Relativistic instant-form approach to the structure of two-body composite systems

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An approach to the electroweak properties of two-particle composite systems is developed. The approach is based on the use of the instant form of relativistic Hamiltonian dynamics. The main feature of this approach is the method of construction of the matrix element of the electroweak current operator. The electroweak current matrix element satisfies the relativistic covariance conditions and in the case of the electromagnetic current also the conservation law automatically. The properties of the system as well as the approximations are formulated in terms of form factors. The approach makes it possible to formulate relativistic impulse approximation in such a way that the Lorentz covariance of the current is ensured. In the electromagnetic case the current conservation law is also ensured. Our approach gives good results for the pion electromagnetic form factor in the whole range of momentum transfers available for experiments at present time, as well as for the lepton decay constant of pions.

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I. INTRODUCTION

The construction of correct quantitative methods of calculation for structure of composite particles is an important line of investigation in particle physics. In nonrelativistic dynamics there exist different correct methods that use model or phenomenological interaction potentials. However, in the case of high energy one needs to develop relativistic methods. It is worth noting that now the experiments on accelerators, in particular, JLab, are performed with such an accuracy that the treatment of traditionally "nonrelativistic" systems (e.g. the deuteron) requires one to take into account relativistic effects. Relativistic effects are important also in the treatment of composite systems of light quarks. However, the relativistic treatment of hadron composite systems is a rather complicated problem. Let us note that the use of the methods of the field theory in this case encounters serious difficulties. For example, it is well known that perturbative QCD cannot be used in the case of quark bound states (see, e.g., Refs. [1,2]).

In the present paper we will use the relativistic constituent model that describes the hadron properties at the quark level in terms of degrees of freedom of constituent quarks. The constituent quarks are considered as extended objects, the internal characteristics of which (mean square radius, anomalous magnetic moments, form factors) are parameters of the model. As a relativistic variant of the constituent model we choose the method of relativistic Hamiltonian dynamics (RHD) (see, e.g., Refs. [3–6] and references therein).

The RHD method as a relativistic theory of composite systems is based on the direct realization of the Poincaré algebra on the set of dynamical observables on the Hilbert space. The RHD theory of particles lies between local field theoretic models and nonrelativistic quantum mechanical models.

Contrary to field theory, RHD deals with a finite number of degrees of freedom from the very beginning. This is certainly a kind of a model approach. The preservation of the Poincaré algebra ensures the relativistic invariance. So, the covariance of the description in the frame of RHD is due to the existence of the unique unitary representation of the inhomogeneous group $SL(2,C)$ on the Hilbert space of composite system states with a finite number of degrees of freedom [7].

The mathematics of RHD is similar to that of nonrelativistic quantum mechanics and permits one to assimilate the sophisticated methods of phenomenological potentials and can be generalized to describe three or more particles. The idea of this approach—RHD—was originated by Dirac. In Ref. [8] he considered different ways of describing the evolution of classical relativistic systems—different forms of dynamics. Dirac defined three main forms of dynamics: point (PF), instant (IF), and light–front (FF) dynamics. RHD is based on the simultaneous action of two fundamental principles, relativistic invariance and the Hamiltonian principle, and presents the most adequate tool to treat the systems with finite number of degrees of freedom.

Our aim is to construct a relativistic invariant approach to the electroweak structure of two-particle composite systems. The main problem here is the construction of the current operators [9–13]. It seems to us that RHD is the most adequate method for our purpose. The use of RHD enables one to separate the main degrees of freedom and thus to construct convenient models.

We use one of the forms of RHD, namely a version of the IF. Our approach has a number of features that distinguish it from other forms of dynamics and other approaches in the frames of IF.

(a) The electroweak current matrix element satisfies automatically the relativistic covariance conditions and in the

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case of the electromagnetic current also the conservation law.

(b) We propose a modified impulse approximation (MIA). It is constructed in a relativistically invariant way. This means that our MIA does not depend on the choice of the coordinate frame, and this contrasts principally with the "frame-dependent" impulse approximation usually used in the instant form (IF) of dynamics.[1]

(c) Our approach provides the correct and natural nonrelativistic limit ("the correspondence principle" is fulfilled).

(d) For composite systems (including the spin-1 case) the approach guarantees the uniqueness of the solution for form factors and does not use such concepts as "good" and "bad" current components.

It is worth noticing that all known approaches [including the perturbative quantum field theory (QFT)] encounter difficulties while constructing a composite-system current operator satisfying Lorentz-covariance and conservation conditions [9–13].

Similar difficulties arise in the frame of the RHD approach, which is widely used in the theory of electroweak properties of composite quark and nucleon systems [6,10,13–27]. At present time the FF dynamics is the most developed and most used for composite systems [10,13–15,17,18]. However there are some difficulties in the FF RHD approach when the electroweak properties of composite systems are considered. In particular, it was shown [14,28] that the calculated electromagnetic form factors for the systems with the total angular momentum $J=1$ (the deuteron, the ρ meson) vary significantly with the rotation of the coordinate frame. This ambiguity is caused by the breaking of the so-called angle condition [14,28], that is, by the breaking of the rotation invariance of the theory. Some of the difficulties of FF dynamics are discussed in Ref. [29]. A possible way to solve the problem by adding some new (nonphysical) form factors to the electromagnetic current was proposed earlier (see Ref. [30] and references therein).

A different approach to the problem was proposed recently in Ref. [13], where a new method of construction of electromagnetic current operators in the frame of FF dynamics was given. The method of Ref. [13] gives unambiguous deuteron form factors. However, as the authors of Ref. [13] note themselves, their current operator and the one used in Ref. [10] are different, since both of them are obtained from the free one, but in different reference frames, related by an interaction dependent rotation.

Let us consider now the impulse approximation, which is widely used for the description of composite systems. In the IA a test particle interacts mainly with each component separately, that is, the electromagnetic current of the composite system can be described in terms of one-particle currents. In fact, the composite-system current is approximated by the corresponding free-system current. This means that exchange currents are neglected, or, in other words, that there is no three-particle forces in the interaction of a test particle with constituents. It is well known that the traditional IA breaks the Lorentz covariance of the composite-system current and the conservation law for the electromagnetic current (see, e.g., Ref. [4] for details).

To satisfy the conservation law in the frame of the Bethe-Salpeter equation and quasipotential equations, for example, it is necessary to go beyond IA: one has to add the so-called two-particle currents to the current operator. In the case of nucleon composite systems these currents are interpreted as meson exchange currents [11]. In the case of a deuteron this means the simultaneous interaction of virtual γ quanta with proton and neutron. However, in Ref. [31] it is shown that the current conservation law can be satisfied without such processes, although they contribute to the deuteron form factor. It seems that at the present time there is an intention to formulate the IA with transformed conservation properties without dynamical contribution of exchange currents [13,25,30].

In the framework of the point form dynamics the current operator was constructed in Ref. [6]. The current operator in Ref. [6] is Lorentz covariant and the conservation law is fulfilled. The approach is based on the realization of the Wigner-Eckart theorem for the Poincaré group. The main idea is to extract from the current matrix element the relativistic invariant part—the reduced matrix element, i.e., the form factor—and to separate the covariant part. The form factors contain all the dynamical information and the covariant part describes the relativistic transformation properties of the matrix element.

Our approach is a generalization of the method [6] for the case of the instant form dynamics. However, the scenario of the generalization of the Wigner-Eckart theorem is quite different.

The IF of relativistic dynamics, although not widely used, has some advantages. The calculations can be performed in a natural straightforward way without special coordinates. The IF method is particularly convenient for discussing the nonrelativistic limit of relativistic results. This approach is obviously rotational invariant, so the IF approach is the most suitable for spin problems.

