Ordinary and radiative muon capture in liquid hydrogen reexamined

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(Received 4 October 2001; published 11 April 2002)

A simultaneous analysis is made of the measured rates of ordinary muon capture (OMC) and radiative muon capture (RMC) in liquid hydrogen, using theoretical estimates for the relevant atomic capture rates that have been obtained in chiral perturbation theory with the use of the most recent values of the coupling constants. We reexamine the basic formulas for relating the atomic OMC and RMC rates to the liquid-hydrogen OMC and RMC rates, respectively. Although the analysis is significantly influenced by ambiguity in the molecular state population, we can demonstrate that, while the OMC data can be reproduced, the RMC data cannot be explained with the use of realistic values of the inputs; the degree of difficulty becomes even more severe when we try to explain the OMC and RMC data simultaneously.

DOI: 10.1103/PhysRevC.65.048501 PACS number(s): 23.40.−s

Ordinary and radiative muon capture (OMC and RMC) on a proton

$$\mu^- + p \rightarrow n + \nu_\mu, \quad \mu^- + p \rightarrow n + \nu_\mu + \gamma \tag{1}$$

are fundamental weak-interaction processes in nuclear physics and a primary source of information on g_P, the induced pseudoscalar coupling constant of the weak nucleon current, see, e.g., [1,2]. The most accurate existing measurements of the OMC and RMC rates have been carried out using a liquid-hydrogen target, which unfortunately makes the analysis of the data sensitive to the molecular transition rates in liquid hydrogen. We denote by Λ_{liq} the OMC rate in liquid hydrogen. The experimental value obtained by Bardin *et al.* [3] is

$$\Lambda_{liq}^{exp} = 460 \pm 20 \ \ (\mathrm{s}^{-1}). \tag{2}$$

As for RMC, Jonkmans *et al.* [4] measured the absolute photon spectrum for $E_\gamma \geqslant 60$ MeV and deduced therefrom the partial RMC branching ratio, R_γ, which is the number of RMC events (per stopped muon) producing a photon with $E_\gamma \geqslant 60$ MeV. The measured value of R_γ is [4,5]

$$R_\gamma^{exp} = (2.10 \pm 0.22) \times 10^{-8}. \tag{3}$$

Surprisingly, the value of g_P deduced in [4,5] from the RMC data is ~1.5 times larger than the partially conserved axial-vector current (PCAC) prediction [6]. By contrast, the value of g_P deduced in [7] from the OMC data is in good agreement with the PCAC prediction.

On the theoretical side, the early estimation of g_P was made using PCAC. Heavy-baryon chiral perturbation theory (HBχPT), a low-energy effective theory of QCD, allows us to go beyond the PCAC approach, but the results of detailed HBχPT calculations [8] up to next-to-next-to-leading order (NNLO) essentially agree with those obtained in the PCAC approach. Thus the theoretical framework for estimating g_P is robust. The key quantities in analyzing OMC and RMC are the atomic rates, Λ_s and Λ_t, where Λ_s (Λ_t) is the capture rate for the hyperfine singlet (triplet) state of the μ-p atom.[1] The atomic rates for OMC and RMC have also been estimated in the framework of HBχPT [9–15]. The expressions obtained in HBχPT have been found to be essentially in agreement with those of the earlier papers [16–20]. It has also been confirmed that the chiral expansion converges rapidly, rendering estimates of the OMC and RMC rates obtained in χPT extremely robust. As for the numerical results, however, the earlier estimates of the atomic OMC rates, e.g. [16,17], need to be revised because some values of the input parameters (g_A, $g_{\pi N}$, etc.) used in those estimates are now obsolete. In Ref. [10], we provided updated estimates of Λ_s^{OMC} and Λ_t^{OMC} based on HBχPT (up to NNLO). A notable finding in [10] is that the use of the recent larger value of the Gamow-Teller coupling constant g_A gives a value of Λ_s^{OMC}, which is significantly larger than the older value commonly quoted in the literature, see Refs. [10,15].

