Axial current conservation in nonrelativistic nuclear physics: The nonlinear σ model

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We analyze partially conserved axial current in the nonlinear realization of chiral symmetry in nuclear physics. We construct the two-nucleon (meson-exchange) axial currents and associated pion emission and absorption operators and compare them with those derived earlier in the linear σ model. We show the absence of necessity of meson-exchange currents in the nonlinear model, in contrast with the linear one.

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I. INTRODUCTION

Theoretical studies of axial meson-exchange current (MEC) have a long history: their perhaps most important applications have been to neutrino reactions with astrophysical significance, such as the $pp \rightarrow De^+\nu_e$. More recently a new class of nuclear weak neutral current (WNC) reactions has been measured in the form of parity violating inelastic electron-deuteron scattering which also calls for axial MEC. The latter process involves both vector [electromagnetic (EM) and WNC] and axial vector currents. Whereas one can define ''model-independent'' EM meson exchange current [1], it is less clear if that can be done for axial exchange current. The *raison d'être* for the axial MEC seems weaker than the one for EM current: The exact conservation of the EM current is underwritten by the local gauge invariance of QED, to be contrasted with the ''merely'' *partially conserved* axial current (PCAC). PCAC, on the other hand, is a consequence of both *spontaneously* and *explicitly* broken chiral symmetry of the strong interactions, which even in the case of a single nucleon and in the limit of no explicit breaking, i.e., in the chiral limit, is rather complicated and not fully understood. Specifically, there are two distinct ways chiral symmetry can be realized: (i) the linear and (ii) the nonlinear realization. The jury is still out on the question of which realization is the ''right'' one or if the question is a meaningful one.

There is a nonlinear ''unitary'' transformation between the relativistic chiral Lagrangians in the two realizations, which often obfuscates manifest differences between them. This (unitary) ''equivalence theorem'' holds only at the level of *exact* solutions to the two Lagrangians, which in reality (of course) are not available. But if there is ''equivalence'' at the relativistic level, there ought to be also ''equivalence'' at the nonrelativistic level, especially because both the linear and nonlinear πNN interactions reduce to the same nonrelativistic interaction. We wish to explore this question here.

Some indications as to the answer already exist, however: In *any* (no matter how good) approximation to the linear and nonlinear Σ models, differences are bound to show up. These differences have been clearly exhibited in the case of relativistic nuclear bound-state axial current matrix elements (MEs), as derived in the Bethe-Salpeter (BS) formalism [2] in the two models: (1) the nonlinear theory is chirally invariant both in the impulse approximation (IA) (one-nucleon current only) and with MEC (two-nucleon current), whereas (2) the linear realization requires both to maintain chiral symmetry. The latter result has been confirmed in the nonrelativistic (NR) limit: In an earlier publication [3] a systematic study of axial current (partial) conservation or, equivalently, of chiral symmetry was begun in the traditional nonrelativistic (Schrödinger equation) approach to nuclear physics. There it was found that PCAC in models with linear realization of chiral symmetry puts constraints not only on the form of the axial current operator, which requires both IA and MEC, but also on the nuclear wave function, by way of fixing the potential entering the nuclear Schrödinger equation.

In view of the aforementioned ''equivalence'' of the two theories, it seems reasonable to expect that the same kind of constraint will carry over into the nonlinear (NL) realization in the NR limit. We shall show in this paper, however, that this conjecture is incorrect. This begs the question: what is the cause of these differences? In this paper we offer an answer to this question by way of extending the NR analysis of Ref. [3] to the nonlinear realization of chiral symmetry. We show that even at the nonrelativistic level there are dramatic differences between the two realizations of chiral symmetry, i.e., between the two standard versions of the σ model, as there are also in the relativistic case, ''equivalence theorem'' notwithstanding.

This paper falls into five sections. After the Introduction, in Sec. II, we try to define chiral symmetry in nonrelativistic systems. In Sec. III we construct one- and two-nucleon axial currents that respect chiral symmetry and PCAC at the level of *nuclear* matrix elements, starting from the NL σ model. In Sec. IV we compare the results with those of the linear σ model and discuss the differences. In Sec. V we summarize and draw the conclusions. In the Appendix we define the nonlinear σ model in its relativistic and nonrelativistic forms.

II. CHIRAL SYMMETRY IN NONRELATIVISTIC NUCLEAR SYSTEMS

Any quantum mechanical symmetry consists of three parts: (1) an invariance (''gauge'') transformation, (2) the corresponding Nöther currents, and (3) the Nöther charges form (''close'') the (Lie) algebra of the invariance (Lie)

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group. In a relativistic field theory the linear chiral transformation is defined with the help of the γ_5 matrix. As there is no such thing as a γ_5 matrix in nonrelativistic quantum mechanics, there can be no corresponding transformation and symmetry, so we are faced with the general question of defining linearly realized chiral symmetry in nonrelativistic nuclear physics. We can do that only by a nonrelativistic reduction of the relativistic currents, while having to forgo or lose the invariance; see Ref. [3]. Nonlinear realization of chiral symmetry, on the other hand, does not depend on the γ_5 matrix, but only on the *dynamical* pion field's nonlinear transformation properties. The emphasis is here on the word "dynamical," as a static pion field does not possess a canonical momentum, so the Nöther charges vanish identically. Thus, nonlinearly realized chiral symmetry can be defined even in the nonrelativistic limit, as long as the pion field is dynamical. However, nonrelativistic nuclear physics does not ordinarily involve dynamical pions, only their "remnants" in the form of the static two-body one-pion-exchange potential (OPEP). So, once again, chiral symmetry disappears in nonrelativistic nuclear physics. In the following we shall make these remarks quantitative. This lack of a (unique) definition of the (nonrelativistic) chiral transformation is the major difference of PCAC from EM current conservation ("gauge invariance").

