# Influence of the N = 50 neutron core on dipole excitations in <sup>87</sup>Rb

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Dipole excitations in the semimagic N=50 nucleus <sup>87</sup>Rb were investigated at the Stuttgart Dynamitron facility using bremsstrahlung with an end-point energy of 4.0 MeV. The widths  $\Gamma$  or the reduced excitation probabilities  $B(\Pi 1)\uparrow$  of 18 states were determined for the first time. The magnetic dipole excitations are well reproduced in the framework of the shell model, however, these calculations cannot describe the observed electric dipole excitations. The  $1/2^+$  state at 3060 keV is proposed to be the weak coupling of an  $f_{5/2}$  proton hole to the 3<sup>-</sup> octupole vibrational state in the N=50 core <sup>88</sup>Sr. The relatively strong E1 transition from that state to the ground state is explained as mainly the neutron  $h_{11/2} \rightarrow g_{9/2}$  transition. The breakup of the N=50 core and neutron excitations into the  $h_{11/2}$  shell are essential to describe electric dipole excitations, but neutroncore excitations do not play an important role for the structure of magnetic dipole excitations.

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#### I. INTRODUCTION

The properties of the semimagic N=50 nuclei between <sup>86</sup>Kr (Z=36) and <sup>92</sup>Mo (Z=42) are essentially determined by the proton subshell closures at Z=38, 40, and 50 due to the successive filling into the  $p_{3/2}$ ,  $p_{1/2}$ , and  $g_{9/2}$  shells. This is reflected in the pronounced changes of the excitation energies of the first  $2^+$  and  $3^-$  states shown in Fig. 1. The presence of these three proton shells and the gaps between them complicate the structure of the N=50 nuclei. This is experienced, e.g., by the difficulties to describe dipole and quadrupole excitations in the odd-mass N=50 nucleus <sup>89</sup>Y by a weak coupling of one proton or one proton hole to the even-mass N=50 core nuclei <sup>88</sup>Sr or <sup>90</sup>Zr, respectively, as confirmed by our investigation of  $^{89}$ Y [1].

A further important question is whether N = 50 core excitations contribute to the structure of low-spin states. This is certainly the case for the 3<sup>-</sup> octupole vibrational state and for the two-phonon 1<sup>-</sup> dipole excitation. However, the twophonon 1<sup>-</sup> excitations  $(2^+_1 \otimes 3^-_1)$  in the N = 50 isotones are known at excitation energies 4.4 MeV  $\leq E_x \leq 4.9$  MeV [2]. Therefore, the corresponding fragments  $(2_1^+ \otimes 3_1^- \otimes \text{particle})$ are expected beyond the energy limit of the present experiments and are not discussed here. The fragmentation of magnetic and electric dipole strengths in N = 50 nuclei is only scarcely investigated experimentally [3] and the known dipole excitations in these nuclei are barely reproduced within nuclear structure models. Photon scattering, in which preferentially dipole excitations take place, is a good tool to obtain information on the structure of low-spin states.

The highest angular momentum observed up to now in the N = 50 nucleus <sup>87</sup>Rb is that of the proton  $g_{9/2}$  single-particle state. The lifetimes of only two excited states are reported [4,5]. A total of 21 dipole excitations are known up to 4 MeV. Of them  $\gamma$  deexcitation has been observed only for 12 [4]. Until now, <sup>87</sup>Rb was investigated in particle transfer reactions [6–9], in decay experiments [10,11], in the (n,n')reaction [12], with the Coulomb excitation [13] as well as with inelastic proton [14] and  $\alpha$ -particle scattering [5].

Following results were obtained in the study of N=50nuclei in the vicinity of <sup>87</sup>Rb:

(i) Neither <sup>88</sup>Sr ( $p_{3/2}$  orbit filled) nor <sup>90</sup>Zr ( $p_{1/2}$  orbit



FIG. 1. Excitation energies of the first  $2^+$  and  $3^-$  states in the N = 50 nuclei with  $34 \le Z \le 44$ .

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filled) represent good inert doubly closed cores. The ground state of <sup>88</sup>Sr shows proton  $f_{5/2}$  and  $p_{3/2}$  as well as proton  $g_{9/2}$  admixtures. The proton  $f_{5/2}$  orbit is significantly empty also in <sup>86</sup>Kr.

(ii) There is an indication of a sudden and large change in the proton distribution once N = 50 is broken open.

(iii) In most cases the core-particle coupling picture fails for the description of excited states in the odd-mass N=50nuclei, i.e., a microscopic model like the shell model with configuration mixing has to be used to understand the structure of excited states in <sup>87</sup>Rb.

The aim of the present investigation is to study the fragmentation of dipole strengths in the N = 50 nucleus <sup>87</sup>Rb by means of a nuclear resonance fluorescence experiment. The experimental results will be compared with the results of shell-model calculations, where also excitations of the N= 50 neutron core are considered. Moreover, the applicability of the weak coupling model to dipole excitations in <sup>87</sup>Rb will be discussed. The role of neutron degrees of freedom for the structure of low-spin states in <sup>87</sup>Rb is investigated.

### **II. EXPERIMENTAL METHODS AND RESULTS**

Nuclear resonance fluorescence (NRF), photon scattering off bound states, represents the most sensitive technique to study low-lying electric and magnetic dipole excitations in heavy nuclei (Ref. [3] and references therein). Excitation energies  $E_x$ , the integrated scattering cross sections  $I_s$ , ground-state transition widths  $\Gamma_0$ , and branching ratios  $\Gamma_0/\Gamma$ can be extracted from the spectra of the scattered photons. These quantities can be transformed into reduced transition probabilities B(E1), B(M1), or lifetimes  $\tau$ .

