

Resonance states with the complex absorbing potential method

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We present a study of the complex absorbing potential (CAP) method to solve resonance states in a nuclear physics problem. Compared to atomic physics cases, nuclear potentials have a short-range property, and the resonances generally have large decay widths. We find that an appropriate functional form of the CAP method is necessary for an accurate calculation. The functional form dependence of the CAP method and the convergence of the resonance eigenvalues are investigated in a two-body system. We also apply the CAP method to examine a three-body system. Our results are in good agreement with others in the literature.

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I. INTRODUCTION

There has been continuous interest in theoretical studies of resonance phenomena in both atomic physics and nuclear physics. Several methods have been developed over the recent years, including the complex scaling method (CSM) or, under a different name, the complex coordinate rotation method [1], the method that employs the analytical continuation in the coupling constant (ACCC) [2], and the Jost function method (JFM) [3]. The most appealing point of the CSM is that one can solve for resonance states in just the same way as for a bound state. The coordinates (momenta) of the system are rotated in the complex coordinate (momentum) plane. After the complex rotation, the resonance wave function is damped in the asymptotic region, and the complex eigenvalue gives the resonance parameters E_r and $\Gamma/2$, where E_r is the resonance energy, and Γ is the total decay width. The proof for the validity of this method is known as the “ABC theorem” [4]. Successful applications of the CSM have been documented in the literature both for atomic physics [1] and for nuclear physics [5]. In the ACCC method, the resonances can be obtained by using an analytical continuation from bound states. The momentum of the resonance state is obtained by using a Padé approximation in terms of a function of the potential coupling constant. A successful application of the ACCC in nuclear physics was shown in Ref. [6]. Another recent development in the studies of resonances is the JFM [3]. With the help of the definite functional form of the asymptotic, the “Jost function” of the Schrödinger equation can be obtained accurately. Therefore, resonance poles on the complex momentum planes can also be obtained accurately by using the JFM [7,8]. While the CSM is a powerful computational tool, it has some difficulties in solving certain problems. In dealing with a resonance state that has a large decay width, one must use a large rotational angle to locate the resonance pole in the complex energy (momentum) plane. Furthermore, for a “realistic” nuclear potential, the calculation of the matrix elements for the rotated Hamiltonian could be quite complicated [9]. In contrast to atomic physics, typical nuclear potentials have the following char-

acteristics: They are short ranged and have a strong repulsive core. The short-range nature is usually represented by using a Gaussian-type function as a phenomenological nuclear interaction. From the restriction of the analyticity of the Gaussian function, the rotational angle can only be taken less than 45° , and the range of the repulsive core is extended due to the complex rotation. Therefore, the use of large angles in the CSM would lead to computational difficulties for some problems [9].

Recently, a computational procedure has been proposed for solving resonance-state problems: the complex absorbing potential (CAP) method [10–12]. The fundamental principle of this method is similar to that of the CSM. Before adding a CAP to the Hamiltonian, a resonance wave function diverges in the asymptotic region. When one adds a CAP to the original Hamiltonian, the divergent property of the wave function becomes convergent. In the diagonalization of the modified Hamiltonian, which includes the CAP, the divergence of the wave function is “absorbed” by the CAP, and as a result, the eigenfunction of the resonance state converges in the asymptotic region. In addition to the fact that it is necessary to perform the calculation of the matrix elements only once in the CAP method, the matrix elements themselves are treated in real quantities. Furthermore, the CAP method does not introduce any complex rotation of the coordinates (momenta) to the original Hamiltonian. Complex quantities are therefore involved only in the diagonalization of the Hamiltonian. As will be demonstrated in the present investigation, the CAP method is an interesting alternative, as well as a powerful tool for solving the resonance states in few-body systems.

In this paper, we study resonance states for a nuclear physics case by using the CAP method. In Sec. II, we briefly explain the CAP method, and in Sec. III we present the calculations for two- and three-body systems. We will summarize and discuss our present results in Sec. IV.

II. COMPLEX ABSORBING POTENTIAL METHOD

In this section, we briefly explain the basic formulation of the complex absorbing potential method. For details, see Refs. [10–12].

