M}_2$ represent the processes of Fig. 11. The interpretation of the first set of distribution functions is the same as before, i.e., emission and absorption of two particles of energy k. The second term has the interpretation of a gluon spectator exactly identical to that of the quark spectator in the earlier section, but with the Pauli factors replaced with Bose factors. Note that, unlike in the preceding section, there is no factor of 2 preceding the matrix elements, as the spectators are gluons.

C. Photon decay with quark and gluon emission absorption off the same quark line

We now begin the analysis of the last loop-containing cut. This is essentially given by the discontinuities of Eqs. (5.1) and (5.2). Combining these two discontinuities and writing $k=E-k$ in the denominators of the terms from $\mathrm{disc}[\Pi^{\mu}_{\mu}(A)]_a$ (note that we have to double this contribution as it emanates from the second self-energy diagram, which has a symmetry factor of 2 more than the first self-energy diagram), we get

$$\begin{aligned}\mathrm{disc}[\Pi^{\mu}_{\mu}]_6 &= (2\pi i)8e^2g^2\int\frac{dk\,d\theta\,d\phi\,\sin\theta d^3q}{(2\pi)^6 q}\,\delta(E-2k)\\ &\quad\times[1-2\tilde{n}(k)]\Bigg\{\frac{2k\mathcal{N}\mathcal{S}}{[E-s_2E_{k-q}-s_1q-k]}\\ &\quad+\frac{k^2(\mathcal{N}\mathcal{S})'}{[E-s_2E_{k-q}-s_1q-k]}+\frac{k^2\mathcal{N}\mathcal{S}[1+s_2E'_{k-q}]}{[E-s_2E_{k-q}-s_1q-k]^2}\Bigg\}\\ &= (2\pi i)8e^2g^2\int\frac{dk\,d\theta\,d\phi\,\sin\theta d^3q}{(2\pi)^6 q}\,\delta(E-2k)\\ &\quad\times[1-2\tilde{n}(k)]\frac{d}{dk}\Bigg\{\frac{k^2\mathcal{N}\mathcal{S}}{[E-s_2E_{k-q}-s_1q-k]}\Bigg\}. \qquad (7.12)\end{aligned}$$

The above term does not readily admit a physical interpretation, however, the infrared limit will be evaluated with the above expression as the starting point as it is formally correct. To try and obtain a physical interpretation from the expression given above, an integration by parts is performed to obtain the discontinuity as

$$\begin{aligned}\mathrm{disc}[\Pi^{\mu}_{\mu}]_6 &= (2\pi i)8e^2g^2\\ &\quad\times\int\frac{dk\,d\theta\,d\phi\,\sin\theta d^3q}{(2\pi)^6 q}\left\{-\frac{d\,\delta(E-2k)}{dk}\right\}\\ &\quad\times[1-2\tilde{n}(E/2)]\frac{k^2\mathcal{N}\mathcal{S}}{[E-s_2E_{k-q}-s_1q-k]}\\ &= (2\pi i)8e^2g^2\int\frac{d^3k\,d^3q}{(2\pi)^6 q}\,\frac{\mathcal{N}\mathcal{S}}{[k-s_2E_{k-q}-s_1q]}\\ &\quad\times\{2\,\delta'(E-2k)[1-2\tilde{n}(E/2)]\}, \qquad (7.13)\end{aligned}$$

where we have used the property that

$$\begin{aligned}\frac{d\,\delta(E-2k)}{dk} &= 2\frac{d\,\delta(E-2k)}{d(2k)}\\ &= -2\frac{d\,\delta(E-2k)}{d(E)} = -2\,\delta'(E-2k).\end{aligned}$$

Interestingly, as an aside, we note that one may still obtain a physical interpretation of the above term in terms of spectators with retarded propagators. To obtain this, we expand the factor $\mathcal{N}\mathcal{S}/(k-s_2E_{k-q}-s_1q)$ by summing over s_1 and s_2. Here, as expected, we will obtain a part dependent on Bose distribution functions and a part dependent on Fermi distribution functions. We will illustrate the physical interpretation using the part containing the Bose distribution functions. We begin by writing the δ function in Eq. (7.13) using the following regulator:

$$\delta(x) = \lim_{\epsilon\to 0}\frac{\epsilon}{x^2+\epsilon^2}. \qquad (7.14)$$

