^{12}C-^{12}C elastic scattering at intermediate energies

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(Received 10 April 2001; published 29 April 2002)

A microscopic study of ^{12}C-^{12}C elastic scattering at 1.016, 1.449, and 2.4 GeV has been made within the framework of the Coulomb modified Glauber model. The elastic *S*-matrix element has been evaluated considering the first two terms of the nuclear phase expansion series and using the realistic densities for the colliding nuclei. The effect of the phase variation of the *NN* scattering amplitude on the calculated elastic scattering differential cross sections has been studied. We find that a very good description of the experimental data at 1.016 and 1.449 GeV is achieved by considering the second order term of the phase expansion series and the phase variation of the *NN* scattering amplitude. However, at 2.4 GeV, although the second order phase term provides some improvement over the optical limit calculation, still the theoretical situation remains unsatisfactory.

DOI: 10.1103/PhysRevC.65.054607 PACS number(s): 25.70.Bc, 21.30.Fe, 24.10.Cn

I. INTRODUCTION

Over the past 15 years or more, the Glauber multiple scattering model [1–3] has been applied by many authors to analyze elastic scattering differential cross sections for the ^{12}C-^{12}C system at low medium energies [4–8]. Since evaluation of the full Glauber amplitude with realistic descriptions for colliding nuclei is a prohibitively complex task, in most of these applications, the so-called optical limit approximation (OLA) has been employed to evaluate the Glauber elastic *S*-matrix element. In this approximation only the leading term in an expansion of the nucleus-nucleus phase-shift function is considered. This term depends upon the one-body densities of the colliding nuclei while the neglected terms depend upon the two-body and other higher order densities. It is found that the OLA gives a reasonably satisfactory account of the experimental data provided that the calculations are made in the Coulomb modified Glauber model (CMGM) in which the conventional Glauber model is suitably modified to account for the deviation of the colliding particle trajectories due to the Coulomb field [7,9]. More specifically, it is observed that the CMGM calculation agrees well with the experimental data at lower energies but not so well at relatively higher energies especially at large momentum transfers (see, e.g., Ref. [7]). This trend of the predictions of the CMGM calculation may be understood by noting that at lower energies the input *NN* total cross section (σ_{NN}) is quite large. Due to this, the OLA phase-shift function is highly absorptive at lower energies. Moreover, the relatively large Coulomb repulsion at lower energies keeps the nuclear overlap region confined to large impact parameter values. Consequently, the scattering is sensitive mainly to the very low-density regions of the colliding nuclei in which case the contributions of the neglected higher order terms may be negligibly small. As the energy increases, σ_{NN} decreases, making the OLA phase-shift function less absorptive. Also the Coulomb repulsion effect weakens. As a result, the scattering now becomes sensitive to the inner surface region also where neglecting the higher order terms in the phase-shift function expansion, as is done in the OLA, may not be a good approximation. Therefore, it is not unreasonable to expect that the inclusion of some higher order terms in the CMGM scattering calculation may improve the theoretical situation at relatively higher energies.

It may be mentioned that recently, El-Gogary *et al.* [10], in an attempt to improve upon their earlier study [11], have reported a full Glauber series calculation of elastic scattering differential cross sections for some light heavy-ion systems. The calculations have been made using a single Gaussian model for the densities of the colliding nuclei in order to be able to perform a full Glauber series calculation incorporating the c.m. correlation in a consistent manner. Their results for ^{12}C-^{12}C scattering at 1.016 and 1.445 GeV show noticeable disagreement with experimental data especially at large momentum transfers. However, this disagreement may be largely due to the use of the single Gaussian model density, which is quite unrealistic for ^{12}C for describing large momentum transfer observables.

In this work we undertake a study of ^{12}C-^{12}C elastic scattering at 1.016, 1.449, and 2.4 GeV in the CMGM considering the first two terms of the expansion of the nuclear phase-shift function. The calculations have been performed using the realistic ground state density for ^{12}C and invoking the phase variation of the *NN* scattering amplitude. We find that consideration of the second order term in the phase-shift expansion series greatly helps in achieving a good description of the ^{12}C-^{12}C elastic scattering data at 1.016 and 1.449 GeV. However, at 2.4 GeV, despite significant improvement, the theoretical situation cannot be described as satisfactory.

