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Structure of Sn isotopes beyond $N=82$

L. Coraggio, A. Covello, A. Gargano, and N. Itaco
Dipartimento di Scienze Fisiche, Università di Napoli Federico II, and Istituto Nazionale di Fisica Nucleare, Complesso Universitario di Monte S. Angelo, Via Cintia, I-80126 Napoli, Italy
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We have performed shell-model calculations for ^{134,135}Sn using a realistic effective interaction derived from the CD-Bonn nucleon-nucleon potential. Comparison shows that the calculated results for ^{134}Sn are in very good agreement with recent experimental data. This supports confidence in our predictions of the hitherto unknown spectrum of ^{135}Sn.

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The Sn isotopes play an essential role in the shell-model description of nuclear structure, as they provide the opportunity to study the change of nuclear properties on varying the number of neutrons over a large range. From the experimental point of view, there is information for practically all isotopes with valence neutrons in the 50-82 shell, namely, from doubly magic ^{132}Sn down to doubly magic ^{100}Sn. It is, therefore, of great relevance to try to go beyond ^{132}Sn, so as to test the shell-model predictions when having valence neutrons in 82-126 shell. While this is not an easy task, since very neutron-rich Sn isotopes lie well away from the valley of stability, two very recent studies [1,2] have provided some experimental information on ^{134}Sn with two neutrons outside the doubly magic ^{132}Sn core.

During the past few years, we have studied several nuclei around ^{132}Sn [3] in terms of the shell model employing realistic effective interactions derived from modern nucleon-nucleon (NN) potentials. We have, therefore, found it interesting to perform calculations of this kind for ^{134}Sn and also to predict the spectroscopic properties of ^{135}Sn with three neutrons in the 82-126 shell, for which no experimental spectroscopic data is available as yet. In this paper, we present the results of our realistic shell-model calculations starting with a brief description of how they have been obtained.

We consider ^{132}Sn as a closed core and let the valence neutrons occupy the six single-particle levels $h_{9/2}$, $f_{7/2}$, $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, and $i_{13/2}$ of the 82-126 shell. The effective two-body matrix elements for the chosen model space have been derived from the CD-Bonn free NN potential [4]. This derivation has been performed within the framework of a G-matrix folded-diagram formalism, including renormalizations from both core polarization and folded diagrams. A detailed description of our derivation is given in Ref. [5]. We should point out, however, that the effective interaction used in the present work has been obtained including G-matrix diagrams through third order.

Let us now come to the single-particle (SP) energies. The value of $\epsilon_{f_{7/2}}$ has been fixed at -2.455 MeV, as determined from the experimental one-neutron separation energy for ^{133}Sn [6]. As regards the spacings between the six SP levels, they have been taken from the experimental spectrum of ^{133}Sn [7], except that relative to the $i_{13/2}$ level which has not been observed. Our adopted values (relative to the $f_{7/2}$ level) are (in MeV) $\epsilon_{p_{3/2}}=0.854$, $\epsilon_{h_{9/2}}=1.561$, $\epsilon_{p_{1/2}}=1.656$, $\epsilon_{f_{5/2}}=2.055$, and $\epsilon_{i_{13/2}}=2.694$. The energy of the $i_{13/2}$ level has been taken from Ref. [8], where it was estimated from the position of the 2.434 MeV level in ^{134}Sb assumed to be a 10^+ state of $\pi g_{7/2}\nu i_{13/2}$ nature. To verify the consistency of this choice, we have calculated the position of the 10^+ state in ^{134}Sb using the above value of $\epsilon_{i_{13/2}}$ and the effective neutron-proton interaction derived from the CD-Bonn potential. It turns out that our result (2.463 MeV) reproduces quite accurately the experimental excitation energy of this state.

We now present the results of our calculations. The spectrum of ^{134}Sn proposed in the experimental studies of Refs. [1,2] is compared with the theoretical one in Fig. 1. In Tables I and II we report the calculated excitation energies up to about 2.8 and 4.0 MeV for the positive- and negative-parity states, respectively. We hope that these predictions may provide a guidance in the interpretation of the results of future studies on ^{134}Sn.