We describe the dynamics of composite systems (the constituent interaction) in the frame of general RHD axiomatics. However, our approach differs from the traditional RHD by the way of construction of matrix elements of local operators. In particular, our method of describing the electromagnetic structure of composite systems permits the construction of current matrix elements satisfying the Lorentz-covariance condition and the current conservation law.

To construct the current operator in the frame of IF RHD we use the general method of the relativistic invariant parameterization of matrix elements of local operators proposed as long ago as 1963 by Cheshkov and Shirokov [32].

The method of Ref. [32] gives matrix elements of the operators of arbitrary tensor dimension (Lorentz scalar, Lorentz vector, Lorentz tensor) in terms of a finite number of relativistic invariant functions: form factors. The form fac-

[1]It is known that correct impulse approximation (IA) realization in the frame of traditional version of IF dynamics encounters difficulties: the standard IA depends on the choice of the coordinate frame. We show below that IA can be formulated in an invariant way, the composite system form factors being defined by the one-particle currents alone.

tors contain all the dynamical information on the transitions defined by the operator.

In the review Ref. [4] two possible variants of such a representation of matrix elements in terms of form factors are presented—the elementary-particle parametrization and the multipole parametrization. The variant of parametrization given in Ref. [32] is an alternative one. In Ref. [32] the authors propose the construction of matrix elements in a canonical basis so it can be called canonical parametrization. This method was developed for the case of composite systems in Refs. [33,34]. The composite-system form factors in this approach are generally the distributions (generalized functions); they are defined by continuous linear functionals on a space of test functions. Thus, for example, the current matrix elements for composite systems are functionals, generated by some Lorentz-covariant distributions, and the form factors are functionals generated by regular Lorentz-invariant generalized functions. We demonstrate these facts below, in Sec. III, using a simple model as an example.

It is worth noting that the statement that the form factors of a composite system are generalized functions is not something exotic. This feature also appears in the standard nonrelativistic potential theory (see Sec. III E).

Our formalism also gives, in fact, the description of the covariance properties of the operators in terms of many-particle as well as one-particle currents. However, the important feature of our formalism is the fact that form factors or reduced matrix elements describing the dynamics of transitions contain in the IA only the contributions of one-particle currents.

So, our approach to the construction of the current operator includes the following main points:

(1) We extract from the current matrix element of the composite system the reduced matrix elements (form factors) containing the dynamical information on the process. Usually these form factors are generalized functions.

(2) Along with form factors we extract from the matrix element a part that defines the symmetry properties of the current: the transformation properties under Lorentz transformation, discrete symmetries, conservation laws, etc.

(3) The physical approximations that are used to calculate the current are formulated not in terms of operators but in terms of form factors.

In this paper we present the main points of our approach. To make it transparent we consider here only simple systems with zero total angular momenta, so that technical details do not mask the essence of the method. We demonstrate the effectiveness of the approach by calculating the pion electroweak properties. In this case the canonical parametrization is very simple and can be realized without difficulties. The case of more complicated systems requires rather sophisticated mathematics for canonical parametrization of local operator matrix elements and will be considered elsewhere.

The paper is organized as follows. In Sect. II we remind the reader briefly of the basic statements of RHD, especially of IF RHD. The IF wave functions of composite systems are defined. In Sec. III our approach to relativistic theory of two-particle composite systems and their electroweak properties is presented. A simple model is considered in detail: two spinless particles in the S state of relative motion, one of the particles being uncharged. The electromagnetic form factor of the system is derived. The standard conditions for the current operator are discussed. The modified impulse approximation (MIA) is proposed. The results of IA and MIA are compared. The nonrelativistic limit is considered. In Sec. IV the developed formalism is used in the case of the system of two particles with spins 1/2. The pion electromagnetic form factor and the lepton decay constant are derived. The model parameters are discussed and the comparison of the results with the experimental data is given. The results of calculations in IA and MIA are compared and are shown to differ significantly. In Sec. V the conclusion is given.

II. RELATIVISTIC HAMILTONIAN DYNAMICS

In this section some basic equations of RHD are briefly reviewed. We use the so-called instant-form dynamics (IF). In this form the kinematic subgroup contains the generators of the group of rotations and translations in the three-dimensional Euclidean space (interaction independing generators):

$$\hat{\vec{J}}, \quad \hat{\vec{P}}. \tag{1}$$

The remaining generators are Hamiltonians (interaction dependent):

$$\hat{P}^0, \quad \hat{\vec{N}}. \tag{2}$$

The additive inclusion of interaction into the mass square operator (Bakamjian-Thomas procedure [35], see, e.g., Ref. [4] for details) presents one of the possible technical ways to include interaction in the algebra of the Poincaré group:

$$\hat{M}_0^2 \to \hat{M}_I^2 = \hat{M}_0^2 + \hat{U}. \tag{3}$$

Here $\hat{M}_0$ is the operator of invariant mass for the free system and $\hat{M}_I$ that for the system with interaction. The interaction operator $\hat{U}$ has to satisfy the following commutation relations:

$$[\hat{\vec{P}}, \hat{U}] = [\hat{\vec{J}}, \hat{U}] = [\vec{\nabla}_P, \hat{U}] = 0. \tag{4}$$

These constraints (4) ensure that the algebraic relations of the Poincaré group are fulfilled for an interacting system. The relations (4) mean that the interaction potential does not depend on the total momentum of the system nor on the projection of the total angular momentum. This fact is well established for a class of potentials, for example, for separable potentials [36]. Nevertheless, conditions (3) and (4) can be considered as the model conditions. There exists another approach [37] in which a potential depends on the total momentum, but that approach is out of the scope of this paper.

In RHD the wave function of the system of interacting particles is the eigenfunction of a complete set of commuting operators. In IF this set is

$$\hat{M}_I^2, \quad \hat{J}^2, \quad \hat{J}_3, \quad \hat{\vec{P}}. \tag{5}$$

$\hat{J}^2$ is the operator of the square of the total angular momentum. In IF the operators $\hat{J}^2, \hat{J}_3, \hat{\vec{P}}$ coincide with those for the free system. So, in system (5) only the operator $\hat{M}_I^2$ depends on the interaction.

To find the eigenfunctions for the system (5) one has first to construct the adequate basis in the state space of composite system. In the case of the two-particle system (for example, the quark-antiquark system $q\bar{q}$) the Hilbert space in RHD is the direct product of two one-particle Hilbert spaces: $\mathcal{H}_{q\bar{q}} \equiv \mathcal{H}_q \otimes \mathcal{H}_{\bar{q}}$.

As a basis in $\mathcal{H}_{q\bar{q}}$ one can choose the following set of two-particle state vectors:

$$|\vec{p}_1, m_1; \vec{p}_2, m_2\rangle = |\vec{p}_1 m_1\rangle \otimes |\vec{p}_1 m_2\rangle,$$

$$\langle \vec{p}, m | \vec{p}\,' m'\rangle = 2p_0 \delta(\vec{p} - \vec{p}\,')\, \delta_{mm'} . \tag{6}$$

Here $\vec{p}_1$ and $\vec{p}_2$ are three-momenta of particles, m_1 and m_2 are spin projections on the axis z, $p_0 = \sqrt{\vec{p}^{\,2} + M^2}$, and M is the constituent mass.