Thus, it ought to be clear that (approximate) nonrelativistic chiral symmetry depends on the (degree of) approximation to the original relativistic theory. There are (at least) two distinct levels of approximation of relevance to the present discussion: (1) (relativistic or nonrelativistic) dynamic nucleons and dynamic (relativistic or nonrelativistic) pions and (2) nonrelativistic nucleons and static pions or nonrelativistic nucleons alone (no pions). Further, for practical reasons, we shall confine ourselves to the one-meson-exchange potential approximation, which has a well-defined meaning within quantum field theory (QFT). Any substantial deviation from the original QFT, such as the introduction of a mean-field one-body potential, may forfeit the underlying chiral symmetry.

Partial conservation of the nuclear axial current

Partial conservation of axial current demands that the (hadronic) axial current $J^a_{\mu 5}$ satisfy the continuity equation

$$\partial^\mu J^a_{\mu 5} = -f_\pi m_\pi^2 \Pi^a + \cdots \tag{1}$$

or, equivalently,

$$\boldsymbol{\nabla}\cdot\mathbf{J}^a_5(\mathbf{R}) + \frac{\partial \rho^a_5(\mathbf{R})}{\partial t} = -f_\pi m_\pi^2 \Pi^a(\mathbf{R}) + \cdots, \tag{2}$$

where Π^a is the (canonical) pion field operator. In the quantum mechanical framework this can be written as an equation relating the divergence of the three-current and the commutator of the Hamiltonian and the axial charge density:

$$\boldsymbol{\nabla}\cdot\mathbf{J}^a_5(\mathbf{R}) + i[H,\rho^a_5(\mathbf{R})] = -f_\pi m_\pi^2 \Pi^a(\mathbf{R}). \tag{3}$$

This equation is a consequence of the (exact) Heisenberg equations of motion and ought to hold in every reasonable approximation.

Now we specialize to nonrelativistic nuclear physics by limiting ourselves to that subspace of the complete Hilbert space that contains at most two (real) nucleons interacting by exchanging one (virtual) meson at a time. The total Hamiltonian of the nucleus H is the sum of the kinetic and potential energies $H=T+V$ of the nucleons, and the total axial current $\mathbf{J}^a_5(\mathbf{R})$ consists of one- and two-nucleon parts.

1. Linear σ model

As stated above, there is no NR equivalent of the γ_5 matrix; hence, the axial current in the NR limit of the linear Σ model is *not* a Noether current. The nuclear axial charge density ρ^a_5 is given by the sum of nonrelativistic one-nucleon axial charge densities $\rho^a_{5,1\text{-b}}$. The axial current conservation equation is broken up into one- and two-body parts without loss of generality. The divergence of the complete one-body current equals $-i$ times the commutator of the kinetic energy T and the one-body axial charge density

$$\boldsymbol{\nabla}\cdot\mathbf{J}^a_5(1\text{-body}) = -i[T,\rho_5(1\text{-body})] - f_\pi m_\pi^2 \Pi^a(1\text{-body}) \tag{4}$$

is of $\mathcal{O}(M^{-2})$, i.e., zero to leading order in $1/M$, due to similar momentum dependences of the kinetic energy T and the axial charge density ρ^a_5(1-body) operators, where

$$\rho^a_{5,(i)}(\mathbf{p}_i',\mathbf{p}_i) = \frac{\tau^a_{(i)}}{2}\boldsymbol{\sigma}_{(i)}\cdot\left(\frac{\mathbf{p}_i'+\mathbf{p}_i}{2M}\right), \tag{5}$$

as well as to the absence of nondiagonal isospin operators from T. The test of the conservation of the complete nuclear axial current is whether or not the potential V commutes with the one-body axial charge density. It turns out that, as a result of the momentum dependence of the operator ρ^a_5(1-body), only a completely trivial, viz., a spatially everywhere constant, potential commutes with the axial charge. In nuclear physics, therefore, one *always* needs a two-body axial current $\mathbf{J}^a_5(2\text{-body}) = \Sigma^A_{j<k}\mathbf{J}_{5,(jk)}(2\text{-body})$ to compensate for the temporal change of the axial charge density in the linear Σ model (see Ref. [3]):

$$\boldsymbol{\nabla}\cdot\mathbf{J}^a_5(2\text{-body}) = -i[V,\rho^a_5(1\text{-body})0] - f_\pi m_\pi^2 \Pi^a(2\text{-body}). \tag{6}$$

Thus the commutator $[V_{2\text{-b}},\rho^a_5]$ is a nonvanishing object that plays a crucial role in maintaining nuclear PCAC in the linear Σ model. We shall not show the form of the axial two-body current in the linear Σ model, as it can be found in Sec. III B of Ref. [3]. We just note here that both σ and π-exchange currents are involved, as dictated by the one-meson-exchange approximation for the two-body potential and the PCAC condition, Eq. (6).

Finally, one may ask why the one-body mean-field potential, which is commonplace in many nuclear physics application, has been omitted? The answer is that it does not have a chirally invariant relativistic field-theoretical definition from which one could deduce a nonrelativistic version of the potential and the corresponding one-body axial ''meson-exchange'' current. The mean-field approximation generally breaks symmetries (translational, gauge) and the same is true of chiral symmetry. Any axial current made to satisfy PCAC with a one-body potential must necessarily contain an element of arbitrariness.

2. Nonlinear σ model

Nuclear PCAC holds in the nonlinear σ model as well, only this time the one- and two-nucleon parts of the axial current $\mathbf{J}_5^a$ are separately (partially) conserved. This means that all (both one- and two-body) the axial currents in momentum space have the form (see Sec. III)

$$\mathbf{J}_5^a(\mathbf{q}) = \mathbf{j}_5^a(\mathbf{q}) - [\mathbf{q}\cdot\mathbf{j}_5^a(\mathbf{q})](\mathbf{q}^2 + m_\pi^2)^{-1}. \tag{7}$$

This generic form can be often found in the older literature as an *ad hoc* prescription for the construction of the gauge-invariant EM currents, which is, of course, arbitrary. But in the case of the nonlinear Σ model it is a definite prediction and it represents the well-known statement that the chiral Ward identities are trivially satisfied in that model.