We consider the time-independent Schrödinger equation,

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where an analytical potential $W(r)$ is added to the original Hamiltonian H as

$$H(\eta) = H - i\eta W(r). \quad (1)$$

Here η is a positive real number, and $W(r)$ is an absorbing potential for the asymptotic divergence of the resonance wave function. The addition of the potential $W(r)$ to the Hamiltonian has the same kind of effect as that when the complex scaling method is used. The divergent tail of the resonance wave function is “absorbed” by the complex absorbing potential $W(r)$. Hence we obtain a converged eigenfunction in the asymptotic region, as $\phi(r) \rightarrow 0$ for $r \rightarrow \infty$.

In an actual calculation for solving the Schrödinger equation of the modified Hamiltonian (1), it is necessary to determine the functional form of the CAP, $W(r)$. The functional form of $W(r)$ is not arbitrary, and the requirements are summarized in Ref. [10]. The important requirement for $W(r)$ is as follows:

$$\text{Re}[W(r)] \geq 0 \quad \text{for } r \geq 0$$

and

$$\text{Re}[W(r)] \rightarrow \infty \quad \text{for } r \rightarrow \infty. \quad (2)$$

Due to the above requirement, the divergent tail of the resonance wave function is absorbed into the potential $W(r)$ in the asymptotic region. One of the typical functional form of the $W(r)$ is a polynomial type, r^n . Note that, from the other requirement for $W(r)$ in Ref. [10], the exponential-type function $\exp(r^n)$, where n is a positive number, cannot be used as an absorbing potential $W(r)$.

After adding the absorbing potential, we diagonalize the modified Hamiltonian (1) using a basis set $\{\phi_i\}$, and obtain an eigenvalue $E(\eta)$ which depends on the parameter η :

$$\langle \phi_i | H(\eta) | \phi_i \rangle \Rightarrow E(\eta). \quad (3)$$

If we use a “complete” basis set, in principle, we can obtain the exact resonance energy E_{res} by taking $\eta \rightarrow 0$ [10]. However, in practice, we use a “finite” basis set. Then the deviation from the true eigenvalue E_{res} depends on the value of the η , as $E_{\text{fb}}(\eta)$. Hence for the resonance energy $E(\eta)$, there exists an optimal value $\eta = \tilde{\eta}$. We expand the $E(\eta)$ in the manner of a truncated Taylor series of order n , as

$$E^{(n)} = E^{(n)}(\tilde{\eta}) = E_{\text{fb}}(\tilde{\eta}) + \sum_{j=1}^n \frac{(-\tilde{\eta})^j}{j!} \left. \frac{d^j E_{\text{fb}}(\eta)}{d\eta^j} \right|_{\eta=\tilde{\eta}}. \quad (4)$$

The optimal value of $\tilde{\eta}$ will be found by satisfying

$$|E^{(n)}(\tilde{\eta}) - E_{\text{res}}| = (\text{minimum}). \quad (5)$$

Although the order of the expansion n can be taken as large enough, it is sufficient practically to be taken up to the first order, $n = 1$, as

$$|\eta \cdot dE_{\text{fb}}(\eta)/d\eta|_{\eta=\tilde{\eta}} = (\text{minimum}). \quad (6)$$

This truncation of the first order provides sufficiently converged results, as shown in Refs. [10–12]. Also, from a computational point of view, such a truncation has an economical efficiency. Therefore, in the CAP method, we search for an $\tilde{\eta}$ which satisfies Eq. (6).

III. CALCULATION AND RESULTS

In this section, we show the calculational results for two- and three-body systems using Gaussian-type potentials, which are usually employed as effective nuclear potential models. Nuclear potentials have a particular nature which is absent in the atomic physics case. Typically, the nuclear potential is a relatively short-range interaction in comparison to the Coulomb interaction. Hence the power of the convergence is stronger than the Coulomb interaction ($\sim 1/r$), and the tail of the potential converges as an exponential ($\sim e^{-r}$), or, in effective model cases, a Gaussian ($\sim e^{-r^2}$).

In choosing a suitable CAP, the behavior of the potential tail plays a major role. Hence it is worthwhile to examine first a suitable form of the CAP, after which the resonance states can be calculated by using this CAP.

A. A two-body system

First we work on a two-body system, since many other accurate calculational methods are available for the purpose of comparison. Here we compare the CAP results to those obtained by using another method, the Jost function method [3], which is one of the accurate calculational method for studying resonance states.