In this regulation scheme, we obtain

$$\delta'(x) = -2\left[\frac{\delta(x)}{x+i\epsilon}+i\pi\delta^2(x)\right]. \qquad (7.15)$$

Substituting the above relation in Eq. (7.13) we get

$$\begin{aligned}\mathrm{disc}[\Pi^{\mu}_{\mu}]_6 &= (2\pi i)16e^2g^2\int\frac{d^3k\,d^3q}{(2\pi)^6 q}(-2)\,\delta(E-2k)\\ &\quad\times[1-2\tilde{n}(E/2)]\frac{\mathcal{N}\mathcal{S}}{[E-s_2E_{k-q}-s_1q-k]}\\ &\quad\times\left\{\frac{1}{(E-2k)+i\epsilon}+i\pi\delta(E-2k)\right\}. \qquad (7.16)\end{aligned}$$

We now write unity in the form of an integral as

$$\begin{aligned}1 &= \int_{\infty}^{\infty}d\omega^0\frac{1}{2}\{\delta(\omega^0+k)+\delta(\omega^0+k)\}\\ &= \int_{\infty}^{\infty}d\omega^0 k\,\delta(\omega^{02}-k^2). \qquad (7.17)\end{aligned}$$

Substituting Eq. (7.17) in Eq. (7.16), we obtain

$$\begin{aligned}\mathrm{disc}[\Pi^{\mu}_{\mu}]_6 &= (2\pi i)16e^2g^2\int d\omega^0(E/2)\,\delta(\omega^{02}-[E/2]^2)\int\frac{d^3k\,d^3q}{(2\pi)^6 q}(-2)\,\delta(E-2k)[1-2\tilde{n}(E/2)]\\ &\quad\times\frac{\mathcal{N}\mathcal{S}}{[E-s_2E_{k-q}-s_1q-k]}\left\{\frac{1}{(E-2k)+i\epsilon}+2\pi i\tilde{n}(\omega^0)\,\delta(E-2k)-2\pi i[\tilde{n}(\omega^0)-1/2]\,\delta(E-2k)\right\}\\ &= (2\pi i)(-16e^2g^2)\int d\omega^0\int\frac{d^3k\,d^3q}{(2\pi)^6 q}\,\delta(\omega^{02}-[E/2]^2)\,\delta(E-2k)[1-2\tilde{n}(E/2)]\\ &\quad\times\frac{E\mathcal{N}\mathcal{S}}{[E-s_2E_{k-q}-s_1q-k]}\left\{\frac{1}{(E-2k)+i\epsilon}+2\pi i\tilde{n}(\omega^0)\,\delta(E-2k)-2\pi i[\tilde{n}(\omega^0)-\theta(-\omega^0)]\,\delta(E-2k)\right\}. \qquad (7.18)\end{aligned}$$

In the above we have simply added and subtracted the factor $2\pi i\tilde{n}(\omega^0)\delta(E-2k)$ inside the curly brackets and rewritten 1/2 as $\theta(-\omega^0)$, as the rest of the integrand is an even function of ω^0.

We now introduce the three-vector part of ω as

$$\delta(\omega^{02}-[E/2]^2)\delta(E-k-k)=\int d^3\omega\,\delta(\omega^{02}-(E/2)^2)\delta(E-k-k)\delta^3(-\vec{k}-\vec{\omega})$$

$$=\int d^3\omega\,\delta(\omega^{02}-|\vec{\omega}|^2)\delta(E-k-|\vec{\omega}|)\delta^3(-\vec{k}-\vec{\omega}). \quad (7.19)$$

As stated previously we now concentrate on the part of $\mathcal{NS}/[k-s_2E_{k-q}-s_1q]$, which depends on the Bose distribution function. This gives

$$\mathrm{disc}[\Pi^{\mu}_{\mu}]_6=i\int\frac{d^4\omega}{(2\pi)^4}\int\frac{d^3kd^3q}{(2\pi)^6 2q2k}2\pi\delta(\omega^{02}-|\vec{\omega}|^2)(2\pi)^4\delta^4(\mathbf{p}-\mathbf{k}-\mathbf{w})[\{1-\tilde{n}(k^0)\}\{1-\tilde{n}(\omega^0)\}-\tilde{n}(k^0)\tilde{n}(\omega^0)]$$