The axial current continuity equation in the momentum space becomes

$$\mathbf{q}\cdot\mathbf{J}_5^a(\mathbf{q}) = \mathcal{O}(m_\pi^2), \tag{8}$$

which is equivalent to the configuration space equation

$$\boldsymbol{\nabla}\cdot\mathbf{J}_5^a(\mathbf{R}) = -f_\pi m_\pi^2 \Pi^a(\mathbf{R}). \tag{9}$$

This, in turn, is equivalent to the continuity equation (3) with the commutator $[H,\rho_5^a(\mathbf{R})]=0$ set equal to zero. As discussed above, that is only possible if $\rho_5^a=0$, the result proved in the Appendix. Thus, nuclear PCAC is independent of the nuclear Hamiltonian H in the nonrelativistic nonlinear Σ model, and there is no need for consistency between the nuclear axial current and nuclear dynamics (Hamiltonian and wave functions). This is a consequence of the vanishing nucleon axial charge in this model.

3. Comparison

The nuclear axial current matrix elements in the nonlinear σ model satisfy PCAC even when the nuclear wave functions used in the calculation are *not* solutions to the Schrödinger equation with the corresponding one-pion-exchange potential, in contrast to the linear realization of chiral symmetry. The same conclusion holds for the nuclear pion production amplitude. Thus we have found a lack of need of consistency between the pion creation and absorption operators and the nuclear Hamiltonian.

This finding is in accorance with the results found in the same model, within the relativistic Bethe-Salpeter approach [2]. Moreover, the result agrees with the original philosophy of the nonlinear effective Lagrangian approach, viz., to achieve PCAC without cancellations between different diagrams and independently of the (nuclear) dynamics [4]. Now we see that this program has also the feature that chiral symmetry does not survive in the static pion limit.

A detailed comparison of the axial MEC derived in the linear (see Ref. [3]) and nonlinear Σ models (see below) indicates a clear functional difference between the two. No amount of adjusting of parameters can turn one into another. That, however, does not necessarily mean that the difference is observable in experiment: the common wisdom at the moment holds that the dominant axial MEC is induced by intermediate Δ resonance state, which does not appear in the simplest versions of either of these two models.

III. NUCLEAR AXIAL CURRENT IN THE NONRELATIVISTIC NONLINEAR Σ MODEL

One can take the nonrelativistic reduction of the relativistic Lagrangian Eq. (A1) to any given order in $1/M$, of course with different detailed results, but always with the same generic structure. The deciding factor here is the presence or absence of the time-dependent pion field: for static pion fields there is no conjugate momentum and hence no axial charge algebra, irrespective of other properties of the Lagrangian. Thus, e.g., the nucleons may remain relativistic, as described by the Dirac equation (though their interactions with pions would necessarily break Lorentz symmetry due to the static, i.e., Lorentz-variant pions) and still their axial charge algebra would not close. Such a ''semirelativistic'' model seems unnatural and we replace it with one with nonrelativistic nucleons and static pions.

In the following we look at the nonlinear σ model axial currents in a nonrelativistic setting by first expanding the relativistic Lagrangian Eq. (A1) in powers of $\boldsymbol{\pi}^2/f_\pi^2$, then making a nonrelativistic reduction, and finally applying Nöther's theorem to obtain the axial current. Thus we find a set of nonrelativistic axial currents that are partially conserved *independently of each other*.[1] This is the distinguishing feature of the nonlinear realization of chiral symmetry.

A. Model and its axial Nöther current

We expand the Lagrangian (A1) to leading order in the nucleon mass and the second nontrivial order in $1/f_\pi$, while keeping a *static* pion field, i.e., expand to $\mathcal{O}(f_\pi^{-2})$ and set $\dot{\boldsymbol{\pi}}=0$, and find

$$\begin{aligned}\mathcal{L} = {} & \psi^\dagger\left[i\partial_t + \frac{\vec{\nabla}^2}{2M}\right]\psi - \frac{1}{2}[(\vec{\nabla}\boldsymbol{\pi})^2 + m_\pi^2\boldsymbol{\pi}^2] \\ & + \left(\frac{f}{m_\pi}\right)\psi^\dagger\boldsymbol{\tau}\cdot(\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi})\psi \\ & + \left(\frac{1}{2f_\pi}\right)^2\psi^\dagger\boldsymbol{\tau}^a\left[\frac{\overleftrightarrow{\nabla}}{2M} + i\,\frac{\vec{\sigma}\times\overleftrightarrow{\nabla}}{2M}\right]\psi\cdot(\boldsymbol{\pi}\times\vec{\nabla}\boldsymbol{\pi})^a + \cdots,\end{aligned} \tag{10}$$

[1]Weinberg [5] was addressing precisely this aspect when he emphasized the non-γ_5 nature of the nonlinear realization of chiral symmetry.

where $\overleftrightarrow{\nabla}=\vec{\nabla}-\overleftarrow{\nabla}$. The Lagrangian (10) is only invariant under chiral transformations, Eq. (A3), to first order in $1/f_\pi$. This means that the associated Nöther current is *not exactly conserved even in the chiral limit*, the remnant being of finite order in $1/f_\pi$, in this case of $\mathcal{O}(f_\pi^{-2})$.