One can choose another basis where the motion of the two-particle center of mass is separated and where three operators of the set (5) are diagonal:

$$|\vec{P}, \sqrt{s}, J, l, S, m_J\rangle,$$

$$\langle \vec{P}, \sqrt{s}, J, l, S, m_J | \vec{P}\,', \sqrt{s'}, J', l', S', m_{J'}\rangle = N_{\mathrm{CG}} \delta^{(3)}(\vec{P} - \vec{P}\,')\, \delta(\sqrt{s} - \sqrt{s'})\, \delta_{JJ'} \delta_{ll'} \delta_{SS'} \delta_{m_J m_{J'}},$$

$$N_{\mathrm{CG}} = \frac{(2P_0)^2}{8k\sqrt{s}}, \quad k = \frac{1}{2}\sqrt{s - 4M^2}. \tag{7}$$

Here $P_\mu = (p_1 + p_2)_\mu$, $P_\mu^2 = s$, $\sqrt{s}$ is the invariant mass of the two-particle system, l is the orbital angular momentum in the center-of-mass frame (c.m.), $\vec{S}^2 = (\vec{S}_1 + \vec{S}_2)^2 = S(S+1)$, S is the total spin in the c.m., and J is the total angular momentum with the projection m_J.

The basis (7) is connected with the basis (6) through the Clebsh-Gordan (CG) decomposition for the Poincaré group (see, e.g., Ref. [34]):

$$|\vec{P}, \sqrt{s}, J, l, S, m_J\rangle = \sum_{m_1 m_2} \int \frac{d\vec{p}_1}{2p_{10}} \frac{d\vec{p}_2}{2p_{20}} |\vec{p}_1, m_1; \vec{p}_2, m_2\rangle \times \langle \vec{p}_1, m_1; \vec{p}_2, m_2 | \vec{P}, \sqrt{s}, J, l, S, m_J\rangle. \tag{8}$$

Here

$$\langle \vec{p}_1, m_1; \vec{p}_2, m_2 | \vec{P}, \sqrt{s}, J, l, S, m_J\rangle = \sqrt{2s}[\lambda(s, M^2, M^2)]^{-1/2} 2P_0 \delta(P - p_1 - p_2) \times \sum_{\tilde{m}_1 \tilde{m}_2} \langle m_1 | D^{1/2}(p_1 P) | \tilde{m}_1\rangle \langle m_2 | D^{1/2}(p_2 P) | \tilde{m}_2\rangle \times \sum_{m_l m_S} \langle \tfrac{1}{2} \tfrac{1}{2} \tilde{m}_1 \tilde{m}_2 | S m_S\rangle Y_{l m_l}(\vartheta, \varphi) \langle S l m_s m_l | J m_J\rangle.$$

Here $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + bc + ac)$, Y_{lm_l} is a spherical harmonic, ϑ and φ are the spherical angles of the vector $\vec{p} = (\vec{p}_1 - \vec{p}_2)/2$ in the c.m., $\langle S m_S | \tfrac{1}{2} \tfrac{1}{2} \tilde{m}_1 \tilde{m}_2\rangle$ and $\langle J m_J | S l m_S m_l\rangle$ are the CG coefficients for the group SU(2), and $\langle \tilde{m} | D^{1/2}(P, p) | m\rangle$ is the three-dimensional spin rotation matrix to be used for correct relativistic invariant spin addition.

It is on the vectors (7) and (8) that the Poincaré-group representation is realized in the vector state space of two free particles. The vector in representation is determined by the eigenvalues of the complete commuting set of operators:

$$\hat{M}_0^2 = \hat{P}^2, \hat{J}^2, \hat{J}_3 . \tag{9}$$

The parameters S and l play the role of invariant parameters of degeneracy.

As in the basis (7) the operators $\hat{J}^2, \hat{J}_3, \hat{\vec{P}}$ in system (5) are diagonal, one needs to diagonalize only the operator $\hat{M}_I^2$ in system (5) in order to obtain the system wave function. The eigenvalue problem for the operator $\hat{M}_I^2$ in the basis (7) has the form of nonrelativistic Schrödinger equation (see, e.g., Ref. [4]).

The corresponding composite-particle wave function has the form

$$\langle \vec{P}\,', \sqrt{s'}, J', l', S', m_J' | p_c\rangle = N_C \delta(\vec{P}\,' - \vec{p}_c)\, \delta_{JJ'} \delta_{m_J m_J'} \varphi^{J'}_{l'S'}(k'), \tag{10}$$

$$N_C = \sqrt{2p_{c0}} \sqrt{\frac{N_{CG}}{4\,k'}}.$$

$|p_c\rangle$ is an eigenvector of the set (5); $J(J+1)$ and m_J are the eigenvalues of $\hat{J}^2$ and $\hat{J}_3$, respectively [Eqs. (5) and (9)].

The two-particle wave function of relative motion for equal masses and total angular momentum and total spin fixed is

$$\varphi^J_{lS}(k(s)) = \sqrt[4]{s}\, u_l(k) k, \tag{11}$$

and the normalization condition has the form

$$\sum_l \int u_l^2(k) k^2 dk = 1. \tag{12}$$

Let us note that for composite quark systems one uses sometimes instead of Eq. (12) the following one:

$$n_c \sum_l \int u_l^2(k) k^2 dk = 1. \tag{13}$$

Here n_c is the number of colors. The wave function (11) coincides with that obtained by ''minimal relativization'' in Ref. [38]. The normalization factors in Eq. (11) in this case correspond to the relativization obtained by the transformation to relativistic density of states

$$k^2 dk \to \frac{k^2 dk}{2\sqrt{(k^2+M^2)}}. \tag{14}$$

The formalism of this section is used in the next one to present the method of calculation of electroweak properties of composite systems. Particularly, the method of construction of electroweak current operators is described.

III. THE NEW RELATIVISTIC INSTANT-FORM APPROACH TO THE ELECTROWEAK STRUCTURE OF TWO-BODY COMPOSITE SYSTEMS

In this section we present our approach to electroweak properties of relativistic two-particle systems. To demonstrate how one describes the electromagnetic properties of composite systems in our version of the RHD instant form we first use the following simple model. We consider the system of two spinless particles in the S state of relative motion, one particle having no charge. Let us note that a similar model was used in Ref. [4] where the authors gave the description of constituent interaction in the IF of RHD and obtained the mass spectrum. The application of our method in general case follows the scheme of this section. The case of the π meson is investigated in Sec. IV and the $S=1$ case in Ref. [39].

Electromagnetic properties of the system are determined by the current operator matrix element. This matrix element is connected with the charge form factor $F_c(Q^2)$ as follows:

$$\langle p_c | j_\mu(0) | p_c' \rangle = (p_c + p_c')_\mu F_c(Q^2), \tag{15}$$

where p_c' and p_c are four-momenta of the composite system in initial and final states, $Q^2=-t, q^2=(p_c-p_c')^2=t$, and q^2 is the momentum-transfer squared. The form (15) is defined by the Lorentz covariance and by the conservation law only and does not depend on the model for the internal structure of the system.

Equation (15) presents the simplest example of the extraction of a reduced matrix element, that is, the simplest realization of the Wigner-Eckart theorem on the Poincaré group. The four-vector $(p_c+p_c')_\mu$ describes symmetry and transformation properties of the matrix element. The reduced matrix element (the form factor) contains all the dynamical information on the process described by the current. The representation of a matrix element in terms of form factors often is referred to as the parametrization of the matrix element. The scattering cross section for elastic scattering of electrons by a composite system can be expressed in terms of charge form factor $F_c(Q^2)$. So, the form factor can be obtained from experiment and it is interesting to calculate it in a theoretical approach.

In this section we calculate the form factor of our simple composite system using the version of RHD IF based on the approach of the Sec. II. Now let us list the conditions for the operator of the conserved electromagnetic current to be fulfilled in relativistic case (see, e.g., Ref. [12]):

(i) *Lorentz covariance*:

$$\hat{U}^{-1}(\Lambda) \hat{j}^\mu(x) \hat{U}(\Lambda) = \Lambda^\mu_\nu \hat{j}^\nu(\Lambda^{-1}x). \tag{16}$$

Here Λ is the Lorentz-transformation matrix, and $\hat{U}(\Lambda)$ is the operator of the unitary representation of the Lorentz group.