From the structure of our wave functions, it turns out that the 20 states of Table I essentially arise from the configurations $(f_{7/2})^2$, $f_{7/2}p_{3/2}$, $(p_{3/2})^2$, $f_{7/2}h_{9/2}$, and $f_{7/2}p_{1/2}$. The

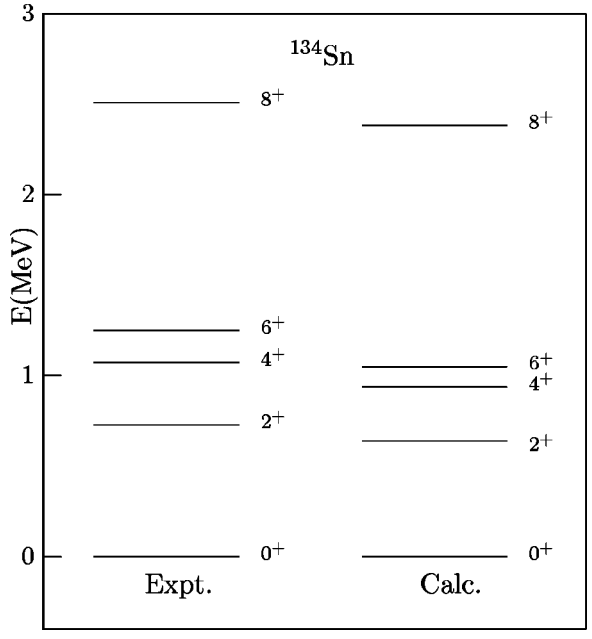

FIG. 1. Experimental and calculated spectra of ^{134}Sn.

TABLE I. Calculated excitation energies for the positive-parity states of ^{134}Sn up to 2.8 MeV.

J	E (MeV)
0	0.0
2	0.640
4	0.936
6	1.050
2	1.501
4	1.765
3	1.890
5	1.944
0	2.167
1	2.304
8	2.382
2	2.490
6	2.512
4	2.515
2	2.545
3	2.635
4	2.703
5	2.733
3	2.740
7	2.744

four members of the $(f_{7/2})^2$ multiplet all lie at an excitation energy smaller than 1.05 MeV and are separated from the other states by a pronounced gap (about 500 keV), above which we predict the existence of four seniority-2 and one seniority-0 states coming from the $f_{7/2}p_{3/2}$ and $(p_{3/2})^2$ configurations, respectively. Just above these states we find the lowest- and highest-spin members (1^+ and 8^+) of the $f_{7/2}h_{9/2}$ multiplet. All the states up to 2.4 MeV excitation energy are dominated by a unique configuration, the percentage of configurations other than the dominant one reaching at most 23% in the 2^+ state at 1.501 MeV. The other members of $f_{7/2}h_{9/2}$ and $(p_{3/2})^2$ multiplets as well as the two states arising from the $f_{7/2}p_{1/2}$ configuration are predicted in the energy region above 2.4 MeV. It should be mentioned that for some of these states a significant admixture of these three configurations is present in our calculated wave functions. As regards the negative-parity states, we have reported in Table II all the eight states arising from the $f_{7/2}i_{13/2}$ configuration. They are practically pure, the percentage of this configuration ranging from 87% to 100%.

TABLE II. Calculated excitation energies for the negative-parity states of ^{134}Sn up to about 4.0 MeV.

J	E (MeV)
3	3.247
5	3.621
4	3.684
7	3.713
9	3.752
6	3.848
8	3.925
10	3.973

TABLE III. Calculated $B(E2)$ (in W.u.) in ^{134}Sn.

$J_i^\pi \rightarrow J_f^\pi$	$B(E2)$
$2^+ \rightarrow 0^+$	1.72
$4^+ \rightarrow 2^+$	1.71
$6^+ \rightarrow 4^+$	0.88
$8^+ \rightarrow 6^+$	0.12

Coming back to the comparison with experiment, we see from Fig. 1 that the calculated level scheme is in good agreement with the proposed one, supporting the interpretation given in Refs. [1,2]. Our calculated value of the ground-state binding energy relative to ^{132}Sn is 5.986 ± 0.064 MeV, to be compared with the experimental one 6.365 ± 0.104 MeV [6]. Note that the error on the calculated value arises from the experimental error on the neutron separation energy of ^{133}Sn.

