Radiative muon capture in hydrogen

E. Truhlík*
Institute of Nuclear Physics, Academy of Sciences of the Czech Republic, CZ–250 68 Řež, Czech Republic

F. C. Khanna†
Theoretical Physics Institute, Department of Physics, University of Alberta, Edmonton, Alberta, Canada T6G 2J1 and TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3

(Received 5 February 2001; published 1 April 2002)

We analyze the radiative capture of the negative muon in hydrogen using amplitudes derived within the chiral Lagrangian approach. Besides the leading and next to leading order terms, given by the well-known Rood-Tolhoek Hamiltonian, we extract from these amplitudes the corrections of the next order in $1/M$ (M is the nucleon mass). In addition, we estimate within the same formalism also the $\Delta(1232)$ isobar excitation effects and processes described by an anomalous Lagrangian. The model we consider allows us to put the Δ isobar off-shell. Our calculations show sensitivity of capture rates and photon spectra to Z, one of the off-shell parameters, related to the $\pi N\Delta$ vertex. We have found that the model can provide the photon spectra, which are in the interval 60 MeV$\leqslant k \leqslant k_{max}$ (k is the photon momentum) close to the experimental one.

DOI: 10.1103/PhysRevC.65.045504 PACS number(s): 23.40.−s, 11.40.Ha, 13.60.−r, 12.39.Fe

I. INTRODUCTION

It is well known [1] that the charged weak interaction of the nucleon with a lepton is described by the weak hadron current

$$J^a_{W,\mu}(q_1)=J^a_{V,\mu}(q_1)+J^a_{A,\mu}(q_1), \tag{1.1}$$

where the vector part is given by the matrix element of the isovector Lorentz four-vector current operator between the nucleon states,

$$\hat{J}^a_{V,\mu}(q_1)=i\left(g_V(q_1^2)\gamma_\mu-\frac{g_M(q_1^2)}{2M}\sigma_{\mu\nu}q_{1\nu}\right)\frac{\tau^a}{2} \tag{1.2}$$

and the axial-vector part is analogously

$$\hat{J}^a_{A,\mu}(q_1)=i\left(-g_A(q_1^2)\gamma_\mu\gamma_5+i\frac{g_P(q_1^2)}{m_l}q_{1\mu}\gamma_5\right)\frac{\tau^a}{2}. \tag{1.3}$$

Here a is the isospin index, m_l is the lepton mass, and the four-momentum transfer is given by $q_{1\mu}=p'_\mu-p_\mu$, where $p'_\mu(p_\mu)$ is the four-momentum of the final (initial) nucleon.

The least known of the four form factors entering the currents, Eqs. (1.2) and (1.3), is the induced pseudoscalar form factor $g_P(q_1^2)$ in the axial-vector current $\hat{J}^a_{A,\mu}$. Actually, its presence in the axial-vector current (1.3) tests our understanding of the basic strong and weak interaction processes, such as the strong πNN vertex and the weak pion decay. Elementary calculations lead to

$$g_P(q_1^2)=-2g_{\pi NN}f_\pi m_l\Delta^\pi_F(q_1^2), \tag{1.4}$$

where $\Delta^\pi_F(q_1^2)$ is the pion propagator, $g_{\pi NN}=13.05$ is the pseudoscalar πNN coupling constant, and $f_\pi=92.4$ MeV is the pion decay constant.

The matrix element of the axial current $\hat{J}^a_{A,\mu}$ should satisfy partial conservation of the axial current (PCAC). It is easy to obtain that

$$\overline{u(p')}q_{1\mu}\hat{J}^a_{A,\mu}u(p)=\overline{u(p')}\left[2Mg_AF_A(q_1^2)-\frac{g_P(q_1^2)}{m_l}q_1^2\right]\gamma_5\frac{\tau^a}{2}u(p). \tag{1.5}$$

It is seen from this equation that if

$$\tilde{g}_P(q_1^2)=-2g_{\pi NN}f_\pi m_l\frac{1}{q_1^2}\left[1+\frac{Mg_A}{g_{\pi NN}f_\pi}F_A(q_1^2)\right] \tag{1.6}$$

is subtracted from $g_P(q_1^2)$, then indeed, PCAC is valid. Here we put

$$g_A(q_1^2)=g_AF_A(q_1^2), \tag{1.7}$$

with $g_A\equiv g_A(0)=-1.267$. In the chiral model [2,3], the axial form factor is of the monopole form

$$F_A(q_1^2)=\frac{m_{a_1}^2}{m_{a_1}^2+q_1^2}, \tag{1.8}$$

where m_{a_1} is the mass of the axial-vector meson $a_1(1260)$. Then for the $\tilde{g}_P(q_1^2)$, Eq. (1.6), we have

$$\tilde{g}_P(q_1^2)=-2g_{\pi NN}f_\pi m_l\frac{1}{q_1^2}\left[q_1^2+\left(1+\frac{Mg_A}{g_{\pi NN}f_\pi}\right)m_{a_1}^2\right]\Delta_F^{a_1}(q_1^2). \tag{1.9}$$

*Email address: truhlik@ujf.cas.cz
†Email address: khanna@phys.ualberta.ca

However, this equation for $\tilde{g}_P(q_1^2)$ cannot be used because of the singularity for $q_1^2=0$, which shows its presence in the hadron radiative part of the radiative muon capture (RMC) amplitude for large photon momentums k and it is close to the physical region for ordinary muon capture (OMC), because of the large value of the axial meson mass. So our model cannot be used beyond the exact Goldberger-Treiman relation and we take

$$\tilde{g}_P(q_1^2) = -2g_{\pi NN} f_\pi m_l \Delta_F^{a_1}(q_1^2), \tag{1.10}$$

which is in agreement with [4].

The best way to search for the effect of the form factor $g_P(q_1^2)$ is the muon capture. In the elementary process of OMC in hydrogen,

$$\mu^- + p \to \nu_\mu + n \tag{1.11}$$

according to Eq. (1.4); the value of the induced pseudoscalar form factor g_P is

$$g_P^{OMC}(p) \equiv g_P(q_1^2 = 0.877 m_\mu^2) = -\frac{2g_{\pi NN} f_\pi m_\mu}{0.877 m_\mu^2 + m_\pi^2} = 6.87 g_A = -8.71, \tag{1.12}$$

and for $\tilde{g}_P(q_1^2)$, Eq. (1.10), we have

$$\tilde{g}_P^{OMC}(p) \equiv \tilde{g}_P(q_1^2 = 0.877 m_\mu^2) = -\frac{2g_{\pi NN} f_\pi}{0.877 m_\mu^2 + m_{a_1}^2} = 0.13 g_A = -0.16, \tag{1.13}$$

which is a correction of $\approx 2\%$ to $g_P^{OMC}(p)$, Eq. (1.12). The resulting value is

$$g_P^{PCAC}(p) = g_P^{OMC}(p) - \tilde{g}_P^{OMC}(p) = -8.55. \tag{1.14}$$

The axial form factor of the nucleon has recently been measured by the $p(e,e'\pi^+)n$ reaction in Ref. [5]. The dipole form of the form factor was used and the extracted axial mass $m_A = 1.077 \pm 0.039$ GeV. For this form factor,

$$\tilde{g}_P(q_1^2) = -2g_{\pi NN} f_\pi m_l \frac{2m_A^2 + q_1^2}{(m_A^2 + q_1^2)^2}, \tag{1.15}$$

and analogously with Eqs. (1.13) and (1.14) we have

$$\tilde{g}_P^{OMC}(p) = 0.34 g_A = -0.43, \tag{1.16}$$

and

$$g_P^{PCAC}(p) = -8.28, \tag{1.17}$$

respectively. Both values of $g_P^{PCAC}(p)$ are in reasonable agreement with the calculations of $g_P^{PCAC}(p)$ within the framework of the heavy baryon chiral perturbation theory (HBChPT) [6–8].

Very flat dependence of the capture rate on g_P in the OMC by the proton, Eq. (1.11), provides its world average value [9] with an error of $\approx 20\%$ and particular experiments have an error larger by a factor of ≈ 2.

Recently, a very precise experimental study of muon capture by ^{3}He [10,11],

$$\mu^- + {}^3\mathrm{He} \to \nu_\mu + {}^3\mathrm{H}, \tag{1.18}$$

yielded the transition rate

$$\Gamma_{exp} = 1494 \pm 4\,\mathrm{s}^{-1}, \tag{1.19}$$

which allowed [12] an extraction of the value of g_P with an accuracy of $\approx 20\%$ from this experiment alone,

$$\frac{g_P}{g_P^{OMC}({}^3\mathrm{He})} = 1.05 \pm 0.19, \tag{1.20}$$

where for the reaction (1.18),

$$g_P^{OMC}({}^3\mathrm{He}) \equiv g_P^{OMC}(q_1^2 = 0.954 m_\mu^2) = 6.68 g_A. \tag{1.21}$$

Analogously with Eq. (1.14), this value of g_P differs slightly from that demanded by PCAC by about the same amount as for the reaction (1.11). A contribution of 20% due to the meson exchange current effect turned out to be essential to get the calculated transition rate

$$\Gamma_{th} = 1502 \pm 32\,\mathrm{s}^{-1}, \tag{1.22}$$

in agreement with the data (1.19). Let us note that a further improvement of the extracted value of g_P is hindered by an uncertainty of $\approx 2\%$ in calculations [12], which will be difficult to improve. The main uncertainty arises from the less known parameters of the Δ excitation processes.

Another interesting tool to extract the value of g_P^{PCAC} is the RMC by the proton,

$$\mu^- + p \to \nu_\mu + \gamma + n. \tag{1.23}$$

As is well known [13], the RMC amplitude contains the pseudoscalar form factor g_P in the form

$$g_P^{L(N)} = -x \frac{2g_{\pi NN} f_\pi m_\mu}{(q^{L(N)})^2 + m_\pi^2} \to x g_P^{OMC}(p) \frac{0.877 m_\mu^2 + m_\pi^2}{(q^{L(N)})^2 + m_\pi^2} \times [1 - (\{q^{L(N)}\}^2 + m_\pi^2) F(\{q^{L(N)}\}^2)], \tag{1.24}$$

where we implemented the correction of Eq. (1.6) and

$$F(\{q^{L(N)}\}^2) = 1/(\{q^{L(N)}\}^2 + m_{a_1}^2) \tag{1.25}$$

for the chiral model [2,3] and

$$F(\{q^{L(N)}\}^2) = (\{q^{L(N)}\}^2 + 2m_A^2)/(\{q^{L(N)}\}^2 + m_A^2)^2 \tag{1.26}$$

for the dipole form of the form factor. As it is seen from Eq. (1.24), the form factor g_P depends either on the square of the four-momentum transfer $q^L = p - p' = \nu + k - \mu = -q_1$ (characterizing the muon radiation process) or on $q^N=\nu-\mu=q^L-k$ (for the hadron radiation). For large photon momentums $k,(q^L)^2\approx+m_\mu^2$, whereas $(q^N)^2\approx-m_\mu^2$, which enhances the hadron radiation amplitude by a factor of ≈ 3. This enhancement makes the reaction (1.23) particularly interesting. On the other hand, the dependence on g_P of the effective form factors g_i, entering the effective RMC Hamiltonian, appears only up to $\mathcal{O}(1/M)$, which makes the isolation of the dependence of the photon spectrum on g_P difficult.[1] The factor x in Eq. (1.24) is used to study the change of the photon spectrum and capture rates by scaling g_P.

The theory of the RMC was elaborated by many authors during the past years (see Refs. [4,13,15–26] and references therein).