Next we apply Nöther's theorem together with the axial transformation properties, Eq. (A3), to find

$$\vec{\mathbf{J}}_5^a=-f_\pi\vec{\nabla}\boldsymbol{\pi}^a+g_A\psi^\dagger\left(\frac{\boldsymbol{\tau}^a}{2}\vec{\sigma}\right)\Psi + \psi^\dagger\left[\frac{(\vec{\nabla}-\overleftarrow{\nabla})}{M}+i\frac{\vec{\sigma}\times\overleftrightarrow{\nabla}}{2M}\right]\left(\boldsymbol{\tau}\times\frac{\boldsymbol{\pi}}{2f_\pi}\right)^a\psi+\cdots. \tag{11}$$

Note that these terms can be divided into different categories depending on how many pion and/or nucleon fields they contain. For example, those containing one pion only, three pions only, etc.; one nucleon only, one nucleon plus one pion, nucleon plus two pions, etc. We shall separate out two such terms: (1) the one-nucleon axial current and (2) the one-nucleon plus one-pion axial current. We cannot do much more with this in configuration space, so we turn to the momentum representation.

B. Axial current vertices and their Ward identities

1. One-nucleon axial current vertex

The complete one-body current vertex receives a contribution from the one nucleon term in the Nöther current (11), as well as one from the pion-pole graph. Since the Goldberger-Treiman (GT) relation $g_AM=g_{\pi NN}f_\pi$ (here $g_A=1.26$) holds in the nonlinear σ model, we can write the one-body axial current vertex as

$$\mathbf{J}_{5,(i)}^a(\mathbf{p}_i',\mathbf{p}_i)=g_A\frac{\tau_{(i)}^a}{2}\left[\boldsymbol{\sigma}_{(i)}-\mathbf{q}\left(\frac{\boldsymbol{\sigma}_{(i)}\cdot\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\right)\right], \tag{12}$$

which separately satisfies the (nonrelativistic) single-nucleon axial Ward-Takahashi identity (WT ID)

$$\begin{aligned}\mathbf{q}\cdot\mathbf{J}_{5,(i)}^a(\mathbf{p}_i',\mathbf{p}_i)&\simeq f_\pi\left(\frac{m_\pi^2}{\mathbf{q}^2+m_\pi^2}\right)g_{\pi NN}\tau_{(i)}^a\left(\frac{\boldsymbol{\sigma}_{(i)}\cdot\mathbf{q}}{2M}\right)\\&=-i\left(\frac{f_\pi m_\pi^2}{\mathbf{q}^2+m_\pi^2}\right)\Gamma_\pi^a(\mathbf{p}_i',\mathbf{p}_i;1\text{-body}).\end{aligned} \tag{13}$$

Note the absence of $[T,\rho_5^a(1\text{-body})]$ on the right-hand side (RHS) of the identity. This is consistent with vanishing of this commutator. The one-body axial current is just the renormalized (by a factor of g_A) version of the linear σ model one. It is commonly assumed that the same holds for the axial charge, as well. The latter assumption, however, is incorrect, as shown in the Appendix.

2. Nucleon-pion axial current vertex

To construct the nucleon-pion axial current vertex in this model we start from the corresponding term in the axial Nöther current, Eq. (11), and add the pion-pole graph

$$\begin{aligned}\mathbf{A}_{\mathrm{I}}^a(\mathbf{p}_1,\mathbf{p}',\mathbf{q},\mathbf{k}^b)=\frac{1}{2f_\pi M}\varepsilon^{abc}\tau^c\Bigg[&\mathbf{p}+\mathbf{p}'+\frac{i}{2}\boldsymbol{\sigma}_{(1)}\times\mathbf{q}\\&-\frac{\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\mathbf{q}\cdot\left(\mathbf{p}_1+\mathbf{p}_1'+\frac{i}{2}\boldsymbol{\sigma}_{(1)}\times\mathbf{k}_1\right)\Bigg],\end{aligned} \tag{14}$$

where three-momentum conservation reads $\mathbf{p}_1+\mathbf{p}_2+\mathbf{q}+\mathbf{k}=0$. This axial current vertex alone maintains chiral symmetry, as can be seen from the corresponding divergence, i.e., the Ward identity, which reads

$$\mathbf{q}\cdot\mathbf{A}_{\mathrm{I}}^a(\mathbf{p},\mathbf{p}',\mathbf{q},\mathbf{k}^b)=\mathcal{O}(f_\pi m_\pi^2). \tag{15}$$

C. The two-nucleon axial current

To construct the partially conserved nonrelativistic axial two-nucleon current in this model we start from the corresponding axial current vertex (14) and attach the free pion "leg" to the second nucleon:

$$\begin{aligned}\mathbf{J}_{5,2\text{-body}}^a(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})=(\vec{\tau}_{(1)}\times\vec{\tau}_{(2)})^a\frac{g_{\pi NN}}{2f_\pi M}\Bigg[&\mathbf{p}_1+\mathbf{p}_1'+\frac{i}{2}\boldsymbol{\sigma}_{(1)}\times\mathbf{k}_1\\&-\frac{\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\mathbf{q}\cdot\left(\mathbf{p}_1+\mathbf{p}_1'+\frac{i}{2}\boldsymbol{\sigma}_{(1)}\times\mathbf{k}_1\right)\Bigg]\\&\times\frac{\boldsymbol{\sigma}_{(2)}\cdot\mathbf{k}_2}{(\mathbf{k}_2^2+m_\pi^2)}+(1\leftrightarrow2),\end{aligned} \tag{16}$$

where $\mathbf{k}_i=\mathbf{p}_i-\mathbf{p}_i'$, $i=1,2$. Three-momentum conservation demands $\mathbf{k}_1+\mathbf{k}_2+\mathbf{q}=0$. This MEC maintains PCAC by itself, as can be seen from the corresponding divergence

$$\mathbf{q}\cdot\mathbf{J}_{5,2\text{-body}}^a(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})=\mathcal{O}(f_\pi m_\pi^2); \tag{17}$$

i.e., PCAC is satisfied by this (individual) two-body current independently of the one-body current and of the commutator $[V,\rho_5^a(\mathbf{R})]$, i.e., as if $[V,\rho_5^a(\mathbf{R})]=0$.