As far as the electromagnetic observables are concerned, only the $B(E2; 6^+ \rightarrow 4^+)$ is known with a measured value of 0.88 ± 0.17 W.u. [1]. To reproduce this experimental $E2$ transition rate an effective neutron charge of $0.70 \pm 0.06e$ is needed in our calculation. This value is significantly smaller than $1.01e$, which was determined by Zhang *et al.* in [1], but comes close to the value $0.62e$ of Ref. [9]. This is essentially due to the fact that in [1] the 4^+ and 6^+ states are interpreted as pure $(f_{7/2})^2$ states. By making use of $e_n^{\rm eff} = 0.70e$, we have calculated the $E2$ rates for transitions involving all the observed states. The values are reported in Table III.

Shell-model calculations on ^{134}Sn have been performed in some previous studies [2,9,10]. These studies used a two-body interaction constructed by Chou and Warburton [10] starting from the Kuo-Herling effective interaction for the ^{208}Pb region [11]. A comparison between the experimental and calculated spectra is shown in Refs. [2,9], but not in [10] since no experimental data was available at that time. It should be mentioned, however, that very different conclusions are given in [2] and [9] as regards the adopted effective interaction. In fact, while the comparison between the theory and experiment of Ref. [2] shows a very good agreement, the calculated excitation energies reported in [9] are all too high. Note that the results of Ref. [9] coincide with those of [10], both studies making use of the same set of SP energies, which is different from that of Ref. [2]. However, we have verified that the differences in the SP energies do not completely account for the results obtained in [2]. This point has already been evidenced in both [2] and [9], but no explanation was given.

Based on the good agreement between theory and experiment obtained for ^{134}Sn, we have found it interesting to predict some spectroscopic properties of ^{135}Sn. This nucleus is expected to have a $\frac{7}{2}^-$ ground state [12,13] with a binding energy of 8.393 ± 0.401 MeV [14], as derived from the systematic trend. No other experimental information on its spectrum is presently available. In Table IV we report the calculated excitation energies of ^{135}Sn up to about 1.5 MeV. Our

TABLE IV. Calculated excitation energies for states of ^{135}Sn up to about 1.5 MeV.

J^π	E (MeV)
$\frac{7}{2}^-$	0.0
$\frac{5}{2}^-$	0.226
$\frac{3}{2}^-$	0.356
$\frac{11}{2}^-$	0.611
$\frac{3}{2}^-$	0.643
$\frac{9}{2}^-$	0.706
$\frac{15}{2}^-$	0.911
$\frac{9}{2}^-$	1.093
$\frac{7}{2}^-$	1.192
$\frac{1}{2}^-$	1.221
$\frac{3}{2}^-$	1.264
$\frac{5}{2}^-$	1.295
$\frac{7}{2}^-$	1.298
$\frac{9}{2}^-$	1.331
$\frac{5}{2}^-$	1.430

calculations confirm the $\frac{7}{2}^-$ nature of the ground state and predict the value 8.396 ± 0.078 MeV for the binding energy. As regards the excited states, we find that the six members of the $f^3_{7/2}$ multiplet all lie at an excitation energy smaller than 1.0 MeV. From Table IV it can be seen that in this energy region we predict the existence of a second $\frac{3}{2}^-$ state (0.643 MeV) of seniority-1 nature, which is dominated by the $f^2_{7/2}p_{3/2}$ configuration. All the levels between 1.0 and 1.5 MeV arise from the $f^2_{7/2}p_{3/2}$ configuration with only two exceptions, the $\frac{1}{2}^-$ and $\frac{9}{2}^-$ states at 1.221 and 1.331 MeV, respectively. The latter is, in fact, the seniority-1 state of the $f^2_{7/2}h_{9/2}$ configuration while the former contains almost the same percentage of $f^2_{7/2}p_{3/2}$ and $f^2_{7/2}p_{1/2}$ configurations.