The nuclear Hamiltonian, suitable for use in nuclear physics calculations of the RMC processes, was provided first by Rood and Tolhoek [19]. It contains the leading and the next to leading order terms in $1/M$ derived from the conserved RMC amplitude given by a set of Feynman diagrams. Christillin and Servadio [21] rederived in an elegant way the RMC amplitude obtained earlier by Adler and Dothan [4] using the low energy theorems. This amplitude is written in terms of elastic weak form factors and the pion photoproduction amplitude, up to terms linear in k and q. It was also found [21] that higher order terms cannot be obtained using this method. Recently, this amplitude was produced [3] from a chiral Lagrangian of the $N\pi\rho\omega a_1$ system. It satisfies the corresponding continuity equations and the consistency condition exactly. Higher order terms follow without any restriction. It was shown that the leading order terms coincide with those given by the low energy theorems. However, higher order terms differ, which is given by a different prescription to pass towards higher energies.

The above mentioned set of the relativistic Feynman diagrams was used by Fearing [22] to calculate the photon energy spectrum for the reaction (1.23). This work was later extended by Beder and Fearing [26] by considering also the contribution from the Δ excitation processes. A recent comparison of the TRIUMF experiment [27,28] with the Beder-Fearing calculations provided a value of $g_P^{OMC}(p)$ that is enhanced by $\approx 50\%$ in comparison with the value of Eq. (1.12), which corresponds to using g_P from Eq. (1.24) with $x=1.5$. This is the so-called "g_P puzzle."

In connection with the presence of the factor x in Eq. (1.24), it should be noted that (i) referring only to the change of $g_P^{OMC}(p)$ is confusing—as it is seen from Eq. (1.24), the whole form factor g_P is scaled; (ii) in the experiment of Refs. [27,28], the high energy part of the photon spectrum is measured, then increasing x simulates processes enhancing this part of the spectrum.[2]

In our opinion, the variation of x can be considered as a tool to study the uncertainty in our knowledge of g_P due to a restricted experimental accuracy. Any real difference from $x=1$ would mean violation of PCAC.

Searching for the processes enhancing the high energy part of the photon spectrum has recently been performed within the concept of HBChPT by several authors [29–33]. Ando and Min [31] considered one-loop order correlations to the tree approximation and confirmed the existing discrepancy. Bernard, Hemmert, and Meissner (BHM) calculated [29] both ordinary and radiative muon capture on the proton in an effective field theory of pions, nucleons, and Δ isobars by using the small scale expansion [34]. According to [29], the most probable explanation of the problem is a combination of many small effects. Besides the photon spectra, BHM present the numerical results also for the singlet (Λ_s) and triplet (Λ_t) capture rates. This will enable us to compare our calculations with those by BHM in greater detail. Here we only note the difference of $\approx 10\%$ in Λ_t. As we shall see later, about half of this difference arises from the use of an approximate equation for the neutrino energy in Ref. [29].

In Ref. [32], a possible explanation of the discrepancy was the suggestion that a fraction of the spin 3/2 orthomolecular $p\mu p$ state in liquid hydrogen can exist. The analysis of the experimental photon spectrum [27,28] yielded 10–20 % of this state. However, this is in sharp contrast with the existing calculations [35,36], which give a zero fraction of this state. As noted very recently in Ref. [33],[3] a new analysis restricts the fraction of spin 3/2 orthomolecular $p\mu p$ state to at most 5%.

Finally, let us comment on Ref. [37], where Cheon and Cheoun reported on the derivation of an additional term from a chiral model, which does not appear in the standard approach to the RMC on the proton and which generates a large contribution to the photon spectrum. As it was shown in Ref. [38], Ref. [37] suffers from two flaws. First, the derivation of this term contains an algebraic error due to an incorrect application of the covariant derivative in Eq. (14). After removing it, the effect is reduced by a factor of ~ 5. The second flaw in Ref. [37] is related to the introduction of the pseudovector πNN coupling by the vertex $\mathcal{L}_1$ of Eq. (18), which yields the desired term. However, the equivalent passage from one type of πNN coupling to another one is guaranteed only by the Foldy-Dyson unitary transformation. As shown in Ref. [38], when this transformation is applied to a chiral model with the pseudoscalar πNN coupling, the pseudovector πNN coupling appears in the resulting Lagrangian, which does not contain the incriminating term, however. Besides, the presence of this term in the RMC amplitude violates the Ward-Takahashi identity derived in Ref. [3]. A later attempt to improve the situation [39] suffers from the same shortcomings. Reference [37] was also criticized in Ref. [40].

This situation makes the expectation of the result from the next TRIUMF experiment on helium,

[1]In Ref. [14], it has been proposed to isolate the effect due to the hadron radiative amplitude in a very difficult polarization experiment.

[2]See also the discussion in Ref. [29].

[3]See also a discussion in Ref. [14].

$$\mu^- + {}^3\mathrm{He} \rightarrow \nu_\mu + \gamma + {}^3\mathrm{H}, \tag{1.27}$$

somewhat tense. However, one should keep in mind also complications analogous to those in reaction (1.18), and non-negligible meson exchange current effects are to be expected, which makes the analysis much more difficult. It is clear that the Beder-Fearing relativistic formalism is not applicable in calculations with the realistic $3N$ wave functions, and a consistent nonrelativistic approach should be developed. Here we make an independent step in this direction by performing the nonrelativistic reduction of the amplitudes derived in Ref. [3] from a chiral Lagrangian of the $N\pi\rho\omega a_1$ system. As a result, we get an effective Hamiltonian, which is close to that obtained by Rood and Tolhoek [19] but not identical with it. We also apply the constructed effective Hamiltonian to compute both the capture rates and the photon energy spectra for the reaction (1.23) and for various spin states. Our reduction provides more terms of the order $(1/M)$ and $(1/M^2)$ than Rood and Tolhoek present. Added to the leading order terms they should reproduce, with a good accuracy, the results given in Ref. [22].[4] Another set of terms $\mathcal{O}(1/m_\rho^2) \approx \mathcal{O}(1/M^2)$ is produced by reduction of additional relativistic amplitudes following from our chiral Lagrangian. We shall call it hard pion (HP) correction. Numerically, it enhances the photon spectra by 2–4 %.

Next we include the Δ isobar using again the formalism of chiral Lagrangians developed in Refs. [2,42], which we extend by adopting results of Refs. [43–45]. Then the resulting $N\Delta\pi\rho a_1$ Lagrangian consists of three terms and is characterized by three couplings and four arbitrary parameters A,X,Y,Z. In its turn, each term contains a tensor of the form

$$\Theta_{\mu\nu}(B) = \delta_{\mu\nu} + \left[\frac{1}{2}(1+4B)A + B\right]\gamma_\mu\gamma_\nu, \quad B = X,Y,Z, \tag{1.28}$$

which ensures the independence of the Δ contribution to the S matrix on the parameter A. The choice $A=-1$ simplifies the Δ propagator considerably. The parameters X,Y,Z, which reflect the off-shell ambiguity of the massive spin 3/2 field, were found [43–47] by analyzing the data on pion photoproduction.[5] The values of these parameters depend on how the pion photoproduction amplitude is unitarized. This model does not require the use of the Breit-Wigner form of the Δ propagator.

In the calculations of the Δ excitation effect in the reaction (1.23), Beder and Fearing [26] took a model for needed vertices with $\Theta_{\mu\nu} = \delta_{\mu\nu}$, the Breit-Wigner form of the Δ propagator, and the needed $\gamma N\Delta$ coupling from Ref. [44], thus introducing an inconsistency into calculations.

The $\pi N\Delta$ and $\gamma N\Delta$ vertices including the off-shell parameters X,Y,Z were discussed in Ref. [49] and the $\pi N\Delta$ vertex of the form of Eq. (1.28) was considered also in the small scale expansion [34]. However, the dependence of the results on these parameters was not exploited in any of the calculations performed within the framework of HBChPT.

Let us note that besides the adopted model [43–45], other models [50,51] were developed to describe the production of pions on protons by the electromagnetic interaction. All these models consider the same nonresonant Lagrangian of the $N\pi\rho\omega$ system, but differ principally in the treatment of the Δ isobar and in the method of unitarization of the πN amplitude.

We have also analyzed the contribution due to amplitudes constructed from an anomalous Lagrangian of the $\pi\rho\omega a_1$ system [52,53]. We have found that the influence of this contribution on the photon energy spectrum is not significant. An earlier estimate of a contribution that arises from the Wess-Zumino-Witten part of the anomalous Lagrangian was reported in Ref. [54].

One can find in the literature an attempt to study the form factor g_P in the reaction of electroproduction of charged soft pions off the proton [29,55],

$$e + p \rightarrow e' + \pi^+ + n. \tag{1.29}$$

The starting point of this attempt is the soft pion production amplitude given as [56]

$$f_\pi M_\lambda^{nj}(q,k) \overset{q\to 0}{\rightarrow} iq_\mu \left\langle p' \left| \int d^4y e^{-iqy} T[\hat{J}^n_{A,\mu}(y)\hat{J}^j_{V,\lambda}(0)] \right| p \right\rangle + \varepsilon^{njm}\langle p' | \hat{J}^m_{A,\lambda}(0) | p\rangle. \tag{1.30}$$

The matrix element of the time-ordered product of the two currents is related to the RMC amplitude by the time reversal. The form factor g_P is contained on the right-hand side of Eq. (1.29) in the matrix element of the axial current. If one admits that in the soft pion limit only the nucleon Born terms contribute to the divergence of the current-current amplitudes, then one has the pion production amplitude that can provide information on g_P. However, when one of the currents is axial, a contribution to the divergence of the current-current amplitudes from the pion pole term in the t-channel survives even in the soft pion limit [57–60]. A part of this contribution cancels the induced pseudoscalar term in the axial current and the remaining part is just the pion pole production amplitude, as one can expect intuitively. Then in the soft pion regime, the reaction (1.29) is suitable to study the weak axial nucleon form factor $F_A(k^2)$ and the electromagnetic form factor (the electromagnetic radius) of the charged pion, but not to extract any information on g_P.

In our opinion, the RMC reactions and particularly reactions (1.23) and (1.27) are at present the only available tools to study the form factor g_P as a function of the momentum transfer.

In order to compare our effective form factors with the results of Ref. [19], we define in Sec. II the effective Hamiltonian analogously and we consider the velocity independent part only. Then in Sec. III, we present the results for the form factors following from our amplitudes [3] up to $\mathcal{O}(1/M^2)$. Further, we deal with the contribution to g_i's from the Δ

[4]These corrections up to the order $(1/M^2)$ were discussed in Ref. [41].

[5]This model describes well also the latest data on the π^0 electroproduction on the proton [48].

excitation amplitudes of our model and we compare our effective weak $N\Delta$ vertex with that used in Ref. [26]. Finally, we discuss the RMC amplitudes stemming from the anomalous Lagrangian.

In Sec. IV, we give the numerical results for the capture rates and present various photon spectra. Without the Δ isobar effect included, our triplet capture rate is close to that calculated earlier by Fearing [22]. However, it agrees with that calculated very recently by BHM only within 10%. Half of this discrepancy can be attributed to an incorrect integration over the phase volume in Ref. [29].

Besides other results, we have found for the spectra, corresponding to the mixture of muonic states in the experiments of [27,28], and for the interval 60 MeV$\leqslant k \leqslant k_{max}$ that (i) inclusion of the on-shell Δ isobar provides an enhancement of $\approx$3.3–8.7%, which is $\approx$1/8–1/5 of the enhancement needed to explain the experimental spectrum; (ii) Putting the Δ isobar off-shell and using the values of the off-shell parameters from the interval fixed in the pion photoproduction [44–47] yields an enhancement up to $\approx$7–14%,[6] which is up to $\approx$1/3 of the enhancement supposed by the data; (iii) the choice $Y=1.75, Z=-1.95$ leads to an enhancement $\approx$25–43%, which turns out to be of the right size to describe the experimental photon spectrum.

In obtaining the above mentioned results, we kept the induced pseudoscalar form factors $g_P^{L,N}$, given in Eq. (1.24), as predicted by the PCAC ($x=1$).