To make comparison between theory and experiment, one needs to relate the theoretically calculated atomic OMC and RMC rates to Λ_{liq} and R_γ, respectively. For convenience, we refer to this relation as the *A-L* (atom-liquid) formula. Bakalov *et al.* [21] made a detailed study of the *A-L* formula, and they gave an explicit expression for Λ_{liq} [see Eq. (56c) in Ref. [21]]. In our previous work [10] we analyzed Λ_{liq} using the *A-L* formula of Bakalov *et al.* and found that the best available estimates of the atomic capture rates based on HBχPT would lead to a value of Λ_{liq} significantly larger than Λ_{liq}^{exp}. We also reported that, by introducing a molecular state mixing parameter ξ considered by Weinberg [22], it was possible to reproduce Λ_{liq}^{exp} and R_γ^{exp} simultaneously. However, the *A-L* formula of Bakalov *et al.* does not correspond to the experimental condition of OMC; to compare with Λ_{liq}^{exp}, the time sequence of the actual measurement should be considered [3]. In this work we reexam-

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[1]The rates Λ_s and Λ_t are generic symbols for OMC and RMC. When we need to distinguish OMC and RMC, we use the symbols $\Lambda_{s,t}^{OMC}$ and $\Lambda_{s,t}^{RMC}$.

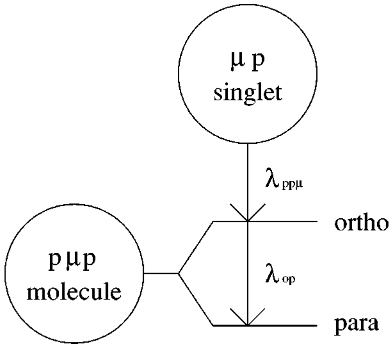

FIG. 1. Atomic and molecular states relevant to muon capture in liquid hydrogen; $\lambda_{pp\mu}$ is the transition rate from the atomic singlet state to the ortho p-μ-p molecular state, and λ_{op} is that from the ortho to para molecular state.

ine Λ_{liq} and R_γ by incorporating into our analysis the experimental conditions as well as the updated estimates of the atomic capture rates.

To evaluate Λ_{liq} and R_γ from the calculated atomic OMC and RMC rates, we need to know the temporal behavior of the various μ-capture components (capture from the atomic states and capture from p-μ-p molecular states). Figure 1 schematically depicts various competing atomic and molecular processes occurring in liquid hydrogen. A muon stopped in liquid hydrogen quickly forms a muonic atom (μ-p) in the lowest Bohr state. The atomic hyperfine-triplet state ($S=1$) decays extremely rapidly to the singlet state ($S=0$), with a transition rate $\lambda_{10}\simeq 1.7\times 10^{10}\ \mathrm{s}^{-1}$. In the liquid-hydrogen target a muonic atom and a hydrogen molecule collide with each other and form a p-μ-p molecule with the molecule predominantly in its ortho state. We denote by $\lambda_{pp\mu}$ the transition rate from the atomic singlet state to the ortho p-μ-p molecular state. The ortho p-μ-p state further decays to the para p-μ-p molecular state. This rate is denoted by λ_{op}. Let $N_s(t)$, $N_{om}(t)$, and $N_{pm}(t)$ represent the numbers of muons at time t in the atomic singlet, ortho-molecular, and para-molecular states, respectively. They satisfy coupled kinetic equations, see Eq. (54a) in Ref. [21]. To integrate these coupled differential equations, we need to know the initial conditions.

For illustration purposes, let us consider a case in which there is one muon in the singlet state at $t=0$; i.e., $N_s(0)=1$ and $N_{om}(0)=N_{pm}(0)=0$. We then have

$$N_s(t)=e^{-\lambda_2 t}, \quad N_{om}(t)=\frac{\lambda_{pp\mu}}{\lambda_2-\lambda_3}(e^{-\lambda_3 t}-e^{-\lambda_2 t}),$$

$$N_{pm}(t)=\frac{\lambda_{op}\lambda_{pp\mu}}{(\lambda_3-\lambda_4)(\lambda_2-\lambda_4)}e^{-\lambda_4 t} - \frac{\lambda_{op}\lambda_{pp\mu}}{(\lambda_2-\lambda_3)(\lambda_3-\lambda_4)}e^{-\lambda_3 t} + \frac{\lambda_{op}\lambda_{pp\mu}}{(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)}e^{-\lambda_2 t}, \tag{4}$$

