Staggering behavior of the low-lying excited states of even-even nuclei in a Sp(4,*R*) classification scheme

S. Drenska,[1] A. Georgieva,[1] and N. Minkov[1,2]
[1]*Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria*
[2]*RCNP Osaka University, 10-1 Mihogaoka, Ibaraki City, Osaka 567-0047, Japan*

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We implement a high order discrete derivative analysis of the low-lying collective energies of even-even nuclei with respect to the total number of valence nucleon pairs N in the framework of F-spin multiplets appearing in an algebraic sp(4,R) classification scheme. We find that for the nuclei of any given F multiplet the respective experimental energies exhibit a $\Delta N=2$ staggering behavior and for the nuclei of two united neighboring F multiplets well pronounced $\Delta N=1$ staggering patterns are observed. Those effects have been reproduced successfully through a generalized sp(4,R) model energy expression and explained in terms of the steplike changes in collective modes within the F multiplets and the alternation of the F-spin projection in the united neighboring multiplets. On this basis we suggest that the observed $\Delta N=2$ and $\Delta N=1$ staggering effects carry detailed information about the respective systematic manifestation of both high order α-particle-like quartetting of nucleons and proton (neutron) pairing interaction in nuclei.

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I. INTRODUCTION

The systematic behavior of the low-lying collective states of even-even nuclei has been extensively studied in different phenomenological and microscopic model approaches [1–4]. In particular, some algebraic classification schemes provide a convenient way to systematize the general collective characteristics of these states in terms of the basic nuclear structure quantities, such as the nuclear mass number, the proton or neutron numbers, as well as the total number of neutron and proton pairs in nuclear valence shells [1,5,6]. These quantities and their combinations can naturally be treated as the quantum numbers of the irreducible representations associated with the respective groups of symmetry. In this framework the low-lying collective energy levels exhibit various kinds of smooth systematic dependence on the different quantum numbers, which can be evaluated empirically [1,5].

From another point of view, however, the current studies of various fine effects in the structure of collective interactions and the respective energy spectra suppose the presence of quite a complicated behavior of nuclear collective characteristics. Thus, the application of the discrete approximation of a high order derivative of given nuclear characteristics as a function of particular physical quantity reveals various kinds of staggering effects that carry detailed information about the fine properties of nuclear interactions and the respective high order correlations in the collective dynamics of the system.

Typical examples are the $\Delta I=1$ staggering effects (with I being the angular momentum) in the collective γ bands [7,8] and nuclear octupole bands [9], the $\Delta I=1$, 2, and 4, staggering in superdeformed nuclear bands [10–13] and the $\Delta I=2$ staggering in the ground-state bands (GSB) of normally deformed nuclei [14].

Another example is the odd-even staggering in nuclear binding energies [15–17] or the behavior of the nuclear masses [18] as a function of the atomic number A, for which the influence of different pairing, α-like quartetting [19] and mean-field effects are considered to play a role.

In the above aspect, a natural question is whether the application of a high order discrete derivative analysis to the low-lying collective states in wide range of nuclear chart would provide any staggering behavior of the respective energies as functions of the quantum numbers of an appropriate classification scheme. The answer of this question could open a new fine tool in the interpretation and the systematic study of nuclear collective interactions and their symmetries.

The purpose of the present work is to examine the above problem with respect to the low-lying collective states of even-even nuclei in the framework of an algebraic Sp(4,R) classification scheme [20]. The basic assumption of this scheme is that the structure of nuclear proton and neutron valence shells can be characterized in a unique way by the quantum numbers of the irreducible representations (irreps) of the algebra su(2) and the ladder representations of the algebra u(1,1) which are subalgebras of the boson representation of Sp(4,R), considered as a classification group. As a result the even-even nuclei of each major (proton and neutron) shell can be classified into sets of F-spin multiplets [6], where any nucleus is uniquely identified by the total number of valence bosons N (pairs of protons and neutrons) and the third projection F_0 of the F spin. It is known that in each F-spin multiplet the energies of the low-lying yrast states exhibit a smooth and periodic behavior as functions of N. On this basis, a unified theoretical description of GSB energies in even-even nuclei has been obtained [5].

In the present paper, we apply the high order discrete derivative analysis to the both experimental and theoretical 2_1^+ energies as functions of the quantum number N in the framework of the respective F multiplets appearing in the Sp(4,R) scheme. The same analysis is also applied to the next higher 4^+ and 6^+ levels of the yrast bands of the classified nuclei and some typical results are presented. As it will be seen the experimental data exhibit considerable $\Delta N=2$

staggering behavior in all considered F multiplets (with N even or odd) and also very well pronounced $\Delta N=1$ staggering effect in the cases when two neighboring F multiplets are united into one single multiplet (with both even and odd N). In addition, we extend the study in some isotopic chains, where a tedious $\Delta N=1$ staggering effect can be observed also, but its general magnitude is quite small compared to the patterns observed in the F-spin multiplets.

We will see that these phenomena are reproduced theoretically by a generalized phenomenological energy expression derived in the framework of the classification scheme. On this basis we are able to provide an adequate physical interpretation of the observed staggering patterns in terms of the related collective model characteristics and the high order α-like quartetting and pairing nucleon interactions as well.

In Sec. II we briefly outline the algebraic base of the Sp(4,R) classification scheme. The high order discrete derivative formalism and its meaning in the context of the present theory is given in Sec. III. Experimental and theoretical staggering patterns with $\Delta N=2$ in the F multiplets and with $\Delta N=1$ in the combined (neighboring) F multiplets and isotopic chains are presented in Sec. IV. A detailed analysis of their form and amplitudes in the different multiplets is also given. In Sec. V the obtained results are analyzed in terms of the model characteristics of the classification scheme. Some comments and conclusions are given in Sec. VI.

II. LOW-LYING COLLECTIVE STATES IN THE Sp(4,R) CLASSIFICATION SCHEME

Symplectic algebras are used in physical problems, where the number of particles (bosons or fermions) changes in a pairwise fashion [21]. In this particular case the Sp(4,R) classification scheme is based on the use of two kinds of bosons, proton (π) and neutron (ν) bosons, interpreted as pairs of the respective nucleons, confined in the valence orbits. The boson representation of the algebra sp(4,R) is generated through the respective creation and annihilation operators as follows [5]:

$$\pi^\dagger\pi^\dagger,\quad \nu^\dagger\nu^\dagger,\quad \pi^\dagger\nu^\dagger,\quad \pi\nu,\quad \nu\nu,\quad \pi\pi,$$

$$N_\pi=\pi^\dagger\pi,\quad N_\nu=\nu^\dagger\nu,\quad F_+=\pi^\dagger\nu,\quad F_-=\nu^\dagger\pi. \qquad (1)$$

The following subset of the above operators (1)

$$F_+,\quad F_-,\quad F_0=\tfrac{1}{2}(N_\pi-N_\nu) \qquad (2)$$

generates a su$_F(2)$ algebra so that a F spin quantum number is assigned to the considered boson system with a mathematical structure similar to that of the isospin.