We also note that taking the parameters of the model for the case (ii) above, an uncertainty of 20% in g_P^{PCAC}, and a 5% admixture of the $S=3/2$ orthomolecular $p\mu p$ state, also provides a photon spectrum close to the experimental one.

Our conclusions are presented in Sec. V.

II. EFFECTIVE HAMILTONIAN FOR RMC

In presenting the effective Hamiltonian, we follow Rood and Tolhoek [19]. Then the velocity independent part is

$$
\begin{aligned}
H_{eff}^{(0)} &= \frac{1}{\sqrt{2}m_\mu}(1-\vec{\sigma}_l\cdot\hat{\nu})\tilde{H}_{eff}^{(0)} \\
&= \frac{1}{\sqrt{2}m_\mu}(1-\vec{\sigma}_l\cdot\hat{\nu})[g_1(\vec{\sigma}_l\cdot\vec{\varepsilon})+g_2(\vec{\sigma}\cdot\vec{\varepsilon})+g_3 i(\vec{\sigma}\cdot\vec{\varepsilon}\times\vec{\sigma}_l)+g_4'(\vec{\sigma}_l\cdot\vec{\varepsilon})(\vec{\sigma}\cdot\hat{k})+g_4''(\vec{\sigma}_l\cdot\vec{\varepsilon})(\vec{\sigma}\cdot\hat{\nu})+g_5'(\vec{\sigma}_l\cdot\hat{k})(\vec{\varepsilon}\cdot\hat{\nu}) \\
&\quad +g_6'(\vec{\varepsilon}\cdot\hat{\nu})+g_7' i(\vec{\sigma}\cdot\hat{k}\times\vec{\varepsilon})+g_7'' i(\vec{\sigma}\cdot\hat{\nu}\times\vec{\varepsilon})+g_8'(\vec{\sigma}_l\cdot\hat{k})(\vec{\sigma}\cdot\vec{\varepsilon})+g_8''(\vec{\sigma}_l\cdot\hat{\nu})(\vec{\sigma}\cdot\vec{\varepsilon})+g_9'(\vec{\sigma}_l\cdot\vec{\sigma}) \\
&\quad +g_{10}'(\vec{\sigma}\cdot\hat{k})(\vec{\varepsilon}\cdot\hat{\nu})+g_{10}''(\vec{\sigma}\cdot\hat{\nu})(\vec{\varepsilon}\cdot\hat{\nu})+g_{11}'(\vec{\sigma}_l\cdot\hat{k})(\vec{\sigma}\cdot\hat{k})(\vec{\varepsilon}\cdot\hat{\nu})+g_{11}''(\vec{\sigma}_l\cdot\hat{k})(\vec{\sigma}\cdot\hat{\nu})(\vec{\varepsilon}\cdot\hat{\nu})].
\end{aligned} \tag{2.1}
$$

Here $\vec{\sigma}_l(\vec{\sigma})$ are the lepton (nucleon) spin Pauli matrices and $\hat{\nu}(\hat{k})$ is the unit vector in the direction of the neutrino (photon) momentum vector $\vec{\nu}(\vec{k})$. Not all the form factors are independent. Using equations

$$\vec{\varepsilon}_\lambda = -i\lambda(\hat{k}\times\vec{\varepsilon}_\lambda), \quad \vec{\varepsilon}_\lambda = \frac{1}{\sqrt{2}}(\hat{i}-\lambda\hat{j}), \tag{2.2}$$

one gets redefinitions,

$$
\begin{aligned}
&g_2 \to g_2 - \lambda(g_7' + y g_7''), \quad g_{10}' \to g_{10}' + \lambda g_7'', \\
&g_8' \to g_8' + \lambda g_3, \quad g_4' \to g_4' - \lambda g_3,
\end{aligned} \tag{2.3}
$$

where $y=(\hat{\nu}\cdot\hat{k})$. The last two terms in Eq. (2.1) are new in comparison with [19].

III. CONTRIBUTION TO $H_{eff}^{(0)}$ FROM THE AMPLITUDES OF THE CHIRAL LAGRANGIAN OF THE $N\Delta\pi\rho\omega A_1$ SYSTEM

Here we discuss our amplitudes and contributions to g_i's. We start by presenting briefly the part of the RMC amplitude derived earlier in Ref. [3] without Δ's, referring for details to Sec. 3 of that paper. Then we deal with the amplitudes describing the Δ excitation processes and we compare our effective vertices with those of Ref. [26]. Finally, we discuss the amplitudes stemming from the anomalous Lagrangian.

A. The RMC amplitude without Δ's

Besides the muon radiative part $M^a(k,q)$, the amplitude $T^a(k,q)$ [3] consists of three terms representing the hadron radiative amplitude

$$
\begin{aligned}
T^a(k,q) = \frac{eG}{\sqrt{2}}\{ & M^a(k,q) + l_\mu(0)\epsilon_\nu(k)[M_{\mu\nu}^{B,a}(k,q) \\
& + M_{\mu\nu}^a(\pi;k,q) + M_{\mu\nu}^a(a_1;k,q)]\}.
\end{aligned} \tag{3.1}
$$

The amplitude $M_{\mu\nu}^{B,a}(k,q)$ consists of the nucleon Born terms and of some related contact amplitudes. The amplitude $M_{\mu\nu}^a(\pi;k,q)$ contains the mesonic amplitude $M_{\mu\nu}^{mc,a}(\pi;k,q)$

[6]The enhancement of $\approx$7–14% was obtained for $Y=1.75$, $Z=-0.8$.

and all contact terms where the electroweak vertex is connected with the nucleon by the pion line. The amplitude $M^{a}_{\mu\nu}(a_1;k,q)$ has graphically a similar structure as the amplitude $M^{a}_{\mu\nu}(\pi;k,q)$ with the pion line changed for the a_1 meson one. These amplitudes satisfy separately continuity equations when contracted with the four-momentum transfer q_μ of the weak vertex.

Since our model respects vector dominance and PCAC, the sum of the hadron radiative amplitudes satisfies exactly the following Ward-Takahashi identities:

$$q_\mu[M^{B,a}_{\mu\nu}+M^{a}_{\mu\nu}(\pi)+M^{a}_{\mu\nu}(a_1)] = if_\pi m_\pi^2 \Delta_F^\pi(q)\mathcal{M}^{a}_{\pi,\nu}+i\varepsilon^{3ab}\overline{u(p')}\hat{j}^{b}_{W,\nu}(q_1)u(p), \tag{3.2}$$

$$k_\nu[M^{B,a}_{\mu\nu}+M^{a}_{\mu\nu}(\pi)+M^{a}_{\mu\nu}(a_1)]=i\varepsilon^{3ab}\overline{u(p')}\hat{j}^{b}_{W,\mu}(q_1)u(p). \tag{3.3}$$

Besides, the monopole electroweak form factors with $m_V=m_\rho$ and $m_A=m_{a_1}$ appear naturally in our amplitudes. Let us note that Eq. (3.3) guarantees the gauge invariance of the model. The consistency condition [21] for our amplitudes is

$$\Delta_F^\pi(q)k_\nu\mathcal{M}^{a}_{\pi,\nu}=\Delta_F^\pi(q_1)i\varepsilon^{3ab}M^{b}_{\pi}. \tag{3.4}$$

Here $\mathcal{M}^{a}_{\pi,\nu}$ is the radiative pion absorption amplitude, M^b_π is the pseudoscalar πNN vertex, and $q=k+q_1$.

The leading amplitudes are the nucleon Born terms $M^{B,a}_{\mu\nu}(k,q)$ [corresponding to $M(b),M(c),M(d)$ in Ref. [19]],[7] the amplitudes $M^{a}_{\mu\nu}(\pi,1),M^{a}_{\mu\nu}(\pi,2)$, and $M^{B,a}_{\mu\nu}(5)$ [the sum of them corresponds to $M(e)$ in Ref. [19]], and the mesonic amplitude $M^{m.c.,a}_{\mu\nu}$ [in correspondence with $M(f)$ in Ref. [19]]. As discussed above, besides these amplitudes, other contact terms appear.

The low energy theorems allow one [4,21], by applying current conservation and PCAC to a general amplitude, to determine consistently the amplitude for the RMC in terms of elastic weak form factors and pion photoabsorption amplitude, up to terms linear in k and q. As shown in Ref. [21], higher order terms cannot be predicted. Since our amplitudes satisfy exactly current conservation and PCAC, we can obtain terms of any desired order. We now present the expansion of our nonresonant amplitudes up to $\mathcal{O}(1/M^2)$.

1. Corrections up to $\mathcal{O}(1/M)$

The nonrelativistic reduction of these amplitudes yields the following contributions up to $\mathcal{O}(1/M)$ to the form factors g_i:

$$g_1=-\lambda_+g_V^L\left[1+\frac{\vec{s}}{2M}\cdot\hat{k}\right]-g_V^N\eta+g_A^N\lambda\,\eta\mu_V,$$

$$g_2=-\lambda_+g_A^L-g_P^N\eta+g_V^N\lambda\,\eta\mu_V-g_A^N\eta,$$

$$g_3=-\lambda_+g_A^L+g_M^N\eta+g_V^N\eta-g_A^N\lambda\,\eta\mu_S,$$

$$g_4=\lambda_+g_A^L-g_V^N\lambda\,\eta\mu_V+g_P^L\left[\lambda_-+\frac{\nu}{m_\mu}(1-y)\lambda_+\right],$$

$$g_4'=g_4\frac{k}{2M},\quad g_4''=g_4\frac{\nu}{2M},$$

$$g_5=\lambda_+g_V^L+g_M^N\eta\mu_V+g_V^N\eta\mu_V+g_M^N\lambda\,\eta,\quad g_5'=g_5\frac{\nu}{2M},$$

$$g_6=\lambda_+g_V^L+(g_P^N+g_A^N)\lambda\,\eta\mu_V+g_M^N\eta-g_V^N\eta(1+2\mu_n),$$

$$g_6'=g_6\frac{\nu}{2M},$$

$$g_7=\lambda_+(g_V^L+g_M^L)+(g_P^N+g_A^N)\lambda\,\eta\mu_V-g_V^N\eta,$$

$$g_7'=g_7\frac{k}{2M},\quad g_7''=g_7\frac{\nu}{2M},$$

$$g_8=-\lambda_+(g_V^L+g_M^L)-g_V^N\lambda\,\eta\mu_S,\quad g_8'=g_8\frac{k}{2M},$$

$$g_8''=g_8\frac{\nu}{2M},$$

$$g_9=\lambda_+(g_V^L+g_M^L)+(g_V^N+g_M^N)\lambda\,\eta\mu_S-2g_A^N\eta\mu_n,$$

$$g_9'=g_6\frac{\nu}{2M},\quad g_{10}=g_P^N\eta\frac{4M\nu}{m_\pi^2+(q^L)^2}+\lambda_+g_P^L\frac{\nu}{m_\mu},$$

$$g_{10}'=g_{10}\frac{k}{2M},\quad g_{10}''=g_{10}\frac{\nu}{2M},$$

$$g_{11}=\lambda_+g_P^L\frac{\nu}{m_\mu},\quad g_{11}'=g_{11}\frac{k}{2M},\quad g_{11}''=g_{11}\frac{\nu}{2M}. \tag{3.5}$$

Here our notations mostly follow Ref. [19],

$$\vec{s}=\vec{k}+\vec{\nu},\quad \eta=\frac{m_\mu}{2M},\quad \lambda_\pm=\frac{1}{2}(1\pm\lambda).$$

In addition we have

$$\mu_V=1+\mu_p-\mu_n\equiv1+\kappa_V,\quad \mu_S=1+\mu_p+\mu_S\equiv1+\kappa_S. \tag{3.6}$$

Besides the obvious momentum dependence of the form factors $g_P^{L(P)}$ given in Eq. (1.24), all other nucleon weak vector and axial-vector form factors are assumed to have either the monopole momentum dependence, which naturally appears in our model, with $m_V=m_\rho$ and $m_A=m_{a_1}$, or, for the sake of comparison, the dipole one with $m_V=0.843$ GeV and $m_A=1.077$ GeV [5].