where $\lambda_2=\lambda_0+\lambda_{pp\mu}+\Lambda_s^{OMC}+\Lambda_s^{RMC}$, $\lambda_3=\lambda_0+\lambda_{op}+\Lambda_{om}^{OMC}+\Lambda_{om}^{RMC}$, $\lambda_4=\lambda_0+\Lambda_{pm}^{OMC}+\Lambda_{pm}^{RMC}$. Here λ_0 is the muon natural decay rate. Λ_{om}^{OMC} and Λ_{pm}^{OMC} are the OMC rates in the ortho-molecular and para-molecular states, respectively; similarly for Λ_{om}^{RMC}, and Λ_{pm}^{RMC}.[2] These rates are given by

$$\Lambda_{om}^F=2\gamma_O(\tfrac{3}{4}\Lambda_s^F+\tfrac{1}{4}\Lambda_t^F), \quad \Lambda_{pm}^F=2\gamma_P(\tfrac{1}{4}\Lambda_s^F+\tfrac{3}{4}\Lambda_t^F), \tag{5}$$

where F stands for "OMC" or "RMC," and $2\gamma_O=1.009$, $2\gamma_P=1.143$ [21].

At this point we discuss the numerical values of $\lambda_{pp\mu}$ and λ_{op}. The former shows a wide scatter in the literature, ranging from $\lambda_{pp\mu}=(1.89\pm 0.20)\times 10^6\ \mathrm{s}^{-1}$ to $(2.75\pm 0.25)\times 10^6\ \mathrm{s}^{-1}$ [23]. In this work, for the sake of definiteness, we employ the averaged value $\lambda_{pp\mu}=2.5\times 10^6\ \mathrm{s}^{-1}$ (the main point of our argument is not affected by this choice). This value is comparable to the muon decay rate $\lambda_0=0.455\times 10^6\ \mathrm{s}^{-1}$. As regards λ_{op}, there is a significant difference between the experimental and theoretical values; $\lambda_{op}^{exp}=(4.1\pm 1.4)\times 10^4\ \mathrm{s}^{-1}$ [7] as compared with $\lambda_{op}^{th}=(7.1\pm 1.2)\times 10^4\ \mathrm{s}^{-1}$ [21].

The dominant state for the OMC and RMC measurements is the ortho-molecular state as is evident from Eq. (4). In both measurements, data collection starts at $t=t_i\neq 0$, and it is essential to incorporate this aspect into the A-L formula (see below). Furthermore, in the OMC experiment the time dependence of the population of each state plays an important role.

The discussion so far is common for both OMC and RMC, but we now turn to the individual discussion of each case. In the OMC experiment (see Fig. 4 in Ref. [3]), μ^- beams arrive at the target on the average in a 3-μs-long burst with a repetition rate of 3000 Hz. The data collection typically starts 1 μs after the end of the 3-μs-long beam burst, and the measurement lasts until 306 μs after the end of the beam burst. As mentioned, the cascade processes leading to the μ-p ground state and the transition between the atomic hyperfine states are extremely fast. One therefore can safely ignore a time lag between the muon arrival time and the time at which the μ-p atomic hyperfine-singlet state is formed. To proceed with the consideration of OMC, we assume that the average time intervals of Ref. [3] cited above are actual time intervals. Then, provided all the muons arrive at the same time, we can choose with no ambiguity that arrival time as the origin of time ($t=0$) and let $t=t_i$, the starting time for data collection, refer to that origin. However, the finite duration ($t_b=3\ \mu$s) of the beam burst causes uncertainty in the value of $t=t_i$ to be used in Eq. (4); t_i can be anywhere between 1.0 μs and 4.0 μs. To account for this muon pulse duration time t_b, we assume for simplicity that the beam pulse has a rectangular shape. Then, at time t the average number of residual muons are

[2]Since Λ_s^{RMC}, Λ_{om}^{RMC}, Λ_{pm}^{RMC} are very small, they can be ignored in the calculation of $N_{s,om,pm}(t)$. In evaluating the RMC rate itself, however, we need these capture rates; see Eq. (9) below.