In Fig. 2 we show the behavior of the experimental energies [15,16] of the lowest-lying states with $J^\pi = \frac{3}{2}^-, \frac{5}{2}^-, \frac{7}{2}^-, \frac{9}{2}^-, \frac{11}{2}^-$, and $\frac{15}{2}^-$ in the $N=85$ isotones with $52 \leqslant Z \leqslant 64$, as well as our predictions for $Z=50$. From this figure we see that our calculated energies are quite consistent with the systematic trends. Note that they refer to states arising from the $f^3_{7/2}$ configuration, which, as mentioned above, we have found to be the lowest-lying ones. In the heavier isotones the lowest-lying states with $J^\pi = \frac{3}{2}^-, \frac{5}{2}^-, \frac{7}{2}^-, \frac{11}{2}^-$, and $\frac{15}{2}^-$ have been interpreted as having this nature, whereas the $\frac{9}{2}^-$ states were supposed to arise from the $f^2_{7/2}h_{9/2}$ configuration [16]. However, only for the $Z \geqslant 60$ isotones, which were studied by means of transfer reaction experiments,

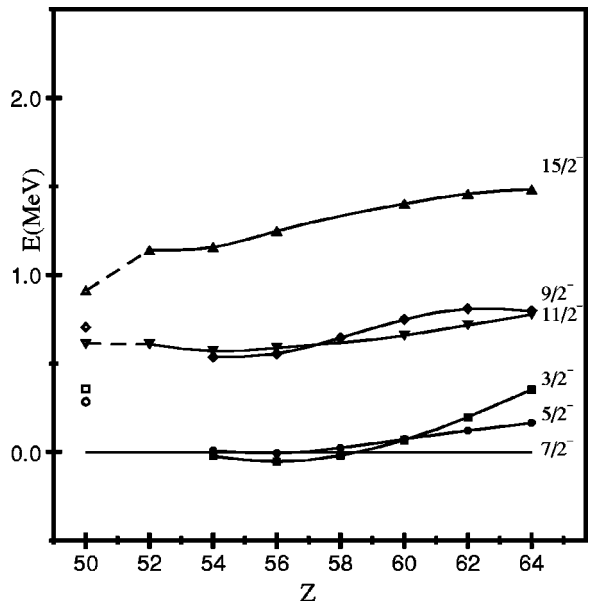

FIG. 2. Energy systematics for the levels of the $(f_{7/2})^3$ multiplet in the $N=85$ isotones. All energies are relative to the $\frac{7}{2}^-$ state. Data for $Z \geqslant 52$ (solid symbols) are taken from [15,16] while for $Z=50$ our calculated values are reported (open symbols).

there is evidence of the nature of these states, while no information is available for the lighter isotones. In this situation, one cannot exclude the possibility that the lowest-lying $\frac{9}{2}^-$ state changes its nature on approaching the proton shell closure. In fact, as a result of our calculations we find that the $\frac{9}{2}^-$ state belonging to the $f^2_{7/2}h_{9/2}$ configuration and that arising from the $f^2_{7/2}p_{3/2}$ configuration lie at about 600 and 400 keV above the first one, respectively. In this context it is worth mentioning that for ^{137}Te, with two protons in the 50-82 shell, we predict three $\frac{9}{2}^-$ states in an even smaller energy range (from about 0.6 to 1.0 MeV), the lowest-lying one containing almost the same percentage of the three-neutron configurations $f^3_{7/2}$ and $f^2_{7/2}p_{3/2}$. As regards the second and third $\frac{9}{2}^-$ states, they are dominated by $f^3_{7/2}$ and $f^2_{7/2}h_{9/2}$ configurations, respectively. Two excited states with $J^\pi = \frac{11}{2}^-$ and $\frac{15}{2}^-$ have been observed in ^{137}Te (see Fig. 2) at 0.608 and 1.141 MeV, respectively. These energies are very well reproduced by our calculations (0.597 and 1.072 MeV).

In summary, we have shown that our realistic effective interaction derived from the CD-Bonn NN potential leads to a very good description of the experimental data that recently became available for ^{134}Sn. We are therefore confident in the predictive power of our calculations and hope that this work may stimulate further experimental efforts to gain information on the structure of Sn isotopes beyond $N=82$.

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