[7]For notations see Sec. 3 of Ref. [3].

2. Corrections up to $\mathcal{O}(1/M^2)$

Here we have two groups of contributions. The first one arises from the expansion of the amplitudes considered above by one order or more in $1/M$, which leads to

$$\left(\frac{2M}{\eta}\right)\Delta g_1 = g_M^N(k-\mu_V\vec{\nu}\cdot\hat{k}) - (g_V^N\mu_S - g_A^N\lambda\,\eta\mu_n)(\vec{s}\cdot\hat{k}),$$

$$\left(\frac{2M}{\eta}\right)\Delta g_2 = -g_M^N\lambda(\vec{\nu}\cdot\hat{k}) + 2\mu_n(g_A^N+\lambda g_V^N - g_P^N)(\vec{s}\cdot\hat{k}),$$

$$\left(\frac{2M}{\eta}\right)\Delta g_3 = -g_M^N k + 2g_A^N\lambda\,\mu_n(\vec{s}\cdot\hat{k}) - 2g_V^N\mu_n(\vec{\nu}\cdot\hat{k}),$$

$$\left(\frac{2M}{\nu\eta}\right)\Delta g_4'' = -g_M^N\lambda\,\mu_S - 2g_V^N\lambda\,\mu_n,$$

$$\left(\frac{M}{\nu\eta}\right)\Delta g_8'' = g_V^N\lambda\,\mu_n,$$

$$\left(\frac{M}{\nu\eta}\right)\Delta g_{10}' = (g_P^N - g_A^N)\mu_n. \qquad (3.7)$$

The main part of the contribution to the photon spectrum arises from the terms proportional to g_A^N and g_V^N in Δg_2. These terms appear due to the neutron recoil induced by the time component of the weak current. Actually, the terms $\Delta g_4'', \Delta g_4''$, and g_{10}' contribute up to $\mathcal{O}(1/M^3)$. We have verified that they change the singlet capture rate by $\approx 10\%$ and the triplet capture rate by $\approx 0.8\%$, which is the reason to keep them. They also arise presumably from the neutron recoil.

The second group of corrections $\mathcal{O}(1/m_\rho^2)\approx\mathcal{O}(1/M^2)$ (the HP correction) stems from some contact terms present in the hadron radiative part of the amplitude (3.1). It is discussed in Sec. 4 of Ref. [3]. Here we quote the results of the nonrelativistic reduction

$$\Delta g_1 = -2g_V^L\left(\frac{2M}{m_\rho}\right)^2\eta\frac{k}{2M}, \quad \Delta g_2 = -\frac{g_A^N}{2}\left(\frac{2M}{m_\rho}\right)^2\eta\frac{2k+y\nu}{2M}. \qquad (3.8)$$

B. The RMC amplitude with Δ's

We derive the RMC amplitudes arising due to the Δ excitations from chiral Lagrangians [42–45]. They correspond to the standard nucleon Born terms with the Δ isobar instead of nucleon in the intermediate state. The needed Lagrangian reads

$$\mathcal{L}^M_{N\Delta\pi\rho a_1} = \frac{f_{\pi N\Delta}}{m_\pi}\bar{\Psi}_\mu\vec{T}\mathcal{O}_{\mu\nu}(Z)\Psi\cdot(\partial_\nu\vec{\pi}+2f_\pi g_\rho\vec{a}_\nu) - g_\rho\frac{G_1}{M}\bar{\Psi}_\mu\vec{T}\mathcal{O}_{\mu\eta}(Y)\gamma_5\gamma_\nu\Psi\cdot\vec{\rho}_{\eta\nu}+\text{H.c.} \qquad (3.9)$$

Here $\vec{T}$ is the operator of the transition spin. Another possible term in the $\rho N\Delta$ vertex is suppressed by one order in $1/M$ and it does not contribute in any sizable manner [26]. We take the operator $\mathcal{O}_{\mu\nu}(B)$ in the following form [43–45]:

$$\mathcal{O}_{\mu\nu}(B) = \delta_{\mu\nu} + C(B)\gamma_\mu\gamma_\nu, \qquad (3.10)$$

$$C(B) = \tfrac{1}{2}(1+4B)A + B. \qquad (3.11)$$

A choice $A=-1$ simplifies considerably [43] the propagator of the Δ.

The coupling constant $f_{\pi N\Delta}$ is not well known and the values for $f^2_{\pi N\Delta}/4\pi$ from the interval between 0.23 and 0.36 can be found in the literature [12]. From the dispersion theory [61], $f^2_{\pi N\Delta}/4\pi\approx 0.30$ and $f^2_{\pi N\Delta}/4\pi\approx 0.35$ from the decay width [62]. Also a good fit to the 33 phase shift was obtained in Refs. [44,45] by using $f^2_{\pi N\Delta}/4\pi\approx 0.314$. The new data on pion photoproduction prefer $f^2_{\pi N\Delta}/4\pi\approx 0.371$ [47]. The ranges of the other relevant parameters of the model are [45–47]

$$-0.8\leqslant Z\leqslant 0.7, \quad -1.25\leqslant Y\leqslant 1.75, \quad 1.97\leqslant G_1\leqslant 2.65. \qquad (3.12)$$

Our radiative amplitude with the Δ excitation can be written analogously with the nucleon Born term $M^{B,a}_{\mu\nu}(1)$ [3] as

$$M^{\Delta,a}_{\mu\nu} = -\overline{u(p')}[\{\hat{J}_{W,\mu\alpha}(-q)\}^+ S_F^{\alpha\gamma}(Q)\hat{j}_{em,\nu\gamma}(k)(T^+)^a T^3 + \{\hat{J}_{em,\nu\gamma}(-k)\}^+ S_F^{\gamma\alpha}(P)\hat{j}_{W,\mu\alpha}(q)(T^+)^3 T^a]u(p). \qquad (3.13)$$

Here the weak $N\Delta$ vertex reads

$$\hat{J}_{W,\mu\alpha}(q) = \hat{J}_{V,\mu\alpha}(q) - \hat{J}_{A,\mu\alpha}(q), \qquad (3.14)$$

with the vector part defined as

$$\hat{J}_{V,\mu\alpha}(q) = i\left(\frac{G_1}{M}\right)m_\rho^2\Delta_F^\rho(q)(q_\beta\delta_{\mu\lambda}-q_\lambda\delta_{\mu\beta})\mathcal{O}_{\alpha\beta}(Y)\gamma_5\gamma_\lambda \qquad (3.15)$$

and the axial-vector part of the form

$$\hat{J}_{A,\mu\alpha}(q) = \left(\frac{f_\pi f_{\pi N\Delta}}{m_\pi}\right)[m_{a_1}^2\Delta^{a_1}_{\mu\lambda}(q) - q_\mu q_\lambda\Delta_F^\pi(q)]\mathcal{O}_{\alpha\lambda}(Z). \qquad (3.16)$$

Further, the electromagnetic $\gamma N\Delta$ vertex is

$$\hat{J}_{em,\nu\gamma}(k) = -\hat{J}_{V,\nu\gamma}(k,k^2=0) = -i\left(\frac{G_1}{M}\right)(k_\beta\delta_{\nu\lambda}-k_\lambda\delta_{\nu\beta})\mathcal{O}_{\gamma\beta}(Y)\gamma_5\gamma_\lambda. \qquad (3.17)$$

Finally, $S_F^{\alpha\gamma}(p)$ is the Δ isobar propagator. With the choice $\mathcal{O}_{\alpha\beta}=\delta_{\alpha\beta}$, our amplitudes, Eqs. (3.14)–(3.16), coincide in

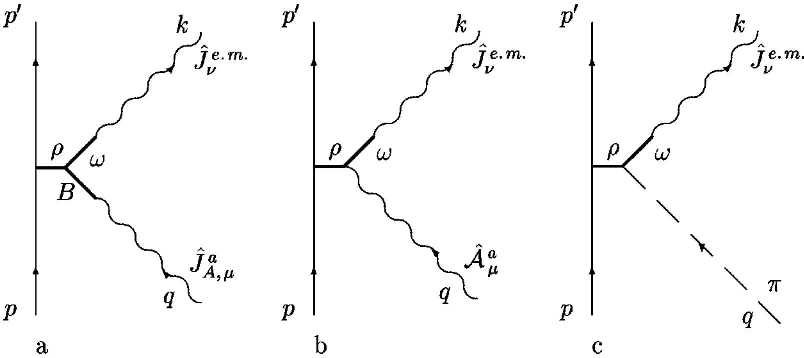

FIG. 1. (a),(b) The radiative hadron amplitudes obtained from the anomalous Lagrangian of the $\pi\rho\omega a_1$ system, Eq. (3.23); in (a) $B=\pi$ or a_1. (c) The associated radiative pion absorption amplitude. These amplitudes satisfy PCAC.

form with those obtained in Ref. [26] from the study of the weak N–Δ vertex in the reaction $\nu d \to \mu^- \Delta^{++} n$.[8]

For the divergence of the resonant amplitude $M^{\Delta,a}_{\mu\nu}$ from Eq. (3.13) we have

$$q_\mu M^{\Delta,a}_{\mu\nu} = i f_\pi m_\pi^2 \Delta^\pi_F(q^2) M^{\Delta,a}_{\pi,\nu}. \tag{3.18}$$

Here the associated resonant radiative pion absorption amplitude $M^{\Delta,a}_{\pi,\nu}$ is

$$M^{\Delta,a}_{\pi,\nu} = -\overline{u}(p')[\{\hat{M}^{\Delta}_{\pi,\alpha}(-q)\}^+ S_F^{\alpha\gamma}(Q)\hat{J}_{em,\nu\gamma}(k)(T^+)^a T^3 + \{\hat{J}_{em,\nu\gamma}(-k)\}^+ S_F^{\gamma\alpha}(P)\hat{M}^{\Delta}_{\pi,\alpha}(q)(T^+)^3 T^a]u(p) \tag{3.19}$$

and the $\pi N \Delta$ vertex reads

$$\hat{M}^{\Delta}_{\pi,\alpha}(q) = i\frac{f_{\pi N\Delta}}{m_\pi} q_\lambda \mathcal{O}_{\alpha\lambda}(Z). \tag{3.20}$$

We now present the contributions from the amplitude (3.13) to the form factors g_i. They are

$$\Delta g_1 = \frac{2}{3}\lambda(C_- - C_+)C\{1+(1-R)[C(Y)+C(Z) +2(2+R)C(Y)C(Z)]\},$$

$$\Delta g_2 = \frac{1}{3}(C^+ + C_-)C(1-R)[-(1+2R)+2(1-2R)C(Y) +2(1-R)C(Z)+4(2-R)C(Y)C(Z)] +(C^+ + C_-)C\frac{g_P^N}{6Mg_A^N}\langle -[(1-R)(1+2R)k+y\nu] +2(1-R)\{[(1-2R)k+y\nu]C(Y)+[(1-R)k+y\nu] \times C(Z)+2[(2-R)k+(2+R)y\nu]C(Y)C(Z)\}\rangle,$$

$$\Delta g_3 = \frac{2}{3}\lambda(C_+ + C_-)C\{1+(1-R)[C(Y)+C(Z) +2(2+R)C(Y)C(Z)]\},$$

$$\Delta g_4' = -\Delta g_8' = (C_+ + C_-)C,$$

$$\Delta g_6 = -\lambda\frac{\nu}{3M}\frac{g_P^N}{g_A^N}(C_+ - C_-)C\{1+(1-R)[C(Y) +C(Z)+2(2+R)C(Y)C(Z)]\},$$

$$\Delta g_{10}' = (C_+ + C_-)C\frac{\nu}{6M}\frac{g_P^N}{g_A^N} \times\{1-2(1-R)[C(Y)+C(Z) +2(2+R)C(Y)C(Z)]\}, \tag{3.21}$$

where

$$C = -\frac{4}{3}\frac{f_\pi f_{\pi N\Delta}}{m_\pi}G_1\eta k, \quad R = M/M_\Delta,$$

$$C_+^{-1} = \left\{(M_\Delta - M) + \frac{2M}{M_\Delta + M}\left[m_\mu - \nu + \frac{m_\mu}{2M} \times\left(2\nu - m_\mu - \frac{2\nu}{m_\mu}(\nu + yk)\right)\right]\right\},$$

$$C_-^{-1} = \left\{(M_\Delta - M) + \frac{2M}{M_\Delta + M}\left[\nu - m_\mu + \frac{m_\mu}{2M}(2\nu - m_\mu)\right]\right\}. \tag{3.22}$$

According to the concept developed in Refs. [43–45], we take the mass of the Δ isobar real.