(ii) *Invariance under translation*:

$$\hat{U}^{-1}(a) \hat{j}^\mu(x) \hat{U}(a) = \hat{j}^\mu(x-a). \tag{17}$$

Here $\hat{U}(a)$ is the operator of the unitary representation of the translation group.

(iii) *Current conservation law*:

$$[\hat{P}_\nu \hat{j}^\nu(0)] = 0. \tag{18}$$

In terms of matrix elements $\langle \hat{j}^\mu(0) \rangle$ the conservation law can be written in the form

$$q_\mu \langle \hat{j}^\mu(0) \rangle = 0. \tag{19}$$

Here q_μ is four-vector of the momentum transfer.

(iv) *Current-operator transformations under space-time reflections*:

$$\hat{U}_P(\hat{j}^0(x^0,\vec{x}), \hat{\vec{j}}(x^0,\vec{x})) \hat{U}_P^{-1} = (\hat{j}^0(x^0,-\vec{x}), -\hat{\vec{j}}(x^0,-\vec{x})),$$

$$\hat{U}_R \hat{j}^\mu(x) \hat{U}_R^{-1} = \hat{j}^\mu(-x). \tag{20}$$

In Eq. (20) $\hat{U}_P$ is the unitary operator for the representation of space reflections and $\hat{U}_R$ is the antiunitary operator of the representation of space-time reflections $R=PT$.

(v) *Cluster separability condition*: If the interaction is switched off, then the current operator becomes equal to the sum of the operators of one-particle currents.

(vi) *The charge is not renormalized by the interaction*: The electric charge of the system with interaction is equal to the sum of the constituent electric charges.

In this paper the explicit equations for the form factors are obtained taking into account all the listed conditions.

A. Electromagnetic properties of the system of free particles

Let us consider first the simple two-particle system described in the beginning of Sec. III. The electromagnetic current $j_\mu^{(0)}(0)$ of the two-particle free system can be calculated in the representation given by the basis (6) or in the representation given by the basis (7). In the first case the operator has the form $j_\mu^{(0)} = j_{1\mu} \otimes I_2$. Here $j_{1\mu}$ is the electromagnetic current of the charged particle and I_2 is the unity operator in the Hilbert space of states of the uncharged particle:

$$\langle \vec{p}_1;\vec{p}_2|j^{(0)}_\mu(0)|\vec{p}_1\,';\vec{p}_2\,'\rangle=\langle \vec{p}_2|\vec{p}_2\,'\rangle\langle \vec{p}_1|j_{1\mu}(0)|\vec{p}_1\,'\rangle. \quad (21)$$

The matrix element of the one spinless particle current in the free case contains only one form factor—the charge form factor of the charged particle $f_1(Q^2)$:

$$\langle \vec{p}_1|j_{1\mu}(0)|\vec{p}_1\,'\rangle=(p_1+p_1')_\mu f_1(Q^2). \quad (22)$$

So, the electromagnetic properties (15) of the system of two free particles are defined by the form factor $f_1(Q^2)$, containing all the dynamical information on elastic processes described by the matrix element (21) [4]. Particularly, the charge of the system is defined by the value of this form factor at $Q^2\to 0$:

$$\lim_{Q^2\to 0} f_1(Q^2)=f_1(0)=e_c\,. \quad (23)$$

e_c is the system charge.

Now let us write the electromagnetic-current matrix element for the two-particle free system in the basis (7) where the center-of-mass motion is separated:

$$\langle \vec{P},\sqrt{s},|j^{(0)}_\mu(0)|\vec{P}\,',\sqrt{s'}\rangle. \quad (24)$$

Here the variables which take zero values are omitted: $J=S=l=0$. One can consider the matrix element (24) as a matrix element of an irreducible tensor operator on the Poincaré group and one can use the Wigner-Eckart theorem, i.e., the canonical parametrization [32–34] giving a technical realization of this theorem. Thus, one can write the matrix element (24) in the form

$$\begin{aligned}\langle \vec{P},\sqrt{s}|j^{(0)}_\mu(0)|\vec{P}\,',\sqrt{s'}\rangle&=A_\mu(s,Q^2,s')\langle\sqrt{s}||g_0(Q^2)||\sqrt{s'}\rangle\\&=A_\mu(s,Q^2,s')g_0(s,Q^2,s'). \quad (25)\end{aligned}$$

It is easy to understand the motivation for the parametrization (25) for our simple system. The four-vector A_μ describes the transformation properties of the matrix element and the invariant function $g_0(s,Q^2,s')$ contains the dynamical information on the process. We will refer to $g_0(s,Q^2,s')$ as to free two-particle form factor. For more complicated systems the parametrization corresponding to the Wigner-Eckart theorem for the Poincaré group can be performed using a special mathematical techniques as described in the papers [32,34,39].

So $A_\mu(s,Q^2,s')$ is defined by the current transformation properties (the Lorentz covariance and the conservation law):

$$A_\mu=\frac{1}{Q^2}[(s-s'+Q^2)P_\mu+(s'-s+Q^2)P'_\mu]. \quad (26)$$

Thus, in the basis (7) the electromagnetic properties of the free two-particle system are defined by the free two-particle form factor $g_0(s,Q^2,s')$. So, in both representations [defined by the basis (6) as well as by the basis (7)] we pass from the description of the system in terms of matrix elements to that in terms of Lorentz-invariant form factors.

One can see that Eqs. (21) and (25) describe electromagnetic properties in terms of only one form factor. Both of these descriptions are, certainly, equivalent from the physical point of view. Let us consider the difference between these descriptions. As we will show below by direct calculation the free two-particle form factor $g_0(s,Q^2,s')$ is not an ordinary function but has to be considered in the sense of distributions in variables s,s', generated by a locally integrable function. So, $g_0(s,Q^2,s')$ is a regular generalized function. All the properties of $g_0(s,Q^2,s')$ have to be considered as the properties of a functional given by the integral over the variables s,s' of the function $g_0(s,Q^2,s')$ multiplied by a test function. As test functions it is sufficient to take a large class of smooth functions that give the uniform convergence of the integral. In particular, the limit (23) giving the total charge of the system through the two-particle form factor is now the weak limit:

$$\lim_{Q^2\to 0}\langle g_0(s,Q^2,s'),\phi(s,s')\rangle. \quad (27)$$

Here $\phi(s,s')$ is a function from the space of test functions. The precise definition of the functional will be given below.

At the first glance it seems that the description of the two-particle free system in terms of the form factor $g_0(s,Q^2,s')$ is too complicated. However, so is the reality, as we will see later in the Sec. III E. In fact, this kind of description is used implicitly for a long time in nonrelativistic theory of composite systems, without calling things by their proper names. It is this kind of description that makes it possible to construct the electromagnetic current operator with correct transformation properties for interacting systems.

The locally integrable function $g_0(s,Q^2,s')$ can be easily obtained by use of CG decomposition (8) for the Poincaré group. Using Eq. (8) we obtain for Eq. (25):

$$\begin{aligned}&\langle \vec{P},\sqrt{s}|j^{(0)}_\mu(0)|\vec{P}\,',\sqrt{s'}\rangle\\&\quad=\int\frac{d\vec{p}_1}{2\,p_{10}}\frac{d\vec{p}_2}{2\,p_{20}}\frac{d\vec{p}_1\,'}{2\,p'_{10}}\frac{d\vec{p}_2\,'}{2\,p'_{20}}\langle \vec{P},\sqrt{s},|\vec{p}_1;\vec{p}_2\rangle\\&\qquad\times\langle \vec{p}_1;\vec{p}_2|j^{(0)}_\mu(0)|\vec{p}_1\,';\vec{p}_2\,'\rangle\langle \vec{p}_1\,';\vec{p}_2\,'|\vec{P}\,',\sqrt{s'}\rangle. \quad (28)\end{aligned}$$

To calculate the free two-particle form factor one has to use Eqs. (21), (22), and (25) and the explicit form of CG coefficients (8) for quantum numbers of the system. As the particles of the system under consideration are spinless, now Eq. (8) does not contain D functions.