The operator $N=N_\pi+N_\nu$ commutes with all the operators (2) acting in this way as a first order invariant of an $u(2)\supset su_F(2)\otimes u_N(1)$ [6,22]. The space H, in which the boson representation of sp(4,R) acts is reducible and the operator $(-1)^N$ splits it into two irreducible spaces H_+ and H_-, corresponding to N even or odd, respectively. Acting on these spaces the operator of the total number of particles N reduces them to a direct sum of a totally symmetric unitary irreps of su$_F(2)$, labeled by $N=0,2,4,\ldots,$ for H_+ and $N=1,2,3,\ldots,$ for H_-. The operator F_0 (2) does not differ essentially from the first order Casimir operator of the noncompact $u(1,1)\subset sp(4,R)$. Hence it reduces the spaces H_+ and H_- to the ladder series defined by its fixed eigenvalues. The states from a given ladder, are defined by the eigenvalue of F_0 and are obtained by the subsequent action of the operator $\pi^\dagger\nu^\dagger$ on the lowest weight state. The operator F_0 reduces each u(2) representation (with fixed value of N) to the representations of $u_\pi(1)\oplus u_\nu(1)$ labeled by N_π and $N_\nu(1)$, respectively. The same is obtained by reducing the u(1,1) ladders with the operator N.

In terms of the well known and rather successful language of IBM-2 [23], the algebraic operators (1) obtain the physical meaning of basic nuclear characteristics inherent for the Sp(4,R) classification scheme. Thus the eigenvalues of the operators $N_\pi=\frac{1}{2}(N_p-N_p^{(1)})$ and $N_\nu=\frac{1}{2}(N_n-N_n^{(1)})$ are naturally interpreted as the numbers of valence proton and neutron pairs for a nucleus of a given shell. Here $N_p^{(1)}$ and $N_n^{(1)}$ are the proton and neutron numbers, respectively, of the double magic nucleus at the beginning of the shell, considered as the "vacuum state" for the symplectic representations. A basic point in the present classification scheme is that the major shells, which are defined by their bordering magic numbers $(N_p^{(1)},N_n^{(1)}|N_p^{(2)},\,N_n^{(2)})$ (with $N_p^{(2)}>N_p^{(1)}$ and $N_n^{(2)}>N_n^{(1)}$) are mapped on two Sp(4,R) multiplets $(N_p^{(1)},N_n^{(1)}|N_p^{(2)},\,N_n^{(2)})_\pm$, with N being even (+) or odd (−). In these terms the total number of valence bosons, N, and the third projection of the F spin, F_0, are exactly the operators that reduce the sp(4,R) spaces to a definite vector in one of the $u_\pi(1)\oplus u_\nu(1)$ subspaces, corresponding to a given nucleus with fixed N and F_0. In this way the nuclei of a given major shell are naturally classified into F_0 multiplets (with fixed F_0 and N taking all its possible values [6]), unified in each of the symplectic spaces H_+ and H_-. This is illustrated in Table I, where part of the even-even nuclei from the rare-earth region are ordered in the H_+ space. The $F_0=0,1,2$ multiplets form the column of the table and nuclei with the same N are in each of the rows. The nuclei belonging to each of these multiplets differ by $\Delta N=2$, an α-like cluster (two protons and two neutrons) created by the operator $\pi^\dagger\nu^\dagger$ (1). It is important to remark that another kind of $\Delta N=2$ nuclear sequences [24] can be defined in the symplectic spaces H_+ and H_- by the action of the operators $\pi^\dagger\pi^\dagger$ (or $\nu^\dagger\nu^\dagger$) on the minimal weight states of each F_0 multiplet. These sequences correspond to the even-even nuclei from a given isotonic (or isotopic) chain, which differ by four protons (or neutrons) with F_0 differing by $\Delta F_0=+1$ (or -1).

Furthermore, if the anticommutation relations of the boson creation and annihilation operators $\pi^\dagger,\pi$ and $\nu^\dagger,\nu$ [25] are formally introduced, one can easily see that sp(4,R) is the even part of a superalgebra, which has the single boson operators as odd generators. These operators practically relate the even and odd symplectic multiplets. On this basis any two neighboring F_0 multiplets from H_+ and H_-, can be united into one single odd-even F_0 multiplet. An example of the ordering of nuclei in such combined multiplet

TABLE I. Nuclei from multiplets with $F_0=0,1,2$ from the shell $(50,82|82,126)_+$ mapped on the even H_+ (N even) subspace of Sp(4,R). The values of the parameter ω are given in parentheses.

	F_0		
N	2	1	0
0			^{132}Sn
2		^{136}Xe	^{136}Te
4	^{140}Ce	^{140}Ba(26)	^{140}Xe(17)
6	^{144}Nd(27)	^{144}Ce(19)	^{144}Ba(8)
8	^{148}Sm(24)	^{148}Nd(13)	^{148}Ce(6)
10	^{152}Gd(13)	^{152}Sm(4)	^{152}Nd(1)
12	^{156}Dy(4)	^{156}Gd(2)	^{156}Sm(1)
14	^{160}Er(4)	^{160}Dy(1)	^{160}Gd(1)
16	^{164}Yb(4)	^{164}Er(1)	^{164}Dy(1)
18	^{168}Hf(3)	^{168}Yb(1)	^{168}Er(1)
20	^{172}W(3)	^{172}Hf(2)	^{172}Yb(1)
22	^{176}Os(3)	^{176}W(2)	^{176}Hf(1)
24	^{180}Pt	^{180}Os(3)	^{180}W(2)
26	^{184}Hg	^{184}Pt(4)	^{184}Os(3)
28	^{188}Pb	^{188}Hg	^{188}Pt(8)
30		^{192}Pb	^{192}Hg
32			^{196}Pb

$F_0=\{1,\frac{3}{2}\}$ from the rare-earth shell is presented on Table II. In these multiplets the neighboring nuclei are determined by the subsequent alternative action of the operators $\pi^\dagger(\Delta F_0=+\frac{1}{2})$ and $\nu^\dagger(\Delta F_0=-\frac{1}{2})$ and thus differ alternatively by $\Delta F_0=\pm\frac{1}{2}$. In addition the nuclei belonging to such odd-even F_0 multiplets differ by $\Delta N=1$. In the same way the isotonic and isotopic nuclear chains of even-even nuclei with $\Delta N=1$ can be obtained by the respective consecutive action of $\pi^\dagger(\Delta F_0=+\frac{1}{2})$ and $\nu^\dagger(\Delta F_0=-\frac{1}{2})$ without a change in the sign of ΔF_0.