In the two-body system, we consider the one-dimensional radial Schrödinger equation

$$-\frac{\partial^2}{\partial r^2} \phi(r) + \frac{l(l+1)}{r^2} \phi(r) + V(r) \phi(r) = E \phi(r), \quad (7)$$

where the convention is $\hbar^2/2\mu = 1$, so as to be arbitrary units. We use the same type of potential of Ref. [6], a two-range Gaussian type potential

$$V(r) = -8\lambda \exp[-(r/2.5)^2] + 2 \exp[-(r/5)^2], \quad (8)$$

where λ is a strength parameter. By changing λ , an eigenvalue of the Hamiltonian for Eq. (8) becomes a resonance state. Since the potential is of Gaussian type, which has a very short-range interaction, we use a Gaussian basis set oriented at the center, for solving the Schrödinger equation (7):

$$\phi_i(r) = N(b_i) \exp(-r^2/2b_i^2). \quad (9)$$

Here b_i is the width parameter of the Gaussian, and $N(b_i)$ is the normalization constant.

We investigate the dependency on the functional form of the absorbing potential. In previous studies of atomic physics [10,11], the absorbing potentials $W(r)$ were taken as r^n -type functions, in which the powers of the potential were $n = 2$ and 6 in Refs. [10] and [11], respectively. The calculation of Ref. [11] showed that the power of the absorbing potential ($n = 6$) is sufficient to “absorb” the divergence of the reso-

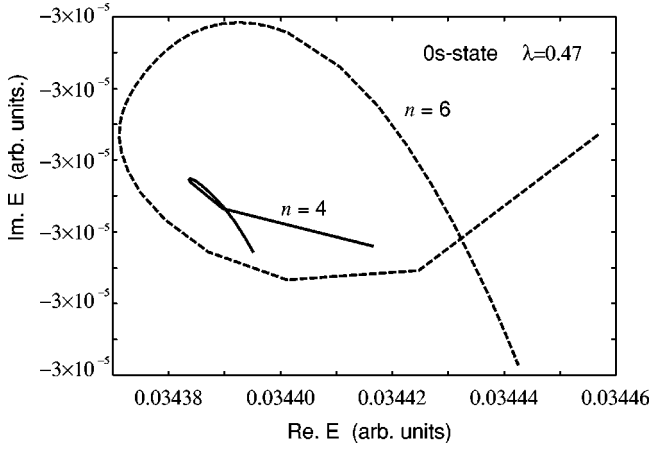


FIG. 1. η trajectory of the resonance pole in the complex energy plane for the $0s$ state. The solid line was obtained by using the power of $n=4$, and the dashed line by using $n=6$.

nance wave function. In atomic physics, a typical ratio between the total decay width Γ and the resonance energy E_r is

$$\Gamma/E_r \sim 10^{-9} - 10^{-2}. \quad (10)$$

The divergence comes from the imaginary part of the complex momentum $k = \sqrt{2\mu(E_r - i\Gamma/2)}$, where μ is the reduced mass of the system. Therefore, when the ratio Γ/E_r is small, the divergence of the resonance wave function becomes weak. However, in nuclear physics, a typical ratio is

$$\Gamma/E_r \sim 10^{-3} \quad (\text{to the same order}), \quad (11)$$

which is considerably large compared to the atomic physics case. An absorbing potential with power $n=6$ could be too strong to absorb the divergence of the resonance wave function in this case. Thus, to investigate the dependence of the shape on the absorbing potential, we calculate the resonance state using two different absorbing powers $n=6$ and 4.

The η trajectories are obtained by changing the coefficient of the absorbing potential η for each power of $W(r)$, $n=6$ and 4, respectively. We use a potential strength of $\lambda = 0.47$ in this calculation. Figure 1 shows the two trajectories on the complex energy plane. They exhibit different behaviors for the change of η . From an empirical point of view in the CAP method, the minimum of $|\eta dE(\eta)/\eta|$ appears around the ‘‘cusp’’ of the trajectory. However a cusp does not show up clearly in the $n=6$ case. We calculated the minimum of $|\eta dE(\eta)/\eta|$ and obtained the resonance energy. The results are summarized in Table I. Here we use the N

TABLE I. The resonance energy obtained by the complex absorbing potential (CAP) method and the Jost function method (JFM).

n	Re E (arb. units)	Im E (arb. units)	η
6	3.439×10^{-2}	-3.134×10^{-5}	4.2×10^{-9}
4	3.438×10^{-2}	-5.267×10^{-5}	3.5×10^{-7}
JFM	3.439×10^{-2}	-5.157×10^{-5}	