In NRF experiments using continuous bremsstrahlung, the total scattering intensity  $I_s$  for a decay of the photoexcited state to the ground state, integrated over the resonance and the full solid angle, is given by

$$I_{s} = g \left( \pi \frac{\hbar c}{E_{\gamma}} \right)^{2} \frac{\Gamma_{0}^{2}}{\Gamma}, \qquad (1)$$

where  $\Gamma_0$  is the partial decay width of the photoexcited state with spin *J* to the ground state with spin  $J_0$ , and  $\Gamma$  is the total width. The so-called "spin factor"  $g = (2J+1)/(2J_0+1)$ represents the statistical weight. The integrated scattering cross section  $I_s$  is proportional to the reduced excitation probabilities  $B(E1)\uparrow$  or  $B(M1)\uparrow$ ,

$$B(\Pi 1)\uparrow = gB(\Pi 1)\downarrow = \frac{9}{16\pi} \left(\frac{\hbar c}{E_{\gamma}}\right)^3 (g\Gamma_0).$$
(2)

Unfortunately, in the case of odd-mass target nuclei, the angular distributions of the scattered photons are rather isotropic. Therefore, in general, for nuclei with ground-state spins  $J_0 > 1/2$  no unambiguous spin assignments to the photoexcited states are possible in present day NRF experiments, which is true also for <sup>87</sup>Rb with a ground-state spin and parity of  $J^{\pi} = 3/2^{-}$ . This implies that the spin factor g is unknown. In this case only the product  $g\Gamma_0^2/\Gamma$  or  $g\Gamma_0$  for measured or known decay branching ratios can directly be

extracted from the measured scattering intensities. Furthermore, the vanishing anisotropy in the angular distributions leads to rather low polarizations of the scattered photons and prevents parity assignments that are possible for even-even nuclei by polarization measurements [15]. The formalism describing photon scattering experiments is outlined in more detail in [3,16,17].

The present NRF experiments on <sup>87</sup>Rb were performed at the bremsstrahlung facility installed at the Stuttgart Dynamitron accelerator [3]. The bremsstrahlung end-point energy was 4.0 MeV. The DC electron currents used in the present experiments had to be limited to about 250  $\mu$ A because of the thermal capacity of the radiator target. The scattering target consisted of a <sup>87</sup>Rb<sub>2</sub>CO<sub>3</sub> sample of 2.507 g total mass with a relative enrichment of 99.2% in <sup>87</sup>Rb. The highly hygroscopic rubidium carbonate was pressed to pills of 16 mm diameter and heated up to 160 °C under vacuum to extract the water. The target material was sandwiched with <sup>27</sup>Al discs (0.7639 g; diameter 16 mm), serving for the photon flux calibration [18]. The scattered photons were detected by three high-resolution HPGe  $\gamma$ -ray spectrometers installed at angles of about 90°, 127°, and 150° with respect to the incoming bremsstrahlung beam. The efficiencies of all three detectors amounted to about 100% each, relative to a standard 7.6 cm $\times$ 7.6 cm NaI(Tl) detector. The energy resolution was typically about 2 keV at a photon energy of 1.3 MeV and about 3 keV at 3 MeV. Therefore, the uncertainties of the excitation energies quoted in Table I are less than 1 keV. The total effective time of data collection was about 80 h. Figure 2 shows the spectrum of photons scattered off <sup>87</sup>Rb detected under a scattering angle of 90° in the energy range 2-4 MeV. The experimental results are summarized in Table I.

As already discussed in Sec. I, the nucleus <sup>87</sup>Rb has been studied in many different reactions. Preferably low-spin states were excited in all of them. Therefore, the accepted set of known levels given in Ref. [4] can be assumed to be nearly complete, and nearly the same states should be excited in the NRF experiment. One unit of angular momentum, and to a much lesser extent two units, are transferred to the target nucleus via NRF. Considering  $J^{\pi} = 3/2^{-1}$  for the ground state of <sup>87</sup>Rb, the excited states are, therefore, expected to have  $J \leq 5/2$ , but 7/2 cannot be completely excluded. Thus, a level observed in the present NRF experiment is assumed to correspond to a known state given in Ref. [4], if the excitation energies agree within the experimental errors and if the conditions for the angular momentum are realized. In the following, some levels are discussed.

The 845 keV state is reported [4] to have spin and parity  $J^{\pi} = 1/2^{-}, 3/2^{-}$ . Taking into account the results of all different experiments, we assign  $J^{\pi} = 1/2^{-}$  to this level (Table I).

The 2378 keV level given in Table I cannot be the 2379 keV state [4] observed in the (p,p') reaction because of the transferred angular momentum of L=(5). It must be the 2387 keV level [4]. The parity of this state is unknown. Further on  $J^{\pi}=1/2^+$  can be excluded, because in this case the  $\gamma$  branch to the  $J^{\pi}=5/2^-$ ,  $E_x=403$  keV level would be an M2 transition and the 2378 keV level would not have been observed in our experiment.

TABLE I. Experimental results of the present  $(\gamma, \gamma')$  experiment on <sup>87</sup>Rb. The excitation energy  $E_x$ , integrated cross section  $I_s$ , angular momentum and parity  $J^{\pi}$ , value  $g\Gamma_0^2/\Gamma$ , branching ratio  $\Gamma_0/\Gamma$ , total width  $\Gamma$ , and the reduced excitation probability  $B(M1)\uparrow$  or  $B(E1)\uparrow$  are given. Isotropic angular correlation and pure M1 and E1 transitions have been assumed.

