Perturbative calculation of the excluded volume effect for nuclear matter in a relativistic mean-field approximation

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Considering the finite volume of nucleons, a Lagrangian density is given. The first-order self-energy of the nucleon and the equation of state of nuclear matter are calculated in the framework of the relativistic mean-field approximation. Our results indicate that the effects of the volume of nucleons are not negligible.

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I. INTRODUCTION

Although quantum field theory has succeeded in many areas of modern physics, people have met some difficulties when quantum field theory is applied to nuclear systems. First, nuclear matter and all nuclei are many-body systems, and their ground states are states where the Fermi sea is filled with interacting nucleons, not the "vacuum." Second, the nucleons are extended objects, but they are treated as "points" in the quantum field theory. These approximations might cause some troubles in the research of nuclear physics. Thus people have tried to exclude the whole volume ocuppied by nucleons from their configuration space, then consider nucleons as "pointlike" particles moving in the mean field [1–8]. Since the nucleon volume is not relativistic invariant, in Refs. [9,10] a relativistic consistent volume effect of nucleons is included in their calculation of the properties of nuclear matter. However, all the previous methods of treating the finite volume effects are mainly from the consideration of the geometry of the participating particles.

Now in this paper, we incorporate the excluded volume effect into the Lagrangian density of nuclear matter. Since the volume of nucleons is about 10% of the space in the saturation nuclear matter, we regard the excluded volume effect as a perturbation and the first-order self-energy is calculated in the framework of the relativistic mean-field approximation. At last, a set of parameters is obtained so that the saturation properties of nuclear matter can be reproduced well.

II. THE EXCLUDED VOLUME EFFECT IN A LAGRANGIAN DENSITY

To take into account of the excluded volume effect, the whole volume of nucleons v_0N is substracted from the volume of nuclear matter V so that the effective space available for nucleon motion is $V-v_0N$, where v_0 is the volume of a nucleon and N is the total number of nucleons in the nuclear matter.

The nucleons in nuclear matter are constantly moving, constrainted by Pauli principle. Thus the length of each nucleon decreases in the direction of its movement by the Lorentz contraction and the effective space for nucleons is

$$V[1-v_0\rho_s'(N)], \tag{1}$$

where $\rho_s'(N)$ is the scalar density of nucleons. The box normalization factor becomes

$$\frac{1}{\sqrt{V[1-v_0\rho_s'(N)]}}, \tag{2}$$

and the corresponding field is written as

$$\psi' = \sqrt{1-v_0\rho_s'(N)}\,\psi, \tag{3}$$

where ψ is the nucleon field in the effective space $V[1-v_0\rho_s'(N)]$. From now on, we use the quantities with a prime for the ones in the original space and the quantities without a prime for the quantities in the effective space. The relation between the scalar densities of nucleons in the two spaces is

$$\rho_s'(N) = [1-v_0\rho_s'(N)]\rho_s(N). \tag{4}$$

From Eq. (4),we obtain

$$\rho_s'(N) = \frac{\rho_s(N)}{1+v_0\rho_s(N)} = \rho_s(N)[1-v_0\rho_s(N)+v_0^2\rho_s^2(N)+\cdots]. \tag{5}$$

If the v_0^2 terms and beyond are neglected, the above equation becomes

$$\rho_s'(N) = \rho_s(N)[1-v_0\rho_s(N)]. \tag{6}$$

Since $\rho_s'(N)$ and $\rho_s(N)$ are the scalar density operators of nucleons in the original space and the effective space respectively, namely, $\rho_s'(N)=\overline{\psi'}\,\psi'$, $\rho_s(N)=\overline{\psi}\psi$, then we can obtain

$$\overline{\psi'}\,\psi' = (1-v_0\overline{\psi}\psi)\overline{\psi}\psi. \tag{7}$$