IV. COMPARISON

The functional forms of the axial one-body currents are identical (up to the overall multiplicative constant) in the two models; the forms of the two-body axial currents are entirely different (so much so that they cannot be meaningfully compared). The dominant axial MEC is believed to be induced by the Δ resonance and is transverse, i.e., unconstrained by the continuity equation, in either formalism. It remains to be seen if this ambiguity in axial MEC can be settled by a judiciously chosen experiment.

Of course, one must remember that we have constructed only the leading term and lightest meson axial MEC. If one were to continue this process to include heavier mesons, one might find further significant differences that would ultimately lead to observable consequences.

This point is not purely academic: the standard model is a gauge theory with exactly conserved weak left isospin current; the phenomenological effective field theories that purport to describe it always violate this symmetry (e.g., by the pion mass term). There must be an observable consequence of this nonconservation, though it is not known today.

The nuclear axial current matrix elements in the nonlinear σ model satisfy PCAC even when the nuclear wave functions used in the calculation are *not* solutions to the Schrödinger equation with the corresponding one-pion-exchange potential, in contrast to the linear realization of chiral symmetry. The same holds for the nuclear pion production amplitude. Thus we have found a lack of necessity of consistency between the pion creation and absorption operators and the nuclear Hamiltonian.

This finding is in accordance with the results found in the same model, within the relativistic Bethe-Salpeter approach [2]. Moreover, the result agrees with the original philosophy of the nonlinear effective Lagrangian approach, viz., to achieve PCAC without cancellations between different diagrams and independently of the (nuclear) dynamics [4]. Now we see that this program has also the (originally unwanted) feature that chiral symmetry does not survive in the static pion limit.

A. Chiral algebra closure

At this stage one ought to make sure that the distinction between PCAC and chiral symmetry is clear: the former is a necessary precondition for the latter, whereas the latter also demands closure of the chiral charge algebra. The chiral charge algebra is a prerequisite for many chiral low-energy transistors (LETs), though not all.[2] Specifically, the Tomozawa-Weinberg (TW) relations demand it (see Adler's original derivation). Thus, the model Lagrangian (10) does *not* lead to TW relations in nuclei even though it satisfies PCAC. Closure of the chiral charge algebra is often taken for granted, however, although the above example warns against it.

1. Nonlinear Σ model

Of course, in the nonlinear Σ model with only nucleons the axial charge vanishes (see the Appendix) and there is no point in talking about chiral algebra closure at all. But we wish to address this question in the nonlinear model with static pions when there might be a vestige of an axial charge operator. Indeed, Eq. (A5) together with $\dot{\boldsymbol{\pi}}=0$ leads to

$$\rho_5^a=\Psi^\dagger\left(\frac{\boldsymbol{\tau}\times\boldsymbol{\pi}}{4f_\pi}\right)^a\Psi, \tag{18}$$

which in turn leads to

$$\begin{aligned}[\rho_5^a(0,\mathbf{x}),\rho_5^b(0,\mathbf{y})]&=i\varepsilon^{abc}\delta(\mathbf{x}-\mathbf{y})\,\boldsymbol{\pi}^c\Psi^\dagger\left(\frac{\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{8f_\pi^2}\right)\Psi\\ &\neq i\varepsilon^{abc}\delta(\mathbf{x}-\mathbf{y})\rho^c(0,\mathbf{x}).\end{aligned} \tag{19}$$

Clearly, there is no closure here. This is merely a manifestation of point (3) in the Appendix, viz., that in the nonlinear Σ model the nucleon axial charge does not close the chiral algebra by itself, i.e., without the pionic part. Hence no chiral LET that depends on closure can hold in this model and in the static pion approximation.

2. Linear Σ model

The nonrelativistic linear σ model has its own problems, however: the chiral algebra does not close either. More specifically, whereas the commutators (A10a) and (A10c) still hold, the double axial charge commutator (A10b) does not. In detail,

$$\begin{aligned}[\rho_5^a(0,\mathbf{x}),\rho_5^b(0,\mathbf{y})]&=i\varepsilon^{abc}\delta(\mathbf{x}-\mathbf{y})\Psi^\dagger\frac{\tau^c}{2}\left(\frac{\mathbf{p}}{M}\right)^2\Psi\\ &\neq i\varepsilon^{abc}\delta(\mathbf{x}-\mathbf{y})\rho^c(0,\mathbf{x}),\end{aligned} \tag{20}$$

the difference being the $(\mathbf{p}/M)^2$ factor. This discrepancy is a manifestation of the relativistic nature of the linear realization of chiral symmetry and of the nonrelativistic approximation used here. For this reason here, just as in the static pion nonlinear Σ model, we do not expect the Tomozawa-Weinberg and other related chiral LETs to be fulfilled for nuclei.

V. SUMMARY AND CONCLUSIONS

We showed explicitly that the (spatial parts of the) axial current in the nonlinear σ model satisfies PCAC separately at the one- and two-nucleon levels without constraints from the Hamiltonian. Nor is there compulsion to introduce two-body axial current. In this sense the nonlinear σ model is profoundly different from the linear one. Therefore one cannot define model-independent axial currents along the same lines as in the EM case [1]. This is in agreement with an earlier study of the relativistic Bethe-Salpeter approach to nuclear systems [2].

Although PCAC is satisfied at the operator level, chiral symmetry is *not* preserved, as the axial charges do not close the chiral algebra, a necessary condition for many, if not all, chiral low-energy theorems. The reason for this is that the one-nucleon elastic matrix element of the axial charge vanishes, $\langle\rho_5^a\rangle_{\mathrm{N}}=0$, in the first-order perturbative approximation to the nonlinear σ model, in agreement with general results of chiral symmetry and those of heavy-baryon chiral perturbation theory (χPT). This implies that the nucleon axial charge operator vanishes in nonrelativistic nuclear models based on the nonlinear realization of chiral symmetry with static or fully integrated out pions (such as the potential models).