It is convenient to integrate in Eq. (28) using the coordinate frame with $\vec{P}\,'=\vec{0},\vec{P}=(0,0,P)$. As the result we obtain the following relativistic invariant form for the function $g_0(s,Q^2,s')$:

$$g_0(s,Q^2,s')=\frac{(s+s'+Q^2)^2Q^2}{2\sqrt{(s-4M^2)(s'-4M^2)}}\times\frac{\vartheta(s,Q^2,s')}{[\lambda(s,-Q^2,s')]^{3/2}}f_1(Q^2). \qquad (29)$$

Here $\vartheta(s,Q^2,s')=\theta(s'-s_1)-\theta(s'-s_2)$, and θ is the step function. The result, naturally, does not depend on the choice of the coordinate frame:

$$s_{1,2}=2M^2+\frac{1}{2M^2}(2M^2+Q^2)(s-2M^2)\mp\frac{1}{2M^2}\sqrt{Q^2(Q^2+4M^2)s(s-4M^2)}.$$

The functions $s_{1,2}(s,Q^2)$ give the kinematically available region in the plane (s,s') (see Ref. [33]).

One can see that the free two-particle form factor (29) $g_0(s,Q^2,s')$ has in fact to be interpreted in terms of the distributions: The ordinary limit as $Q^2\to 0$ is zero because of the cutting ϑ functions and the static limit exists only as the weak limit (27).

Let us calculate this limit. Let us define the functional giving regular generalized function as a functional in $\mathbb{R}^2$ as follows:

$$\langle g_0(s,Q^2,s'),\phi(s,s')\rangle=\int d\mu(s,s')g_0(s,Q^2,s')\phi(s,s'). \qquad (30)$$

Here

$$d\mu(s,s')=16\sqrt[4]{ss'}\,\theta(s-4\,M^2)\,\theta(s'-4\,M^2)d\mu(s)d\mu(s'),$$

$$d\mu(s)=\frac{1}{4}k\,d\sqrt{s}. \qquad (31)$$

The θ functions in these formula give the physical region of possible variations of the invariant mass squares in the initial and final states explicitly. The measure (31) is due to the relativistic density of states (11) and (14). $\phi(s,s')$ is a function from the test function space. So, for example, the limit of $g_0(s,Q^2,s')$ as $Q^2\to 0$ (the static limit) has the meaning only as the weak limit [compare with Eq. (23)]:

$$\lim_{Q^2\to 0}\langle g_0,\phi\rangle=\langle e\,\delta(\mu(s')-\mu(s)),\phi\rangle. \qquad (32)$$

It is this weak limit that gives the electric charge of the free two-particle system. If the test functions are normalized to the relativistic density of states, then the right-hand side (r.h.s.) of the Eq. (32) is equal to the total charge of the system.

B. Electromagnetic structure of the system of two interacting particles

Now let us consider the electromagnetic structure of our simple model (15) in the case of interacting particles. As we have mentioned in Sec. II when constructing the bases (6) and (7) in the frame of RHD the state vector $|p_c\rangle$ belongs to the direct product of two one-particle spaces. We can write the decomposition of this vector with $J=l=S=m_J=0$ in the basis (7). Now Eq. (15) has the form

$$\int\frac{d\vec{P}d\vec{P}'}{N_{\rm CG}N'_{\rm CG}}d\sqrt{s}\,d\sqrt{s'}\langle p_c|\vec{P},\sqrt{s}\rangle\langle\vec{P},\sqrt{s}|j_\mu(0)|\vec{P}',\sqrt{s'}\rangle\times\langle\vec{P}',\sqrt{s'}|p'_c\rangle=(p_c+p'_c)_\mu F_c(Q^2). \qquad (33)$$

Here $\langle\vec{P}',\sqrt{s'}|p'_c\rangle$ is the wave function in the sense of the instant form of RHD (10).

Using Eq. (10) we obtain for Eq. (33):

$$\int\frac{N_cN'_c}{N_{CG}N'_{CG}}d\sqrt{s}d\sqrt{s'}\varphi(s)\varphi(s')\langle\vec{p}_c,\sqrt{s}|j_\mu(0)|\vec{p}'_c,\sqrt{s'}\rangle=(p_c+p'_c)_\mu F_c(Q^2). \qquad (34)$$

We have omitted in the wave function (11) the variables with zero values: $J=S=l=0$.

Let us discuss the possibility of using the Wigner-Eckart theorem (or the canonical parametrization) in the case of the matrix element $\langle\vec{p}_c,\sqrt{s}|j_\mu(0)|\vec{p}'_c,\sqrt{s'}\rangle$ in Eq. (34). In the previous cases the state vectors and the operators entering matrix elements transformed following one and the same representation of the nonuniform group SL$(2,C)$ [7]. Let us perform the Lorentz transformation of the current operator:

$$\hat{U}^{-1}(\Lambda)j^\mu(0)\hat{U}(\Lambda)=\tilde{j}^\mu(0). \qquad (35)$$

We obtain

$$\langle p|\tilde{j}^\mu(0)|p'\rangle=\langle p|\hat{U}^{-1}(\Lambda)j^\mu(0)\hat{U}(\Lambda)|p'\rangle=\langle\Lambda p|j^\mu(0)|\Lambda p'\rangle. \qquad (36)$$

This means that the transformation properties of the current four-vector (16) can be described using four-momenta of the initial and final states, i.e., one can use the canonical parametrization.

In the matrix element in the integrand of Eq. (34) the state vectors and the operator transform following the different representations of the group SL$(2,C)$. The current operator describes the transitions in the system of two interacting particles and transforms following the representation with the generators of Lorentz boosts depending on the interaction (5). The state vectors belong to the basis (7) and physically describe the system of two free particles and so transform following a representation with generators that do not depend on the interaction (9). So, if one considers the matrix element $\langle\vec{p}_c,\sqrt{s}|j_\mu(0)|\vec{p}'_c,\sqrt{s'}\rangle$ (that is, the interaction current between free states) *per se* , not in the context of the decomposition (34), one cannot use the Wigner-Eckart theorem.

However, one must consider this matrix element as a generalized function, that is, as having meaning only as the integrand in Eq. (34). Let us show that in this case one can use the Wigner-Eckart theorem.

The set of the free two-particle states $|\vec{P},\sqrt{s}\rangle$ is complete:

$$\hat{I}=\int\frac{d\vec{P}}{N_{\mathrm{CG}}}d\sqrt{s}|\vec{P},\sqrt{s}\rangle\langle\vec{P},\sqrt{s}|. \quad (37)$$

Using Eqs. (10) and (37) we obtain

$$\begin{aligned}\int & \frac{N_cN_c'}{N_{\mathrm{CG}}N_{\mathrm{CG}}'}d\sqrt{s}d\sqrt{s'}\varphi(k)\varphi(k') \\ &\times\langle\vec{p}_c,\sqrt{s}|\hat{U}^{-1}(\Lambda)j_\mu(0)\hat{U}(\Lambda)|\vec{p}_c',\sqrt{s'}\rangle \\ &=\langle p_c|\hat{U}^{-1}(\Lambda)\hat{I}j_\mu(0)\hat{I}\hat{U}(\Lambda)|p_c'\rangle \\ &=\langle\Lambda p_c|\hat{I}j_\mu(0)\hat{I}|\Lambda p_c'\rangle \\ &=\int\frac{N_cN_c'}{N_{\mathrm{CG}}N_{\mathrm{CG}}'}d\sqrt{s}d\sqrt{s'}\varphi(k)\varphi(k') \\ &\times\langle\Lambda\vec{p}_c,\sqrt{s}|j_\mu(0)|\Lambda\vec{p}_c',\sqrt{s'}\rangle. \end{aligned} \quad (38)$$

So, we have an analog of Eq. (36) *in the sense of distributions* and we can use the Wigner-Eckart theorem in the intergrand. One can speak about the Wigner-Eckart theorem in weak sense. Now the problem of canonical parametrization of the matrix element (34) can be solved if one considers the equality (34) as the equality of two functionals.