That is to say, when the volume of nucleons is excluded, the field operators in the Lagrangian should be changed in the following way:

$$\bar{\psi}O\psi \rightarrow (1-v_0\bar{\psi}\psi)\bar{\psi}O\psi, \tag{8}$$

where O is an operator, such as 1, γ^μ, etc. The corresponding Lagrangian is written as

$$\mathcal{L}=(1-v_0\bar{\psi}\psi)\bar{\psi}(i\gamma_\mu\partial^\mu - m - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - \frac{1}{4}\omega_{\mu\gamma}\omega^{\mu\gamma} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu, \tag{9}$$

where

$$U(\sigma)=\frac{1}{2}m_\sigma^2\sigma^2+\frac{1}{3}g_2\sigma^3+\frac{1}{4}g_3\sigma^4. \tag{10}$$

The Lagrangian in the relativistic mean-field approximation is

$$\mathcal{L}_{\mathrm{RMF}}=(1-v_0\bar{\psi}\psi)\bar{\psi}(i\gamma_\mu\partial^\mu - m - g_\sigma\sigma_0 - g_\omega\gamma^0\omega_0)\psi - \frac{1}{2}m_\sigma^2\sigma_0^2 - \frac{1}{3}g_2\sigma_0^3 - \frac{1}{4}g_3\sigma_0^4 + \frac{1}{2}m_\omega^2\omega_0^2, \tag{11}$$

where σ_0 and ω_0 are the expectation values of meson fields in the ground states of nuclear matter. Since the radius of a nucleon in nuclear matter is usually taken as 0.63 fm, the whole volume of nucleons is only about 10% of the volume of the saturated nuclear matter so that the effect of nucleon volume can be treated as a perturbation. The Lagrangian density $\mathcal{L}_{RMF}$ can be divided into nonperturbative and perturbative parts, e.g.,

$$\mathcal{L}_{RMF}=\mathcal{L}_0+\mathcal{L}_I, \tag{12}$$

with

$$\mathcal{L}_0=\bar{\psi}(i\gamma_\mu\partial^\mu - m - g_\sigma\sigma_0 - g_\omega\gamma^0\omega_0)\psi - \frac{1}{2}m_\sigma^2\sigma_0^2 - \frac{1}{3}g_2\sigma_0^3 - \frac{1}{4}g_3\sigma_0^4 + \frac{1}{2}m_\omega^2\omega_0^2 \tag{13}$$

and

$$\mathcal{L}_I=-v_0\bar{\psi}\psi\bar{\psi}(i\gamma_\mu\partial^\mu - m - g_\sigma\sigma_0 - g_\omega\gamma^0\omega_0)\psi \tag{14}$$

in the interaction picture defined by the nonperturbative Lagrangian density $\mathcal{L}_0$. The equation of motion for the nucleon field can be easily obtained from $\mathcal{L}_0$:

$$(i\gamma_\mu\partial^\mu - m - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu)\psi=0. \tag{15}$$

The perturbative Hamilton in the interaction picture is

$$\mathcal{H}_I=-v_0\bar{\psi}\psi[\bar{\psi}(-i\vec{\gamma}\cdot\vec{\nabla}+m^*+g_\omega\gamma^0\omega_0)\psi], \tag{16}$$

where $m^*=m+g_\sigma\sigma_0$. The first order of the evolution operator $U(t_2,t_1)$ is

$$U(t_2,t_1)=1-i\int_{t_1}^{t_2}dt\int d^3x\mathcal{H}_I(\vec{x},t). \tag{17}$$

When the effect of nucleon volume is considered, the corresponding Green function is

$$\begin{aligned} G&=\langle 0|T\{\psi(x_2)\bar{\psi}(x_1)U(+\infty,-\infty)\}|0\rangle \\ &=\langle 0|T\{\psi(x_2)\bar{\psi}(x_1)\}|0\rangle \\ &\quad -v_0\int_{-\infty}^{+\infty}dt\int d^3x\langle 0|T\{\psi(x_2)\bar{\psi}(x_1)\bar{\psi}(x)\gamma_0\psi(x)\bar{\psi}(x) \\ &\quad \times(-\vec{\gamma}\cdot\vec{\nabla}-im^*-ig_\omega\gamma^0\omega_0)\psi(x)\}|0\rangle. \end{aligned} \tag{18}$$