TABLE I. Coupling constants and the atomic capture rates (s^{-1}) used in the present analysis.

g_A	$g_{\pi N}$	Λ_s^{OMC}	Λ_t^{OMC}	Λ_s^{RMC}	Λ_t^{RMC}
1.267	13.40	695	11.9	0.891×10^{-3}	20.1×10^{-3}

$$\overline{N}_\mu(t)\equiv\frac{1}{t_b}\int_0^{t_b}dt' N_\mu(t-t'), \tag{6}$$

where $N_\mu(t)=N_s(t)+N_{om}(t)+N_{pm}(t)$. The OMC experiment [3] counts the number of electrons produced by $\mu^-\to e^-\overline{\nu}_e\nu_\mu$, and Λ_{liq} is deduced from the difference between the muon decay rate in liquid hydrogen and that in vacuum; the latter is determined from the number of positrons produced in $\mu^+\to e^+\nu_e\overline{\nu}_\mu$. We use the expression of Ref. [3] (and $t_i=4$ μs)

$$\Lambda_{liq}\equiv\left(\frac{\int_{t_i}^{\infty}dt\frac{d\overline{N}_e}{dt}}{\int_{t_i}^{\infty}dt(t-t_i)\frac{d\overline{N}_e}{dt}}\right)-\lambda_0, \tag{7}$$

where $\overline{N}_e(t)$ is the averaged number of electrons produced at time t and $d\overline{N}_e(t)/dt=\lambda_0\overline{N}_\mu(t)$. Here we have used the fact that the duration of the measuring time (306 μs) is long enough to be treated as ∞.

On the other hand, for the RMC experiment [4,5], the muons essentially arrive one by one and the data taking begins at $t_i=365$ ns. We therefore can neglect the beam burst duration time in the RMC case, and we obtain

$$R_\gamma=\frac{N_\gamma(\infty)-N_\gamma(t_i)}{N_\mu(t_i)}. \tag{8}$$

Here $N_\gamma(t)$ is the number of photons obtained by integrating the photon spectrum over the interval, $60\leqslant E_\gamma\leqslant 99$ MeV, and the production of photons in RMC is determined by

$$\frac{dN_\gamma(t)}{dt}=\Lambda_s^{RMC}N_s(t)+\Lambda_{om}^{RMC}N_{om}(t)+\Lambda_{pm}^{RMC}N_{pm}(t), \tag{9}$$

where $N_\gamma(0)=0$.

We give the numerical values of inputs to be used in what follows. Table I presents the values of the coupling constants and the atomic capture rates. The OMC and RMC rates for the hyperfine-singlet and -triplet states have been calculated in HBχPT up to NNLO [10,14] and with the use of the most recent values of the coupling constants discussed in [10].

We estimate Λ_{liq} by using the atomic OMC rates in Table I. Besides the *A-L* formula in Eq. (7), we consider two others for the sake of comparison; these two *A-L* formulas are that of Bardin *et al.* [7] and that of Bakalov *et al.* [21]. For the ortho-para transition rate we employ either λ_{op}^{exp} or λ_{op}^{th}. The use of λ_{op}^{exp} leads to $\Lambda_{liq}=460$ s^{-1} with Eq. (7) and $\Lambda_{liq}=459$ s^{-1} with the *A-L* formula of Bardin *et al.* These values agree with $\Lambda_{liq}^{exp}=460\pm20$ s^{-1}. Meanwhile, if we employ λ_{op}^{th}, we obtain $\Lambda_{liq}\simeq421$ s^{-1} with Eq. (7), and $\Lambda_{liq}\simeq419$ s^{-1} with the formula of Bardin *et al.* Thus, Λ_{liq} is highly sensitive to λ_{op}. On the other hand, the use of the *A-L* formula of Bakalov *et al.* [21] gives too large a value for Λ_{liq} regardless of whether we use λ_{op}^{exp} or λ_{op}^{th}; $\Lambda_{liq}=532$ s^{-1} for λ_{op}^{exp}, and $\Lambda_{liq}=518$ s^{-1} for λ_{op}^{th}. As mentioned before, the *A-L* formula of Bakalov *et al.*, which was adopted in our previous work [10], corresponds to the choice of $t_i=0$, and this choice does not simulate the experimental condition.

TABLE II. Λ_{liq} (s^{-1}) and R_γ ($\times10^{-8}$) calculated for various values of ξ and for the choice of $\lambda_{op}=\lambda_{op}^{exp}$ or λ_{op}^{th}.