C. The RMC amplitude from an anomalous Lagrangian of the $\pi\rho\omega a_1$ system

We have considered so far the amplitudes where a natural parity does not change in any vertex. The natural parity of a particle is defined as $P(-1)^J$, where P is the intrinsic parity and J is the spin of the particle. Some amplitudes of this kind relevant for the process under study are presented in Fig. 1. The starting point is an anomalous Lagrangian of the $\pi\rho\omega a_1$ system [53,52] constructed within the approach of hidden local symmetries [64,65]. The electromagnetic interaction in such a system was first considered in Ref. [66] and the relevant constants $\tilde{c}_i$ were extracted from the data as well. The

[8]For a recent study of this reaction see [63].

weak interaction was incorporated explicitly in Ref. [52] and the refit of the constants to the modern data [67] was made in Ref. [53].

The Lagrangian reads

$$\mathcal{L}_{an}=2ig_\rho\varepsilon_{\kappa\lambda\mu\nu}\left[(\partial_\kappa\omega_\lambda)(g_\rho\vec{\rho}_\mu-e\vec{\mathcal{V}}_\mu)+\left(g_\rho\omega_\kappa-\frac{1}{3}e\mathcal{B}_\kappa\right)\times(\partial_\lambda\vec{\rho}_\mu)\right]\left[\tilde{c}_7\left(\frac{1}{f_\pi}\partial_\nu\vec{\pi}+e\vec{\mathcal{A}}_\nu\right)+\tilde{c}_8\left(\frac{1}{2}e\vec{\mathcal{A}}_\nu-g_\rho\vec{a}_\nu\right)\right]$$
$$+2ie\varepsilon_{\kappa\lambda\mu\nu}\left[\left(\frac{1}{3}\partial_\kappa\mathcal{B}_\lambda\right)(g_\rho\vec{\rho}_\mu-e\vec{\mathcal{V}}_\mu)+\left(g_\rho\omega_\kappa-\frac{1}{3}e\mathcal{B}_\kappa\right)(\partial_\lambda\vec{\mathcal{V}}_\mu)\right]\left[\tilde{c}_9\left(\frac{1}{f_\pi}\partial_\nu\vec{\pi}+e\vec{\mathcal{A}}_\nu\right)+\tilde{c}_{10}\left(\frac{1}{2}e\vec{\mathcal{A}}_\nu-g_\rho\vec{a}_\nu\right)\right], \tag{3.23}$$

where besides the meson fields, the external vector isoscalar $\mathcal{B}_\mu$ and isovector $\vec{\mathcal{V}}_\mu$ and axial-vector isovector $\vec{\mathcal{A}}_\mu$ fields are also included. The constants $\tilde{c}_i$ are [53]

$$\tilde{c}_7=8.64\times10^{-3},\quad \tilde{c}_8=-1.02\times10^{-1},$$
$$\tilde{c}_9=9.23\times10^{-3},\quad \tilde{c}_{10}=1.29\times10^{-1}. \tag{3.24}$$

The axial RMC amplitudes arising from the anomalous Lagrangian $\mathcal{L}_{an}$, Eq. (3.23), are

$$M^{an,a}_{\mu\nu}(1)=i\frac{g_\rho^2}{3}\varepsilon_{\eta\nu\beta\sigma}k_\eta q_\sigma q_\mu\Delta^\pi_F(q)\Delta^\rho_{\beta\lambda}(q_1)\overline{u}(p')\times\left(\gamma_\lambda-\frac{\kappa_V}{2M}\sigma_{\lambda\alpha}q_{1\alpha}\right)\tau^a u(p),$$
$$M^{an,a}_{\mu\nu}(2)=-i\frac{g_\rho^2}{3}\varepsilon_{\eta\nu\beta\mu}k_\eta\Delta^\rho_{\beta\lambda}(q_1)\overline{u}(p')\times\left(\gamma_\lambda-\frac{\kappa_V}{2M}\sigma_{\lambda\alpha}q_{1\alpha}\right)\tau^a u(p) \tag{3.25}$$

and they correspond to the processes presented in Figs. 1(a) and 1(b). Together with the radiative pion absorption amplitude of Fig. 1(c)

$$M^{an,a}_{\pi,\nu}=-\frac{g_\rho}{3f_\pi}\varepsilon_{\eta\nu\beta\sigma}k_\eta q_\sigma\Delta^\rho_{\beta\lambda}(q_1)\overline{u}(p')\times\left(\gamma_\lambda-\frac{\kappa_V}{2M}\sigma_{\lambda\alpha}q_{1\alpha}\right)\tau^a u(p), \tag{3.26}$$

the amplitudes (3.25) satisfy PCAC,

$$q_\mu[M^{an,a}_{\mu\nu}(1)+M^{an,a}_{\mu\nu}(2)]=if_\pi m_\pi^2\Delta^\pi_F(q)M^{an,a}_{\pi,\nu}. \tag{3.27}$$

The contribution from the anomalous amplitudes (3.25) to the total hadron radiative amplitude (3.1) for the reaction (1.23) is given as

$$T^a_{an}=\frac{eG}{\sqrt{2}}l_\mu(0)\varepsilon_\nu(\tilde{c}_7+\tilde{c}_9)[M^{an,a}_{\mu\nu}(1)+M^{an,a}_{\mu\nu}(2)]. \tag{3.28}$$

In Ref. [8], a contribution arising from the Wess-Zumino-Witten anomalous Lagrangian (AL) was estimated. Graphically, it corresponds to our Fig. 1(b) with the pion instead of the ρ meson and with the vector interaction $\hat{\mathcal{V}}^a_\mu$ instead of the axial one. The associated amplitude depends on the momentum transfer q^L and, therefore, it does not possess the enhancement factor ≈ 3 for large photon momentums. For illustration, we present the contribution to one of the form factors

$$\Delta g_4'=-\eta\frac{k^2}{8\pi^2f_\pi^2}\frac{\lambda k+y\nu}{2m_\mu}g_P^L, \tag{3.29}$$

all other contributions have a similar structure. It is also seen that these contributions are $\mathcal{O}(1/M^3)$ because $8\pi^2f_\pi^2\sim M^2$.

In our case, it is the amplitude related to the graph of Fig. 1(a) that is q^N dependent. We present from the calculated contributions to g_i arising from the amplitudes (3.25), only that for the form factor g_2, the others are suppressed by one order in $1/M$,

$$\Delta g_2=-\frac{g_\rho^2}{3g_A}\left(\frac{2M}{m_\rho}\right)^2(1+\kappa_V)\eta\frac{k}{2M}\frac{\vec{s}^2}{4M^2}g_P^N+\frac{g_\rho^2}{3}\left(\frac{2M}{m_\rho}\right)^2\times(1+\kappa_V)\eta\frac{\vec{k}\cdot\vec{s}}{4M^2}. \tag{3.30}$$

For comparison, we keep also the g_P^N-dependent contribution. Using the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation $2f_\pi^2g_\rho^2=m_\rho^2$, we can rewrite this contribution in the form

$$-\frac{1+\kappa_V}{g_A}\eta\frac{k}{M}\frac{\vec{s}^2}{12f_\pi^2}g_P^N.$$

Taking into account that $12f_\pi^2\approx M^2/10$ we can see that the g_P^N–dependent contribution is larger than the g_P^L-dependent one for large values of k, by a factor ≈ 20. However, it is not enough to influence the photon spectrum, because of an additional factor $(\tilde{c}_7+\tilde{c}_9)$ in the amplitude Eq. (3.28) (see below). It is seen from Eq. (3.30) that the first term on the right-hand side is suppressed in comparison with the second one arising from the contact amplitude $M^{an,a}_{\mu\nu}(2)$. As we shall see later, the second term contributes to the triplet capture rate by an amount $\approx -0.2\%$.

Let us note that the sum of the vector RMC amplitudes arising from the anomalous Lagrangian, Eq. (3.23), is zero with a good accuracy.

One can obtain a more general result for Δg_2, Eq. (3.30), by using the change

$$1/3\to g_{\rho1}/g_{\omega1},\quad \kappa_V\to g_{\rho2}/g_{\rho1},$$

which corresponds to the model used in Refs. [44,45] for describing the pion photoproduction amplitude in the t channel. In this model, the $\pi\rho\gamma$ and $\pi\omega\gamma$ amplitudes are effectively the same as those obtained from our anomalous Lagrangian, Eq. (3.23), and the ρNN and ωNN vertices contain four constants $g_{\rho 1}, g_{\rho 2}, g_{\omega 1}$, and $g_{\omega 2}$, which are the free parameters obtained together with other free parameters of the model from a fit to the data.

Compared with the sets of the form factors given in Eqs. (3.7) and (3.8), the g_i's, Eq. (3.30), are even larger. However, due to the values of $\tilde{c}_i$, Eq. (3.24), the factor $(\tilde{c}_7+\tilde{c}_9) \approx 1.8\times 10^{-2}$ makes the contribution from the amplitude T^a_{an}, Eq. (3.28), small.

IV. RESULTS

Using the Hamiltonian $H^{(0)}_{eff}$, Eq. (2.1), and the sets of the form factors g_i, Eqs. (3.5), (3.7), (3.8), and (3.21), we have calculated the capture rates and the photon energy spectra for the RMC in a muon-hydrogen system described by a spin density matrix $\rho_\zeta(\zeta=s,t)$,

$$\frac{d\Lambda_\zeta}{dk}=\frac{1}{4\pi^3}(\alpha^2 G_F \cos\theta_c m/m_\mu)^2 m M_n k \times\int_{-1}^{+1} dy \frac{\nu_0^2}{W+k(y-1)} \mathrm{Tr}\{(1-\vec{\sigma}\cdot\hat{\nu}) \times \tilde{H}^{(0)}_{eff}\rho_\zeta[\tilde{H}^{(0)}_{eff}]^+\}. \tag{4.1}$$

Here α is the fine structure constant, G_F is the Fermi constant, $\cos\theta_c$ is the Cabibbo angle, m is the reduced mass of the μp system, the neutrino energy is determined by the energy conservation,

$$\nu_0=\frac{W}{W+k(y-1)}(k_{max}-k) \approx\left[1+\frac{k}{M_p}(1-y)+\frac{k}{M_p^2}(y-1) \times[m_\mu+k(y-1)]\right](k_{max}-k), \tag{4.2}$$

where the maximum photon energy is given as

$$k_{max}=\frac{W^2-M_n^2}{2W}, \quad W=M_p+m_\mu, \tag{4.3}$$

and $M_{p(n)}$ is the proton (neutron) mass. The singlet and triplet spin density matrices are [36,68]

$$\rho_s=\frac{1}{4}(1-\vec{\sigma}\cdot\vec{\sigma}_l), \quad \rho_t=\frac{1}{4}\left(1+\frac{1}{3}\vec{\sigma}\cdot\vec{\sigma}_l\right). \tag{4.4}$$

We have also calculated the capture rates and spectra for the orthomolecular and paramolecular $p\mu p$ states and for the mixture of muonic states relevant to the TRIUMF experiment [27,28]. The orthomolecular (Λ_o) and paramolecular (Λ_p) capture rates are given in terms of Λ_s and Λ_t as [36]

$$\Lambda_o=0.756\Lambda_s+0.253\Lambda_t, \quad \Lambda_p=0.286\Lambda_s+0.857\Lambda_t, \tag{4.5}$$

and the capture rate Λ_T, relevant to the TRIUMF experiment [27,28], is

$$\Lambda_T=0.061\Lambda_s+0.854\Lambda_o+0.085\Lambda_p. \tag{4.6}$$

Now we present numerical results for the capture rates.