After some straightforward derivation the first order self-energy of nucleon can be expressed as

$$\Sigma=I\Delta m+\gamma_0\Sigma_0-\vec{\gamma}\cdot\vec{\Sigma}, \tag{19}$$

where

$$\Delta m=v_0\int\frac{d^3k}{(2\pi)^3}\left(\frac{3}{2}E^*(k)+g_\omega\omega_0+\frac{3}{2}\frac{{m^*}^2}{E^*(k)}\right), \tag{20}$$

$$\Sigma_0=-v_0\int\frac{d^3k}{(2\pi)^3}\left(m^*-\frac{g_\omega\omega_0}{E^*(k)}m^*\right), \tag{21}$$

$$\vec{\Sigma}=-v_0\int\frac{d^3k}{(2\pi)^3}\left(\frac{3}{2}\frac{m^*}{E^*(k)}\right)\vec{k}_1 \tag{22}$$

with $E^*(k)=\sqrt{\vec{k}^2+{m^*}^2}$. In the first order approximation, the Green function reads as

$$G(k_1)=G^0(k_1)-iG^0(k_1)\Sigma G^0(k_1) \tag{23}$$

with

$$G^0(k_1)=\left(\frac{i}{\not{k}_1-m^*+i\varepsilon}\right)_{\beta\alpha}. \tag{24}$$

This Green function can be rewritten as follows:

$$\begin{aligned} G(k_1)&=\left(\frac{i}{\not{k}_1-m^*-\Sigma+i\varepsilon}\right) \\ &=\left(\frac{i}{\gamma_0(\varepsilon^*(k_1)-\Sigma_0)-\vec{\gamma}\cdot(\vec{k}_1-\vec{\Sigma})-(m^*+\Delta m)+i\varepsilon}\right) \\ &=\left(\frac{i}{\gamma_0(\varepsilon^*(k_1)-\Sigma_0)-\vec{\gamma}\cdot\vec{k}_1X-(m^*+\Delta m)+i\varepsilon}\right), \end{aligned} \tag{25}$$

namely,

TABLE I. The parameters in the calculation of the relativistic mean-field approximation.

	ITP (this model)	NL3 [11]	QRZ [12]
M (MeV)	939	939	939
m_σ (MeV)	550	508.194	532
m_ω (MeV)	783	782.501	780
g_σ	12.30	10.217	7.676
g_ω	6.45	12.868	7.036
g_2 (fm^{-1})	-17.30	-10.431	28.140
g_3	28.00	-28.885	0.012
r_c (fm)	0.63	0.00	0.62
Nuclear matter properties			
ρ_0 (fm^{-3})	0.145	0.148	0.147
E/A (MeV)	16.727	16.299	15.687
K (MeV)	288.65	271.76	241
m^*/m	0.695	0.60	0.83

$$G(k_1)=\left(\frac{iX^{-1}}{\gamma_0\left(\dfrac{\varepsilon^*(k_1)-\Sigma_0}{X}\right)-\vec{\gamma}\cdot\vec{k}_1-\left(\dfrac{m^*+\Delta m}{X}\right)+i\varepsilon}\right), \tag{26}$$

in which

$$X=1+\frac{3}{8}v_0\rho_s(N). \tag{27}$$

Now we can see that the perturbation interaction causes the corrections of mass, energy, and wave function of the nucleon, which are expressed by Δm, Σ_0, and X^{-1}. The effective mass m^* and energy of a nucleon $\varepsilon^*(k_1)$ should be changed as