$E_x$ (keV)	$I_s$ (eV b)	$J^{\pi \; \mathrm{a}}$	$g\Gamma_0^2/\Gamma$ (meV)	$\Gamma_0/\Gamma$	Г (meV)	$B(M1)^{\uparrow b} \ (\mu_N^2)$	$B(E1)^{\uparrow c}$ (10 <sup>-4</sup> $e^2 \text{ fm}^2$ )
845	12.9(17)	$1/2^{-d}$	2.39(32)	1.0	4.78(64)	0.34(5)	
1390	3.13(65)	$(3/2)^{-}$	1.57(33) <sup>e</sup>	0.81 <sup>a</sup>	2.40(50)	$6.2(13) \times 10^{-2}$	
1463	1.51(55)	$(1/2)^{-}$	0.84(31)	0.87 <sup>a</sup>	2.23(81)	$2.7(10) \times 10^{-2}$	
1578	0.64(44)	1/2-,3/2-	0.42(28)	0.105(8) <sup>a</sup>		$8.8(58) \times 10^{-2}$	
1741	13.6(10)	$(3/2, 5/2)^{-}$	10.7(8)	0.67(3) <sup>a</sup>		0.26(2)	
2014	1.01(32)	$(1/2, 3/2, 5/2)^{-d}$	1.07(34)	1.0		$1.1(4) \times 10^{-2}$	
2284	3.65(36)	$(1/2, 3/2, 5/2)^{-d}$	4.95(48)	0.67(6)		$5.4(5) \times 10^{-2}$	
2378	4.04(38)	(1/2,3/2,5/2) <sup>d</sup>	5.95(56)	0.70(6)			
2398	29.0(17)	1/2-,3/2-	43.4(26)	0.85(2)		0.32(2)	
2555	12.1(8)	3/2+,5/2+	20.6(14)	0.977(2) <sup>a</sup>			12.1(8)
2811	1.14(21)	3/2+,5/2+	1.57(33)	0.54(2) <sup>a</sup>			1.3(3)
$3005^{\text{f}}$	4.58(41)	$(1/2, 3/2, 5/2)^{+ d}$	10.8(10)	1.0			3.8(4)
3043	0.37(12)	(1/2,3/2,5/2) <sup>d</sup>	0.89(29)	1.0			
3055	0.66(13)	3/2,5/2,7/2(-)	1.60(32)	0.78(3) <sup>a</sup>		$6.2(12) \times 10^{-3}$	
3060	3.86(36)	$1/2^{+}$	9.39(87)	1.0	18.7(17)		3.1(3)
3309	9.47(73)	3/2+,5/2+	27.0(21)	0.908(8) <sup>a</sup>			7.8(6)
3338	0.94(18)	1/2-,3/2-	2.73(51)	0.34(14)		$1.9(3) \times 10^{-2}$	
3702	31.5(22)	$(1/2, 3/2, 5/2)^{(-) d}$	113(15)	0.93(3)		0.21(1)	
3767	1.24(36)	1/2-,3/2-	4.6(13)	1.0		$7.4(21) \times 10^{-3}$	
3837	15.4(22)	$1/2^{+}$	59.2(86)	1.0	117(11)		10.0(15)

<sup>a</sup>Taken from [4].

<sup>b</sup> $B(M1)\uparrow$  as calculated from  $g\Gamma_0^2/\Gamma$ ; 1 W.u. $(M1)=1.79~\mu_N^2$ .

 $^{c}B(E1)\uparrow$  as calculated from  $g\Gamma_{0}^{2}/\Gamma$ ; 1 W.u. $(E1)=1.27 e^{2} \text{ fm}^{2}$ .

<sup>d</sup>Angular momentum from this work.

<sup>e</sup>The calculated angular correlation value has been considered.

<sup>f</sup>Superimposed by a <sup>27</sup>Al line.



FIG. 2. Spectrum of photons scattered off <sup>87</sup>Rb, measured at a scattering angle of 90°. Lines marked with the transition energy in keV belong to <sup>87</sup>Rb. Also calibration lines (<sup>27</sup>Al) and background lines are marked. SE, single escape peak; DE, double escape peak. The small 3055 keV line is well separated from the 3060 keV peak, but is not visible with the present scale.

The 3055 and 3060 keV levels (Table I) are assigned to the levels in Ref. [4], which have the same excitation energy. In the (p,p') reaction a level at 3058 keV has been observed with an energy error of 8 keV, which is supposed to be a doublet [14]. Therefore, the transferred angular momentum could not be deduced. This level has not been adopted in the compilation [4]. Considering the uncertainty of 8 keV, the two states observed in the doublet could be the states at 3055 and 3060 keV as well.

The 3702 keV level is tentatively assigned to the 3692 keV level adopted in Ref. [4], although the experimental errors of the excitation energies do not overlap. Therefore, a tentative assignment of the negative parity is given. The 3702 keV state identified in the present experiment cannot be the 3707 keV level [4] found in the (p,p') reaction, because an L = 4 transfer to this state has been found [14].

The 3767 keV level is assigned to a level with the same excitation energy adopted in Ref. [4]. A doublet at 3773 keV has been found in the (p,p') reaction [14], but no angular momentum could be deduced. This doublet has not been adopted in Ref. [4]. One of the doublet members may be the 3767 keV state.