Using Eqs. (11) and (31) we can rewrite Eq. (34) in the form of the functional in $\mathbb{R}^2$:

$$\begin{aligned}\int & d\mu(s,s')u(k(s))J_\mu(\vec{p}_c,\sqrt{s};\vec{p}_c',\sqrt{s'})u(k(s')) \\ &=(p_c+p_c')_\mu F_c(Q^2), \\ J_\mu(\vec{p}_c,\sqrt{s};\vec{p}_c{}',\sqrt{s'})&=\frac{N_cN_c'}{N_{\mathrm{CG}}N_{\mathrm{CG}}'}\langle\vec{p}_c,\sqrt{s}|j_\mu|\vec{p}_c{}',\sqrt{s'}\rangle. \end{aligned} \quad (39)$$

The l.h.s. in Eq. (39) contains a functional in $\mathbb{R}^2$ generated by the Lorentz-covariant function (current matrix element). Let us denote

$$\psi(s,s')=u(k(s))u(k'(s')). \quad (40)$$

The functional in the l.h.s. of Eq. (39) is given on the set of test functions $\psi(s,s')$ through an integral in $\mathbb{R}^2$ and defines a Lorentz-covariant (regular) generalized function with the values in the Minkowski space (see, e.g., Ref. [40]). Here Q^2 is a parameter. The test-function space can be (in general) larger than Eq. (40). However, the uniconvergence of Eq. (39) has to be guaranteed.

Let us write the matrix element in the form analogous to Eq. (25):

$$J_\mu(\vec{p}_c,\sqrt{s};\vec{p}_c',\sqrt{s'})=B_\mu(s,Q^2,s')G(s,Q^2,s'). \quad (41)$$

The covariant part in Eq. (41) [as well as in Eq. (25)], the vector $B_\mu(s,Q^2,s')$, is supposed to be an ordinary smooth function and the invariant part $G(s,Q^2,s')$ is generalized function. In fact, $G(s,Q^2,s')$ is the reduced matrix element containing the information on the process. This kind of representation of a Lorentz-covariant generalized function as a product of a Lorentz-covariant ordinary smooth function and a Lorentz-invariant generalized function was described in Ref. [40].

Using Eq. (41) we can rewrite Eq. (39) in the following form:

$$\begin{aligned}\int & d\mu(s,s')\psi(s,s')B_\mu(s,Q^2,s')G(s,Q^2,s') \\ &=(p_c+p_c')_\mu F_c[\psi](Q^2). \end{aligned} \quad (42)$$

To obtain the vector B_μ let us require Eq. (42) to be covariant in the sense of distributions, that is, to be valid for any test function $\psi(s,s')$ in any fixed frame. The variation of the test function in the functional (42) means in fact, following Eq. (40), the variation of the wave function of the internal motion. Under such a variation the vector in the r.h.s. of Eq. (42) is unchanged as it is constructed with four-vectors describing the motion of the system as a whole, independent of the internal constituent motion. As to the form factor in the r.h.s. it varies under the test function variation. So, under a variation of the test function the r.h.s. of Eq. (42) remains to be collinear to the vector $(p_c+p_c')_\mu$. At the same time, under arbitrary variation of the test function the vector in the l.h.s. in general changes the direction. So, for the validity of the equality (42) with arbitrary test function it is sufficient to require that the following equation

$$B_\mu(s,Q^2,s')=(p_c+p_c')_\mu \quad (43)$$

holds. This choice of the vector B_μ in Eq. (43) ensures that the l.h.s. of Eq. (39) satisfies the condition of Lorentz covariance for the current as well as the condition of current conservation.

Let us discuss the physical meaning of the representations (41) and (43) for the matrix element. As this representation is explicitly Lorentz covariant and also satisfies the current conservation law, then it means that the current operator for the composite system contains the contribution not only of one-particle currents but of two-particle currents, too (see, e.g., Ref. [4]):

$$j=\sum_k j^{(k)}+\sum_{k<m}j^{(km)}. \quad (44)$$

Here the first term is the sum of one-particle currents and the second of two-particle currents. In the case of our simple model each sum in Eq. (44) contains only one term. It is well known that if one approximates $j(x)\approx\Sigma_k j^{(k)}(x)$, then the current operator in IF dynamics does not satisfy the condition of Lorentz covariance and the conservation law [4]. So, from the physical point of view, the covariant part of the current matrix element (43), which defines the transformation properties of the current in Eq. (39) is given by Eq. (44) and contains the contributions of one- and two-particle currents.

The invariant part of the decomposition (41) is the form factor or the reduced matrix element $G(s,Q^2,s')$ and contains the information on the dynamics of the scattering of test particle by each of the constituents [the first term in Eq. (44)], i.e., by the free two-particle system, as well as by two constituent simultaneously (the second term). So, the form factor contains the contribution of the free system form factor (29) and the contribution of some exchange currents analogous to meson currents in nucleon systems [9]:

$$G(s,Q^2,s')=g_0(s,Q^2,s')+G_c(s,Q^2,s'). \tag{45}$$

Here G_c is the reduced matrix element containing the contribution (44) of two-particle currents.

Using Eqs. (11), (31), (40), (43) one can obtain from Eq. (42) the scalar equation of the following form:

$$\int d\sqrt{s}d\sqrt{s'}\,\varphi(s)G(s,Q^2,s')\varphi(s')=F_c(Q^2). \tag{46}$$

The representation (46) for the charge form factor of the system is quite general.

Let us note that one can use the described formalism in the general case of composite systems with nonzero total angular momentum J (the detailed consideration is given in Ref. [39]). In this case the current matrix element in the decomposition like Eq. (34) is a matrix with matrix indices being the total angular momentum projections m_J' and m_J in the initial and final states. We decompose this matrix element in the set of the linear independent matrices:

$$D^J(p_c,p_c')[\Gamma_\mu(p_c')p_c^\mu]^n, \quad n=0,1,\ldots,2J. \tag{47}$$

Here $\Gamma_\mu(p_c')$ is the spin four-vector defined with the use of the Pauli-Lubanski vector (see, e.g., Ref. [34], and also Sec. IV A).

The set of matrices (47) is the set of Lorentz scalars (scalars and pseudoscalars). The decomposition contains the four-vectors analogous to B_μ [Eq. (41)].

We used the described approach to consider the systems with $J=1$ (ρ meson, deuteron) and obtained a good description of the experimental data [39]. Now let us proceed with the approximate calculation of the form factor (46).

C. Modified impulse approximation

The problem of the calculation of the form factor $G(s,Q^2,s')$ [Eq. (46)] including exchange currents is a very difficult problem. We propose an approximation that is a kind of analog of relativistic impulse approximation. We propose to omit the contribution of the two-particle currents to the form factor $G(s,Q^2,s')$.

However, we will not change the covariant part B_μ of the current matrix element in Eq. (41), so that this covariant part will contain the contribution of the two-particle currents and so that the transformation properties of the matrix element will not be changed. So we approximately change the generalized function $G(s,Q^2,s')$ in Eqs. (41) and (45) for the generalized function $g_0(s,Q^2,s')$ [Eqs. (25) and (29)], which describes, as we have shown before, the electromagnetic properties of the free two-particle system. Nevertheless, the matrix element (33) and (41) as a whole will contain the contributions of two-particle currents, although not the full contribution but one that ensures its correct transformation properties.