All the even and odd symplectic multiplets, constructed in this way, are given in the tables of [5]. Moreover, it has been shown that the low-lying yrast energies exhibit a smooth behavior in each F_0 multiplet, which allows their unified description in terms of the nuclear collective characteristics inherent for the Sp$(4,R)$ classification scheme. On this basis the following generalized energy expression has been proposed [5]:

$$E_{yrast}(h_k,I,\omega)=\alpha\{h_k\}I(I+\omega). \quad (3)$$

The second term of Eq. (3) represents the generalized collective interaction characterized by the geometrical parameter ω. The values of this parameter for the nuclei presented in Tables I and II are given in parentheses. The latter has the physical meaning of a measure for the interplay between the vibrational ($\omega>20$) and the rotational ($\omega=1$) collective modes (3) and reflects the respective changes in the nuclear shape. Since in heavy nuclei the shape is changed from almost spherical at the beginning of given shell to axially deformed in the midshell region and back towards spherical at its closure, the parameter ω changes, respectively, from $\omega>20$ to $\omega=1$ and then again to $\omega>1$. The values of ω have

TABLE II. The nuclei from odd-even multiplet with $F_0=\{1,\frac{3}{2}\}$ from the shells $(50,82|82,126)_\pm$, with the values of the parameter ω in parentheses.

	H_+	H_-
N/F_0	1	$\frac{3}{2}$
2	^{136}Xe	
3		^{138}Ba
4	^{140}Ba(26)	
5		^{142}Ce(30)
6	^{144}Ce(19)	
7		^{146}Nd(20)
8	^{148}Nd(13)	
9		^{150}Sm(13)
10	^{152}Sm(4)	
11		^{154}Gd(4)
12	^{156}Gd(2)	
13		^{158}Dy(2)
14	^{160}Dy(1)	
15		^{162}Er(3)
16	^{164}Er(1)	
17		^{168}Yb(2)
18	^{168}Yb(1)	
19		^{170}Hf(2)
20	^{172}Hf(2)	
21		^{174}W(3)
22	^{176}W(2)	
23		^{178}Os(3)
24	^{180}Os(3)	
25		^{182}Pt(4)
26	^{184}Pt(4)	
27		^{186}Hg
28	^{188}Hg	
29		^{199}Pb
30	^{192}Pb	

been determined [5] for each of the classified nuclei by using the experimental yrast energy ratios $(E_{I+2})/E_I$, which are known to provide a reliable criteria [26] of nuclear collectivity. So, the assigned integer values of ω for each of the classified nuclei (fixed values of N and F_0) determine its type of collectivity.

The first term in Eq. (3), $\alpha\{h_k\}$, is a dynamical (inertial) coefficient determined as a function of the six quantum numbers h_k, $k=1,2,\ldots,6$, which are specific for each nucleus in a given shell. Generally, the variables h_k depend on the number of protons N_p and neutrons N_n and the four magic numbers N_p^1, N_n^1, N_p^2, $N,_n^2$ and can be expressed in terms of the classification quantum numbers N and F_0. Thus for a fixed nuclear shell the inertial parameter is given by the expression

$$\alpha\{h_k\}=A_1+A_2N+A_3F_0+A_4N^2+A_5F_0^2+A_6NF_0, \quad (4)$$

where $A_i, i=1,2,\ldots,6$ are phenomenological parameters determined overall for all nuclear shells [5].

In general, the geometrical parameter ω has its specific values for each of the classified nuclei. The inertial parameter $\alpha\{h_k\}$ is a function only of N in each F_0 multiplets, and of the two classification numbers in the combined odd-even and isotopic multiplets.

III. HIGH ORDER DISCRETE DERIVATIVE ANALYSIS OF COLLECTIVE ENERGIES IN THE Sp(4,*R*) SCHEME

As it became clear from Sec. II, the even-even nuclei of given F_0 multiplet appearing in the considered Sp(4,R) classification scheme are uniquely determined by the subsequent even (or odd) values of the valence boson number N. This allows us to apply the high order discrete derivative analysis to the low-lying GSB energies $E(N)$ for a given angular momentum $I=2,4,6$ in any considered F_0 multiplet, as a function of the quantum number N, by analogy to the staggering analysis of rotational band energies applied in terms of the angular momentum I [8,9,27].

Since for any given F_0 multiplet (see Table I) the values of the quantum number N of the neighboring members differ by $\Delta N=2$ (α particle), the following quantity can be introduced:

$$Stg(2N)=\tfrac{1}{16}[6\Delta_2E(N)-4\Delta_2E(N-2)-4\Delta_2E(N+2)+\Delta_2E(N+4)+\Delta_2E(N-4)], \quad (5)$$

where

$$\Delta_2E(N)=E(N+2)-E(N). \quad (6)$$

The point function $Stg(2N)$ is proportional to the discrete approximation of the fourth derivative of the function $\Delta_2E(N)$, and obviously, to the fifth derivative of the energy $E(N)$,

$$Stg(2N)=\tfrac{1}{32}\{10E(N+2)+5E(N-2)+E(N+6)-[10E(N)+5E(N+4)+E(N-4)]\}. \quad (7)$$

As such, it naturally obtains zero values for any polynomial form of $E(N)$ of power less or equal to 4. Therefore, any nonzero values of the quantity $Stg(2N)$ will imply a higher order functional dependence. Moreover, any staggering (zigzagging) behavior of the $Stg(2N)$ values will suggest the presence of quite complicated nonpolynomial dependence of the lowest collective energy and the respective nuclear interactions on the quantum number N.

On the above basis it is expected that the application of the quantity $Stg(2N)$ to the low-lying collective nuclear states within the F_0 multiplets of Sp(4,R) could provide a detailed information about the possible influence of α-particle-like high order quartetting interaction ($\pi^\dagger\nu^\dagger\pi\nu$) of nucleons on the systematic behavior of collective excitation energies.

We should remark that the application of the function $Stg(2N)$ for the energies of the nuclei in the isotopic and isotonic chains is restricted, since even in the larger shells there is not enough number of points (observed excitation energies). Actually this is due to the circumstance that the available data are separated in the two spaces H_+ and H_-. The above restriction is naturally released in the extended regions of exotic nuclei (such as with $N_n=N_p$ and close to the drip lines), where the number of newly obtained experimental data grows continuously.

Now, we can extend the framework of the fine systematic analysis of low-lying collective energies by considering the unified odd-even F_0 multiplets (see Table II). Within these multiplets the neighboring members differ by $\Delta N=1$ (a proton or neutron pair) in a way that the increasing N corresponds to a *subsequent alternative* adding of proton and neutron pairs to the valence shells. In this case the systematic behavior of the excitation energies can be characterized analogously to Eq. (5) by a discrete approximation of the fourth derivative of the energy difference for $\Delta N=1$,

$$Stg(1N)=\tfrac{1}{16}[6\Delta_1E(N)-4\Delta_1E(N-1)-4\Delta_1E(N+1)+\Delta E_1(N+2)+\Delta_1E(N-2)], \quad (8)$$

where

$$\Delta_1E(N)=E(N+1)-E(N). \quad (9)$$

The use of the same order in the discrete derivative approximations (5) and (8) for the F_0 multiplets with $\Delta N=2$ and the combined odd-even and isotopic multiplets with $\Delta N=1$, respectively, allows one to treat the α-particle quartetting and the different kinds of nucleon pairing correlations in nuclei on the same footing, as well as to compare quantitatively their fine systematic influence on nuclear collectivity. Also, in this way the present classification scheme allows the standard analysis of the $\Delta N=1$ staggering effect in isotopic chains of even-even nuclei. In the isotonic chains this analysis is limited due to the Coulomb restriction on the possible proton numbers, but nevertheless it seems to be perspective in view of the current progress in the experimental investigations of proton rich nuclei.