A. Capture rates

Here we present the results for the capture rates calculated for the interval $0\leqslant k\leqslant k_{max}$ in various models. If not stated otherwise, we use the monopole form factors and we put $x=1$ in Eq. (1.24).

(a) We first discuss the results obtained in the model with the Δ isobar kept on-shell. We give the singlet and triplet capture rates in greater detail in order to see explicitly various contributions,

$$\Lambda_s\times 10^3=0.40(0)+1.65(-1)+1.29(-2)+0.11(\mathrm{HP}) -0.07(\mathrm{AL})+0.05(\Delta)=3.43(3.51)\,\mathrm{s}^{-1}, \tag{4.7}$$

$$\Lambda_t\times 10^3=43.7(0)+53.1(-1)+3.7(-2)+0.9(\mathrm{HP}) -0.2(\mathrm{AL})+2.2(\Delta)=103.0(103.4)\,\mathrm{s}^{-1}. \tag{4.8}$$

Here on the right-hand sides of Eqs. (4.7) and (4.8), the number $n(n=0,-1,-2)$ in the brackets means the order of the contribution $\mathcal{O}(1/M^n)$ and HP (AL) and Δ mean the contributions from the hard pion form factors (3.8) [from the form factors (3.30)] and from the form factors (3.21) due to the Δ isobar excitation processes, respectively. These were calculated using the parameters

$$\frac{f^2_{\pi N\Delta}}{4\pi}=0.371, \quad G_1=2.525, \quad Y=Z=-0.5. \tag{4.9}$$

The choice of the parameters Y and Z is such that only the terms proportional to $\delta_{\mu\nu}$ in Eq. (1.28) contribute (the Δ is on-shell). The contribution of the Δ excitation to Λ_t is $\approx 2\%$, which is in agreement with BHM. The numbers in the brackets on the right-hand sides of Eqs. (4.7) and (4.8) are obtained using Eq. (1.24) for $g_P^{L(N)}$ without the correction $\tilde{g}_P^{L(N)}$ included.

For the other capture rates we have

$$\Lambda_o=28.7\times 10^{-3}\ \mathrm{s}^{-1}, \quad \Lambda_p=89.3\times 10^{-3}\ \mathrm{s}^{-1}, \quad \Lambda_T=32.3\times 10^{-3}\ \mathrm{s}^{-1}. \tag{4.10}$$

For the dipole form factors, analogously with Eqs. (4.7) and (4.8) we obtain

$$\Lambda_s=3.28(3.50)\times 10^{-3}\ \mathrm{s}^{-1}, \quad \Lambda_t=101.5(102.5)\times 10^{-3}\ \mathrm{s}^{-1}. \tag{4.11}$$

Let us compare our results with available calculations. The singlet and triplet capture rates without the Δ isobar excitation were calculated earlier by Opat [18] and Fearing [22]. Opat obtained $\Lambda_s = 4.96\times 10^{-3}\ \mathrm{s}^{-1}$ and $\Lambda_t = 90.0\times 10^{-3}\ \mathrm{s}^{-1}$, while Fearing's calculations provide $\Lambda_s = 3.23\times 10^{-3}\ \mathrm{s}^{-1}$ and $\Lambda_t = 99.8\times 10^{-3}\ \mathrm{s}^{-1}$. As it can be seen from Eqs. (4.7) and (4.8), Fearing's results are close to ours.

Very recent calculations [29] yield $\Lambda_s = (2.90-3.10)\times 10^{-3}\ \mathrm{s}^{-1}$ and $\Lambda_t = (112-114)\times 10^{-3}\ \mathrm{s}^{-1}$. Having in mind that Λ_s results as the difference of two large and almost equal numbers, the agreement between our value, Eq. (4.7), of Λ_s and the BHM value can be considered as satisfactory. However, the difference of $\approx 10\%$ between the triplet capture rates is too large. Half of this discrepancy can be understood by checking the integration over the phase volume. We use for the neutrino momentum, Eq. (4.2) with k_{max} from Eq. (4.3), while BHM employed for the k_{max} equation (4.37), which is, in our notations,

$$k_{max} = m_\mu\left(1+\frac{m_\mu}{2M_N}\right)\left(1+\frac{m_\mu}{M_N}\right)^{-1} \approx m_\mu\left(1-\frac{m_\mu}{2M_N}+\frac{m_\mu^2}{2M_N^2}\right), \tag{4.12}$$

where $M_N = (M_p+M_n)/2$ is the nucleon mass. From Eq. (4.3), one obtains $k_{max} = 99.15$ MeV, while from Eq. (4.12) one has $k_{max} = 100.3$ MeV and instead of $\Lambda_t = 103.0\times 10^{-3}\ \mathrm{s}^{-1}$ one obtains $\Lambda_t = 108.0\times 10^{-3}\ \mathrm{s}^{-1}$, which is larger by $\approx 5\%$. Using in Eq. (4.2) for ν_0 the expansion (4.12) for k_{max}, one obtains

$$\nu_0 = m_\mu - k - \frac{m_\mu^2}{2M_N} + \frac{k}{M_N}(1-y)(m_\mu - k) + \frac{1}{2M_N^2}[m_\mu + k(y-1)][m_\mu^2 + 2k(m_\mu - k)(y-1)]. \tag{4.13}$$

In Ref. [29], Eq. (4.39) is used for ν_0. It retains terms up to $\mathcal{O}(1/M_N)$,[9] which yields in our case $\Lambda_t = 107.2\times 10^{-3}\ \mathrm{s}^{-1}$. But the source of the remaining difference of $\approx 5\%$ between the results for the Λ_t is not clear.

(b) The results of calculations without the Δ isobar excitation effect are

$$\Lambda_s = 3.38\times 10^{-3}\ \mathrm{s}^{-1}, \quad \Lambda_t = 100.8\times 10^{-3}\ \mathrm{s}^{-1}, \quad \Lambda_T = 31.6\times 10^{-3}\ \mathrm{s}^{-1}. \tag{4.14}$$

(c) We now present the capture rates for the same case as in (b), but for the value of the parameter $x = 1.5$. The rates are

$$\Lambda_s = 8.35\times 10^{-3}\ \mathrm{s}^{-1}, \Lambda_t = 116.6\times 10^{-3}\ \mathrm{s}^{-1}, \quad \Lambda_T = 39.8\times 10^{-3}\ \mathrm{s}^{-1}. \tag{4.15}$$

The strong dependence of the capture rates on g_P was already known to Opat [18].

(d) The capture rates, calculated for the same parameters as in (a), but with $Y = 1.75$ and $Z = -0.8$, are

$$\Lambda_s = 3.28\times 10^{-3}\ \mathrm{s}^{-1}, \quad \Lambda_t = 106.0\times 10^{-3}\ \mathrm{s}^{-1}, \quad \Lambda_T = 33.0\times 10^{-3}\ \mathrm{s}^{-1}. \tag{4.16}$$

In this case, the effect of the Δ isobar excitation is, for the values of Y and Z allowed by the inequalities of Eq. (3.12), maximal and it is $\approx 5\%$ for Λ_t and $\approx 4\%$ for Λ_T, as follows from comparing Eqs. (4.14) and (4.16).

(e) The capture rates, calculated as in (d), but with $Z = -1.95$, are

$$\Lambda_s = 5.93\times 10^{-3}\ \mathrm{s}^{-1}, \quad \Lambda_t = 116.7\times 10^{-3}\,\mathrm{s}^{-1}, \quad \Lambda_T = 38.1\times 10^{-3}\,\mathrm{s}^{-1}. \tag{4.17}$$

These capture rates are close to those calculated in the case (c) without the Δ isobar excitation effect, but for $x = 1.5$. It follows that in order to achieve the same enhancement in the rates as by scaling g_P by 50%, one should use the values of the parameter Z that are outside of the interval (3.12) found in the pion photoproduction processes. However, it is well known [43–47] that the values of the off-shell parameters X,Y,Z depend strongly on whether the pion production amplitude is unitarized or not and on the method of unitarization. On the other hand, our RMC amplitudes with Δ's are related by the continuity equation (3.18) only to the nonunitarized radiative pion absorption amplitude, and the restriction to use the off-shell parameters X,Y,Z only from Eq. (3.12) may not be mandatory in our case. But it should also be noted that the need for a too large absolute value of the parameter Z to explain the data can be a consequence of the presence of some effects such as possible systematic errors in the experiment [27,28] or molecular phenomena, which are not yet fully understood.

Let us note that our choice of the parameters of the model is not optimal. In order to extract an optimal set of these parameters from the data, one should use a minimization procedure.

Similar enhancement in the capture rates as in the case (e) can be achieved by considering the Δ isobar on-shell, but by taking $f_{\pi N\Delta}^2/4\pi \approx 20$ or $G_1 \approx 20$, which is an amplification of ≈ 7 in the $\pi N\Delta$ coupling and of ≈ 8 in the constant G_1, which is much more than the enhancement factor of ≈ 2.5 needed to change the parameter $Z = -0.8$ in the case (d) to $Z = -1.95$ in the case (e). So our calculations show that the capture rates for the RMC in hydrogen are sensitive to the change in the off-shell parameter Z. We show in the following section that the photon spectra also possess this feature.

[9]There is a factor 2 missing in the denominator of the third term on the right-hand side of this equation.

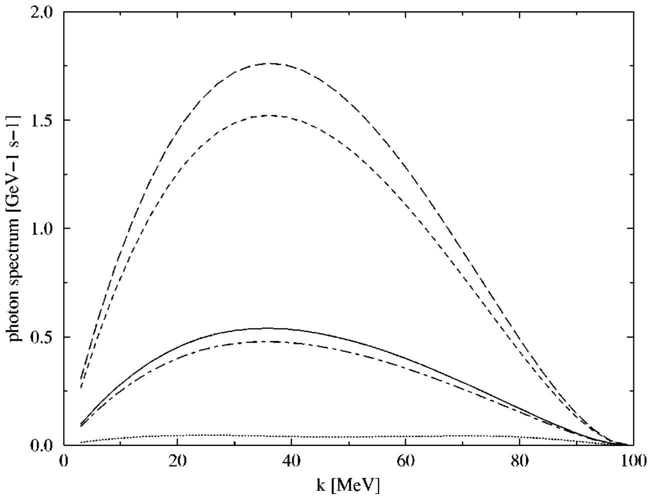

FIG. 2. Photon spectra calculated in model (a) of the preceding section, the Δ isobar excitation effect is included; the dotted, long-dashed, dot-dashed, and dashed curves correspond, respectively, to the singlet, triplet, ortho, and para $p\mu p$ molecule spin combinations, the solid curve corresponds to the mixture of muonic states relevant to the TRIUMF experiment [27,28].