II. FORMALISM

According to the Glauber multiple scattering model the elastic *S*-matrix element $S_{\rm el}(b)$, in the impact parameter space for the collision of a projectile nucleus of mass number *B* from a target nucleus of mass number *A*, described, respectively, by the ground state wave functions Φ_0 and Ψ_0, is given by (see, e.g., Refs. [3,12,13])

$$S_{\rm el}(b)=\left(\Phi_0\Psi_0\left|\prod_{i=1}^{A}\prod_{j=1}^{B}[1-\Gamma(\mathrm{b}-\mathbf{s}_i+\mathbf{s}'_j)]\right|\Psi_0\Phi_0\right), \tag{1}$$

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where $\mathbf{b}$ is the impact parameter, $\mathbf{s}_i$ and $\mathbf{s}'_j$ are, respectively, the projections of the target and projectile nucleon coordinates $\mathbf{r}_i$ and $\mathbf{r}'_j$ in the impact parameter plane, and Γ is the nucleon-nucleon profile function, which is the two dimensional Fourier transform of the *NN* scattering amplitude $f(q)$.

It is well known that evaluation of $S_{\rm el}(b)$ as given by Eq. (1) is a difficult task for realistic descriptions of colliding nuclei. However, there exist several schemes in the literature to evaluate it approximately [12–15]. Here we follow the method developed by Franco and Varma [13], which seems more appropriate for the present study. In this method one introduces a nucleus-nucleus phase-shift function $\chi(b)$ through the relation

$$S_{\rm el}(b)=e^{i\chi(b)} \tag{2}$$

and expands $\chi(b)$ as

$$\chi(b)=\sum_{m=1}^{\infty}\chi_m(b). \tag{3}$$

The first term in this expansion, namely, $\chi_1(b)$, gives the so-called OLA. It depends upon the one-body densities of the colliding nuclei and it is this term that has been used in most of the applications of the Glauber model to heavy-ion scattering at medium energies as stated before. The next term $\chi_2(b)$ depends upon the two-body densities and we will include it in the present analysis. These terms may be expressed as [13,15]

$$i\chi_1(b)=-AB\overline{\Gamma}(b), \tag{4}$$

$$i\chi_2(b)=-\frac{1}{2}[AB\overline{\Gamma}(b)]^2+\frac{AB}{2}[(B-1)G_{21}(b)+(A-1)\times G_{12}(b)+(A-1)(B-1)G_{22}(b)], \tag{5}$$

where

$$\overline{\Gamma}(b)=\frac{1}{ik}\int_0^{\infty}dq\,qJ_0(qb)f(q)F_A(q)F_B(q), \tag{6}$$

$$G_{12}(b)=\left(\frac{1}{2\pi ik}\right)^2\int d\mathbf{q}_1d\mathbf{q}_2e^{-i(\mathbf{q}_1+\mathbf{q}_2).\mathbf{b}}f(q_1)f(q_2)\times F_A^{(2)}(\mathbf{q}_1,\mathbf{q}_2)F_B(-\mathbf{q}_1-\mathbf{q}_2), \tag{7a}$$

$$G_{21}(b)=\left(\frac{1}{2\pi ik}\right)^2\int d\mathbf{q}_1d\mathbf{q}_2e^{-i(\mathbf{q}_1+\mathbf{q}_2).\mathbf{b}}f(q_1)f(q_2)\times F_A(\mathbf{q}_1+\mathbf{q}_2)F_B^{(2)}(-\mathbf{q}_1,-\mathbf{q}_2), \tag{7b}$$

$$G_{22}(b)=\left(\frac{1}{2\pi ik}\right)^2\int d\mathbf{q}_1d\mathbf{q}_2e^{i(\mathbf{q}_1+\mathbf{q}_2).\mathbf{b}}f(q_1)f(q_2)\times F_A^{(2)}(\mathbf{q}_1,\mathbf{q}_2)F_B^{(2)}(-\mathbf{q}_1,-\mathbf{q}_2). \tag{7c}$$