IV. $\Delta N=2$ AND $\Delta N=1$ STAGGERING PATTERNS FOR THE LOW-LYING COLLECTIVE ENERGIES: EXPERIMENT AND THEORY

We have applied the functions $Stg(2N)$ (5) and $Stg(1N)$ (8) for the analysis of the experimental data [28–30] on the lowest collective energies of even-even nuclei classified in the respective symplectic multiplets. In all the considered cases these functions are oscillating with respect to the zero of their scale, with a changing amplitude, i.e., exhibit staggering patterns, which will be analyzed below.

The function (5) has been used in all possible cases, in which the F_0 multiplets of the considered Sp(4,R) classification scheme [5], contain more than eight nuclei. For all of them we observe $\Delta N=2$ staggering behavior of the 2_1^+ energy. Some typical staggering patterns including either odd or even F_0 multiplets are shown on Fig. 1.

We see that for the different F_0 multiplets the general scale of the staggering amplitude depends essentially on the particular valence shells. Thus, for the two multiplets $F_0=3/2$ [Fig. 1(a)] and $F_0=0,1,2$ [Fig. 1(c)], belonging to the

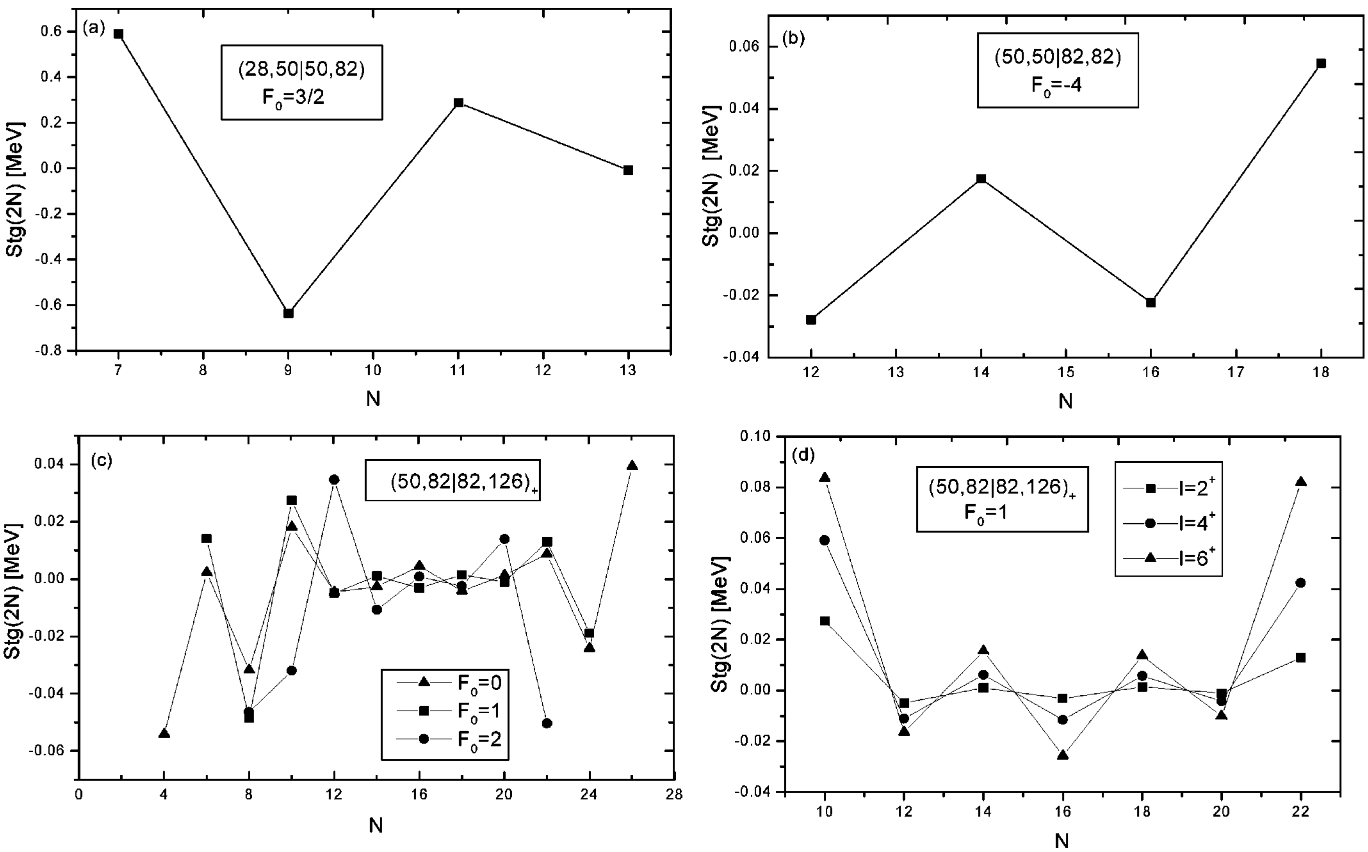

FIG. 1. The experimental values of $Stg(2N)$, Eq. (5), for the F_0 multiplets: (a) $F_0=3/2$ from $(28,50|50,82)_-$; (b) $F_0=-4$ from $(50,50|82,82)_+$; (c) $F_0=0,1,2$ from $(50,82|82,126)_+$; (d) $F_0=1$ from $(50,82|82,126)_+$ for $I=2,4,6$.

essentially different (light and heavy) valence shell configurations $(28,50|50,82)_-$ and $(50,82|82,126)_+$, respectively, the staggering amplitude generally differs by more than one order in magnitude. While for the lighter shells $(28,50|50,82)_-$ the function $Stg(2N)$ oscillates between -0.8 and 0.6 MeV [Fig. 1(a)], for the heavier (rare-earth) shells it drops in the interval $[-0.06,0.04]$ MeV [Fig. 1(c)]. Some intermediate staggering magnitude in the interval $[-0.03,0.05]$ MeV is observed for the multiplet $F_0=-4$ [Fig. 1(b)], which belongs to the "intermediate" valence space $(50,50|82,82)_+$.

As a result, we find that the general $\Delta N=2$ staggering scale tends to be large for the light nuclear valence shells and essentially smaller, by more than one order in magnitude, for the heavy nuclear regions. However, we should remark that the more detailed comparison is restricted due to the fact that the "length" of the F_0 multiplets increases towards the heavier valence shells and, as it will be pointed out below, more complicated structure of the staggering patterns appears there.

So, in the region of heavy nuclei we find that for any particular F_0 multiplet the $\Delta N=2$ staggering function strongly varies in dependence on the quantum number N. In Fig. 1(c) we show three well developed (long enough) patterns of the multiplets $F_0=0,1,2$ of the rare earth valence shells $(50,82|82,126)_+$, illustrated in Table I. We find that in the beginning of any multiplet $(N=4-12)$ as well as near its end $(N>20)$, a considerable staggering amplitude of about 0.05 MeV is observed, while in the middle of the multiplet $(N=15–20)$, which corresponds to the effective middle of the valence shells, the staggering effect is essentially suppressed to practically zero amplitude. Also, we remark that in some cases [see the multiplet $F_0=2$ in Fig. 1(c)] irregularities in the zigzagging curve with subsequent changes in the phase of the staggering pattern are observed. This is a possible indication for the presence of a subshell closure at $Z=64$ in the ^{152}Gd isotope at $N=10$ of the multiplet. The classification permits one to take into account the subshells, but in this work these are not considered, although their features are seen on the experimental staggering patterns.