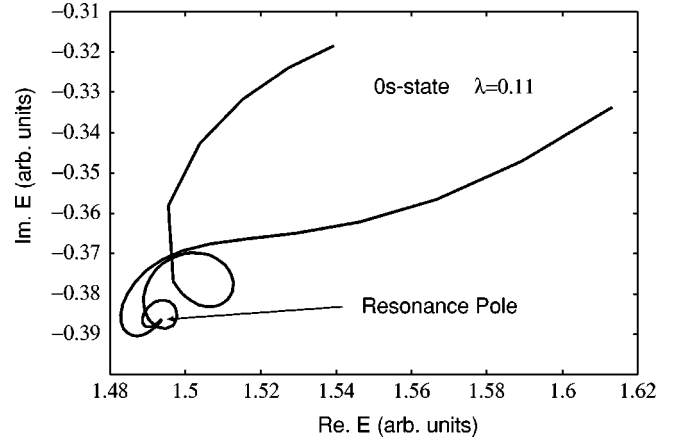


FIG. 2. η trajectory of the resonance energy with changing η for the $0s$ state where $\lambda=0.11$.

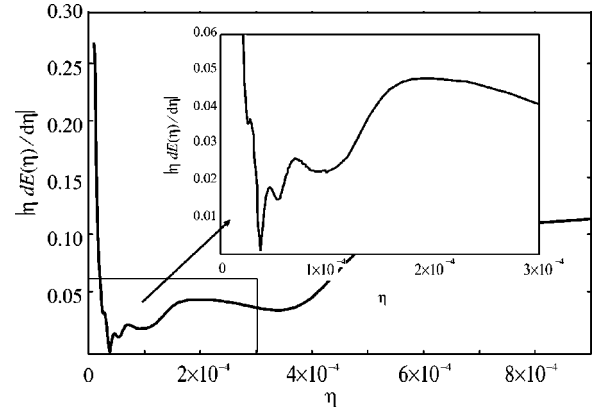


FIG. 3. Calculation of $|\eta dE(\eta)/d\eta|$ in search of its minima.

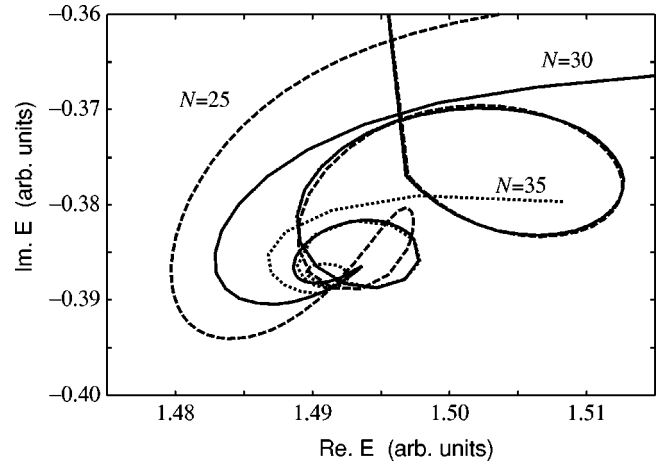
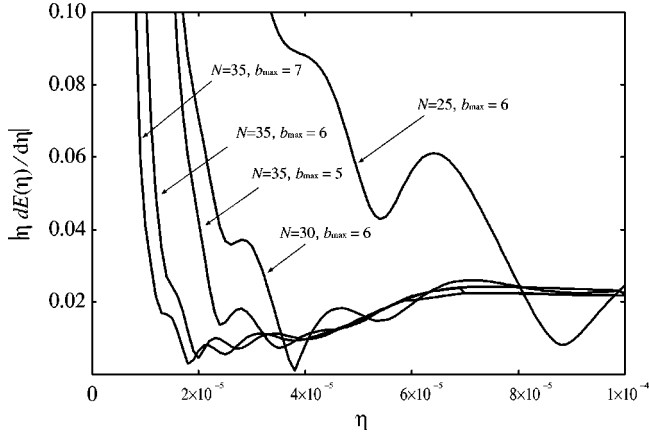


FIG. 4. η trajectory for the resonance eigenvalues with different basis set sizes.

FIG. 5. Minima of $|\eta dE(\eta)/d\eta|$ for different b_{\max} .