Let us note that our approximation does not contradict general statements (see Ref. [4]) that state that to obtain a correct description of electromagnetic current of composite system that satisfy the Lorentz-covariance condition and the current conservation law, one has to take into account many-particle currents. Thus, in our approximation the scalar equality (46) transforms into an approximate scalar equality that corresponds, from the physical point of view, to the relativistic impulse approximation. In the developed mathematical formalism we have not broken the Lorentz covariance of the current nor the current conservation law. Let us point out that to calculate the form factor we do not use a special current component as it is done in other mathematical formulations of RHD (see, e.g., Ref. [10]). Let us remark that from the physical point of view, the form factor $g_0(s,Q^2,s')$ contains the contributions of one-particle currents only [see Eqs. (25), (28), and (29)] and in this sense our approximation corresponds to the known impulse approximation. In order to emphasize that our approximation differs from the usual IA we will refer to it as the modified impulse approximation (MIA). The form factor of the composite system in MIA has the form

$$F_c(Q^2)=\int d\sqrt{s}d\sqrt{s'}\,\varphi(s)g_0(s,Q^2,s')\varphi(s'). \tag{48}$$

We do not discuss in this paper the problem of going beyond the limits of MIA and of obtaining corrections to $g_0(s,Q^2,s')$ in Eqs. (45) and (48). This means that if considering, for example, nucleon systems, we do not take into account the meson current.

Let us consider now the fulfilling of the conditions (i)–(vi) for the electromagnetic current. The conditions (i)–(iii) are satisfied by construction. For example, the fulfilling of (i) and (iii) is ensured by the correct transformation properties of the four-vectors in Eqs. (25), (41), and (43). Condition (iv) is satisfied immediately as the form factor $g_0(s,Q^2,s')$ in Eq. (25) and the form factor $G(s,Q^2,s')$ in Eq. (41) are scalars in our simple model.[2]

The condition of cluster separability (v) needs a more detailed consideration. At large distances (or if the interaction is switched off) the contribution of two-particle currents has to go to zero: $G_c(s,Q^2,s')\to 0$ in Eq. (45). This means that in the form (45) the form factor $G(s,Q^2,s')$ has to trans-

[2]The currents that do not conserve the parity also can be considered in our formalism. In that case one can separate not only the scalar part of the current matrix element but the pseudoscalar part, too. This case is considered elsewhere.

form into $g_0(s,Q^2,s')$. Let us remark that the condition of cluster separability is fulfilled in MIA, too, as in this approximation the use of $g_0(s,Q^2,s')$ instead of $G(s,Q^2,s')$ is assumed from the very beginning. When the interaction is switched off the generalized function $g_0(s,Q^2,s')$ for the free two-particle system acts on a larger space of test functions than Eq. (40). As $g_0(s,Q^2,s')$ contains only the one-particle current contributions (28) the condition (v) is satisfied and the composite-system current becomes the sum of the one-particle currents. The condition on the charge to be nonrenormalizable also is fulfilled directly in MIA because the weak limit (32) does exist on test functions (40). So, our prescription for the construction of the current in MIA satisfies all the conditions for the current operator.

Let us note that Eq. (48) for the composite-system form factor is analogous to the equations obtained in the framework of the dispersion approach [33,41–43] (see also Ref. [44,45]) based on the analytic properties of the scattering amplitudes, matrix elements, and form factors in the complex energy plane.

As the dispersion approach is rather correctly derived in the frame of QFT [46], this fact can be considered as a possible link between QFT and RHD. The establishment of such a link is one of the unsolved problems of RHD [4].

Let us note that an immediate application of the approach to quark systems is difficult to realize because of the fact of quark confinement. However, there are some investigations based on similar ideas where the form factors of hadrons as constituent-quark bound states are considered in the frame of the dispersion technique of the integral over composite-particle mass [45].

D. MIA versus IA

Let us compare the approximation MIA with the well known IA. To do this let us first calculate the form factor in IF RHD not using the canonical parametrization. In particular, let us formulate the IA in terms of operators as it is formulated usually (not in terms of form factors). Let us decompose the matrix element (15) through the complete set of states (6):

$$\langle p_c|j_\mu(0)|p_c'\rangle=\int\frac{d\vec p_1 d\vec p_2}{2p_{10}2p_{20}}\frac{d\vec p_1\,' d\vec p_2\,'}{2p_{10}'2p_{20}'}\langle p_c|\vec p_1;\vec p_2\rangle \times\langle\vec p_1;\vec p_2|j_\mu|\vec p_1\,';\vec p_2\,'\rangle\langle\vec p_1\,';\vec p_2\,'|p_c'\rangle. \quad (49)$$

Here $\langle\vec p_1;\vec p_2|p_c\rangle$ is wave function of constituents in composite system. If the current matrix element in Eq. (49) is taken in the IA approximation (44) and contains one-particle currents only, then Eq. (49) is self-contradictory [4].

To write the form factor in terms of wave functions (10) one has to perform the CG decomposition of the basis (6) in terms of the basis (7) in the wave functions (49) and to use the explicit form for CG coefficients (8) for the quantum numbers of the system:

$$\langle\vec p_1;\vec p_2|p_c\rangle=\sqrt{\frac{2}{\pi}}\langle\vec P,\sqrt{s},J,l,S,m_J|p_c\rangle. \quad (50)$$

The current matrix element in Eq. (49) has the form (21). The one-particle currents are expressed through the form factors (22).

Equation (49) is an equality for two four-vectors. Taking different components of this equality and exploiting the δ functions in integrals, one can calculate the form factor of the composite system. The result of the calculation of the form factor in this way is not unambiguous. In particular, it depends on the actual choice of the component of the current (49) to be used in the calculation. Moreover, the result depends on the coordinate frame chosen to perform the integration in Eq. (49). This is the general feature of the IA in the usual formulation of the IF RHD (see, e.g., Ref. [4]).

Let us write the final result of the calculation of the form factor from the equation for the null component of the current and perform the integration in the coordinate frame where $\vec p_c\,'=\vec 0,\vec p_c=(0,0,p)$. If now we write the integral in terms of the invariant variables s,s', the obtained form factor has the form:

$$F_c(Q^2)=\frac{M_c}{4}\frac{\sqrt{2(2M_c^2+Q^2)}}{4M_c^2+Q^2}\int\sqrt{\frac{s}{s'}} \times\frac{d\sqrt{s}\,d\sqrt{s'}}{\sqrt{(s-4M^2)(s'-4M^2)}} \times\frac{(s+s'+Q^2)^4Q^2}{[\lambda(s,-Q^2,s')]^{3/2}}\frac{1}{(s\,s')^{1/4}} \times\frac{\theta(s,Q^2,s')}{\sqrt{s'}(s+Q^2)}\varphi(s)\varphi(s')f_1(Q^2). \quad (51)$$

Equation (51) differs from Eq. (48), obtained with the use of the two-particle free form factor. In the case of wave functions satisfying the conditions (11) and (12), the form factor (51) satisfies the normalization: $F_c(0)=e_c$. Let us note that the form factor obtained in this way from the third current component in Eq. (49) does not satisfy this condition.