$$\times[\{1+n(q)\}+n(q)](-64e^2g^2)E^2\left[\frac{k(E-k-q)-\vec{k}\cdot(\vec{k}-\vec{q})}{(E-k-q)^2-|\vec{k}-\vec{q}|^2}+\frac{k(E-k+q)-\vec{k}\cdot(\vec{k}-\vec{q})}{(E-k+q)^2-|\vec{k}-\vec{q}|^2}\right]\{\tilde{\Delta}_R(\omega)\}$$

$$=i\int\frac{d^4\omega}{(2\pi)^4}\int\frac{d^3kd^3q}{(2\pi)^6 2q2k}2\pi\delta(\omega^{02}-|\vec{\omega}|^2)(2\pi)^4\delta^4(\mathbf{p}-\mathbf{k}-\mathbf{w})[\{1-\tilde{n}(k^0)\}\{1-\tilde{n}(\omega^0)\}-\tilde{n}(k^0)\tilde{n}(\omega^0)]$$

$$\times[\{1+n(q)\}+n(q)][m^{\mu *}\mathcal{M}_{1,\mu}+m^{\mu *}\mathcal{M}_{2,\mu}], \quad (7.20)$$

where $i\not{\omega}\tilde{\Delta}_R(\omega)$ is the retarded propagator. One may note that the integrand in the above equation is simply the interference matrix elements of the first $[\mathcal{M}_1=\epsilon^{\mu}(p)\mathcal{M}_{1,\mu}]$ and second diagram $[\mathcal{M}_2=\epsilon^{\mu}(p)\mathcal{M}_{2,\mu}]$ of Fig. 12 and the Born term $[m=\epsilon_{\mu}(p)m^{\mu}]$ with a gluon spectator. A similar interpretation may be obtained for the the third and fourth diagrams of Fig. 12 in terms of quark spectators. However, as the above equation is not mathematically well defined, it will not be used in evaluating the infrared limit. Equation (7.12) will be used instead.

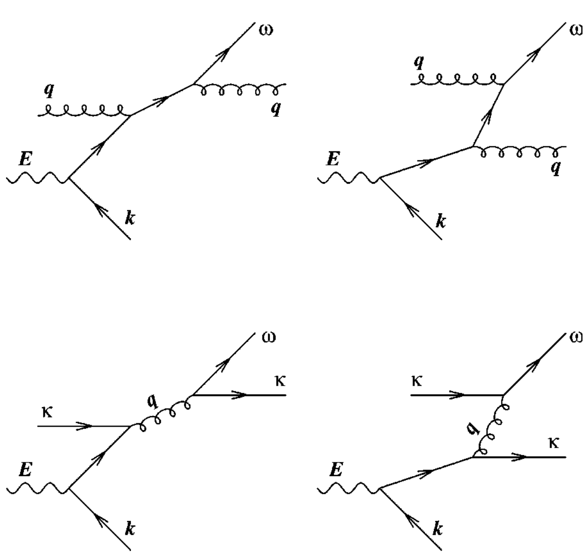

FIG. 12. Photon decay into a $q\bar{q}$ pair. The quark then emits or absorbs a quark or a gluon.

VIII. INFRARED BEHAVIOR

We now examine closely the infrared and collinear singularity structure of the terms enumerated in the two sections above. We will examine the infrared behavior in the limit of heavy dilepton production from a plasma of massless quarks, i.e., $E \gg T$. There are essentially five terms.

F: photon gluon production denotes the reaction $q+\bar{q}\rightarrow g+\gamma$.
C: Compton-like reaction between a gluon and quark/antiquark $g+q\rightarrow\bar{q}+\gamma$.
D: denotes the three-body fusion to form the photon $g+q+\bar{q}\rightarrow\gamma$.
V: denotes photon formation from vertex corrected quarks/antiquarks.
S: denotes photon formation from self-energy corrected quarks/antiquarks.

The full imaginary part of the two loop self-energy may be schematically written as

$$2\,\mathrm{Im}\,\Pi^{\mu}_{\mu}=-\frac{8e^2g^2}{(2\pi)^3}\int dw\{n(w)[F(w)+D_g(w)+V_g(w)+S_g(w)]+\tilde{n}(w)[C(w)+D_q(w)+V_q(w)+S_q(w)]\}. \quad (8.1)$$

The first four terms represent the part of the terms mentioned above that are proportional to the gluon distribution function. The last four terms are those proportional to the quark/antiquark distribution function. This is essentially the same notation as used by the authors of [2]. We now compute these contributions in turn.