=20 basis set only for checking the dependency of the power of n . In both cases, resonance energies calculated by using the CAP method have some small deviation from that of the JFM calculation, since we use a small number of the basis set ($N=20$). However, it is obvious, from Fig. 1 and Table I, that the absorbing potential of the $n=4$ case is suitable much more than that of the $n=6$ case.

To see the applicability for the calculation of a broader resonance, we investigate the $0s$ state of the $\lambda=0.11$ case. In Fig. 2, we can see a clear cusp in the η trajectory of the resonance pole. Here we use the power of the absorbing potential $n=4$. The number of the basis is $N=30$, and the maximum size of the length parameter in the basis function is $b_{\max}=6.0$ fm. As seen from Fig. 3, we obtain the optimal value of $\tilde{\eta}$, and the resonance energy is deduced to be $1.494-i0.3864$ (arb. units). The result is close to the result calculated by using the JFM: $1.490-i0.3888$ (arb. units).

We also investigate the dependence on the basis set number N and the maximum size of the length parameter b_{\max} . Figure 4 shows each respective η trajectory obtained by using the basis set number $N=25, 30$, and 35 with a fixed value $b_{\max}=6$ fm, respectively. We see that the η trajectories have a strong dependence on the basis set number N . The minima of $|\eta dE(\eta)/d\eta|$ are also obtained, and are shown in Fig. 5.

The resonance energies calculated for the $\lambda=0.11$ case are summarized in Table II. It is shown that the dependence on the length parameter b_{\max} is small when the basis set

TABLE II. Resonance energies for the $0s$ state with $\lambda=0.11$ obtained by using basis sets with different N and b_{\max} .

N	b_{\max} (arb. units)	Re E (arb. units)	Im E (arb. units)	η
25	6.0	1.497	-0.3803	8.8×10^{-5}
30	6.0	1.494	-0.3864	3.8×10^{-5}
35	6.0	1.490	-0.3871	2.7×10^{-5}
35	7.0	1.491	-0.3882	1.8×10^{-5}
35	5.0	1.492	-0.3861	3.5×10^{-5}
JFM		1.489	-0.3888	

TABLE III. Resonance energies for the $1s$ state with $\lambda=0.51$ obtained by using different basis sets.

Basis	Re E (arb. units)	Im E (arb. units)	η
<i>spd</i>	1.055	-0.3458	2.8×10^{-5}
<i>spdf</i>	1.059	-0.3700	4.5×10^{-5}
<i>spd-f-mod.</i>	1.061	-0.3533	3.4×10^{-5}
<i>spd-fg-mod.</i>	1.063	-0.3636	4.1×10^{-5}
<i>spd-fgh-mod.</i> First-minima	1.075	-0.3518	3.6×10^{-5}
<i>spd-fgh-mod.</i> Second-minima	1.043	-0.3582	6.1×10^{-5}
JFM	1.046	-0.3582	

number N is large. In each case, except for $N=25$, the agreement with the other calculational result (the JFM calculation) is good to within three or four digits. Hence we consider that this is sufficient for most nuclear physics cases.

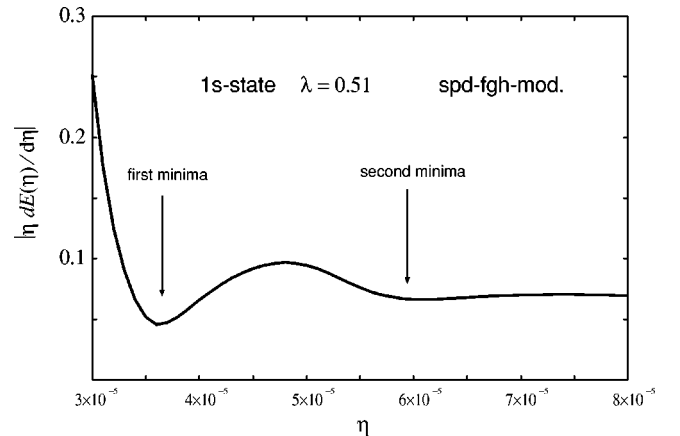
So far, we have only used a simple Gaussian basis set for the $0s$ state of the system. However, it would be necessary to employ a larger basis set to investigate the first-excited state. In order to take into account the correct nodal behavior of the basis set, we multiply an r^{λ_i} term by the basis function:

$$\phi_i(r) = N(b_i)(r/b_i)^{\lambda_i} \exp(-r^2/2b_i^2). \quad (12)$$

Here we take λ_i up to 5, and label the wave functions of $\lambda_i=0,1,\dots,5$ as $s-, p-, \dots, h$ -wave types, respectively. To investigate the dependence of the basis set, we calculate the $1s$ state with the potential strength of $\lambda=0.51$.