Further, in Fig. 1(d) we illustrate the $\Delta N=2$ staggering patterns obtained for the experimental levels with angular momentum $I=2,4,6$ within the long $F_0=1$ multiplet. The different curves are characterized with the same phases of the oscillation, while the amplitude increases with the increasing of the angular momentum, conserving the same dependence on the quantum number N.

Now, let us consider the fine structure of the odd-even F_0 multiplets obtained by uniting two neighboring F_0 multiplets from the spaces H_+ and H_-. The respective 2_1^+ energy sequences are obtained by switching between the subsequent members of the two considered multiplets. The unified odd-even F_0 multiplets contain much larger number of nuclei than the single multiplets, which allows one to extend the analysis by applying the discrete function (8) to all the nuclear shells, including the first $(28,28|50,50)_{\pm}$ and the last (open) $(82,126|126,\ldots)_{\pm}$ major shells. In all of them we found that the experimental energies exhibit rather well pronounced $\Delta N=1$ staggering patterns. Some typical examples illustrating different combinations of odd-even multiplets from all the shells are shown in Fig. 2.

First of all, we see that the general scale of the $\Delta N=1$ staggering is larger by almost one order of magnitude than the scale in the $\Delta N=2$ staggering patterns of the respective separate F_0 multiplets. For example, compare Fig. 1(c) and Fig. 2(c), where the staggering patterns of the energies of the nuclei from Tables I and II, respectively, are plotted.

Further, we find that the trend of the decrease of the staggering amplitude with the increase of the shell dimension is observed with no exceptions in this case, e.g., in the $\Delta N=2$ case. This can be seen, for the odd-even multiplets with $F_0=\{-5/2,-2\}$ from the shell $(28,28|50,50)_{\pm}$ in Fig. 2(a) and from the shell $(82,126|126,\ldots)_{\pm}$ [30] in Fig. 2(d), where the maximal staggering amplitude drops from 0.2 MeV in the first one, to 0.003 MeV in the last one. This decrease is gradual through the consecutive shells [Fig. 2(a–d)].

As in the $\Delta N=2$ case, we observe that in the beginning and the end of any particular unified odd-even multiplets the staggering amplitude is larger, than in the middle of it. It is, however, important to remark that in the middle of the unified F_0 multiplets in the "lighter"shells $(28,28|50,50)_{\pm}$, $(28,50|50,82)_{\pm}$, and $(50,50|82,82)_{\pm}$ the decrease in the staggering magnitude is not very well expressed, as could be seen for the multiplets $F_0=\{-5/2;-2\}$ in Fig. 2(a), $F_0=\{1,3/2\}$ in Fig. 2(b) and $F_0=\{-4;-7/2\}$ in Fig. 2(e), respectively. In the middle of all the combined F_0 multiplets in the rare-earth $(50,82|82,126)_{\pm}$ region the staggering amplitude is almost vanishing, as illustrated for the $F_0=\{1,3/2\}$ multiplet in Fig. 2(c).

The application of Eq. (8) to the energies of the next exited states $I=4,6$ in the combined odd-even multiplets, shows as in the $\Delta N=2$ staggering, that all the typical features of the energy behavior of the 2_1^+ states, outlined above are even enhanced with the increase of the angular momentum. An example for one of the longest combined multiplets $F_0=\{-4;-7/2\}$ from the shell $(50,50|82,82)_{\pm}$ is given in Fig. 2(e).

The nuclear isotonic chains obtained in the scheme are too short, so the $Stg(1N)$ function is investigated only in the isotopic chains of even-even nuclei, obtained by the consecutive action of the boson operators ν^+ (pairs of neutrons). It does not exhibit a pronounced staggering effect. An exception is presented in Fig. 3, for the Os isotopes, but the maximal amplitude in this case is two orders smaller (0.006 MeV) than the odd-even $\Delta N=1$ staggering in the same shell [Fig. 2(c)].

Hence, we apply the staggering filters (5) and (8) on the theoretically obtained by means of Eq. (3) energies of the low-lying collective states of even–even nuclei in all the multiplets, in which experimental $\Delta N=2$ and $\Delta N=1$ staggering was observed. We found that the behavior of Eqs. (5) and (8) is reproduced rather well by the theoretical energies. This result is illustrated for 2_1^+ energies from the multiplet $F_0=0$ of the shell $(50,82|82,126)_+$ from Table I and for the odd-even $F_0=\{0,1/2\}$ multiplet of the same shell in Figs. 4(a) and 4(b), respectively. We see that in both cases with $\Delta N=2$ and $\Delta N=1$ the specific systematic dependence of the staggering magnitude on the quantum number N is reproduced. Moreover in the ends of the multiplets the larger amplitudes are reproduced together with the correct signs. The theoretical amplitude is a bit smaller than the experimental one and its irregularity is more clear in the region of the well deformed nuclei. As illustrated in Figs. 4(c–e) the staggering behavior of the energies of the states with $I=2,4,6$, respectively, in the combined multiplet $F_0=\{1/2,1\}$ from the shell $(50,82|82,126)$ is reproduced by the generalized formula (3), including the sign and typical features of the amplitude for the higher states. This means, that all the presented analysis for the first exited 2_1^+ states is appropriate for all the higher yrast states, which were included in the fitting procedure, when Eq. (3) was deduced in [5]. The slight discrepancies in the theoretical reproduction of the staggering amplitudes are due to the accuracy of the phenomenological description and the restriction to integer values of ω.

V. MODEL ANALYSIS

A relevant model interpretation of the presented investigation can be obtained on the basis of the geometrical factor ω, nonexplicitly depending on N and F_0, defining the nuclei in the multiplet. Its phenomenological values [5] vary in the limits $\omega=1$–30 and specify the character of the collective motion. In decreasing order, these values consequently characterize the collective excitations of nearly spherical, vibrational nuclei [large values of $\omega\approx15$–30; U(5) limit of IBM1]; the transitions region (with fast decrease of ω) including the γ-soft nuclei with $\omega\approx5$; [O(6) limit] and the region of well deformed, rotational nuclei [$\omega=1$; SU(3)]. In the lighter shells [Figs. 1(a), 1(b), 2(a), 2(e)] we have a transition only from spherical to γ-soft nuclei and back to almost the same value of ω, which corresponds to the transition from the U(5) to the O(6) limit of the IBM1 [31] (only one side of the Casten's triangle [32]). The typical model distribution of 2_1^+-energy levels between the different ω modes of collective motion is illustrated in Fig. 5 for the long $F_0=0$ multiplet of the shell $(50,82|82,126)_+$ in the first column of Table I. There the thin curves represent various ω-fixed modes of the $E_{yrast}(h_k,2,\omega)$ as functions of the valence pairs number N, while the thick curve connects its phenomenologically determined values, taken from Table I. We see that in the beginning of the multiplet $(N=4)$ the theoretical 2_1^+ energies start from the high $\omega=17$ "vibrational" curve, but with the increase of N they rapidly dump down (through $\omega=8,6$) to the "rotational iso-line" with $\omega=1$ that "receives" all levels in the middle region $N=10$–22. Further, in the end of the multiplet (shell closure region $N=24$–30) several jumps up to the transitional $(\omega=3,6,8)$ collective modes are observed.