The 3837 keV level is assigned to the 3834 keV level

TABLE II. Gamma-decay branching ratios  $\Gamma_1/\Gamma$  obtained in the  $(\gamma, \gamma')$  experiment on <sup>87</sup>Rb.  $E_i$  and  $E_f$  denote the energies of the initial and final states, respectively.

$\frac{E_{\gamma}}{(\text{keV})}$	$E_i$ (keV)	$E_f$ (keV)	$\Gamma_1/\Gamma$
1881	2284	403	0.33(6)
1975	2378	403	0.30(6)
1995	2398	403	0.08(1)
1553	2398	845	$0.07(2)^{a}$
1760	3338	1578	0.66(14)
2312	3702	1390	0.07(3)

 ${}^{a}\Gamma_{2}/\Gamma.$ 

adopted in Ref. [4]. Because of the large energy difference it cannot be the reported 3824 keV state.

All states with known angular momentum observed in the NRF experiment—except the one at 3055 keV—are dipole excited levels. Therefore, the levels at 2014, 2284, 2378, 3005, 3043, and 3702 keV should be ascribed very likely also to dipole excitations and we assign to them tentatively J = (1/2, 3/2, 5/2) as given in Table I. For the same reason, the J = 7/2 assignment to the level at 3055 keV seems to be very unlikely.

The widths  $\Gamma$  or the reduced excitation probabilities  $B(\Pi 1)\uparrow$  of 18 states given in Table I were measured for the first time. Fourteen levels of the hitherto known 21 dipole excitations with  $J \leq 5/2$  in the energy range  $E_x \leq 4$  MeV [4] have been observed in our  $(\gamma, \gamma')$  experiment. The known dipole excitations at 403, 1893, 2414, 2530, 2961, 3692, and 3974 keV were not observed in our investigation. The reason may be the typical detection limits of  $B(E1)\uparrow\approx 1 \times 10^{-5} e^2 \text{ fm}^2$  and  $B(M1)\uparrow\approx 0.9\times 10^{-3} \mu_N^2$  for NRF experiments at the Stuttgart bremsstrahlung facility [19].

One new level at 3043 keV was found. For the levels at 2014, 2284, 2398, 3005, 3043, 3060, 3338, 3702, 3767, and 3837 keV  $\gamma$  deexcitation was observed for the first time and for the states given in Table II  $\gamma$  decays to excited levels have been found. For the energy range  $0.5 \le E_x \le 3.3$  MeV, practically the same dipole excitations as in the former (p,p') study [14] have been observed in our  $(\gamma,\gamma')$  experiment. In the NRF investigation, additionally the 1578 and 3043 keV levels have been observed, in the (p,p') experiment additionally the states at 2961 and 3099 keV were found.

Twelve of the observed 20 dipole excitations are M1 transitions, six are E1 excitations [4]. For the 845, 1741, 2398, and 3702 keV states large  $B(M1)\uparrow$  values have been found, for the 2555 and 3837 keV levels large  $B(E1)\uparrow$  values were measured.

#### **III. SHELL-MODEL CALCULATIONS**

Up to now no shell-model calculation including excitations of the N=50 neutron core has been performed for <sup>87</sup>Rb. The shell-model space used in the present calculations includes the active proton orbitals  $\pi(0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2})$  and neutron orbitals  $\nu(1p_{1/2}, 0g_{9/2}, 1d_{5/2})$  relative to a hypothetic <sup>66</sup>Ni core. The orbitals are denoted by the radial quantum number that gives the number of nodes of the radial wave function excluding the nodes at the origin and at infinity. Since an empirical set of effective interaction matrix elements for this model space is not available yet, various empirical sets have been combined with the matrix elements of a modified surface-delta interaction. Details of this procedure are described in Refs. [20,21]. The effective interaction in the proton shells was taken from Ref. [22]. In that work, the residual interaction and the single-particle energies of the proton orbitals were deduced from a least-squares fit to 170 experimental level energies in N = 50 nuclei with mass numbers between 82 and 96. The data given in Ref. [23] have been used for the proton-neutron interaction between the  $\pi(1p_{1/2}, 0g_{9/2})$  and the  $\nu(1p_{1/2}, 0g_{9/2})$  orbitals. These data were derived from an iterative fit to 95 experimental level energies of N=48, 49,and 50 nuclei. The matrix elements of the neutron-neutron interaction of the  $\nu(1p_{1/2}, 0g_{9/2})$  orbitals have been assumed to be equal to the isospin T=1 component of the protonneutron interaction given in Ref. [23]. For the  $(\pi 0 f_{5/2}, \nu 0 g_{9/2})$  residual interaction, the matrix elements proposed in Ref. [24] have been used.