The results are summarized in Table III, and the η trajectories are shown in Fig. 6. For the calculation of the $f-, g-$ and h -wave types, we label them using the name “mod.” to distinguish from the normal parametrization of the Gaussian width parameter b_i . In the normal parametrization, the basis set parameter b_i is taken from almost 0 to b_{\max} . On the other hand, in the “mod” case, b_i is taken from b_{\max} to $2 \times b_{\max}$, by reason of convergence.

In the calculation of *spd-fgh-mod.*, it can be seen from Fig. 6 that there are two minima of almost equal depth in $|\eta dE(\eta)/d\eta|$. If we take the second minimum, we obtain a

FIG. 6. Minima of $|\eta dE(\eta)/d\eta|$ for the $1s$ state in the *spd-fgh-mod.* with $\lambda=0.51$.

result much closer to that of the JFM calculation. The appearance of these two minima with a comparable depth indicates that it is necessary to improve the functional form of the CAP and is also necessary to take into account the second-order correction in Eq. (4).

B. Three-body system

To summarize the results in the two-body case so far, the CAP calculation showed a good agreement with the other calculational results, where we employed the JFM, within several digits. Therefore, it has been shown that the CAP method is a powerful tool for use in searching for resonance poles in the complex energy plane, only performing the calculations of the matrix elements of real quantities.

Hence, it is worthwhile to investigate its applicability for solving the resonance states in many-body systems, and we proceed to the study of a three-body system. As our first step, we consider a three-boson system with equal neutron masses for simplicity. Other calculational results are available for this system [6,9]. Here, we use the potential to be the same form as in Ref. [6], a two-range Gaussian,

$$V(r) = -120 \exp(-r^2) + 3\lambda \exp[-(r/3)^2] \quad (\text{MeV}), \quad (13)$$

and our convention is to take $\hbar^2/m = 41.47 \text{ MeV fm}^2$. λ is a strength parameter. We calculate the three-boson resonance state by changing the strength parameter λ .

To solve the resonance state of this system, we use a combination of the Gaussian basis, which is centered at the origin, as follows:

$$\phi_i(r, R) = N(b_i)N(d_i) \exp(-r^2/2b_i^2) \exp(-R^2/2d_i^2) \cdot \mathcal{Y}_{l,L}^{JM}. \quad (14)$$

Here r and R are the Jacobi coordinate for the particles 1 to 2, and 1-2 to 3, respectively, and $\mathcal{Y}_{l,L}^{JM}$ is the angular momentum part combined as

$$\mathcal{Y}_{l,L}^{JM} = [Y_l \otimes Y_L]_{JM}, \quad (15)$$

where l and L are corresponding angular momenta for the coordinates r and R , respectively. Due to the absence of a spin-dependence force in the interaction, and because we are treating the boson case, we do not consider the spin part of

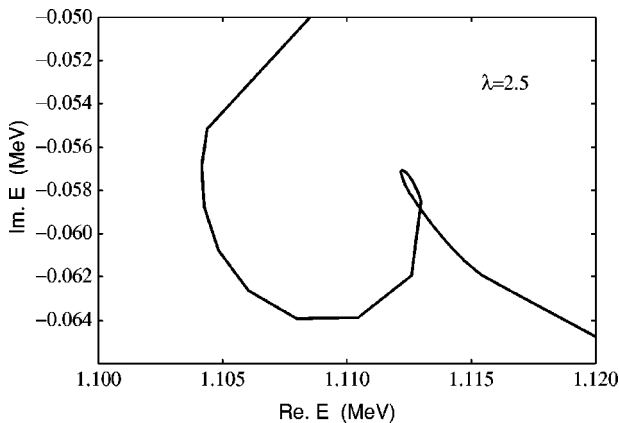


FIG. 7. η -trajectory of the complex eigenvalue for $\lambda=2.5$ in a three-body system.