In the above expressions $F_{A(B)}(q)$ and $F^{(2)}_{A(B)}(\mathbf{q}_1,\mathbf{q}_2)$ are the one- and two-body form factors of the target (projectile) nucleus,

$$F_{A(B)}(q)=\int d\mathbf{r}e^{i\mathbf{q}.\mathbf{r}}\rho_{A(B)}(r), \tag{8}$$

$$F^{(2)}_{A(B)}(\mathbf{q}_1,\mathbf{q}_2)=\int d\mathbf{r}_1d\mathbf{r}_2e^{i(\mathbf{q}_1.\mathbf{r}_1+\mathbf{q}_2.\mathbf{r}_2)}\rho^{(2)}_{A(B)}(\mathbf{r}_1,\mathbf{r}_2), \tag{9}$$

where $\rho_{A(B)}(r)$ and $\rho^{(2)}_{A(B)}(\mathbf{r}_1,\mathbf{r}_2)$ are the ground state one- and two-body densities of the target (projectile) nucleus, $f(q)$ the *NN* scattering amplitude, and $\mathbf{q}$ the momentum transfer.

The quantity $f(q)$ is one of the main ingredients of the Glauber model calculation and it is related to the *NN* profile function $\Gamma(b)$ that enters into the expression for $S_{\rm el}(b)$ as given by Eq. (1) as

$$f(q)=\frac{ik}{2\pi}\int d\mathbf{b}e^{\mathbf{q}.\mathbf{b}}\Gamma(b). \tag{10}$$

It is generally taken to be of the form [16]

$$f(q)=\frac{ik\sigma_{NN}(1-i\alpha_{NN})}{4\pi}e^{-(\beta_{NN}+i\gamma_{NN})q^2/2}, \tag{11}$$

where σ_{NN} is the *NN* total cross section, α_{NN} is the ratio of the real to the imaginary parts of $f(0)$, and β_{NN} and γ_{NN} are the slope and phase variation parameters, respectively. At present our knowledge of the phase variation parameter that introduces an overall q-dependent phase in the scattering amplitude is very poor, therefore in scattering calculations it has been mostly ignored or treated in a phenomenological manner.

III. EVALUATION OF $S_{\rm el}(b)$

In this work we will evaluate $S_{\rm el}(b)$ as given by Eqs. (1)–(3) by considering only the first two terms χ_1 and χ_2 of the expansion for $\chi(b)$ as given by Eq. (3). To evaluate χ_1 and χ_2 one needs the nuclear form factors $F_{A(B)}(q)$ and $F^{(2)}_{A(B)}(\mathbf{q}_1,\mathbf{q}_2)$. For a light nucleus such as ^{12}C the former may be obtained from the charge form factor after applying corrections for the finite charge distribution of the proton. It is convenient to use the form factor in the following sum-of-Gaussian parametrization form:

$$F_v(q)=\sum_{j=1}^{N_v}c_{vj}e^{-b_{vj}q^2},\quad v=A,B, \tag{12}$$

where c_{vj} and b_{vj} are the fitting parameters, and N_v is the number of Gaussian terms required to reproduce the realistic form factor.

As regards the nuclear two-body form factor $F^{(2)}_v(\mathbf{q}_1,\mathbf{q}_2)$, it differs from the product of two one-body form factors $F_v(q_1)F_v(q_2)$ because of the presence of correlations in nuclei. Generally speaking, three types of correlations have been discussed in connection with nuclear scattering, namely, center-of-mass (c.m.), Pauli, and dynamical short-range correlations [17]. Their effects on nucleon-nucleus and nucleus-nucleus scattering have been studied by several authors (see, e.g., Refs. [15,17,18]). It is found that for light nuclei the c.m. correlation is most important. Considering only this correlation the two-body form factor may be written as [15]

$$F_v^{(2)}(\mathbf{q}_1,\mathbf{q}_2)=\exp\left(\frac{\mathbf{q}_1\cdot\mathbf{q}_2}{2v\,\alpha_{Mv}^2}\right)F_v(q_1)F_v(q_2),\quad v=A,B, \tag{13}$$

where α_{Mv}^2 is the oscillator constant of the nuclear harmonic oscillator model. Substituting Eqs. (11)–(13) in Eqs. (4)–(7), closed expressions for χ_1 and χ_2, which determine $S_{\rm el}(b)$, can be derived easily.