The single-particle energies relative to the <sup>66</sup>Ni core have been derived from the single-particle energies of the proton orbitals given in Ref. [22] with respect to the <sup>78</sup>Ni core and from the neutron single-hole energies of the  $1p_{1/2}, 0g_{9/2}$  orbitals [23]. The transformation of these single-particle energies to those relative to the <sup>66</sup>Ni core has been performed [25] on the basis of the effective residual interactions described above. The obtained values are  $\epsilon_{0f_{5/2}}^{\pi} = -9.106 \text{ MeV}, \quad \epsilon_{1p_{3/2}}^{\pi} = -9.033 \text{ MeV}, \quad \epsilon_{1p_{1/2}}^{\pi} = -4.715 \text{ MeV}, \quad \epsilon_{0g_{9/2}}^{\pi} = -0.346 \text{ MeV}, \quad \epsilon_{1p_{1/2}}^{\nu} = -7.834 \text{ MeV}, \quad \epsilon_{0g_{9/2}}^{\nu} = -6.749 \text{ MeV}, \text{ and}$  $\epsilon_{1d_{5/2}}^{\nu} = -4.144$  MeV. These single-particle energies and the corresponding values of the strengths of the residual interactions have been used to calculate level energies as well as M1 and E2 transition strengths. For the latter, effective g factors of  $g_s^{\text{eff}} = 0.7 g_s^{\text{free}}$  and effective charges of  $e_{\pi} = 1.72 e$ and  $e_{\nu} = 1.44e$  [26], have been applied. The nucleus <sup>87</sup>Rb has nine protons and 12 neutrons in the considered configuration space. To make the calculations feasible, a truncation of the occupation numbers was necessary. At most three protons are allowed to occupy the  $(1p_{1/2}, 0g_{9/2})$  subshell and at most one  $0g_{9/2}$  neutron can be lifted to the  $1d_{5/2}$  orbital. With these restrictions, a configuration space with dimensions smaller than 8700 has been obtained. The calculations were carried out with the code RITSSCHIL [27].

### **IV. DISCUSSION**

# A. Magnetic dipole excitations in <sup>87</sup>Rb

As shown in Fig. 3, the shell model reproduces the right order of the experimentally observed first  $3/2^-$ ,  $5/2^-$ , and  $1/2^-$  states and provides a fair reproduction of the excitation energies. The calculated level density corresponds approximately to the observed one. As shown in Fig. 4, there is a



FIG. 3. Comparison of experimentally observed dipole excited levels in <sup>87</sup>Rb with the results of our shell-model calculations. The angular momenta are given as 2J; solid line, observed in the ( $\gamma$ ,  $\gamma'$ ) experiment (Table I); dashed line, taken from [4]; the lowest five calculated states with J = 1/2, 3/2, and 5/2 are given.

remarkable agreement of the measured and calculated  $B(M1)\downarrow$  values for the transitions of the states with known angular momentum to the ground state. Also the  $B(M1)\downarrow$  value for the  $\gamma$  branch of  $E_{\gamma}=987$  keV from the second  $3/2^-$  state to the first  $5/2^-$  state is well reproduced by the calculations, whereas the  $B(E2)\downarrow$  value for the 1060 keV transition from the second  $1/2^-$  state to the first  $5/2^-$  state deviates by a factor of about 10.

On the basis of this overall agreement we assigned some of the experimental states to the corresponding calculated levels (Fig. 3). This assignment allows the following conclusions on the structure of the observed states on the basis of the calculated wave functions. As an example, the main contributions to the shell-model wave functions of the first and second  $1/2^-$ ,  $3/2^-$ , and  $5/2^-$  states are given in Table III. The discussion below includes all the calculated magnetic dipole excitations shown in Fig. 3.

Dominating proton  $p_{3/2}$ ,  $f_{5/2}$ , and  $p_{1/2}$  single-particle character is found for the  $3/2_1^-$  ground state, the  $5/2_1^-$  and  $1/2_1^-$  states, respectively. The other magnetic dipole excitations are dominated by three protons in the fp shell. In addition to the main contributions given in Table III, the wave functions contain a large number of small components of the types  $\pi(fp)^1$ ,  $\pi(fp)^3$ ,  $\pi(fp)^1\nu(0g_{9/2}^{-1}1d_{5/2}^1)_2$ , or  $\pi(fp)^3\nu(0g_{9/2}^{-1}1d_{5/2}^1)_2$ . Components with neutrons contribute from 3% to 9% to the wave functions, but the excitation of the N=50 core, which is possible in our shell-model cal-



FIG. 4. The depopulation of experimentally observed magnetic dipole excitations in <sup>87</sup>Rb compared with the results of shell-model calculations. The  $B(M1)\downarrow$  value for the 403 keV transition has been taken from [4].

culation, does not allow M1 transitions. Excitations of protons to the  $0g_{9/2}$  shell were not predicted in our calculation.

According to the wave functions, the very strong 845 keV  $1/2_1^- \rightarrow 3/2_1^-$  transition is the proton  $p_{1/2} \rightarrow p_{3/2}$  spin-flip M1 transition, which corresponds to the  $1_1^+ \rightarrow 0_1^+$  transition in <sup>88</sup>Sr [28]. *M*1 transitions between the spin-orbit partners  $p_{1/2}$  and  $p_{3/2}$  are the only allowed *M*1 transitions in the considered shell-model space. A detailed analysis of the wave functions shows that the *M*1 transition  $3/2_2^- \rightarrow 3/2_1^-$  is realized between relatively strong components of the configurations  $\pi(0f_{5/2}^{-2}1p_{3/2}^{-1}) \rightarrow \pi(0f_{5/2}^{-2}1p_{1/2}^{1})$  and  $\pi(1p_{1/2}^1)\nu(0g_{9/2}^{-1}1d_{5/2}^1) \rightarrow \pi(0f_{5/2}^{-2}1p_{1/2}^{-1})$ . The *M*1 transition  $1/2_2^- \rightarrow 3/2_1^-$  is enabled via the relatively strong components  $\pi(0f_{5/2}^{-2}1p_{3/2}^{-1}) \rightarrow \pi(0f_{5/2}^{-2}1p_{1/2}^{-1})$  and  $\pi(1p_{1/2}^{-1}) \rightarrow \pi(0f_{5/2}^{-2}1p_{3/2}^{-1})$ . The *M*1 transition  $1/2_2^- \rightarrow 3/2_1^-$  is enabled via the relatively strong components  $\pi(0f_{5/2}^{-2}1p_{3/2}^{-1}) \rightarrow \pi(0f_{5/2}^{-2}1p_{1/2}^{-1})$  and  $\pi(1p_{1/2}^{-1}) \rightarrow \pi(0f_{5/2}^{-2}1p_{3/2}^{-1})$ . The *M*1 transition  $1/2_2^- \rightarrow 3/2_1^-$  is enabled via the relatively strong components  $\pi(0f_{5/2}^{-2}1p_{3/2}^{-1}) \rightarrow \pi(0f_{5/2}^{-2}1p_{1/2}^{-1})$  and  $\pi(1p_{1/2}^{-1}) \rightarrow \pi(1p_{3/2}^{-2})$ . The relatively small  $B(M1) \downarrow$  value observed for the  $5/2_1^- \rightarrow 3/2_1^-$  transition can be explained by the fact that the wave function for the  $5/2_1^-$  state contains only very small components allowing such transitions.