A. Self energy correction S_g and S_q

The self-energy correction is essentially given by Eq. (7.12), i.e.,

$$\mathrm{disc}[\Pi^{\mu}_{\mu}]_6=(2\pi i)8e^2g^2\int\frac{dkd\theta d\phi\sin\theta d^3q}{(2\pi)^6q}\delta(E-2k)\times[1-2\tilde{n}(k)]\frac{d}{dk}\sum_{s_1,s_2}\frac{k^2\mathcal{N}\mathcal{S}}{[E-s_2E_{k-q}-s_1q-k]}. \tag{8.2}$$

We concentrate, first, on the sum $\mathcal{S}=\Sigma_{s_1,s_2}k^2\mathcal{N}\mathcal{S}/(E-s_2E_{k-q}-s_1q-k)$. This may be expanded as

$$\mathcal{S}=\frac{k}{w}\left[\frac{[1/2-\tilde{n}(w)+1/2+n(q)]\mathbf{w}_+\cdot\mathbf{k}_+}{E-k-w-q}+\frac{[1/2-\tilde{n}(w)-\{1/2+n(q)\}]\mathbf{w}_+\cdot\mathbf{k}_-}{E-k+w-q}+\frac{[-\{1/2-\tilde{n}(w)\}+1/2+n(q)]\mathbf{w}_+\cdot\mathbf{k}_+}{E-k-w+q}+\frac{[-\{1/2-\tilde{n}(w)\}-\{1/2+n(q)\}]\mathbf{w}_+\cdot\mathbf{k}_-}{E-k+w+q}\right]. \tag{8.3}$$

We now concentrate on the terms proportional to $1/2+n(q)$, i.e.,

$$\mathcal{S}_g=k\left[\frac{1}{2}+n(q)\right]\left[\frac{2k(E-k-q)-2\vec{w}\cdot\vec{k}}{(E-k-q)^2-w^2}+\frac{2k(E-k+q)-2\vec{w}\cdot\vec{k}}{(E-k+q)^2-w^2}\right]. \tag{8.4}$$

Introducing the variables $\alpha=E-2k-2q$, $\beta=E-2k+2q$, and $y=\cos\theta$ (where θ is the angle between $\vec{k}$ and $\vec{q}$), we obtain

$$\mathcal{S}_g=k\left[\frac{1}{2}+n(q)\right]\left[2+\frac{(2k-E)\alpha}{E\alpha+2kq(1+y)}+\frac{(2k-E)\beta}{E\beta-2kq(1-y)}\right]. \tag{8.5}$$

Dropping the $\frac{1}{2}$ ahead of the gluon distribution function we obtain the matter part of $\mathcal{S}_g$. Using only this part we obtain (performing the unimportant angle integrations)

$$-\frac{8e^2g^2}{(2\pi)^3}\int dqn(q)S_g(q)=\frac{8e^2g^2}{(2\pi)^3}\int dk[1-2\tilde{n}(k)]\delta(k-E/2)\times\int dqq\int dy\frac{d}{dk}\mathcal{S}_{g,mat}. \tag{8.6}$$

The limits of the y integration are the locations for the onset of collinear singularities, these are shielded by removing a small part of phase space ϵ, i.e., the y integration is performed within the limits $-1+\epsilon\rightarrow 1-\epsilon$. The results will now depend on ϵ. This gives the result as

$$-\frac{8e^2g^2}{(2\pi)^3}\int dqn(q)S_g(q)=\frac{-8e^2g^2}{(2\pi)^3}\int dqn(q)\left[-4q-4q\ln\left(\frac{2}{\epsilon}\right)\right]. \tag{8.7}$$

In the above, the term $2\tilde{n}(E/2)$ has been dropped, as we are interested in the heavy dilepton limit, where $E\gg T$ and as a result $\tilde{n}(E/2)\rightarrow 0$. Thus, we get

$$S_g(w)=-4w-4w\ln\left(\frac{2}{\epsilon}\right). \tag{8.8}$$

We now concentrate on the terms, in Eq. (8.3), which are proportional to the factor $1/2-\tilde{n}(w)$. Following a similar procedure as above we obtain

$$-\frac{8e^2g^2}{(2\pi)^3}\int dw\tilde{n}(w)S_q(w)=\frac{-8e^2g^2}{(2\pi)^3}\int dw\tilde{n}(w)\left[-4w-4w\ln\left(\frac{2}{\epsilon}\right)\right], \tag{8.9}$$

thus, giving us the relation

$$S_q(w)=-4w-4w\ln\left(\frac{2}{\epsilon}\right). \tag{8.10}$$

Note that in both the expressions for S_q and S_g there is a $\ln(\frac{2}{\epsilon})$ term, which diverges as $\epsilon\rightarrow 0$. This is a collinear singularity. We shall allow ϵ to vanish only when all the different contributions to the dilepton rate have been added together.