IV. CALCULATION OF CROSS SECTION

To calculate the elastic scattering differential cross section in the CMGM we, following Ref. [7], replace b in $S_{\rm el}(b)$ by b', which is the distance of closest approach in Rutherford orbits and is given by

$$Kb'=\eta+(\eta^2+K^2b^2)^{1/2}, \tag{14}$$

where K is the c.m. momentum of the nucleus-nucleus system and $\eta=Z_AZ_Be^2/\hbar v$ is the Sommerfeld parameter with Z_A (Z_B) as the target (projectile) charge number and v the projectile velocity.

Next, the diagonal S-matrix element S_l in the angular momentum representation is obtained from $S_{\rm el}(b')$ using Eq. (14) and the correspondence $Kb\leftrightarrow(l+1/2)$:

$$S_l=S_{\rm el}(b')\big|_{Kb'=\eta+[\eta^2+(l+1/2)^2]^{1/2}}. \tag{15}$$

Finally, the elastic scattering differential cross section for the ^{12}C-^{12}C system is calculated using the expression for two identical bosons:

$$\frac{d\sigma}{d\Omega}=|F(\theta)+F(\pi-\theta)|^2 \tag{16}$$

with

$$F(\theta)=F_{\rm Coul}(\theta)+\frac{i}{2K}\sum_{l=0}^{\infty}(2l+1)e^{2i\sigma_l}(1-S_l)P_l(\cos\theta), \tag{17}$$

where θ is the scattering angle, $F_{\rm Coul}$ the Coulomb amplitude, and σ_l the Coulomb phase shift.

V. RESULTS AND DISCUSSION

The formalism discussed in previous sections will now be applied to analyze ^{12}C-^{12}C elastic scattering data at 1.016, 1.449, and 2.4 GeV. The reason for the choice of this data set is that the OLA does not give satisfactory results at these energies. The input required for calculating the cross sections are (i) the parameters c_j and b_j of the sum-of-Gaussian parametrization of the ^{12}C form factor given by Eq. (12), (ii) the oscillator model constant α_M^2 for ^{12}C, and (iii) the NN scattering amplitude parameters σ_{NN}, α_{NN}, β_{NN}, and γ_{NN}. The values of c_j and b_j used in the calculation are $c_1=1.494$, $b_1=0.741$ fm^2, $c_2=-0.494$, and $b_2=0.417$ fm^2. These values have been taken from Ref. [15] and they reproduce the ^{12}C nuclear form factor up to momentum transfer $q\cong2.7$ fm^{-1} very nicely. For α_M^2 we use the value 0.38 fm^{-2} as given in Ref. [19].

As regards the NN scattering parameters, namely, σ_{NN}, α_{NN}, β_{NN}, and γ_{NN}, the first three can, in principle, be obtained from NN scattering experiments. The last one, namely, the phase variation parameter γ_{NN}, gives an overall q-dependent phase to the scattering amplitude and presently little is known about it. Therefore, in most earlier studies it has been ignored to make the Glauber calculation parameter free. One of the aims of the present study is to highlight its importance in Glauber calculations in the energy range under consideration.

The values of the quantity σ_{NN} have been calculated using the parametrization of σ_{pp} and σ_{np} in the energy range of about 10–1000 MeV/nucleon as given in Ref. [20]. The values of σ_{NN} so obtained are very closely the same as used by Hostachy *et al.* [21] from whose work we take the values of the parameter α_{NN}.