The present shell-model space seems to be sufficiently large to describe the magnetic dipole excitations in <sup>87</sup>Rb. Neutron core excitations are not important for the discussed magnetic dipole transitions.

### B. The weak-coupling picture in <sup>87</sup>Rb

Several attempts have been made to describe excited states in <sup>87</sup>Rb by means of a weak coupling of a proton or proton hole in the fp shell to the N = 50 core <sup>88</sup>Sr [10,12,14].

TABLE III. Main components of shell-model wave functions of states in  $^{87}\mathrm{Rb}.$ 

$J^{\pi}$	Contribution of the configuration		
1/21	83% $\pi(1p_{1/2}^1)$ 6% of the type $\pi(fp)^1 \nu(0g_{9/2}^{-1}1d_{5/2}^1)_2$ or $\pi(fp)^3 \nu(0g_{9/2}^{-1}1d_{5/2}^1)_2$		
1/22	61% $\pi(0f_{5/2}^{-1}1p_{3/2}^{-1}1p_{1/2}^{1})$ 9% of the type $\pi(fp)^{1}\nu(0g_{9/2}^{-1}1d_{5/2}^{1})_{2}$ or $\pi(fp)^{3}\nu(0g_{9/2}^{-1}1d_{5/2}^{1})_{2}$		
3/21	$ \begin{array}{c} 82\% \ \pi(1p_{3/2}^{-1}) \\ 1.3\% \ \pi(0f_{5/2}^{-1}1p_{3/2}^{-1}1p_{1/2}^{1})\nu(0g_{9/2}^{-1}1d_{5/2}^{1})_{2} \ ^{a} \\ 3\% \ \text{of the type} \ \pi(fp)^{1}\nu(0g_{9/2}^{-1}1d_{5/2}^{1})_{2} \ \text{or} \\ \pi(fp)^{3}\nu(0g_{9/2}^{-1}1d_{5/2}^{1})_{2} \end{array} $		
3/22	28% $\pi(0f_{5/2}^{-1}1p_{3/2}^{-1}1p_{1/2}^{1})$ 25% $\pi(1p_{3/2}^{-1})$ 7% of the type $\pi(fp)^{1}\nu(0g_{9/2}^{-1}1d_{5/2}^{1})_{2}$ or $\pi(fp)^{3}\nu(0g_{9/2}^{-1}1d_{5/2}^{1})_{2}$		
5/21	71% $\pi(0f_{5/2}^{-1})$ 3% of the type $\pi(fp)^1 \nu(0g_{9/2}^{-1}1d_{5/2}^1)_2$ or $\pi(fp)^3 \nu(0g_{9/2}^{-1}1d_{5/2}^1)_2$		
5/22	72% $\pi(1p_{3/2}^{-2}1p_{1/2}^{1})$ 5% of the type $\pi(fp)^{1}\nu(0g_{9/2}^{-1}1d_{5/2}^{1})_{2}$ or $\pi(fp)^{3}\nu(0g_{9/2}^{-1}1d_{5/2}^{1})_{2}$		
1/21+	56% $\pi(0f_{5/2}^{-2}1p_{3/2}^{-1}1p_{1/2}^{1}0g_{9/2}^{1})$		
3/21+	35% $\pi(0f_{5/2}^{-1}1p_{3/2}^{-2}1p_{1/2}^{1}0g_{9/2}^{1})$		
5/21+	24% $\pi(1p_{3/2}^{-2}0g_{9/2}^{1})$		

<sup>a</sup>This small contribution is important for the explanation of E1 transitions (cf. the text).

Hulstmann *et al.* [14] stated that the "interpretation of the experimental results within the weak-coupling model is impossible." This result is compatible with the conclusions drawn in our investigation of a weak coupling of a  $g_{7/2}$  proton to positive-parity core states in the N=82 cores <sup>140</sup>Ce and <sup>142</sup>Nd [29]. If the coupled particle is an important constituent of the core state, then—because of the Pauli principle—the addition of this particle leads to a rearrangement of the nucleons in the shells and the weak-coupling concept must fail.