B. Vertex correction V_g and V_q

We concentrate first on the term proportional to the fermionic frequency, i.e., V_q. This vertex correction is essentially given by Eqs. (7.1), (7.3), and (7.4). The first two need to be doubled, as mentioned before in Sec. VI. Extracting only the part proportional to the fermionic distribution function $\tilde{n}(w)$, we obtain the V_q integral as

$$-\frac{8e^2g^2}{(2\pi)^3}\int dw\tilde{n}(w)V_q(w)=2(-2\pi i)8e^2g^2\times\int\frac{d^3kd^3w}{(2\pi)^6k^2w^2}\{[1-2\tilde{n}(k)][-2\tilde{n}(w)]\}\times\left[\frac{(\mathbf{k}_+\cdot\mathbf{w}_{s_2})(\mathbf{k}_-\cdot\mathbf{w}_{s_3})s_2}{[E-(s_2-s_3)w][(k+s_3w)^2-q^2]}\right]\delta(E-2k). \tag{8.11}$$

Performing the sum on s_2, s_3 and setting $y=\cos\theta$ (where θ is the angle between $\vec{k}$ and $\vec{w}$), we can perform one of the integrations with the help of a delta function to get

$$-\frac{8e^2g^2}{(2\pi)^3}\int dw\tilde{n}(w)V_q(w)$$
$$=\frac{32e^2g^2}{(2\pi)^3}\int dww[-\tilde{n}(w)]\int_{-1+\epsilon}^{1-\epsilon}dy\frac{1}{2}\left[\left\{\frac{E}{E+2w}+\frac{E}{E-2w}\right\}\frac{1+y}{1-y}+\left\{\frac{w}{E+2w}-\frac{w}{E-2w}\right\}(1+y)\right]. \quad (8.12)$$

Note, once again, that the limits of the final angular integration y signal the onset of collinear singularities. These are, once again, regulated by removing the small part of phase space ϵ. At this point we introduce the condition that the limit of interest is, for dilepton mass, much larger than the temperature, i.e., $E \gg T$. The presence of the distribution function $\tilde{n}(w)$, depending on the energy w severely restricts the contribution from regions where $w \gg T$ to the integral. Thus, the dominant contribution to the integral is from the regions where $w \ll T$ or $w \sim T$. Hence, in the integral we may make the approximation that $w \ll E$ and expand the factors in the square brackets to linear power in w/E. This finally gives

$$-\frac{8e^2g^2}{(2\pi)^3}\int dw\tilde{n}(w)V_q(w)$$
$$=-\frac{8e^2g^2}{(2\pi)^3}\int dw\tilde{n}(w)\left[-8w+8w\ln\left(\frac{2}{\epsilon}\right)\right]. \quad (8.13)$$

Thus we obtain that

$$V_q(w)=-8w+8w\ln\left(\frac{2}{\epsilon}\right). \quad (8.14)$$

Following almost a similar method as above we may obtain V_g from Eq. (7.2) (with an overall factor of 2, as there is another cut that gives an identical contribution) as

$$V_g(w)=4w-\frac{2E^2}{w}\ln\left(\frac{2}{\epsilon}\right). \quad (8.15)$$

Once again, note that both expressions demonstrate a collinear divergence as $\epsilon \to 0$. The term V_g also displays an infrared divergence as $w \to 0$.