This stands in marked contrast to nonrelativistic nuclear dynamics based on the linear σ model and the Bjorken and

[2]Some LETs depend on the spatial components of the currents.

Nauenberg variation thereof [6], which require the existence of two-nucleon or meson-exchange axial currents as well as their consistency with the underlying nuclear Hamiltonian [3]. Nucleons in such models have nonvanishing axial charge operators, which do *not* close the chiral algebra either, due to the nonrelativistic approximation made in their derivation.

Consequently, chiral symmetry in nonrelativistic nuclear physics without pionic degrees of freedom is an imperfect concept, whether realized linearly or nonlinearly. Without the pionic degrees of freedom one cannot even talk about spontaneously broken chiral symmetry.

These results have consequences in at least two fields: (i) In the field of axial meson-exchange current in nuclear weak decays and parity violating leptonuclear scattering. It has been a custom, for more than two decades, to calculate the axial MEC in the form of corrections to the axial charge operator rather than to the current [7]. As the nucleon axial charge has been shown to vanish in the nonlinear Σ model used there, it is manifest that the whole subject will have to be reexamined. (ii) In the so-called effective field theory approach to the two-nucleon problem in which the pions have been "integrated out," i.e., where only (e.g., static contact) NN potentials are used, chiral symmetry is not defined, because the axial charge generators all vanish and thus the chiral charge algebra becomes trivial, i.e., equivalent to the isospin algebra.

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APPENDIX: NUCLEON AXIAL CHARGE IN THE NONLINEAR σ MODEL

We shall show that the single-nucleon elastic matrix element of the axial charge operator vanishes in the nonlinear Σ model. For this purpose we shall need some basic facts about the axial charge in the said model.

1. Lagrangian and the Nöther currents

The Lagrangian density of Weinberg's nonlinear σ model [4] is given by

$$\begin{aligned}\mathcal{L}=&\bar{\psi}[i\partial\!\!\!/-M]\psi+\frac{1}{2}\mathcal{R}[\mathcal{R}(\partial_\mu\boldsymbol{\pi})^2-m_\pi^2\boldsymbol{\pi}^2]+\mathcal{R}\left(\frac{f}{m_\pi}\right)\\&\times(\bar{\psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi)\cdot\partial^\mu\boldsymbol{\pi}\\&+\mathcal{R}\left(\frac{g_{\pi NN}}{2g_AM}\right)^2(\bar{\psi}\gamma_\mu\boldsymbol{\tau}\psi)\cdot(\boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi}),\end{aligned}\tag{A1}$$

where

$$\mathcal{R}=\left[1+\left(\frac{g_{\pi NN}}{2g_AM}\right)^2\boldsymbol{\pi}^2\right]^{-1}=\left[1+\left(\frac{\boldsymbol{\pi}}{2f_\pi}\right)^2\right]^{-1}$$

and

$$\left(\frac{f}{m_\pi}\right)=\left(\frac{g_A}{2f_\pi}\right)=\left(\frac{g_{\pi NN}}{2M}\right).$$

The nonlinear function of the pion fields is to be understood as an expansion in powers of $\boldsymbol{\pi}/f_\pi$ [4,8]. Manifestly, such a series has infinitely many terms, which makes it impossible to use in its entirety with our present methods. Rather, the Feynman rules and the associated Nöther currents are also defined by the power series expansion (which is essentially the method used in chiral perturbation theory).[3] That expansion, however, will be the cause of chiral noninvariance of the expanded Lagrangian and consequently of axial current nonconservation.

The *nonlinear* chiral transformations[4] are

$$\delta_5\boldsymbol{\pi}^a=f_\pi\boldsymbol{\varepsilon}_5^a\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)+\boldsymbol{\pi}^a\left(\frac{\boldsymbol{\varepsilon}_5\cdot\boldsymbol{\pi}}{2f_\pi}\right),\tag{A2}$$

$$\delta_5\Psi=i\boldsymbol{\varepsilon}_5\cdot\left(\frac{\boldsymbol{\tau}\times\boldsymbol{\pi}}{4f_\pi}\right)\Psi,\tag{A3}$$

which leave the Lagrangian (A1) invariant up to the pion mass term. Hence, the associated axial vector Noether current reads

$$\begin{aligned}\mathbf{J}_{\mu5}^a=&\mathcal{R}\left[g_A\bar{\Psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}^a}{2}\Psi+\bar{\Psi}\gamma_\mu\left(\frac{\boldsymbol{\tau}\times\boldsymbol{\pi}}{4f_\pi}\right)^a\Psi-f_\pi\mathcal{R}\partial_\mu\boldsymbol{\pi}^a\right]\\&\times\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)+\boldsymbol{\pi}^a\mathcal{R}\left[g_A\bar{\Psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\Psi\right.\\&\left.-f_\pi\mathcal{R}\partial_\mu\boldsymbol{\pi}\right]\cdot\left(\frac{\boldsymbol{\pi}}{2f_\pi^2}\right)+\mathcal{R}\bar{\Psi}\gamma_\mu\left(\frac{\boldsymbol{\tau}\times\boldsymbol{\pi}}{4f_\pi}\right)^a\Psi\\=&\mathcal{R}\left[g_A\bar{\Psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}^a}{2}\Psi-f_\pi\mathcal{R}\partial_\mu\boldsymbol{\pi}^a\right]\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\\&+\boldsymbol{\pi}^a\mathcal{R}\left[g_A\bar{\Psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\Psi-f_\pi\mathcal{R}\partial_\mu\boldsymbol{\pi}\right]\cdot\left(\frac{\boldsymbol{\pi}}{2f_\pi^2}\right)\\&+\mathcal{R}\bar{\Psi}\gamma_\mu\left(\frac{\boldsymbol{\tau}\times\boldsymbol{\pi}}{2f_\pi}\right)^a\Psi\left(1-\frac{\boldsymbol{\pi}^2}{8f_\pi^2}\right).\end{aligned}\tag{A4}$$

[3]The above form of the nonlinear Lagrangian (A1) differs by the presence of g_A in the denominators of the factors $(g_{\pi NN}/2Mg_A)$ from the standard textbook version [8]. The source of this difference, as emphasized by Weinberg [4], is the need to have both the g_A factor in the axial current and the empirically correct two-pion-nucleon contact interaction. This result can be obtained directly by a chiral rotation and unitary transformation [9] from the hybrid-heterotic σ model of Bjorken and Nauenberg [6]; see also p. 323 in Ref. [10].