Coming to β_{NN}, it is not a very well determined quantity in the energy range under consideration. Some authors (see, e.g., Refs. [7,22]), in their analyses of elastic scattering differential cross-section data, have assumed it to be zero. On the other hand, the value of β_{NN} at 210 MeV/nucleon, taken as the simple average of β_{pp} and β_{np}, for the spin independent part of the NN scattering amplitude as determined by Wallace [23] from the phase shifts, is 1.24 fm^2. Schwaller *et al.* [24] have given a plot of β_{NN} as a function of incident nucleon kinetic energy in the energy range 0.15–2.0 GeV from which it follows that $\beta_{NN}\cong0.8$ fm^2 at 200 MeV/nucleon. Charagi and Gupta [20] in their analysis of the ^{12}C-^{12}C total reaction cross section in the energy range 0.01–1.0 GeV/nucleon have taken $\beta_{NN}=0.42$ fm^2. Ray *et al.* [25] has determined β_{pp} and β_{pn} from the phase shifts of Arndt, Hackman, and Roper (AHR) [26] in the energy range 0.1–2.2 GeV from which it follows that the values of β_{NN} at 0.1, 0.15, and 0.2 GeV are 0.51, 0.58, and 0.62 fm^2, respectively. In view of this we performed exploratory calculations with different values of β_{NN} at the energies concerned. It is found that a value of $\beta_{NN}=0.85$ fm^2 gives satisfactory results in the energy range of our interest. This suggests that perhaps the energy dependence of β_{NN} is quite weak in this energy region. Therefore, in this work, we have used $\beta_{NN}=0.85$ fm^2, which is very close to the value given by Schwaller *et al.* [24]. The values of the parameters σ_{NN}, α_{NN}, and β_{NN} that have been used in our calculation are listed in Table I.

It may be mentioned that the approximation of $f(q)$ by the Gaussian form generally used at intermediate energies for numerical convenience has been borrowed from the description of NN scattering at high energies where the small angle

TABLE I. Values of *NN* scattering amplitude parameters.

E_{lab} (GeV)	E_{lab}/nucleon (MeV)	σ_{NN} (mb)	α_{NN}	β_{NN} (fm^2)
1.016	84.7	60.1	1.33	0.85
1.449	120.7	42.53	1.34	0.85
2.4	200	30.93	0.96	0.85

scattering is peaked in the forward direction so that the slope parameter can be determined to a good degree of accuracy. At low intermediate energies the Gaussian parametrization may not work as well as at high energies, causing some ambiguity in the determination of β_{NN}. However, the study of Ray *et al.* [25] shows that the spin independent part of the *NN* amplitude generated from the AHR phase shifts can be described by the Gaussian parametrization reasonably well at momentum transfers less than 1.5–2.0 fm^{-1} down to at least 100-MeV incident nucleon energy.

In Fig. 1 we present a study of the effect of introducing the phase variation of the *NN* scattering amplitude on the calculation of the scattering cross section in the optical limit approximation. The importance of the phase variation in Glauber model calculations at higher energies was first highlighted by Franco and Yin [16] who found that its consideration greatly helps in reproducing alpha-light ion elastic scattering data at $E_{\mathrm{lab}}=1.05$ GeV per nucleon with $\gamma_{NN}=0.4$ fm^2. Later, Lombard and Maillet [27] and Auger and Lazard [28] who analyzed p-^{4}He elastic scattering data at 1.0 and 0.8 GeV, respectively, also found that consideration of the phase factor with $\gamma_{NN}\approx 0.25$ fm^2 considerably improves the theoretical situation (it may be pointed out that although the authors in Refs. [27,28] have applied an overall phase factor with $\gamma_{NN}\approx 0.4$ fm^2, the effective value of γ_{NN} in their calculation is about 0.25 fm^2 as has been clarified in Ref. [29]). Existence of such a phase factor has also been demonstrated using a simple *NN* potential model [29]. From Fig. 1, it is seen that the phase variation that has been ignored in some earlier studies (see, e.g., Ref. [7]) has a significant effect on the calculated cross section especially at larger angles. The solid lines show the calculated cross section without the phase variation ($\gamma_{NN}=0$), while the other curves show the results for different values of γ_{NN} in the range -0.7–0.7 fm^2. We note that for $\gamma_{NN}>0$, the calculated curve goes down relative to the $\gamma_{NN}=0$ case with increasing γ_{NN}, and for $\gamma_{NN}<0$ the calculated curve goes upward as the absolute value of γ_{NN} increases. In summary a decrease in the value of γ_{NN} pushes the calculated curve upward and also fills the minima. We further note that consideration of the phase variation helps little in getting an improved theoretical situation as far as the calculation of the cross section in the optical limit approximation is concerned. Calculations at 1.449 and 2.4 GeV (not shown) give similar results.