C. Photon formation from quark, antiquark and gluon D_g and D_q

The reverse reaction to this process represents heavy-photon "decay" into a $q\bar{q}\gamma$. Due to this reason, the process is denoted by the letter D [2]. The full decay contribution is given by Eq. (6.8) as

$$\mathrm{disc}[\Pi^2](E-k-q-E_{k-q})$$
$$=-i\int\frac{d^3kd^3qd^3w}{(2\pi)^9 2q2k2w}(2\pi)^4\delta^4(\mathbf{p}-\mathbf{k}-\mathbf{q}-\mathbf{w})$$
$$\times\{[1-\tilde{n}(k)][1+n(q)][1-\tilde{n}(w)]-\tilde{n}(k)n(q)\tilde{n}(w)\}$$
$$\times 32e^2g^2\left[\frac{E-2k}{E-2w}+\frac{E-2w}{E-2k}+2\frac{E(E-2q)}{[E-2w][E-2k]}\right]. \quad (8.16)$$

In the above equation, note that if three of the delta functions are used to set $\vec{w}=-\vec{k}-\vec{q}$, then the remaining delta function imposes the condition that

$$E=k+q+\sqrt{k^2+q^2+2kq\cos\theta}.$$

As mentioned before, we work in the limit $E \gg T$; in this case the delta function can be satisfied by the following regions of phase space.

(a) $k \sim E$, $q \sim E$, and hence $w \sim E$; in this case all the distribution functions $n(q), \tilde{n}(k), \tilde{n}(w) \to 0$ and, thus, so do products of distribution functions.

(b) $k \sim T \ll E$, $q \sim E$, and hence $w \sim E$; in this case $\tilde{n}(k) \sim 1$. However $n(q), \tilde{n}(w) \to 0$, and so do products of distribution functions.

(c) $w \sim T \ll E$, $q \sim E$, and hence $k \sim E$; in this case $\tilde{n}(w) \sim 1$. However $n(q), \tilde{n}(k) \to 0$, and so do products of distribution functions.

(d) $q \sim T \ll E$, $k \sim E$, and hence $w \sim E$; in this case $n(q) \sim 1$. However $\tilde{n}(w), \tilde{n}(k) \to 0$, and so do products of distribution functions.

Contributions from (b) and (c) will give us D_q, (d) will give us D_g, while the contribution from (a) is negligible in comparison. We begin by calculating D_q from the regions (b) and (c) of the phase space. Here we can ignore all combinations of distribution functions containing $n(q)$. As before, we also ignore the vacuum term, concentrating only on the matter contribution. Noting the symmetry in the matrix element under interchange of k and w, we may change variables $w \to k$ in the part of the integrand proportional to the distribution function $\tilde{n}(w)$ to get

$$-\frac{8e^2g^2}{(2\pi)^3}\int dk\tilde{n}(k)D_q(k)$$
$$=\int\frac{d^3kd^3q}{(2\pi)^5 2q2k2w}\delta(E-k-q-w)\{2\tilde{n}(k)\}32e^2g^2$$
$$\times\left[\frac{E-2k}{E-2w}+\frac{E-2w}{E-2k}+2\frac{E(E-2q)}{[E-2w][E-2k]}\right]. \quad (8.17)$$

The argument of the delta function is the equation $g(q) = k+q+w(q)-E=0$. The solution of this equation is at $q = q_s(k,E)$,

$$q_s=\frac{1}{2}\,\frac{E(E-2k)}{E-k(1-y)}. \tag{8.18}$$

The delta function can be written as

$$\delta(g(q))=\frac{\delta(q-q_s)}{|g'(q_s)|}.$$

Substituting this back into the equation for D_q, we can do the dq integration with the above-mentioned delta function. We can then perform the remaining angular integration by removing the small part of phase space ϵ to shield the collinear singularities. Now expanding up to linear order in k as $k\ll E$, we get

$$-\frac{8e^2g^2}{(2\pi)^3}\int dk\tilde{n}(k)D_q(k)$$
$$=-\frac{8e^2g^2}{(2\pi)^3}\int dk\tilde{n}(k)\left[2k+(-2k-E)\ln\left(\frac{2}{\epsilon}\right)\right]. \tag{8.19}$$

Thus we get

$$D_q(w)=2w+(-2w-E)\ln\left(\frac{2}{\epsilon}\right). \tag{8.20}$$

We can now obtain D_g by concentrating on region (d) of the phase space and ignoring all combinations of distribution functions containing $\tilde{n}(k)$ or $\tilde{n}(w)$, this gives us

$$D_g(w)=-2w+\left(2w-2E+\frac{E^2}{w}\right)\ln\left(\frac{2}{\epsilon}\right). \tag{8.21}$$