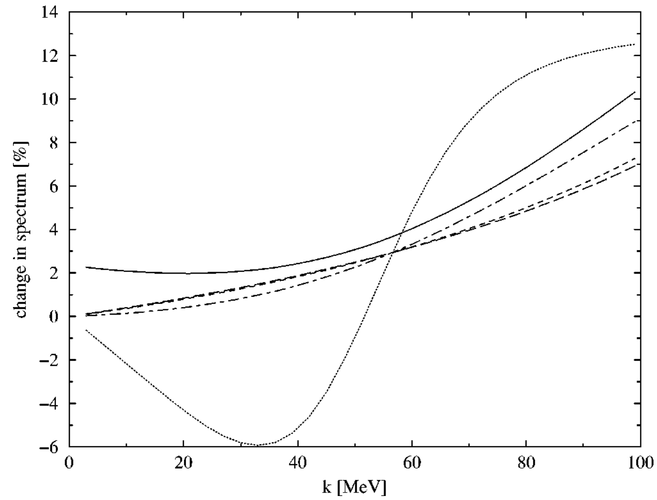

FIG. 3. Relative difference in percent for the spectra calculated with [model (a)] and without [model (b)] the Δ excitation included; the dotted, long-dashed, dot-dashed, and dashed curves correspond, respectively, to the singlet, triplet, ortho, and para $p\mu p$ molecule spin combinations, the solid curve corresponds to the mixture of muonic states relevant to the TRIUMF experiment [27,28].

B. Photon spectra

Photon spectra corresponding to the capture rates calculated in model (a) of the preceding section are presented in Fig. 2. The Δ isobar is included, but the choice of the parameters $Y=Z=-0.5$ is such that the isobar is on-shell. As it is seen from Fig. 2, our spectra are in a close correspondence with those of Fig. 3 of Ref. [26]. However, our spectrum for the triplet state of the μ-p system (long-dashed curve) differs from the analogous spectrum of Fig. 5 of Ref. [29], as it should, because the triplet capture rates differ significantly.

The percentage change in the spectra when the Δ excitation effect is taken into account is presented in Fig. 3. The Δ excitation effect was calculated according model (a) of the preceding section. The case without this effect corresponds to model (b). This change in the spectra due to Δ was first calculated by Beder and Fearing [26]. Our Fig. 3 is in a good agreement with Fig. 4 of Ref. [26], but it differs from the analogous Fig. 6 of Ref. [29].

Similar calculations are presented in Figs. 4 and 5 using instead of model (a) the spectra of the models (d) and (e), respectively. As it is seen, by putting the Δ isobar off-shell, both the singlet and triplet spectra are changed sensibly.

In Fig. 6, we show how the photon spectra, relevant to the mixture of the muonic states for the TRIUMF experiment, depend on the parameters of our model. The dotted curve corresponds to model (b) (no $\Delta, x=1$), the dashed curve is the photon spectrum for case (a) (Δ on-shell, $Y=Z=-0.5$), and the dot-dashed curve is calculated using model (d): Δ is off-shell, the parameters $Y=1.75, Z=-0.8$ are at the boundary of the region (3.12) allowed by the pion photoproduction data [44–47]. In this case, about two times more enhancement is achieved in comparison with the dashed curve. The dependence of the photon spectrum on the change in g_P is illustrated by the long-dashed curve, calculated within model (c) (no $\Delta, x=1.5$). We have found that this curve, normalized to 14.5 counts at $k=60$ MeV, follows closely for $k\geqslant 60$ MeV the solid curve of Fig. 4 [28]. All other curves in this and in the next figure are multiplied by this normalization factor. The long-dashed curve closely follows the solid curve from $k\approx 60$ MeV, which corresponds to model (e): the Δ isobar is off-shell, the parameter $Y=1.75$ is the same as in case (d), the parameter $Z=-1.95$.

Evidently, this picture is in agreement with that obtained by comparing the Beder-Fearing model [26] with the experiment of Refs. [27,28]: the experimental photon spectrum can be satisfactorily described with g_P of the form Eq. (1.24) for

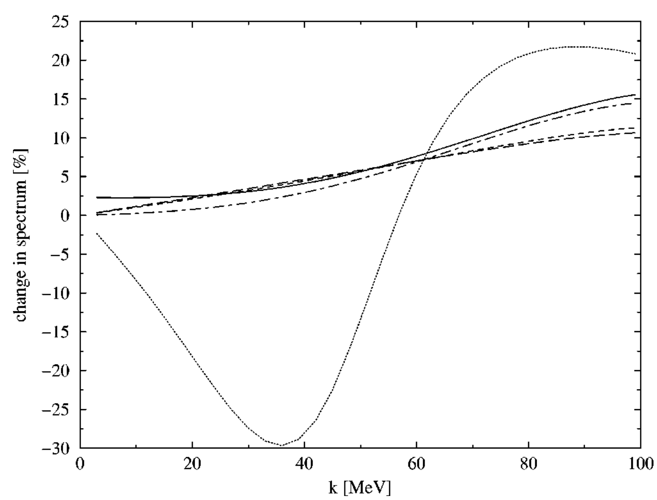

FIG. 4. Relative difference in percent for the spectra calculated with [model (d)] and without [model (b)] the Δ excitation included; the dotted, long-dashed, dot-dashed, and dashed curves correspond, respectively, to the singlet, triplet, ortho, and para $p\mu p$ molecule spin combinations, the solid curve corresponds to the mixture of muonic states relevant to the TRIUMF experiment [27,28].

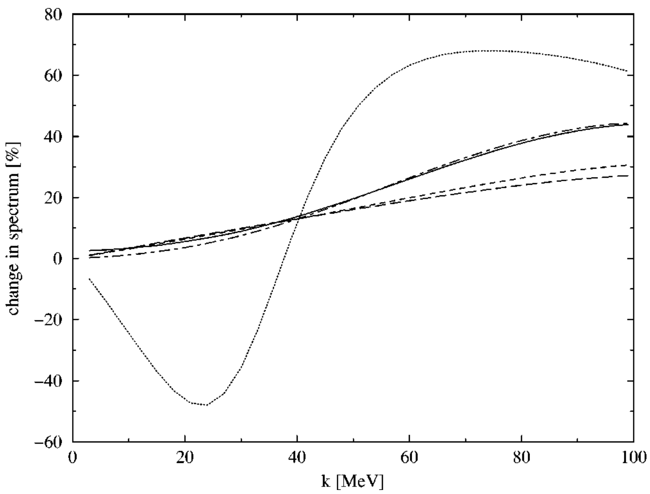

FIG. 5. Relative difference in percent for the spectra calculated with [model (e)] and without [model (b)] the Δ excitation included; the dotted, long-dashed, dot-dashed, and dashed curves correspond, respectively, to the singlet, triplet, ortho, and para $p\mu p$ molecule spin combinations, the solid curve corresponds to the mixture of muonic states relevant to the TRIUMF experiment [27,28].

$x\approx 1.5$ and only a small part of the enhancement could be explained by calculating the spectra by including the on-shell Δ isobar. However, as it is seen from the solid curve, putting the Δ isobar off-shell, one can describe the experimental data quite well.

The numerical analysis of the photon spectra, presented in Fig. 6, shows that in the interval 60 MeV$\leqslant k \leqslant k_{max}$ the effect from the on-shell Δ isobar excitation yields an enhancement of $\approx$3.3–8.7 % of the spectrum, which is 12–20 % of enhancement demanded by the experiment. However, if the Δ isobar is taken off-shell, an enhancement of the spectrum up to $\approx$7–14 % can be obtained, if the off-shell parameters Y,Z are taken from the interval of Eq. (3.12), fixed in the pion photoproduction [44–47], which is $\approx$28–30 % of the

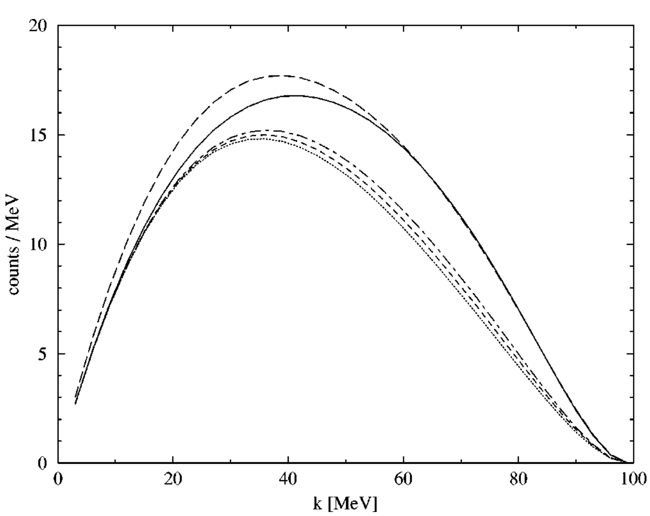

FIG. 6. Influence of the Δ isobar parameters on the photon spectra corresponding to the mixture of muonic states in TRIUMF experiment [27,28]. For the explanation of the curves see text.

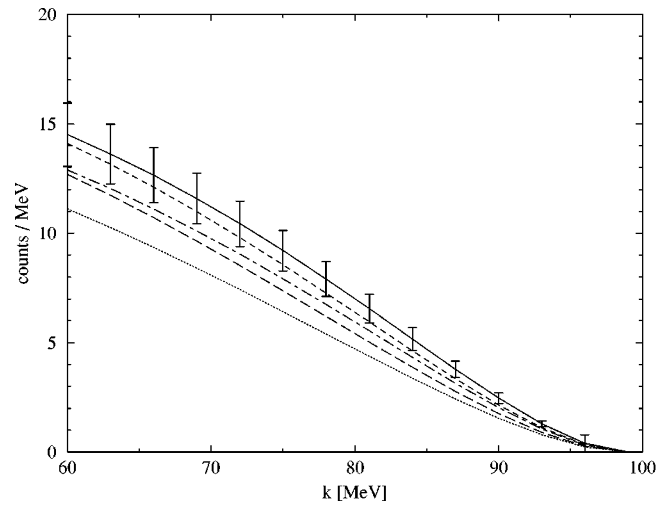

FIG. 7. Dependence of the photon spectra, corresponding to the mixture of muonic states relevant to the TRIUMF experiment [27,28], on various parameters of the calculations; the dotted curve is for reference and corresponds to model (a) (32.3/7.1/196); the long-dashed curve is calculated for model (d), but $\xi = 0.95$ (36.3/8.1/225); the dot-dashed curve is obtained within model (d), but $x = 1.2$ (35.9/8.6/239); the dashed curve is for model (d), but $x = 1.2$ and $\xi = 0.95$ (39.4/9.3/259); the solid curve corresponds to model (c) (39.7/9.9/276). For the details see the text.

enhancement supposed by the data. It is also seen from Fig. 6 that the solid curve, obtained in model (e) with $Y = 1.75$ and $Z = -1.95$ provides the needed enhancement of $\approx$25–43 %.

In Fig. 7, we show the dependence of the calculations on uncertainties in our knowledge of g_P and of the admixture ξ of the $S = 3/2$ orthomolecular $p\mu p$ state. As discussed recently [33], the admixture of the $S = 3/2$ orthomolecular $p\mu p$ state changes the molecular capture rate to [69]

$$\Lambda'_o = \xi\Lambda_o(1/2) + (1-\xi)\Lambda_o(3/2), \qquad (4.18)$$

where $\Lambda_o(1/2)$ is Λ_o of Eq. (4.5) and $\Lambda_o(3/2) = 1.009\Lambda_t$. It was found in [33] that the data on OMC in hydrogen requires $g_P \leqslant 1.2 g_P^{PCAC}$ or $\xi \geqslant 0.95$. This restriction on g_P is in agreement with our Eq. (1.20) for the OMC in ^{3}He. The dependence on the uncertainty in g_P and on ξ is illustrated by the long-dashed, dot-dashed, and dashed curves. The numbers in the brackets are for the capture rate in the full interval $0 \leqslant k \leqslant k_{max}$, for the partial capture rate in the interval 60 MeV$\leqslant k \leqslant k_{max}$, and for the capture rate in counts for this interval, respectively. Otherwise, the unit for the capture rates is 10^{-3} s^{-1}. For the sake of illustration, we assigned a 10% error [28] to a set of ‘‘data’’ represented by 14 points of the spectrum (c). The number of counts 276, related to this curve, is to be compared with the number of counts 286, which can be read off the histogram presented in Fig. 4 [28]. As it is seen, all curves lie already inside the 2σ bound.