The above rather large discrete jumps of energy as a func-

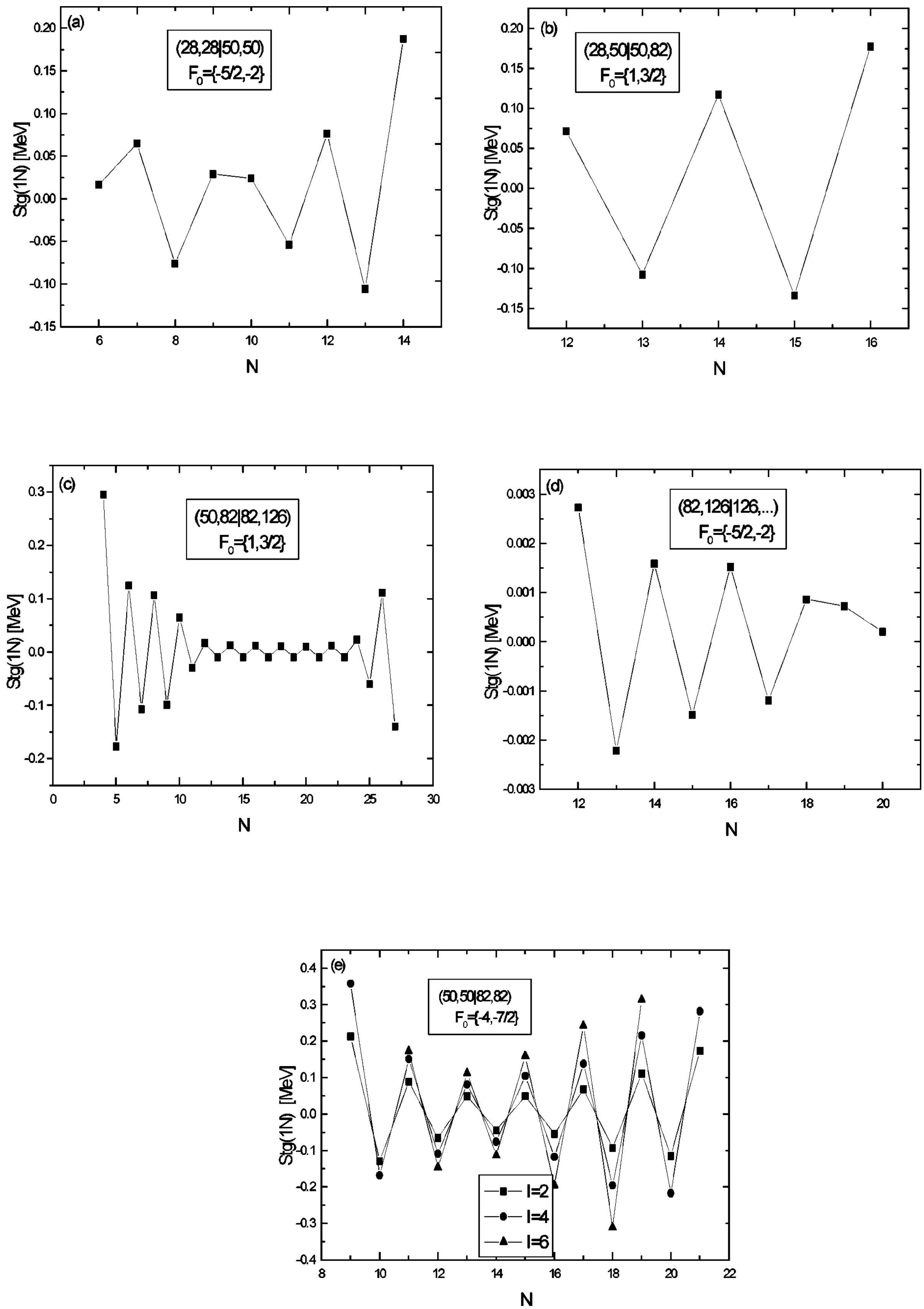

FIG. 2. The experimental values of $Stg(1N)$, Eq. (8), for the odd-even F_0 multiplets: (a) $F_0=\{-5/2,-2\}$ from (28,28|50,50); (b) $F_0=\{1,3/2\}$ from (28,50|50,82); (c) $F_0=\{1,3/2\}$ from (50,82|82,126); (d) $F_0=\{-5/2,-2\}$ from (82,126|126, . . .); (e) $F_0=\{-4,-7/2\}$ from (50,50|82,82) for $I=2,4,6$.

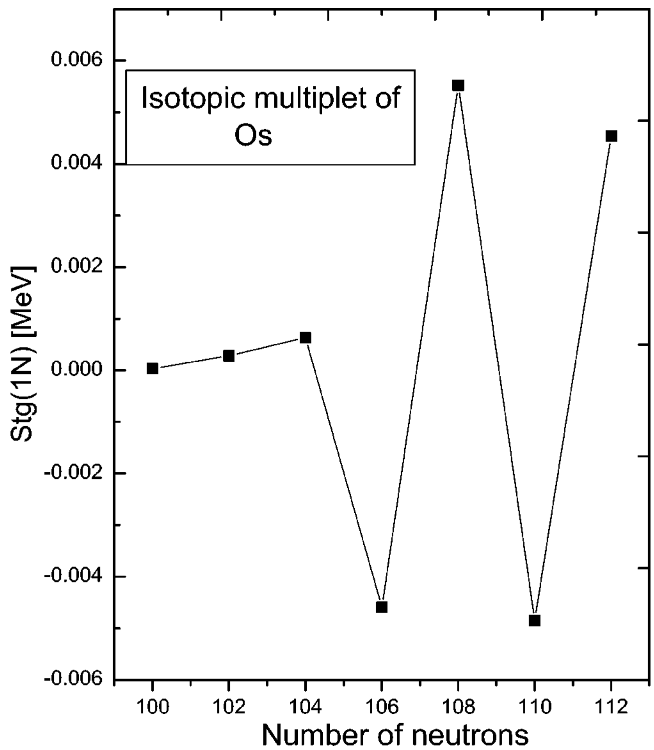

FIG. 3. The experimental values of $Stg(1N)$, Eq. (8), as function of the neutron numbers N_n, obtained for the Os isotopic multiplet from $(50,82|82,126)$.

tion of N between the different ω lines cause the pronounced $\Delta N=2$ staggering effect in the beginning and the end of the theoretically "filled" F_0 multiplet. The small (but nonzero) staggering amplitudes appearing in the middle region can be considered as the effect of the five-point discrete derivative, Eq. (6), which has a "memory" of the preceding states and also "propagates" the information about "what happens" in the ends.