In Fig. 2 we show the effect of including the second order phase-shift function $\chi_2(b)$ on the calculated cross section at 1.016 GeV. The dotted curve shows the calculated cross section in the OLA [i.e., considering only $\chi_1(b)$] with $\gamma_{NN}=0$. The dashed curve shows the effect of including $\chi_2(b)$ in the analysis still with $\gamma_{NN}=0$. It is seen that $\chi_2(b)$ has a large effect on the calculated cross section. It causes the theoretical curve to fall below the experimental values and makes it more oscillatory. However, the decreasing trend of the calculated cross sections at larger angles is similar to the experimental one. Next, the phase variation parameter is varied to find that a very good fit to the data is achieved with $\gamma_{NN}=-0.7$ fm^2 as may be seen from the solid curve. Thus, it may be said that consideration of both the second order phase-shift function and the phase variation are important for a satisfactory description of ^{12}C-^{12}C data at 1.016 GeV. It may be clarified that the value of γ_{NN} found here need not necessarily be the same as one would obtain from the Gaussian parametrization of the *NN* amplitude generated from the phase shifts. This is because the *NN* scattering measurements determine the scattering amplitude, at best, only to within an overall q-dependent phase factor as pointed out by Franco and Yin [16]. This is the reason that an overall q-dependent phase factor was applied in Ref. [28] to the *NN* amplitude

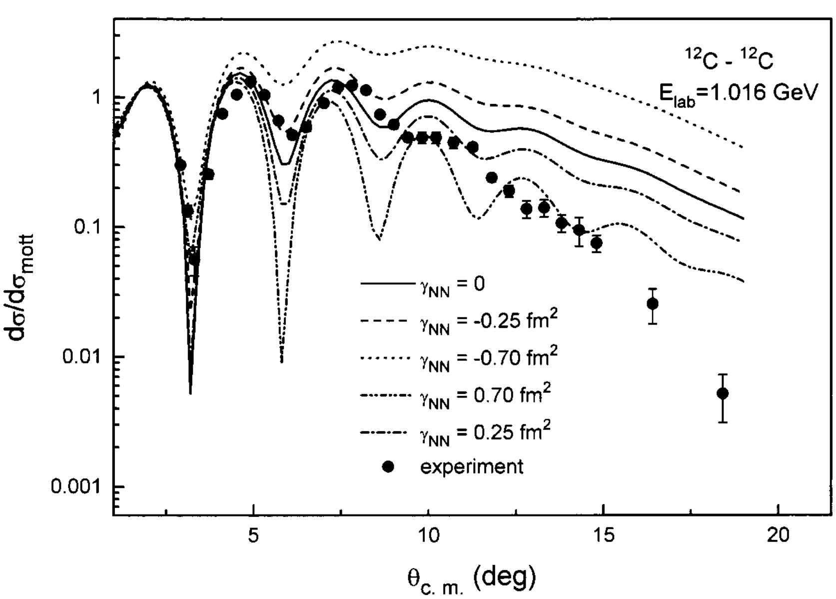

FIG. 1. Effect of phase variation of the *NN* scattering amplitude on the optical limit calculation of ^{12}C-^{12}C elastic scattering differential cross section at 1.016 GeV.