[4]The second power of the pion field in these transformation laws is what gives the nonlinear realization its name.

This axial Nöther current is *not conserved*, but has a divergence of $\mathcal{O}(m_\pi^2)$. It is important to note that the $\mathcal{R}$ factors must remain unexpanded, or else the truncated expansion will be another source of the Lagrangian's (A1) symmetry breaking and of axial current nonconservation, even in the chiral limit $m_\pi \to 0$.

Now note that the axial charge has the form

$$\mathbf{J}_{05}^a = \rho_5^a = -\frac{\boldsymbol{\pi}^a}{2f_\pi}(\boldsymbol{\pi}\cdot\boldsymbol{\Pi}_\pi) - f_\pi\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\boldsymbol{\Pi}_\pi^a - i\Pi_\Psi\left(\frac{\boldsymbol{\tau}\times\boldsymbol{\pi}}{4f_\pi}\right)^a\Psi, \tag{A5}$$

where

$$\boldsymbol{\Pi}_\pi^a = \mathcal{R}^2\dot{\boldsymbol{\pi}}^a - \left(\frac{\mathcal{R}}{2f_\pi}\right)\left[\Psi^\dagger\frac{(\boldsymbol{\tau}\times\boldsymbol{\pi})^a}{2f_\pi}\Psi + g_A\Psi^\dagger\gamma_5\boldsymbol{\tau}^a\Psi\right], \tag{A6a}$$

$$\Pi_\Psi = i\Psi^\dagger \tag{A6b}$$

are the pion and the nucleon field canonical conjugate momenta. There is also the vector (isospin) charge density

$$\mathbf{J}_0^a = \bar{\psi}\gamma_0\frac{\boldsymbol{\tau}^a}{2}\psi - (\Pi_\pi\times\boldsymbol{\pi})^a, \tag{A7}$$

which follows from the conserved polar vector Nöther (isospin) current density

$$\mathbf{J}_\mu^a = \bar{\Psi}\gamma_\mu\frac{\boldsymbol{\tau}^a}{2}\Psi + \mathcal{R}\left\{\boldsymbol{\pi}\times\left[\mathcal{R}\partial_\mu\boldsymbol{\pi} - g_A\bar{\Psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2f_\pi}\Psi - \bar{\Psi}\gamma_\mu\left(\frac{\boldsymbol{\tau}\times\boldsymbol{\pi}}{4f_\pi^2}\right)\Psi\right]\right\}^a, \tag{A8}$$

which is a consequence of the Lagrangian's, Eq. (A1), invariance under the isospin transformations

$$\delta\boldsymbol{\pi} = -\boldsymbol{\varepsilon}\times\boldsymbol{\pi}, \tag{A9a}$$

$$\delta\Psi = i\left(\frac{\boldsymbol{\varepsilon}\cdot\boldsymbol{\tau}}{2}\right)\Psi. \tag{A9b}$$

These two kinds of charges "close" the chiral algebra

$$[\rho^a(0,\mathbf{x}),\rho^b(0,\mathbf{y})] = i\varepsilon^{abc}\rho^c(0,\mathbf{x})\,\delta(\mathbf{x}-\mathbf{y}), \tag{A10a}$$

$$[\rho_5^a(0,\mathbf{x}),\rho_5^b(0,\mathbf{y})] = i\varepsilon^{abc}\rho^c(0,\mathbf{x})\,\delta(\mathbf{x}-\mathbf{y}), \tag{A10b}$$

$$[\rho_5^a(0,\mathbf{x}),\rho^b(0,\mathbf{y})] = i\varepsilon^{abc}\rho_5^c(0,\mathbf{x})\,\delta(\mathbf{x}-\mathbf{y}), \tag{A10c}$$

as long as the pion $\boldsymbol{\pi}$ and nucleon Ψ fields and their associated momenta $\boldsymbol{\pi}_\pi, \Pi_\Psi$ satisfy the canonical (anti)commutation relations

$$[\pi^a(0,\mathbf{x}),\Pi_\pi^b(0,\mathbf{y})] = i\,\delta^{ab}\delta(\mathbf{x}-\mathbf{y}) \tag{A11a}$$

$$\{\Psi^\alpha(0,\mathbf{x}),\Pi_\Psi^\beta(0,\mathbf{y})\} = i\,\delta^{\alpha\beta}\delta(\mathbf{x}-\mathbf{y}). \tag{A11b}$$

Note the following.

(1) The charges (A5),(A7) satisfy ("close") the chiral algebra Eqs. (A10a), (A10b) and (A10c) independently of the form or Lorentz invariance of the underlying Lagrangian. The closure merely depends on the existence of the pion and matter fields and their canonical momenta, as well as on their chiral transformation properties, Eqs. (A3). In other words, even in the nonrelativistic version of the theory the chiral charges still exist and close the chiral algebra, as long as the pion fields are truly dynamical degrees of freedom (DOF). When the pion field loses its dynamical role, e.g., if it becomes a static field with vanishing conjugate momentum, the chiral charges cease to exist.