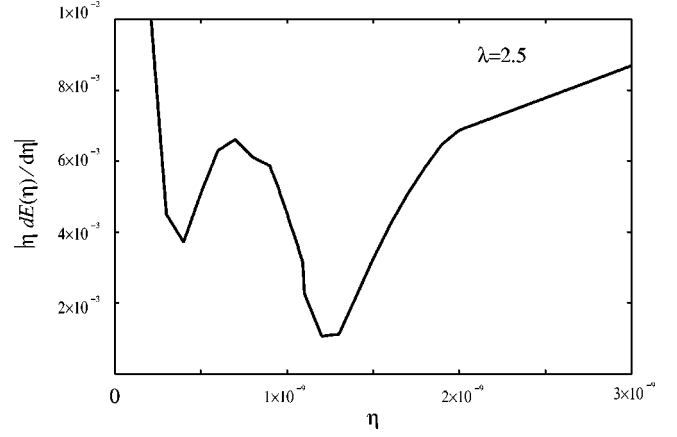


FIG. 8. Minima of $|\eta dE(\eta)/d\eta|$ for $\lambda=2.5$ in a three-body system.

the wave function. To avoid complexity, we start the calculation by only using the total S -wave state and the $L=l=0$ configuration.

In a $L=l=0$ calculation, it is useful to choose the complex absorbing potential as having the same radial dependence for r and R , although, in the general case, the radial dependencies differ between r and R :

$$-i\eta W(r, R) = -i\eta(r^4 + R^4). \quad (16)$$

As performed in the two-body calculation, we change η in order to find the appropriate $\tilde{\eta}$, which makes the minimum of Eq. (6).

This system has one S -wave bound state in the range of $\lambda \leq 2.4$, which becomes a slightly unbound state at $\lambda = 2.5$. We first show the $\lambda = 2.5$ case as an example.

Figure 7 shows the η trajectory of the complex eigenvalue for the $\lambda = 2.5$ case. We can clearly see a cusp in the η trajectory of the eigenvalue. The existence of a cusp indicates the existence of a sharp minimum of Eq. (6). Next we investigate the minimum of $|\eta \cdot dE/d\eta|$. The obtained result for $|\eta \cdot dE/d\eta|$ is shown in Fig. 8. The minima appear at $\eta = 4.7 \times 10^{-11}$ and 1.1×10^{-11} . The deepest minimum is at

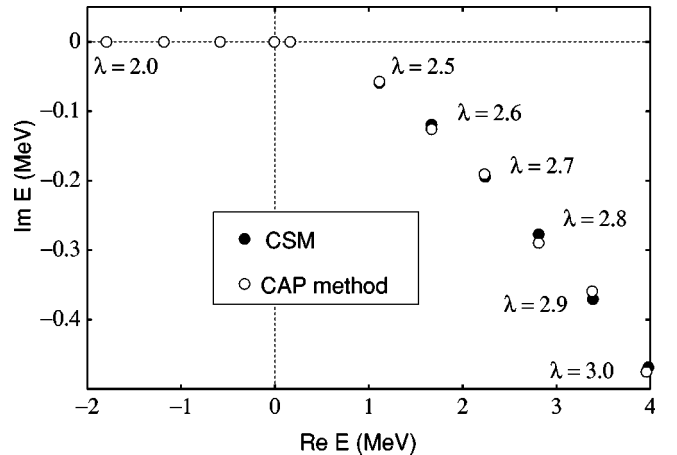


FIG. 9. Resonance states obtained by using the CAP method and the CSM.

$\eta = 1.1 \times 10^{-11}$, and by using this η we obtain the resonance energy, $E_{\text{res}} = 1.11 - i0.0571$ MeV. By changing the λ up to 3, we can calculate the resonance states using the CAP method. The calculation of the complex scaling method is performed for the purpose of comparison. As shown in Fig. 9, the correspondence of the results obtained by these two methods is reasonably good.

The result shown in Fig. 9 shows that the CAP method is applicable for solving a three-body system. The functional form of the complex absorbing potential of Eq. (16) is one of the possibilities. Even though the CAP can be chosen as different radial dependencies for r and R , by using a simple functional form of the CAP we obtained a converging result for resonance states compared with the results obtained by using the CSM.

IV. SUMMARY AND DISCUSSION

In the present work, we have studied resonances in nuclear physics using the complex absorbing potential (CAP) method. In a two-body system, we have obtained accurate results that are in a good agreement with the other accurate results obtained by using the Jost function method (JFM). In a three-body system, we have also obtained a good agreement between the CAP results and those of the CSM. Hence we have demonstrated that the CAP method is capable of producing accurate resonance parameters—the resonance energy E_r and the decay width Γ is not only for two-body systems but also for three-body systems in nuclear physics.