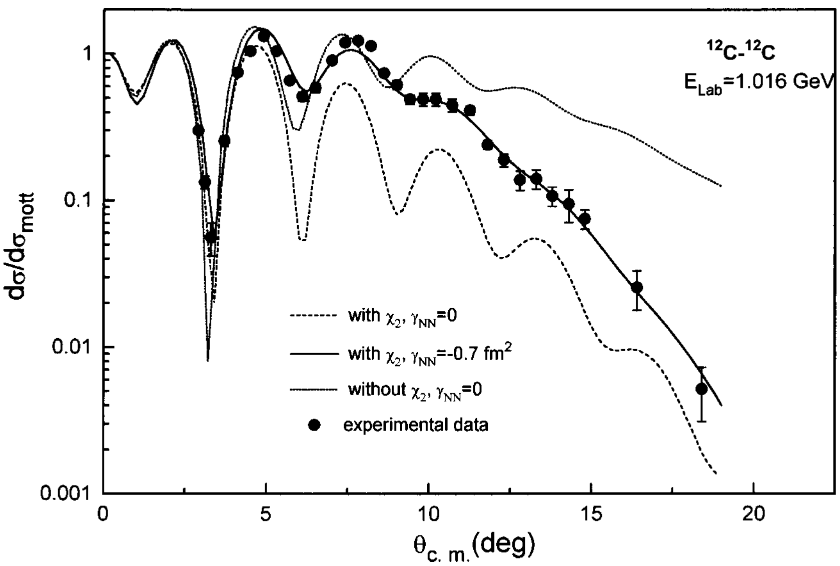

FIG. 2. ^{12}C-^{12}C elastic scattering differential cross sections at 1.016 GeV. Dotted curve: optical limit calculation ignoring the phase variation. Dashed curve: calculation with $\chi_2(b)$ but ignoring the phase variation. Solid curve: calculation including both the second order phase-shift function $\chi_2(b)$ and the phase variation.

generated from the phase shifts.

In Fig. 3 we show the calculated and experimental cross sections at 1.449 and 2.4 GeV. The descriptions of the curves are the same as in Fig. 2. It is seen that at these energies also the second order phase-shift function affects the calculated cross section significantly. As a matter of fact at 1.449 GeV, the calculated cross sections with $\chi_2(b)$, keeping $\gamma_{NN}=0$ (dashed curve), agrees with the experimental data quite satisfactorily except in the region of forward angle minima. Our attempt to get still better agreement with the data by varying γ_{NN} helped only a little as is evident from the solid curve, which is calculated with $\gamma_{NN}=-0.1\ \mathrm{fm}^2$. At 2.4 GeV, al-

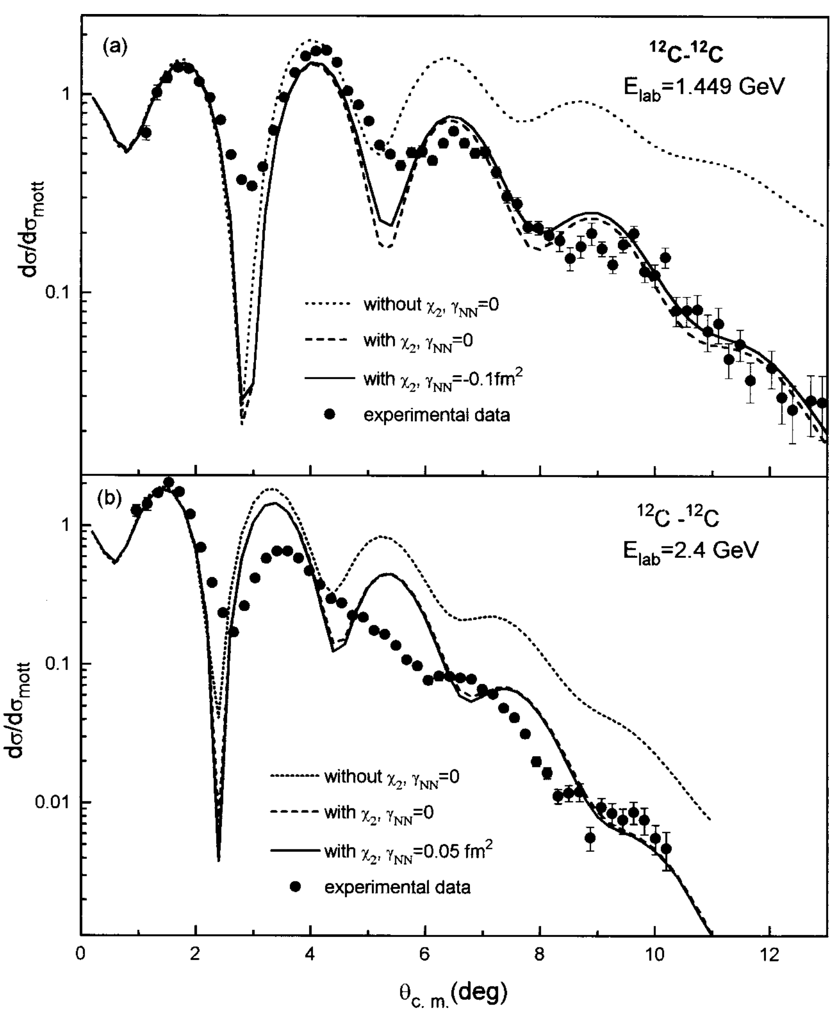

FIG. 3. Same as in Fig. 2 but at lab energies 1.449 and 2.4 GeV.