$$m^*\rightarrow\frac{m^*+\Delta m}{X}, \tag{28}$$

$$\varepsilon^*(k_1)\rightarrow\frac{\varepsilon^*(k_1)-\Sigma_0}{X}. \tag{29}$$

The scalar density $\rho_s(N)$ and vector density $\rho_v(N)$ of nucleons are

$$\rho_v(N)=\frac{4}{(2\pi)^3}\int_0^{k_F}d^3k, \tag{30}$$

$$\rho_s(N)=\frac{4}{(2\pi)^3}\int_0^{k_F}d^3k\frac{m^*}{\sqrt{(\vec{k})^2+m^{*2}}}. \tag{31}$$

We must calculate the effective mass m^* of nucleons self-consistently:

$$\begin{aligned}m^*={}&\frac{1}{X}(m+\Delta m)\\&-\frac{1}{X}C_s^2\left(\frac{4}{(2\pi)^3}\int_0^{k_F}d^3k\frac{Xm^*-\Delta m}{[\vec{k}^2+(Xm^*-\Delta m)^2]^{1/2}}\right)\\&-\frac{1}{X}C_s^2[B(Xm^*-\Delta m-m)^2+C(Xm^*-\Delta m-m)^3],\end{aligned} \tag{32}$$

where $C_s=g_\sigma/m_\sigma$, $B=g_2/g_\sigma^3$, $C=g_3/g_\sigma^4$, and

$$k_F=\left(\frac{3\pi^2}{2}\rho_v(N)\right)^{1/3}. \tag{33}$$

From the equations

$$m_\sigma^2\sigma_0+g_2\sigma_0^2+g_3\sigma_0^3=-g_\sigma\rho_s(N) \tag{34}$$

and

$$m_\omega^2\omega_0=g_\omega\rho_v(N), \tag{35}$$

the values of σ_0 and ω_0 are calculated. At last we acquire the energy density of symmetric nuclear matter

$$\varepsilon=\varepsilon_N+\frac{1}{2}m_\sigma^2\sigma_0^2+\frac{1}{3}g_3\sigma_0^3+\frac{1}{4}g_4\sigma_0^4-\frac{1}{2}m_\omega^2\omega_0^2, \tag{36}$$

where

$$\begin{aligned}\varepsilon_N={}&\frac{4}{(2\pi)^3}\int_0^{k_F}d\vec{k}(\vec{k}^2+m^{*2})^{1/2}+\frac{2}{3\pi^2}k_F^3\Big(g_\omega\omega_0\\&+\frac{1}{4}v_0[m^*\rho_v(N)-g_\omega\omega_0\rho_S(N)]\Big).\end{aligned} \tag{37}$$

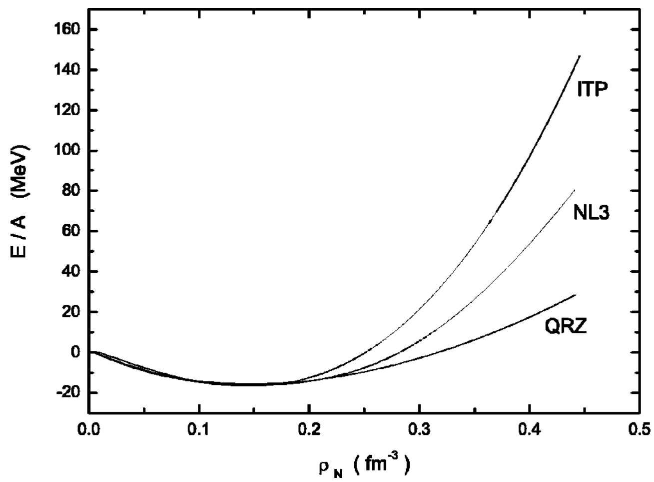

FIG. 1. Average energy per nucleon E/A as a function of the nucleon density ρ_N for nuclear matter with different parameters, (ITP) for perturbative calculation of the excluded volume effect in this model, (NL3) for the relativistic mean-field approximation with NL3 parameter [11], (QRZ) for the results in Ref. [12].