Generally, in an actual calculation, the computational time is devoted almost to the calculations of the matrix elements of the Hamiltonian $\langle H \rangle$. When the CSM is applied, it is necessary to investigate the convergence with respect to the changes of the rotational angle θ . Therefore we need to calculate the matrix elements for each rotation angle. It would take a considerable effort and much computer time to examine the convergence, especially when the calculations of $\langle H \rangle$ are complicated. On the other hand, in the CAP method, it is necessary only once to calculate the matrix elements $\langle H \rangle$. Hence the CAP method is an efficient computational tool for studying resonance states in many-nucleon systems.

In spite of the fact that successful results can be obtained by using the CAP method, there are, however, several factors which are necessary to be considered for using the CAP method. First, for a given physical system, we need to choose an optimized absorbing potential while there is no such necessity in using the CSM and ACCC. In nuclear physics where the range of the mutual interaction between the particles is shorter than the Coulomb interaction, we have found that the practical form of W is of r^4 . By using such a CAP, we have obtained good convergence of the resonance eigenvalues even for broad resonance states. Second, for a practical limitation, calculations using a finite basis set can only be performed. Therefore, we must perform the matrix diagonalization of the Hamiltonian for various η values in order to locate the optimized η , from which the resonance energy and width can be deduced. In contrast, it is not necessary for the ACCC calculation to perform such repeated diagonalization. However, there is an advantage for the CAP method over ACCC. In general, the CAP method is capable of calculating the resonance energy and the total width for a high-lying multichannel resonance in a straightforward manner. The computational process for a multichannel resonance is the same as that for a single-channel resonance. Calculations for high-lying resonances using the ACCC method might be more involved.

In future work, it is of interest and worthwhile to use the CAP method, together with the use of the realistic nuclear (nucleus) interactions, in the studies of the many-body resonant systems such as three-neutron resonances [9], and proton emission [13]. Recently, the CSM was used to investigate nuclear resonances in the framework of the Hartree-Fock (HF) approach [14]. It would be of interest to combine the CAP method and the HF approach to investigate resonances in nuclear physics.

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- [1] Y. K. Ho, Phys. Rep. **99**, 1 (1983), and references therein.
 - [2] V. I. Kukulin and V. M. Krasnopol'sky, J. Phys. A **10**, 33 (1977); V. I. Kukulin, V. M. Krasnopol'sky, and M. Miselkhi, Yad. Fiz. **29**, 818 (1979) [Sov. J. Nucl. Phys. **29**, 421 (1979)].
 - [3] S. A. Sofianos and S. A. Rakityansky, J. Phys. A **30**, 3725 (1997); **31**, 5149 (1998).
 - [4] J. Aguilar and J. M. Combes, Commun. Math. Phys. **22**, 269 (1971); E. Balslev and J. M. Combes, *ibid.* **22**, 280 (1971).
 - [5] See, for example, K. Katō, S. Aoyama, T. Myo, H. Masui, T. Yamada, and K. Ikeda, Few-Body Syst., Suppl. **12**, 118 (2000).
 - [6] N. Tanaka, Y. Suzuki, and K. Varga, Phys. Rev. C **56**, 562 (1997).
 - [7] H. Masui, S. Aoyama, T. Myo, and K. Katō, Prog. Theor. Phys. **102**, 1119 (1999).
 - [8] H. Masui, S. Aoyama, T. Myo, K. Katō, and K. Ikeda, Nucl. Phys. A **673**, 207 (2000).
 - [9] H. Witala and W. Glöckle, Phys. Rev. C **60**, 024002 (1999).
 - [10] U. V. Riss and H.-D. Meyer, J. Phys. B **26**, 4503 (1993); **28**, 1475 (1995); J. Chem. Phys. **105**, 1409 (1996); J. Phys. B **31**, 2279 (1998).
 - [11] S. Sahoo and Y. K. Ho, J. Phys. B **33**, 2195 (2000); **33**, 5151 (2000).
 - [12] S. Sahoo and Y. K. Ho, Chin. J. Phys. (Taipei) **38**, 127 (2000).
 - [13] R. A. Kryger *et al.*, Phys. Rev. Lett. **74**, 860 (1995).
 - [14] A. T. Kruppa, P.-H. Heenen, F. Flocard, and R. J. Liotta, Phys. Rev. Lett. **79**, 2217 (1997).