For the detailed discussion of the weak-coupling model we will use the following energies:  $E_x(2_1^+, {}^{88}\text{Sr}) = 1836 \text{ keV}$ ,  $E_x(3_1^-, {}^{88}\text{Sr}) = 2734 \text{ keV}$ , and the experimental proton single-particle energies  $E_x(p_{3/2}) = 0$ ,  $E_x(f_{5/2}) = 403 \text{ keV}$ ,  $E_x(p_{1/2}) = 845 \text{ keV}$ , and  $E_x(g_{9/2}) = 1578 \text{ keV}$ observed in  ${}^{87}\text{Rb}$ . The coupling of a  $p_{3/2}$  proton hole and a  $p_{1/2}$  proton to the first 2<sup>+</sup> state in  ${}^{88}\text{Sr}$  would give multiplets in  ${}^{87}\text{Rb}$  at about 1800 and 2700 keV, respectively. The main structure of the first 2<sup>+</sup> state in  ${}^{88}\text{Sr}$  is given in Ref. [30] to 73% of  $\pi(1p_{3/2}^{-1}1p_{1/2}^{1})$ , which is the result of a shell-model calculation. Thus, the coupling particle occupies the same shell as the main constituents of the core state. Hence, there is no weak coupling and the weak coupling concept fails to describe the considered states in <sup>87</sup>Rb.

Another possibility to produce magnetic dipole excitations in <sup>87</sup>Rb with J=3/2 or J=5/2 would be a coupling of a  $g_{9/2}$  proton to the 3<sup>-</sup> state in <sup>88</sup>Sr. This weak coupling should be possible because of the positive parity of the coupled particle. But such a multiplet should be expected around 4300 keV, which is beyond the scope of our experiment.

In the following section, we propose the weak-coupling model to be a possible way to describe the  $1/2^+$  state in  ${}^{87}$ Rb at 3060 keV, which deexcites via a relatively strong *E*1 transition to the ground state.

### C. Electric dipole excitations in <sup>87</sup>Rb

In Fig. 3, also the calculated positive-parity states are shown, which can be excited by E1 transitions. According to the shell-model wave functions given in Table III, the positive-parity states are generated by the excitation of one proton to the  $g_{9/2}$  shell. However, these calculated states obviously do not correspond to the positive-parity states observed in the experiment. The calculated states are predicted at too high excitation energies and the level density is too low. Moreover, our shell-model space does not allow E1 transitions, what is in clear discrepancy to the observations. Thus, our shell-model calculations including even the breakup of the N=50 core are not able to describe the electric dipole excitations.

As already stated in the preceding section, the weakcoupling model may be used, if there is no strong overlap between the coupling particle and the core state. In this sense, the following couplings of protons with the first 2<sup>+</sup> or 3<sup>-</sup> states in <sup>88</sup>Sr are allowed and enable the generation of electric dipole excitations in <sup>87</sup>Rb: (i) 2<sup>+</sup>  $\otimes g_{9/2}$  giving a 5/2<sup>+</sup> state at about 3400 keV, (ii) 3<sup>-</sup>  $\otimes p_{1/2}$  giving a 5/2<sup>+</sup> state at about 3600 keV, (iii) 3<sup>-</sup>  $\otimes p_{3/2}$  giving 3/2<sup>+</sup> and 5/2<sup>+</sup> states at about 2700 keV, and (iv) 3<sup>-</sup>  $\otimes f_{5/2}$  giving 1/2<sup>+</sup>, 3/2<sup>+</sup> and 5/2<sup>+</sup> states at about 3100 keV. The first three couplings may be candidates for the experimentally observed 3/2<sup>+</sup> or 5/2<sup>+</sup> states between 2300 and 3400 keV. Since the experimental spin values are not fixed, a unique experiment-to-theory assignment is impossible.

The only coupling resulting in an  $1/2^+$  state is that of the last example given above. Therefore, the first  $1/2^+$  state at 3060 keV may be tentatively explained by the weak coupling of a proton  $f_{5/2}$  hole to the 3<sup>-</sup> octupole vibrational state in the core <sup>88</sup>Sr, giving a  $1/2^+$  state at 3137 keV. The observed electric dipole excitations around 3 MeV may belong to the  $3^- \otimes \pi f_{5/2}$  multiplet.

The experimental value  $B(E1) \downarrow = 6.2 \times 10^{-4} e^2 \text{ fm}^2$  of the 3060 keV ground-state transition in <sup>87</sup>Rb agrees approximately with that of the  $3_1^- \rightarrow 2_1^+$  transition observed in <sup>88</sup>Sr  $[B(E1) \downarrow = 7.67 \times 10^{-4} e^2 \text{ fm}^2 [31]]$ . Therefore, we assume that both *E*1 transitions take place between similar configurations.

The experimental  $3_1^- \rightarrow 2_1^+ B(E1) \downarrow$  value in <sup>88</sup>Sr is well reproduced by calculations within the quasiparticlephonon model (QPM), which is explained in Ref. [32] in more detail. The values  $B(E1,3_1^- \rightarrow 2_1^+) = 3.2$ 

TABLE IV. Structure of the first  $2^+$  and  $3^-$  RPA phonons predicted by QPM calculations for <sup>88</sup>Sr in terms of two-quasiparticle components. The main contributions are shown only.