(2) The requirement for the vector and axial charges to close the chiral algebra can be restated as follows: the charges must have the generic shapes given in Eqs. (A5), (A7). An arbitrary truncation or expansion of the axial charges, Eq. (A5), in powers of $\boldsymbol{\pi}^a/2f_\pi$ will generally destroy closure. Expansion and truncation of the Lagrangian is necessary, as otherwise the model would be intractable. Consequently, the Nöther current must be truncated, too. But the truncation must not change the *generic* form of the axial charge (A5).

(3) Of the two parts, the mesonic and the fermionic, of each charge only the former closes the chiral algebra by itself. The fermionic charges do not form a chiral algebra without the mesonic, i.e., pionic, part, in contrast with the linear realization of chiral symmetry, where each term closes the algebra separately. This is in line with Weinberg's insistence on the "non-γ_5" nature of chiral symmetry in the nonlinear realization and with his interpretation [4,5] of chirality as only being carried by pions.

2. Nucleon axial charge elastic matrix element

Next we shall prove that the single-nucleon elastic matrix element of the axial charge vanishes to lowest-order perturbation theory. To evaluate the first-order perturbation theory one-nucleon matrix element of Eq. (A5), we switch to the interaction representation; i.e., we turn the canonical momenta into free ones and expand $\mathcal{R}$ in powers of $\boldsymbol{\pi}^2/2f_\pi^2$. Thus we find

$$\langle N|\mathbf{J}_{05}^a|N\rangle_{\text{pert}} = -\langle N|\boldsymbol{\pi}^a\left(\frac{\boldsymbol{\pi}\cdot\dot{\boldsymbol{\pi}}}{2f_\pi}\right)|N\rangle - f_\pi\langle N|\left(1-\frac{\boldsymbol{\pi}^2}{2f_\pi^2}\right)\dot{\boldsymbol{\pi}}^a|N\rangle + \left(\frac{1}{4f_\pi}\right)\langle N|\Psi^\dagger(\boldsymbol{\tau}\times\boldsymbol{\pi})^a\Psi|N\rangle = 0, \tag{A12}$$

where all fields and states are in the interaction picture now. Each of the three terms is separately zero, because each contains an odd number of pion fields; hence, one pion field is always between the vacuum states, but the pion field must have zero vacuum expectation value (VEV) to conserve parity.

The same result can also be found in the book by De Alfaro *et al.*, p. 379, Eq. (6.90) [10], where it was derived in a different, Lorentz-invariant way, however. This result is also in agreement with the heavy-baryon χPT, as can be seen from the following. In the heavy-fermion χPT [11], one has

$$\langle N|\mathbf{J}^a_{\mu 5}|N\rangle = g_A \bar{\Psi}\gamma_\mu\gamma_5 \frac{\boldsymbol{\tau}^a}{2}\Psi \to \frac{i}{2}\bar{u}\left(\gamma_5\sigma_{\mu\nu}v^\nu\frac{\boldsymbol{\tau}^a}{2}\right)u, \tag{A13}$$

as the axial current, where $v^\nu=(1,\vec{0})$. Hence, the nucleon axial charge (the $\mu=0$ component of the nucleon axial current) vanishes,

$$\langle N|\rho^a_5|N\rangle = \frac{i}{2}\bar{u}\left(\gamma_5\sigma_{0\nu}v^\nu\frac{\boldsymbol{\tau}^a}{2}\right)u=0, \tag{A14}$$

by the antisymmetry of $\sigma_{\mu\nu}=(i/2)[\gamma_\mu,\gamma_\nu]$.

In another study of nonrelativistic chiral symmetry in nuclear systems [3] we discovered that the spatial part of the axial current in the Bjorken-Nauenberg model is renormalized from unity to $g_A=1.26$, whereas the axial charge (the temporal component of the axial four-current) is *not* [3]. This fact can be interpreted as the vanishing of that part of the axial charge that is associated with the gradient coupling. In this paper it is shown that the same result holds in the nonlinear σ model, but this time the effective nucleon axial charge vanishes altogether, because the interaction is purely pseudovector in that model.

At first sight this result is, if not paradoxical, then at least perplexing, for it stands in marked contrast to the usual assumption made in nonrelativistic nuclear physics that in this model the axial charge is not only nonzero, but renormalized upwards to $g_A=1.26$. Clearly, such nonrelativistic models of the nuclear axial charge will have to be reexamined.

Moreover, it seems to imply that one has lost one subset of generators of the chiral $SU(2)_L\otimes SU(2)_R$ algebra, i.e., that the chiral symmetry is lost and/or undefined. That is not so, as long as the pions are dynamical degrees of freedom: the axial charge still exists in the nonlinear realization of chiral symmetry, but it is carried exclusively by the pions — not by the pions and the nucleons, as in the linear realization.

Hence the elastic matrix element of the nucleon axial charge may vanish, as it indeed does. Other, nonelastic matrix elements need not vanish, or else the total axial charge may then actually be zero. This last case occurs only when the pions become static, or "integrated out," as in ordinary nonrelativistic nucleons-only potential models of nuclear physics.

It is just that in the present case some of the chiral charge elastic matrix elements vanish, contrary to naive expectations. That implies, however, that certain subsectors of the complete Fock space may have unexpected properties. Specifically, the nucleons-only subspace, which may be viewed as the proper Hilbert space of the nonrelativistic nuclear physics, has vanishing axial charge elastic matrix elements. Thus we may say that the *effective* nucleon axial charge density operator in the nucleons only Hilbert subspace vanishes, $\rho^a_{5\mathrm{N}}=0$, in the nonlinear σ model.

Consequently, in those versions of chiral models, such as χPT, in which the pions are static or have been "integrated out," one cannot talk of chiral symmetry anymore because the chiral algebra does not close and/or the axial charges vanish altogether [12,13].

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