At the same time, the change in the geometric factor ω, through its dependence on N, reflects phenomenologically the respective change in the intrinsic nuclear structure and its influence on the character of collective motion. On this basis we can directly interpret the experimental $\Delta N=2$ staggering patterns as a result of rapid change in nuclear collective properties with respect to different intrinsically determined modes, related in this case to the accumulation of α-like clusters, as obvious from the ordering of the nuclei on Table I.

Now, we can directly extend the above considerations with respect to the $\Delta N=1$ staggering patterns corresponding to changes in the intrinsic nuclear structure with alternating kinds of nucleon pairs. In the combined symplectic multiplets the analysis concerning the geometric factor ω and its contribution to the fine systematical behavior of 2_1^+ energies is still valid for their members, but their more gradual change must be taken into account. It provides an analogous explanation of the large $\Delta N=1$ staggering amplitudes in the ends of given odd-even multiplet. However, in this case there is another factor that appears to be decisive especially for the essentially larger staggering scale compared to the $\Delta N=2$ case. In the single F_0 multiplet the difference between two neighboring energies is entirely determined by the quantum number N and the geometric factor ω (all other model quantities and parameters are fixed). In the combined multiplets there is additional dependence on the quantum number F_0, Eq. (4). The alternative change of the F_0 values with $+\frac{1}{2}$ then $-\frac{1}{2}$ in the two neighboring F_0- multiplets provides respective change in the inertial factors $\alpha\{h_k\}$ Eq. (4), and thus *a priori* produces a mutual energy oscillation in the respective energy sequences. Hence, even in the energy difference (9) some staggering is obtained, due to its dependence on the oscillating values of F_0 as seen in Fig. 6.

The latter results in magnifying the $\Delta N=1$ staggering background. Thus, our theoretical analysis suggests that the form and the sensitivity of $\Delta N=1$ staggering effect to the nuclear shell structure are determined by discrete changes of the quantum number ω with respect to N, but its general magnitude is also determined by the fluctuations of the F_0 values, (see as an example Table II). The magnitude and the form of the staggering pattern, together with the respective set of phenomenological ω values provide both, relevant qualitative and quantitative, characteristics of nuclear collectivity in the framework or the $\mathrm{Sp}(4,R)$ classification scheme.

VI. CONCLUSIONS

The investigation and the physical interpretation of the two types $\Delta N=2$ and $\Delta N=1$ staggering patterns reflect the fact of the specific ordering of the even-even nuclei in the two types of multiplets of the $\mathrm{Sp}(4,R)$ classification scheme, in which they differ by an α-like cluster or alternating pairs of protons and neutrons. The results presented so far allow us to outline the following systematic behavior of the experimentally observed low-lying collective energy levels in the framework of the considered $\mathrm{Sp}(4,R)$ model scheme. The energies of the levels classified in any single F_0 multiplet of $\mathrm{Sp}(4,R)$ exhibit $\Delta N=2$ staggering behavior, while the levels of more general odd-even multiplets provide respective $\Delta N=1$ staggering patterns of their fifth order discrete derivative. In both cases the general staggering magnitude is relatively large for the light nuclear valence shells and considerably smaller for the valence shells of heavy nuclei. In addition, both $\Delta N=2$ and $\Delta N=1$ staggering patterns of heavy nuclei are characterized by relatively large amplitudes in the ends and their suppression in the middle region of the respective single and combined odd-even F_0 multiplets. On the other hand, we find that the $\Delta N=1$ staggering effect is by an order of magnitude *stronger* and more *regular* in form than the $\Delta N=2$ staggering. For the odd-even F_0 multiplets the staggering is inherent even for the energy differences of the neighboring states (9), with F_0 integer and half-integer. The weakest staggering effect is observed in the isotopic nuclear chains, where there is no oscillation of the ΔF_0 values.

The application of the $\mathrm{Sp}(4,R)$ classification scheme provides relevant physical interpretation of the experimentally observed $\Delta N=2$ and $\Delta N=1$ staggering effects in terms of the nucleon correlations in nuclear valence shells. The $\Delta N=2$ effect indicates very fine systematic behavior of nuclear