$\overline{J^{\pi}}$	E (keV)	Structure	Contribution (%)
21+	2100	$\nu(0g_{9/2}1d_{5/2})$	11
		$\pi(1p_{3/2}1p_{1/2})$	29
		$\pi(0f_{5/2}1p_{1/2})$	28
		$\pi(0g_{9/2}0g_{9/2})$	9
		$\pi(0f_{5/2}0f_{5/2})$	8
		$\pi(1p_{3/2}1p_{3/2})$	7
$\overline{3_{1}^{-}}$	2850	$\nu(0g_{9/2}0h_{11/2})$	8
		$\pi(1p_{3/2}0g_{9/2})$	66
		$\pi(0f_{5/2}0g_{9/2})$	8

×10<sup>-4</sup>  $e^2 \text{ fm}^2$ ,  $E_x(2_1^+, {}^{88}\text{Sr}) = 1852 \text{ keV}$ , and  $E_x(3_1^-, {}^{88}\text{Sr}) = 2640 \text{ keV}$  result from this model in good agreement with the experimental values. The model wave functions are constructed out of quasiparticle random-phase approximation (RPA) phonons and the wave function of an excited state is taken as a superposition of one-, two-, and three-phonon components. In Table IV, the structure of the first 2<sup>+</sup> and 3<sup>-</sup> RPA phonons for the QPM calculations to  ${}^{88}\text{Sr}$  is presented. The QPM wave function of the first 2<sup>+</sup> state in  ${}^{88}$  Sr consists to 91% of the first  $2_1^+$  RPA phonon and that of the first 3<sup>-</sup> state to 92% of the first  $3_1^- \rightarrow 2_1^+ E1$  transition in  ${}^{88}\text{Sr}$  is dominantly a  $\nu h_{11/2} \rightarrow \nu g_{9/2}$  transition.

Regarding the possible  $3^{-}(^{88}Sr) \otimes \pi f_{5/2}$  structure of the  $1/2^+$  level at 3060 keV in <sup>87</sup>Rb, this state should contain also small  $\nu h_{11/2}$  components, since the  $\nu h_{11/2} \rightarrow \nu g_{9/2}$  transition is the only allowed E1 transition in the considered configuration space. The shell-model wave function of the  $3/2^{-1}$ ground state in <sup>87</sup>Rb contains to 1.3% the configuration  $\pi(0f_{5/2}^{-1}1p_{3/2}^{-1}1p_{1/2}^{1})\nu(0g_{9/2}^{-1}1d_{5/2}^{1})_2$  (Table III). Thus,  $\nu h_{11/2}$  $\rightarrow \nu g_{9/2}$  transitions are possible also for the  $1/2^+(3060 \text{ keV}) \rightarrow 3/2^-$  ground-state transition in <sup>87</sup>Rb, what gives a qualitative explanation for this electric dipole transition as well as for the similarity with the  $3_1^- \rightarrow 2_1^+$  transition in <sup>88</sup>Sr. It is well known that strong E1 transitions may be realized even by very small admixtures to the wave functions. The above discussion shows that the breakup of the N=50 core and the excitation of one neutron to the  $h_{11/2}$ shell are crucial points for the description of electric dipole excitations in the N = 50 nucleus <sup>87</sup>Rb.

The weak coupling of an  $f_{5/2}$  proton to the first 3<sup>-</sup> state in the N = 50 nucleus <sup>86</sup>Kr at 3099 keV would give a  $1/2^+$  state

at about 3500 keV, which could be a possible candidate for the  $1/2^+$  state in <sup>87</sup>Rb at 3837 keV. Unfortunately, the strength of the  $3_1^- \rightarrow 2_1^+$  transition in <sup>86</sup>Kr is unknown and the question remains open whether such a weak coupling is able to reproduce the very strong *E*1 excitation to the 3837 keV state in <sup>87</sup>Rb.

### V. SUMMARY

In the first nuclear resonance fluorescence experiment on the N=50 nucleus <sup>87</sup>Rb, the width  $\Gamma$  of five and the reduced excitation probabilities  $B(\Pi 1)\uparrow$  of 13 dipole excitations have been determined. The  $\gamma$  deexcitation of ten levels and branchings of five levels were observed for the first time. Strong  $B(M1)\downarrow$  and  $B(E1)\downarrow$  values were deduced for the deexcitations of the  $1/2^-$  state at 845 keV and of the  $1/2^+$ state at 3837 keV to the  $3/2^-$  ground state, respectively.

The magnetic dipole excitations are described in the framework of shell-model calculations as one- or threeproton excitations into the fp shell, and neutron excitations of the N=50 core were found to play no important role. The shell-model calculations allowing the excitation of the N = 50 core by lifting one neutron from the  $g_{9/2}$  to the  $d_{5/2}$  shell are not able to reproduce the observed electric dipole excitations.

The weak-coupling model turns out to be not applicable for the description of the observed magnetic dipole excitations, whereas the electric dipole excitations may be described in the framework of a weak-coupling picture. The  $1/2^+$  state at 3060 keV is proposed to result from the coupling of an  $f_{5/2}$  proton hole to the 3<sup>-</sup> octupole vibrational state in the N=50 core <sup>88</sup>Sr. The strong E1 transition of that  $1/2^+$  state to the ground state is explained as the neutron  $h_{11/2} \rightarrow g_{9/2}$  transition. The breakup of the N=50 core and the excitation of one neutron to the  $h_{11/2}$  shell are very important for the description of the electric dipole excitations. Until now there is no explanation for the fast E1 transition from the 3837 keV state to the ground state.

Further experimental and theoretical investigations of dipole excitations in the N=50 nuclei are needed. For the understanding of the structure of these states, the shell-model space has to be expanded by neutron excitations into the  $h_{11/2}$  shell. The inclusion of such excitations is not feasible yet, since the dimension of the corresponding configuration space is beyond our possibilities for diagonalization. QPM calculations for the odd-mass N=50 nuclei could be an alternative.

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