D. Pair annihilation *F* and Compton scattering *C*

The procedure to obtain these is almost exactly identical to the two terms of the preceding section. The total Compton scattering contribution can be obtained from Eq. (6.17) by doubling it, as mentioned in the paragraph immediately following Eq. (6.17). We may, once again, from phase space considerations show that the dominant contribution to Compton scattering occurs from a region where $k\sim T\ll E$(k is the incoming quark or antiquark energy). The leading term of Compton scattering is, thus, proportional to the quark or antiquark distribution function. From similar considerations, the leading term of pair annihilation can be demonstrated to be proportional to the outgoing gluon distribution function. Expanding them up to linear order in the quark or gluon energy w, we get

$$C(w)=2w+(-2w+E)\ln\left(\frac{2}{\epsilon}\right) \tag{8.22}$$

and

$$F(w)=-2w+\left(2w+2E+\frac{E^2}{w}\right)\ln\left(\frac{2}{\epsilon}\right). \tag{8.23}$$

IX. RESULTS

In the first seven sections we evaluated the two different self-energies of the photon at two loops, then evaluated the various cuts of the self-energies that constituted its imaginary part; we then recombined the various cuts and reinterpreted them as physical processes and finally evaluated these terms in the limit of heavy photon emission. In Sec. VIII we have concentrated solely on the thermal or matter part of these expressions. The vacuum part is well known. All the expressions contain collinear singularities (as $\epsilon\to 0$), which for the moment have been shielded by removing the small part of phase space (ϵ) where these singularities occur. Some of the expressions also display infrared singularities as $w\to 0$. Hence, the final integrations over w are yet to be performed. In the following we will combine all these terms and perform this integration.

We now resubstitute the terms *F*, *C*, *D*, *V*, and *S* back in Eq. (8.1) to get the coefficients of the bosonic and fermionic distribution functions as

$$n(w)(F+D_g+V_g+S_g)=n(w)(-4w), \tag{9.1}$$

$$\tilde{n}(w)(C+D_q+V_q+S_q)=\tilde{n}(w)(-8w). \tag{9.2}$$

Thus, we find that when all the cuts are summed, the collinear and infrared singularities cancel. This is in contradistinction with the results of Refs. [8,2], where the infrared singularities cancel but the collinear singularities persist. With these, we get the full imaginary part of the self-energy as

$$\mathrm{Im}\,\Pi^{\mu}_{\mu_{\text{two loop,thermal}}}=-\frac{4e^2g^2}{(2\pi)^3}\left[-\frac{4\pi^2T^2}{3}\right]=\frac{8e^2\alpha_sT^2}{3}. \tag{9.3}$$

We may also derive the Born term and quote the two-loop vacuum contribution (from [17]) as

$$\mathrm{Im}\,\Pi^{\mu}_{\mu_{\text{one loop}}}+\mathrm{Im}\,\Pi^{\mu}_{\mu_{\text{two loop,vacuum}}}=\frac{-3e^2}{2\pi}E^2\left(1+\frac{\alpha_s}{\pi}\right). \tag{9.4}$$

X. DISCUSSIONS AND CONCLUSIONS

In this paper, we have calculated the imaginary part of the two-loop heavy boson retarded self-energy in the imaginary time formalism. We also elucidate the analytic structure of the self-energy by recombining and reinterpreting various cuts of the self-energy as physical processes that occur in the the medium. Cuts with loops have been interpreted as interference terms between $O(\alpha)$ tree scattering amplitudes and the Born term with spectators. At each stage the results from the self-energy cuts were matched by rederiving the amplitudes of the tree-level diagrams. This constitutes an important check of each of the contributions from the self-energy.

Each of the contributions contain infrared and collinear singularities. In the interest of simplicity, we analyzed this singular behavior in the region where the dilepton mass is far greater than the temperature. This allows us to neglect a series of terms that appear subdominant. In each case we retained terms only up to order T^2/E^2. One might argue that this represents a considerable approximation of the result. However the resulting simplification allows us to analyze far simpler and analytically integrable expressions. We would point out that this was precisely the approximation used in [2,8] where a remnant collinear divergence was deduced at $O(T^2/E^2)$. When all the contributions were summed, all infrared and collinear divergences canceled, leaving a finite result $O(T^2/E^2)$. In this sense our results differ slightly from those of Ref. [7] who find a remnant $O(T^4/E^4)$ result. The possible reasons for this discrepancy are many. For example, the authors of Ref. [7] use a complicated finite temperature remormalization prescription, ours is the same prescription as at zero temperature; they apply finite self-energy corrections on the outer legs of all their processes and we do not. However, both calculations (as well as those of Ref. [6]) yield the consistent result that all collinear and infrared divergences cancel in the final rate expression. This is consistent with the Kinoshita-Lee-Nauenberg theorem [18,19], even though a formal proof of the theorem at finite temperature is still elusive. We leave this and other extensions for future investigations.