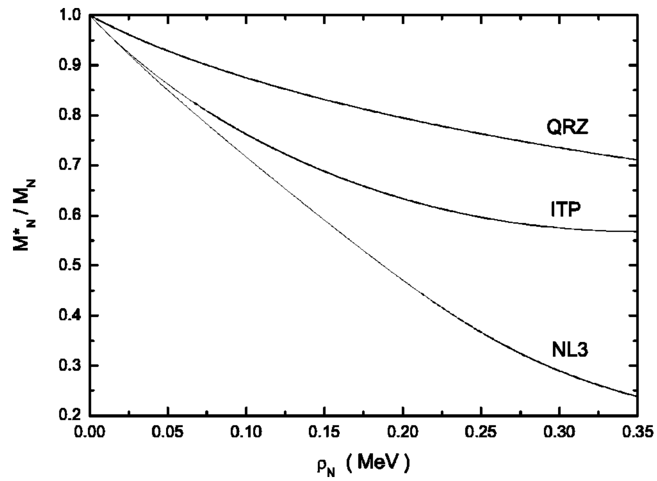

FIG. 2. Effective mass of nucleons M_N^* as a function of the nucleon density ρ_N for nuclear matter with different parameters. The meanings of ITP, NL3, and QRZ are the same as those of Fig. 1.

III. THE EQUATION OF STATE FOR NUCLEAR MATTER

In our calculation the radius of the nucleon in the nuclear matter is taken as 0.63 fm and the four parameters g_σ, g_ω, g_2, g_3 in this model are fixed by fitting the saturation properties of normal nuclear matter. The parameters in this model (labeled by ITP) are listed in Table I. As a comparison, the parameter set NL3, which is often used in the RMF calculation with all the nucleons taken as pointlike particles [11], and QRZ set, which was obtained in Ref. [12] by including the finite volume effect from the geomitry consideration, are also given in Table I. Comparing with the NL3 set, the coupling constant to scalar mesons g_σ of ITP becomes lager. Since the excluded volume effect actually supplies a repulsive force, so this trend is reasonable. For the coupling constant to vector mesons g_ω which is associated with the repulsive interaction between nucleons, its ITP value is smaller than the one of NL3. This is consistent with the result of QRZ set.

With different sets of parameters, the average energy per nucleon as a function of the number density for nuclear matter is shown in Fig. 1. Comparing these results, we can see that the average energy per nucleon of our calculation increases more quickly than those of NL3 [11] and QRZ [12] as $\rho_S(N) > 2\rho_0$ although the values of compression modulus are almost the same at the saturation density. It manifests that the influence of the excluded volume effect of nucleons increases at higher density. The results at even higher densities are not given in the figure since we are not sure if the purturbative model used in this paper is still suitable to calculate the equation of state at those densities.

The effective nucleon mass M_N^* as a function of the nucleon density ρ_N for nuclear matter with different set of parameters is displayed in Fig. 2. It decreases as the number density of nucleons increases. However, a little larger effective nucleon mass is obtained in our model than the result of the relativistic mean-field approximation where the nucleon is treated as ''pointlike'' particles with NL3 parameters but the change is not so significant as indicated by the QRZ curve. The differences between the three models become larger as the density increases. This implies that at higher densities we have not only to take into account of the finite volume effect of nucleons but also have to pay attention to choose a ''better'' model to deal with it.

IV. SUMMARY

In summary, we give a Lagrangian density including the effect of volume of nucleons, then the first order self-energy of the nucleon is derived, and the equation of state of nuclear matter is calculated in the framework of the relativistic mean-field approximation. Our results indicate that the finite volume can cause a considerable influence on some properties of the nuclear matter.

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