V. SUMMARY

In this paper, we have presented the capture rates and the photon energy spectra for the RMC in hydrogen, calculated using the effective Hamiltonian $H^{(0)}_{eff}$, Eq. (2.1), where the form factors g_i are obtained from the amplitudes derived from the chiral Lagrangian of the $N\Delta\pi\rho\omega a_1$ system. The nonresonant part of the Lagrangian contains the normal and anomalous Lagrangians of the $N\pi\rho\omega a_1$ system interacting with the external electromagnetic and weak fields through the associated one-body currents [2,3,42,52,53]. In the expansion of the amplitudes in $1/M$, we keep all terms up to $\mathcal{O}(1/M^2)$.

For the resonant part of our Lagrangian, we have extended the currently used model [2,26,29,42] by adopting results of the model developed by Olsson and Osypowski [43] and by Davidson, Mukhopadhyay, and Wittman [44,45] for the πN scattering and pion photo-production and electroproduction off the nucleon, which allows one to consider the Δ isobar off-shell.

Without the Δ isobar excitation included, our capture rates agree well with these calculated earlier by Fearing [22]. However, the calculated triplet capture rate differs by $\approx 10\%$ from that one derived quite recently within the HBChPT approach in Ref. [29]. About half of this discrepancy can be understood by the use of an approximate equation for the neutrino momentum in Ref. [29]. The origin of the rest of the discrepancy is not clear.

In the model, restricting the Δ isobar on-shell, our spectra are close to those obtained earlier by Beder and Fearing [26]. Our full model, including the Δ isobar off-shell, turns out to be sensitive to the off-shell parameters Y,Z. For the type of the photon spectrum measured in the experiment [27,28], this model provides up to 30% of enhancement needed to explain the experimental photon spectrum, if the parameters Y,Z are taken from the interval of Eq. (3.12), found in the pion photoproduction. This is about two times more than what one can obtain from the model with the Δ isobar on-shell. If, in addition, we take into account the existing 20% uncertainty in the constant g_P^{PCAC} and a 5% admixture of the $S=3/2$ orthomolecular $p\mu p$ state, we find that the generated photon spectrum is close to the experimental one.

In addition, we have found that the choice $Y=1.75,Z=-1.95$ alone also leads to an enhancement, which is of the right size to describe the experimental photon spectrum [27,28], using for the induced pseudoscalar form factor, g_P of Eq. (1.24) without any scaling $(x=1)$. It would be difficult to find any physics behind the scaling of g_P, which would mean a violation of PCAC.

It follows from our results that the too large absolute values of the parameter Z needed to explain the existing data on the photon spectrum can simulate the presence of some effects such as some other systematic errors in the experiment [27,28] besides those that were already taken into account, or molecular phenomena, which may not be well understood at present. Evidently, independent verification of the TRIUMF data would be of great importance. On the other hand, the part of our full model containing the electroweak interaction of the off-shell Δ isobar is widely used to describe successfully the pion photo-production and electroproduction data off the nucleon, which provides a solid basis for confidence in our results. For the reaction of the RMC in hydrogen, Eq. (1.11), it is the only model known so far, providing enough enhancement in the high energy region of the photon spectrum.

In conclusion we note that the reactions of the RMC in hydrogen and ^{3}He are at present the only available effective tools for the study of the form factor g_P as a function of the momentum transfer. Therefore, more efforts in investigation of these reactions, both theoretical and experimental, are highly desirable.

ACKNOWLEDGMENTS

This work was supported by Grant No. GA ČR 202/00/1669. The research of F.C.K. is supported in part by NSERCC. A part of this work was done during the stay of E.T. at the Theoretical Physics Institute of the University of Alberta. He thanks Professor F. C. Khanna for the warm hospitality. The help with computations by R. Teshima is acknowledged. We thank Professor R. Davidson for the correspondence and Dr. O. Dragoun for discussions.

[1] R. J. Blin-Stoyle, *Fundamental Interactions and the Nucleus* (North-Holland, London/American Elsevier/New York, 1973).
[2] E. Ivanov and E. Truhlík, Nucl. Phys. **A316**, 437 (1979).
[3] J. Smejkal, E. Truhlík, and F.C. Khanna, Few-Body Syst. **26**, 175 (1999).
[4] S.L. Adler and Y. Dothan, Phys. Rev. **151**, 1267 (1966); **164**, 2062(E) (1967).
[5] A. Liesenfeld *et al.*, Phys. Lett. B **468**, 20 (1999).
[6] V. Bernard, N. Kaiser, and Ulf-G. Meissner, Phys. Rev. D **50**, 6899 (1994).
[7] V. Bernard, H.W. Fearing, T.R. Hemmert, and Ulf-G. Meissner, Nucl. Phys. **A635**, 121 (1998).
[8] H.W. Fearing, R. Lewis, N. Mobed, and S. Scherer, Phys. Rev. D **56**, 1783 (1997).
[9] G. Bardin *et al.*, Nucl. Phys. **A352**, 365 (1981).
[10] V.V. Vorobyov *et al.*, Hyperfine Interact. **101/102**, 413 (1996).
[11] P. Ackerbauer *et al.*, Phys. Lett. B **417**, 224 (1998).
[12] J.G. Congleton and E. Truhlík, Phys. Rev. C **53**, 956 (1996).
[13] L. Klieb and H.P.C. Rood, Phys. Rev. C **29**, 223 (1984).
[14] Shung-Ichi Ando, Harold W. Fearing, and Dong-Pil Min, Phys. Rev. C **65**, 015502 (2002).
[15] K. Huang, C.N. Yang, and T.D. Lee, Phys. Rev. **108**, 1340 (1957).
[16] G.K. Manacher and L. Wolfenstein, Phys. Rev. **116**, 782 (1959).
[17] G.A. Lobov and I.S. Shapiro, Sov. Phys. JETP **16**, 1286 (1963).
[18] G.I. Opat, Phys. Rev. **134**, B428 (1964).
[19] H.P.C. Rood and H.A. Tolhoek, Nucl. Phys. **A70**, 658 (1965).
[20] D. Beder, Nucl. Phys. **A258**, 447 (1976).
[21] P. Christillin and S. Servadio, Nuovo Cimento Soc. Ital. Fis., A **42A**, 165 (1977).

[22] H.W. Fearing, Phys. Rev. C **21**, 1951 (1980).
[23] M. Gmitro and A.A. Ovchinnikova, Nucl. Phys. **A356**, 323 (1981).
[24] L. Klieb, Nucl. Phys. **A442**, 721 (1985).
[25] M. Gmitro and P. Truöl, Adv. Nucl. Phys. **18**, 241 (1987).
[26] D.S. Beder and H.W. Fearing, Phys. Rev. D **35**, 2130 (1987); **39**, 3493 (1989).
[27] G. Jonkmans *et al.*, Phys. Rev. Lett. **77**, 4512 (1996).
[28] D.H. Wright *et al.*, Phys. Rev. C **57**, 373 (1998).
[29] V. Bernard, T.R. Hemmert, and Ulf-G. Meissner, Nucl. Phys. **A686**, 290 (2001).
[30] T. Meissner, F. Myhrer, and K. Kubodera, Phys. Lett. B **416**, 36 (1998).
[31] S.I. Ando and D.P. Min, Phys. Lett. B **417**, 177 (1998).
[32] S.I. Ando, F. Myhrer, and K. Kubodera, Phys. Rev. C **63**, 015203 (2001).
[33] S.I. Ando, F. Myhrer, and K. Kubodera, nucl-th/0109068.
[34] T.R. Hemmert, B.R. Holstein, and J. Kambor, J. Phys. G **24**, 1831 (1998).
[35] A. Halpern, Phys. Rev. **134**, A34 (1964).
[36] D.D. Bakalov *et al.*, Nucl. Phys. **A384**, 302 (1982).
[37] I.T. Cheon and M.K. Cheoun, nucl-th/9811009.
[38] J. Smejkal and E. Truhlík, nucl-th/9811080.
[39] I.T. Cheon and M.K. Cheoun, nucl-th/9906035.
[40] H.W. Fearing, nucl-th/9811027.
[41] H.W. Fearing and R.S. Sloboda, Nucl. Phys. **A340**, 342 (1980).
[42] F.C. Khanna and E. Truhlík, Nucl. Phys. **A673**, 455 (2000).
[43] M.G. Olsson and E.T. Osypowski, Nucl. Phys. **B87**, 399 (1975).
[44] R. Davidson, N.C. Mukhopadhyay, and R. Wittman, Phys. Rev. Lett. **56**, 804 (1986).
[45] R.M. Davidson, N.C. Mukhopadhyay, and R.S. Wittman, Phys. Rev. D **43**, 71 (1991).
[46] M. Benmerrouche, R.M. Davidson, and N.C. Mukhopadhyay, Phys. Rev. C **39**, 2339 (1989).
[47] R. M. Davidson (private communication).
[48] C. Mertz *et al.*, Phys. Rev. Lett. **86**, 2963 (2001).
[49] V. Bernard, N. Kaiser, and Ulf-G. Meissner, Int. J. Mod. Phys. E **4**, 193 (1995).
[50] D. Drechsel *et al.*, Nucl. Phys. **A645**, 145 (1999).
[51] T. Sato and T.-S.H. Lee, Phys. Rev. C **54**, 2660 (1996).
[52] J. Smejkal, E. Truhlík, and F. C. Khanna, in *Mesons and Light Nuclei '98, Proceedings of the 7th International Conference*, Průhonice, edited by J. Adam, Jr. *et al.* (World Scientific, Singapore, 1999), p. 490; J. Smejkal, E. Truhlík, and F.C. Khanna, nucl-th/0110007.
[53] E. Truhlík, J. Smejkal, and F.C. Khanna, Nucl. Phys. **A689**, 741 (2001).
[54] H.W. Fearing, R. Lewis, N. Mobed, and S. Scherer, Nucl. Phys. **A631**, 735c (1998).
[55] S. Choi *et al.*, Phys. Rev. Lett. **71**, 3927 (1993).
[56] S. L. Adler and R. Dashen, *Current Algebras* (Benjamin, New York, 1968).
[57] N. Dombey and B.J. Read, Nucl. Phys. **B60**, 65 (1973).
[58] E.A. Ivanov and E. Truhlík, Fiz. Elem. Chastits At. Yadra **12**, 492 (1981) [Sov. J. Part. Nucl. **12**, 198 (1981)].
[59] D. Drechsler and L. Tiator, J. Phys. G **18**, 449 (1992).
[60] E. Truhlík, Phys. Rev. C **64**, 055501 (2001).
[61] J. Thakur and L.L. Foldy, Phys. Rev. C **8**, 1957 (1973).
[62] H. Sugawara and F. von Hippel, Phys. Rep. **172**, 1764 (1968).
[63] L. Alvarez-Ruso, S.K. Singh, and M.J. Vicente Vacas, Phys. Rev. C **59**, 3386 (1999).
[64] M. Bando, T. Kugo, and K. Yamawaki, Phys. Rep. **164**, 217 (1988).
[65] Ulf-G. Meissner, Phys. Rep. **161**, 213 (1988).
[66] N. Kaiser and Ulf-G. Meissner, Nucl. Phys. **A519**, 671 (1990).
[67] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
[68] I.M. Shmushkevich, Nucl. Phys. **11**, 419 (1959).
[69] S. Weinberg, Phys. Rev. Lett. **4**, 575 (1960).