ξ	1.00	0.95	0.90	0.85	0.80
$\Lambda_{liq}(\lambda_{op}^{exp})$	460	439	419	399	379
$\Lambda_{liq}(\lambda_{op}^{th})$	421	404	386	369	352
$R_\gamma(\lambda_{op}^{exp})$	1.41	1.55	1.68	1.82	1.95
$R_\gamma(\lambda_{op}^{th})$	1.54	1.67	1.80	1.93	2.06

An estimate of R_γ is obtained from Eq. (8) and the atomic RMC rates given in Table I. With the use of λ_{op}^{exp}, the calculated value of R_γ is significantly smaller than R_γ^{exp} in Eq. (3); $R_\gamma^{exp}/R_\gamma^{th}\approx1.5$. If in Eq. (8) we use λ_{op}^{th} instead of λ_{op}^{exp}, then R_γ is enhanced by about 9% but the increase is not large enough to reconcile R_γ^{th} with R_γ^{exp}. Thus it is not possible to reproduce R_γ^{exp} in the existing theoretical framework with the use of the standard set of input parameters. In addition, we remark that our results indicate that the sensitivity of R_γ to λ_{op} is comparable to that of Λ_{liq}.[3]

Next, we discuss the sensitivity of Λ_{liq} and R_γ to possible changes in the value of the molecular mixing parameter ξ. As discussed by Weinberg [22], the possible mixing of the ortho-molecular p-μ-p spin 3/2 state and spin 1/2 state, parametrized by ξ, may change the molecular capture rates to

$$\Lambda_{om}'^{F}=\xi\Lambda_{om}^F(1/2)+(1-\xi)\Lambda_{om}^F(3/2), \tag{10}$$

where F stands for "OMC" or "RMC"; $\Lambda_{om}^F(1/2)=\Lambda_{om}^F$ [see Eq. (5)] and $\Lambda_{om}^F(3/2)=2\gamma_O\Lambda_t^F$. Although the existing theoretical estimate favors $\xi\simeq1$ [21,25], we treat it here, as we did in Ref. [10], as a parameter to fit the data. In Table II we show Λ_{liq} and R_γ calculated for various values of ξ ($\xi=1.00$, 0.95, 0.90, 0.85, 0.80) and for the cases of $\lambda_{op}=\lambda_{op}^{exp}$ and $\lambda_{op}=\lambda_{op}^{th}$. We can see that, to explain R_γ^{exp}, a large deviation of ξ from unity is needed but this deviation spoils the agreement with Λ_{liq}^{exp}; no value of ξ can explain Λ_{liq}^{exp} and R_γ^{exp} simultaneously. This conclusion should supersede the one given in [10].

Our findings are largely in the nature of reconfirming the conclusions stated in one way or another in the literature, but

[3]The result of a more precise measurement of λ_{op} at TRIUMF [24] will shed much light on this issue.

a coherent treatment of OMC and RMC in liquid hydrogen as described here is hoped to be useful. Our treatment is characterized by the use of the best available atomic capture rates obtained in HBχPT, and by an improved *A-L* formula. As mentioned, the atomic capture rates calculated using a phenomenological relativistic tree-level model [18] are consistent with those of HBχPT [13–15,26] (provided the former uses the updated value of g_A and the PCAC value of g_P). Therefore, the above conclusions are not necessarily unique to HBχPT. However, since HBχPT gives Λ_s and Λ_t with high precision (primarily because the value of g_P is strictly restricted by chiral symmetry), it allows us to draw much sharper conclusions than the phenomenological approach. Although we have presented examples of simulation of the experimental conditions, they are only meant to serve illustrative purposes. Definitive analyses can be done only by the people who carried out the relevant experiments. Finally, we remark that a precise measurement of the OMC rate in hydrogen gas is planned at PSI [23]. This experiment would eliminate the ambiguity of the molecular transition rate discussed in this paper and directly test the HBχPT prediction [10,15].

This work was motivated by Dr. T. Gorringe's criticism (communicated to us by Dr. H.W. Fearing) about the assumption $t_i=0$ made in our earlier work. We are deeply grateful to these two colleagues for that information and also for other illuminating remarks. Thanks are also due to Dr. T.-S. Park for useful discussions. This work was supported in part by the U.S. National Science Foundation, Grant Nos. PHY-9900756 and No. INT-9730847.

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