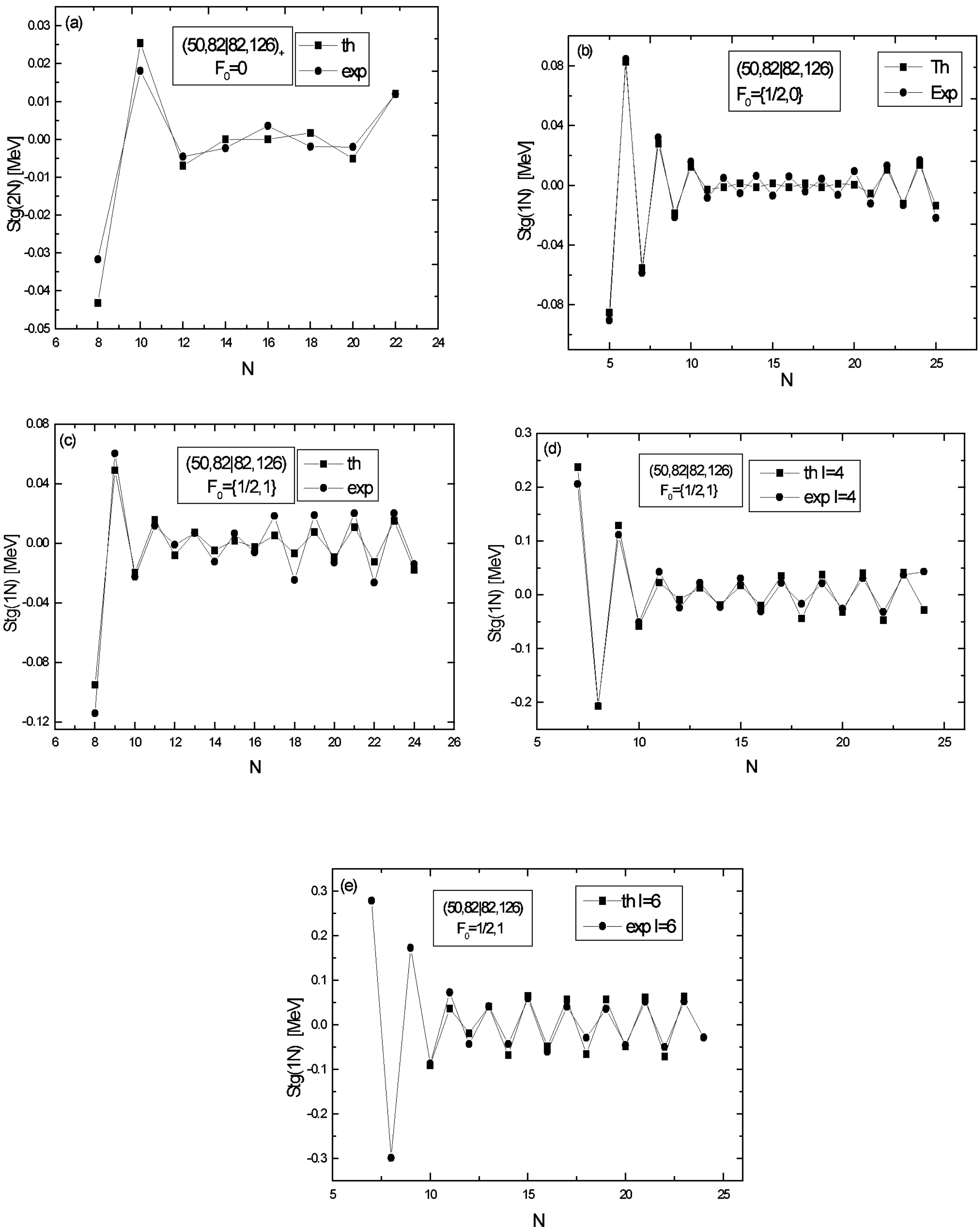

FIG. 4. The experimental and theoretical values (3) of the functions: (a) $Stg(2N)$, Eq. (6), for the multiplet $F_0=0$ from $(50{,}82|82{,}126)_+$; (b) $Stg(1N)$, Eq. (8), for the odd-even multiplet $F_0=\{0,1/2\}$ from (50,82|82,126); (c) same as (b) for the $F_0=\{1,1/2\}$ from (50,82|82,126) for $I=2$; (d) same as (c) for $I=4$; (e) same as (c) for $I=6$.

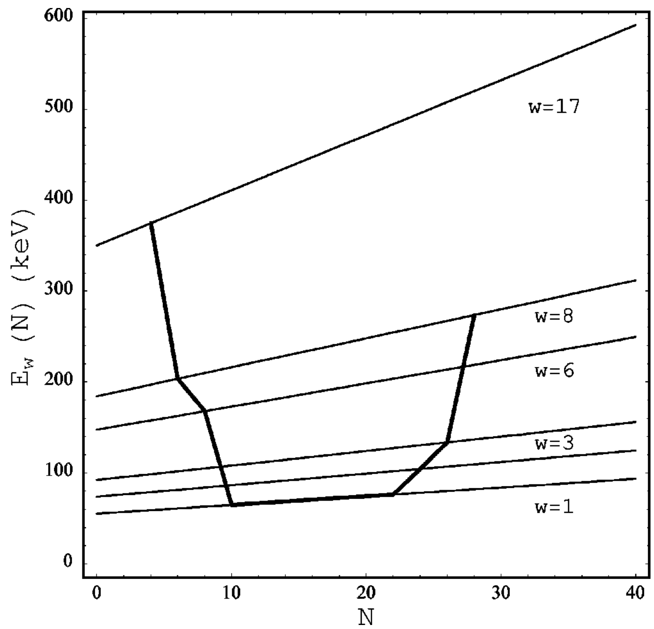

FIG. 5. The theoretical energies $E_{yrast}(N,\omega)$, Eq. (3), for different values of the parameter ω and the real behavior of the energies (thick line) for the specific ω for the multiplet $F_0=0$ from the shell (50,82|82,126).

collectivity associated with the *simultaneous* adding of proton and neutron pairs (α-like clusters) to valence shells, relevant to the effect of four-nucleon interaction. In the $\Delta N=1$ staggering effect the situation is different. Here the filling of valence shells is realized in two ways: (1) In the odd-even multiplets by the alternate adding of proton and neutron pairs, which is reflected by the alternate change of the F_0 values and thus strongly magnifies the staggering effect; and (2) in the isotopic multiplets by adding of only neutron pairs, which gives a very small staggering.

In these two cases the observed effect is due to the pairing correlations, which are of lower order, but its magnitude strongly depends on the charge fluctuations in the consecutive adding of the nucleon pairs.

This analysis provides an indication of the different ways of influence of the different orders of valence nucleon correlations (pairing and quartetting) on the fine systematic characteristics of the nuclear collectivity.

The general formula for the yrast energies $E_{yrast}(h_k,I,\omega)$ obtained for the even-even nuclei, as classified in the symplectic multiplets reproduces rather accurately the experimental $\Delta N=2$ and $\Delta N=1$ staggering. The geometric parameter ω, introduced in [5], not only unifies the description of the energies of the ground-state bands of the spherical, transitional and well-deformed nuclei, but also is the main reason for obtaining the effects of the shape transitions reflected in both the staggering effects. The amplitude of the staggering generally follows the same trend as ω, so it could be considered as relevant characteristic of the type of collectivity of nuclear spectra and the nuclear shape. In the region of the well-deformed nuclei, when $\omega=1$ the staggering effect is strongly suppressed, which is observed in the rare-earth and actinide shells and is the reason for the larger scale in the light ones.

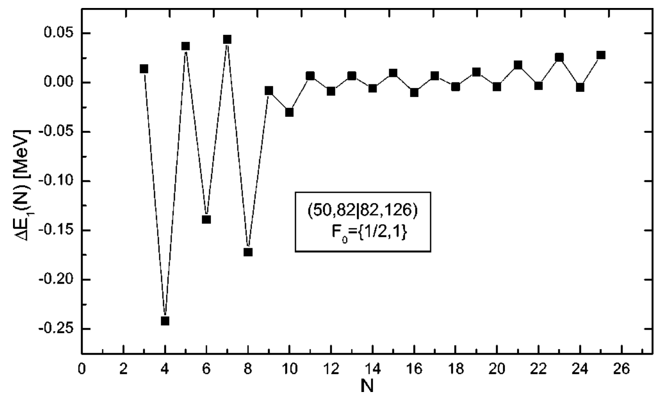

FIG. 6. The experimental values of $\Delta_1 E(N)$, Eq. (9), for the odd-even $F_0=\{1/2,1\}$ multiplet from the shell (50,82|82,126).

The generalizations of the interaction coefficient $\alpha\{h_k\}$, Eq. (4), as functions of the classification quantum numbers, are important for the reproduction of the staggering effects. In the $\Delta N=2$ case F_0 is fixed, as well as in the isotopic multiplets with $\Delta N=1$, while in the combined odd-even multiplets its values oscillate from state to state. This reflects in the larger values of the amplitudes and their regularity in the $Stg(1N)$ function in the combined multiplets.

The investigation in this work, reveals once again that the symplectic classification scheme is rather convenient for a generalization and reproduction of important physical properties of the even-even nuclei. It also has a predicting power, since in the consideration of the staggering $\Delta N=1$ effects in the last shell, we used experimental data [30], which was not included in the fitting procedure in [5]. As the ladder series of the boson representations are infinite dimensional, the symplectic multiplets can be extended to the newly obtained data for exotic nuclei and they can be considered as a new test for this kind of fine structure investigations. Moreover the suggested quantitative analysis could provide a rather fine estimation of the possible energy regions, where low-lying collective excitations of exotic nuclei can be expected. A perspective field of further elaborations is the physical interpretation of the observed irregularities in the staggering behavior.

In conclusion, the above results show that the investigation of the different types of the staggering effects based on a convenient classification scheme is a useful tool to understand not only the collective properties of the lowest states of even-even nuclei, but it also provides various quantitative considerations on the fine structure of the changes in nuclear collectivity.

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