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APPENDIX A: NOTATION

Our notation is categorized by the explicit presence of an apparent Minkowski time $x^0=-i\tau$ and a momentum $q^0=i2n\pi T$ or $i(2n+1)\pi T$ for bosons or fermions, respectively. Our metric is $(1,-1,-1,-1)$. For the case of zero chemical potential our bosonic propagators have the same appearance as at zero temperature, i.e.,

$$i\Delta(q)=\frac{i}{(q^0)^2-|q|^2}. \tag{A1}$$

The Feynman rules are also the same as at zero temperature, with the understanding that we replace the zeroth component of the momentum by $i(2n+1)\pi T$ for a fermion and by an even frequency in the case of a boson. One may, in the case of zero chemical potential, relate this to the familiar case of Ref. [13] by noting that

$$\Delta(q)=\frac{1}{(q^0)^2-|q|^2}=\frac{-1}{(\omega_n)^2+|q|^2}=-\Delta_E(\omega_n,q), \tag{A2}$$

where $\Delta_E(\omega_n,q)$ is the familiar Euclidean propagator presented in the literature [13], [15]. One may immediately surmise the form of the noncovariant propagator $\Delta(|\vec{q}|,x^0)$, the Fourier transform of which is the covariant propagator

$$\begin{aligned}\Delta(q,q^0)&=-\int_0^\beta d\tau e^{-i\omega_n\tau}\Delta_E(|\vec{q}|,\tau)\\&=-i\int_0^{-i\beta}dx^0e^{iq^0x^0}\Delta_E(|\vec{q}|,\tau)\\&=-i\int_0^{-i\beta}dx^0e^{iq^0x^0}\Delta(|\vec{q}|,x^0).\end{aligned} \tag{A3}$$

The full fermionic propagators are

$$S(q,q^0)=(\gamma^\mu q_\mu)(-i)\int_0^{-i\beta}dx^0e^{iq^0x^0}\Delta_\mu(|\vec{q}|,x^0), \tag{A4}$$

where

$$\Delta_\mu(|\vec{q}|,x^0)=\frac{1}{2E_q}\sum_s f_s(E_q)e^{-isx^0(E_q)}. \tag{A5}$$

APPENDIX B: DISCONTINUITY ACROSS A SECOND ORDER POLE

Imagine we have a function of a complex variable $F(z)$ and it is given to be in the form

$$F(z)=\int dx\frac{f(z,x)}{z-x}+\frac{g(z,x)}{(z-x)^2}, \tag{B1}$$

where x is a real variable, integrated on the real axis. Most of the discontinuities that we evaluate can be cast in this general form. This can be rewritten as

$$-F(z)=\int dx\frac{f(z,x)}{x-z}-\frac{g(z,x)}{(x-z)^2}. \tag{B2}$$

The functions $f(z,x)$ and $g(z,x)$ are analytic in x and hence admit a Taylor expansion

$$\begin{aligned}f(z,x)&=f(z,x=z)+\frac{df}{dx}(z,x=z)[x-z]\\&\quad+\frac{1}{2}\frac{d^2f}{dx^2}(z,x=z)[x-z]^2+\cdots.\end{aligned} \tag{B3}$$

Substituting Eq. (B3) in Eq. (B2) we get

$$-F(z)=\int dx\frac{f(z,x=z)}{x-z}+\frac{df}{dx}(z,x=z)+\frac{d^2f}{dx^2}(z,x=z)\times[x-z]+\cdots-\frac{g(z,x=z)}{(x-z)^2}-\frac{dg}{dx}(z,x=z)\times\frac{1}{x-z}-\frac{1}{2}\frac{d^2g}{dx^2}(z,x=z)\frac{1}{(x-z)^2}-\cdots. \quad \text{(B4)}$$

Recalling that only the pure first order poles develop a discontinuity or imaginary part at the pole, we get the imaginary part of Eq. (B4) as

$$\text{disc}[-F(z)]=\int dx2\pi i\,\delta(x-z)\left[f(z,x)-\frac{dg}{dx}(z,x)\right]. \quad \text{(B5)}$$

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