though inclusion of $\chi_2(b)$ with $\gamma_{NN}=0$ brings the theoretical curve much closer to the experimental data, the theoretical situation is still quite unsatisfactory. In this case also, variation of γ_{NN} helped little in getting convincingly better agreement with the experimental data as is evident from the solid curve that corresponds to $\gamma_{NN}=0.04\,\text{fm}^2$. Any other larger or smaller value is found to deteriorate the situation.

The good agreement between the presently calculated cross sections and the experimental data at 1.016 and 1.449 GeV, but not similar agreement at 2.4 GeV, may be interpreted as follows. At 2.4 GeV, which corresponds to 200 MeV/nucleon, the input σ_{NN} is much smaller than its values at the two lower energies. Because of this the ^{12}C-^{12}C system is more transparent at this energy than at 1.016 and 1.449 GeV. At the lower energies the system is transparent only for a small range of large impact parameter values corresponding to the overlap of the surface tails of the two colliding nuclei and is opaque for smaller impact parameter values. At the higher energy the system remains transparent even for relatively smaller impact parameter values. Therefore, the scattering at 2.4 GeV is expected to be more sensitive to the details of the interior region than at the lower energies. Now, if the higher order phase-shift functions neglected in the present analysis were important for relatively smaller impact parameter values, their omission would manifest at the higher energies but not at lower energies. This appears to be the reason for the some disagreement between the present calculation and the experimental data at 2.4 GeV. It is hoped that the consideration of the higher order phase-shift functions that are relatively difficult to evaluate with realistic densities would improve the theoretical situation at this energy. The higher order phase-shift functions, which depend upon the expectation values of the products of several Γ's with respect to the ground state densities of the colliding nuclei, can be more efficiently evaluated using the method developed by Yin, Tan, and Chen [30] and Zhong [31], used in Ref. [10]. Since the method is applicable to uncorrelated densities, the higher order terms can be evaluated in terms of appropriate uncorrelated densities and applying the prescription given in Ref. [13] to finally obtain the c.m. correlation corrected higher order phase-shift functions.

The results of the present analysis suggest that the phase variation parameter γ_{NN} decreases as the projectile energy per nucleon decreases. It has the value of 0.4 fm^2 at 1 GeV [16], 0.25 fm^2 at about 0.8 GeV [25,26], and assumes negligible values at about 0.2 GeV, where it appears to change sign to remain negative down to about 80 MeV. However, more studies at other energies in this energy range are needed to arrive at a definitive conclusion on this matter.

It is generally believed that the ^{12}C nucleus is deformed in its ground state. This aspect has been ignored in the analysis presented above. The effect of deformation on ^{12}C-^{12}C scattering can, in principle, be studied in a realistic way within the framework of the Glauber model using the angular momentum projected wave function of the microscopic deformed model for ^{12}C. However, this is a computationally difficult task as is evident from the work of Abgrall, Labarsouque, and Morand [32] who used such a description of ^{12}C to analyze p-^{12}C elastic scattering data at 1.0 GeV. Interestingly, their results for p-^{12}C scattering show that the elastic scattering and to a lesser extent the transition to the 2^+(4.44-MeV) state are only weakly affected by the multistep contributions. In view of this we do not expect that the consideration of the ground state deformation of ^{12}C in a realistic way would substantially affect the findings of the present study.

In summary, we have made a microscopic study of ^{12}C-^{12}C elastic scattering data at 1.016, 1.449, and 2.4 GeV within the framework of the Coulomb modified Glauber model using realistic densities of the colliding nuclei. A good description of the experimental data at 1.016 and 1.449 GeV is found by considering up to a second order term in the nuclear phase-shift expansion series and the phase variation of the *NN* scattering amplitude. At 2.4 GeV, however, although consideration of the second order phase brings theory much closer to experiment, noticeable discrepancies still exist, suggesting the importance of the neglected higher order phase shifts at this energy. The present study also sheds some light on the energy dependence of the phase variation parameter.

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