Let us compare IA and MIA results and note once again that in MIA we separate (by use of the scheme of canonical parametrization) the covariant part of the current matrix element in Eq. (42) prior to performing any calculations. This covariant part ensures the correct transformation properties of the corresponding decompositions in terms of free-particle states. The difference between Eqs. (48) and (51) is

$$\Delta F_c(Q^2)=\int d\sqrt{s}\,d\sqrt{s'}\,\varphi(s)\varphi(s')g_0(s,Q^2,s') \times[1-R(s,Q^2,s')], \quad (52)$$

$$R(s,Q^2,s')=\frac{M_c}{2}\frac{\sqrt{2(2M_c^2+Q^2)}}{4M_c^2+Q^2}\sqrt{\frac{s}{s'}} \times\frac{(s+s'+Q^2)^2}{(s\,s')^{1/4}}\frac{1}{\sqrt{s'}(s+Q^2)}. \quad (53)$$

The value $R(s,Q^2,s')$ presents an additional factor to one-particle currents that is in reality a two-particle current contribution. This term ensures the Lorentz covariance of the electromagnetic current matrix element and the current conservation law in Eq. (39). Let us note that this additional term contains no dynamical information on the interaction of the test particle with two constituents simultaneously. It does not depend, for example, on the interaction constants for such a process.

So, to summarize, we can write the following schematic equations:

$$(\mathrm{IA})_{\mathrm{Breit}} \neq (\mathrm{IA})_{\mathrm{lab}},$$

$$(\mathrm{MIA})_{\mathrm{Breit}} = (\mathrm{MIA})_{\mathrm{lab}}.$$

It is well known that the standard IA depends strongly on the coordinate frame used for the calculation. The MIA results do not depend on it at all. So, the differences between IA and MIA results for different IA coordinate frames can be rather significant.

Notice that IA and MIA coincide in the nonrelativistic limit. As this takes place, the nonrelativistic limits of form factors, which were obtained from the different current components, are identical. Hence the difference between the IA and MIA is really connected with the breaking of relativistic covariance conditions. We give the quantitative comparison of the form factors obtained in the IA and MIA in the Sec. IV, where the realistic calculation of the pion electromagnetic structure is given.

E. The nonrelativistic limit

The description of composite-system form factors in terms of distributions is not a specific feature of our relativistic approach. A similar formalism is widely used in the nonrelativistic theory of composite systems [47] in depth (although not referring to the mathematics of distributions). In the nonrelativistic limit our approach gives the formalism developed in Ref. [47].

In the nonrelativistic limit the relativistic charge form factor (48) has the following form:

$$F_{\mathrm{NR}}(Q^2) = \int k^2 dk\, k'^{\,2} dk'\, u(k) g_{0\mathrm{NR}}(k,Q^2,k') u(k'), \tag{54}$$

$$g_{0\mathrm{NR}}(k,Q^2,k') = \frac{f_1(Q^2)}{k\,k'\,Q}\,\theta(k,Q^2,k'), \tag{55}$$

$$\theta(k,Q^2,k') = \vartheta\left(k' - \left|k - \frac{Q}{2}\right|\right) - \vartheta\left(k' - k - \frac{Q}{2}\right).$$

Here $g_{0\mathrm{NR}}(k,Q^2,k')$ is the free relativistic form factor obtained from Eq. (29) in the nonrelativistic limit. $f_1(Q^2)$ is the charged-particle form factor. The obtained result coincides with that derived in standard nonrelativistic calculations [47].

Rigorously speaking, Eq. (54) has to be interpreted as a functional in the sense of distributions generated by the function $g_{0\mathrm{NR}}(k,Q^2,k')$ and defined on test functions $u(k)u(k')$. The ordinary function (55) generates a regular generalized function defined generally on the larger class of test functions $\psi(k,k')$ in $\mathbb{R}^2$, providing the uniform convergence of the integral. One needs the uniform convergence to take limits in the integrands.

Let us define the functional in $\mathbb{R}^2$ by the following regular distribution [compare with Eqs. (30)–(31)]:

$$\langle g_{0\mathrm{NR}}(k,Q^2,k'), \psi(k,k')\rangle = \int d\mu(k,k') g_{0\mathrm{NR}}(k,Q^2,k')\psi(k,k'), \tag{56}$$

$$d\mu(k,k') = \theta(k)\theta(k') d\mu(k) d\mu(k'), \quad d\mu(k) = k^2 dk.$$

The function $g_{0\mathrm{NR}}(k,Q^2,k')$, which appears in Ref. [47] quite formally, here has a definite physical meaning and describes the electromagnetic properties of a nonrelativistic free system of two spinless particles in the S state, one of a particle having no charge [compare with $g_0(s,Q^2,s')$ in Eqs. (25), (29), and (30)]. The statical limit $\lim_{Q^2\to 0} g_{0\mathrm{NR}}(k,Q^2,k')$ giving the system charge exists only in the weak sense as the limit of the functional (56):

$$\lim_{Q^2\to 0}\langle g_{0\mathrm{NR}}(k,Q^2,k'), \psi(k,k')\rangle = \langle e_c \delta(\mu(k') - \mu(k)), \psi(k,k')\rangle. \tag{57}$$

On the test functions $\psi(k,k') = u(k)u(k')$ [with $u(k)$ the normalized bound state wave function], the functional (56) defines the bound state form factor in the nonrelativistic IA (54). The weak limit (57) is equal to the system charge:

$$\lim_{Q^2\to 0}\langle g_{0\mathrm{NR}}(k,Q^2,k'), \psi(k,k')\rangle = e_c \int_0^\infty k^2 dk\, u^2(k) = e_c. \tag{58}$$

To go beyond the nonrelativistic IA one has to add some terms to $g_{0\mathrm{NR}}(k,Q^2,k')$. For example, such terms cause the meson exchange currents in two-nucleon systems. So, in standard nonrelativistic theory the dynamical treatment of exchange currents is performed in the same way as in our relativistic approach (45).

To conclude, one can consider our approach to the IA to be a relativistic generalization of nonrelativistic IA, and our equations for form factors in this approximation to be a relativistic generalization of the equations of Ref. [47]. Let us remark that in more complicated systems (e.g., for ρ mesons and deuterons), our relativistic form factors also have correct nonrelativistic limits that coincide with Ref. [47].

IV. THE ELECTROWEAK STRUCTURE OF PIONS

Now we apply the method of previous sections to the calculation of the electroweak structure of pions. There exist many experimental data on pions, so the effectiveness of the method can be checked by the comparison with the data (see, e.g., Ref. [15] and references therein).

A. The electromagnetic form factor of pions

The pion is spinless, so the electromagnetic current matrix element has the form (15) with $p_c \to p_\pi, F_c(Q^2) \to F_\pi(Q^2)$. In the frame of the composite-quark model, the pion is considered as the bound state of u and $\bar{d}$ quarks. We assume that quark masses are equal: $m_u = m_d = M$.

To calculate in MIA the composite-system form factor one needs to construct first the free two-particle form factor (25), (29), and (48). Contrary to the simple model of the preceding section now we consider the system of two charged particles with spins $\frac{1}{2}$. This gives the following complications. First, Eq. (21) for the current operator of the free system is now transformed to the form

$$j_\mu^{(0)}(0) = j_{1\mu} \otimes I_2 \oplus j_{2\mu} \otimes I_1 . \tag{59}$$

Here $j_{(1,2)\mu}$ is the electromagnetic current of particles and $I_{(1,2)}$ is the unity operator in the one-particle state Hilbert spaces. Equation (59) can be rewritten in terms of matrix elements:

$$\begin{aligned}&\langle \vec{p}_1, m_1; \vec{p}_2, m_2 | j_\mu^{(0)}(0) | \vec{p}_1{}', m_1'; \vec{p}_2{}', m_2' \rangle \\ &\quad = \langle \vec{p}_2, m_2 | \vec{p}_2{}', m_2' \rangle \langle \vec{p}_1, m_1 | j_{1\mu} | \vec{p}_1{}', m_1' \rangle + (1 \leftrightarrow 2).\end{aligned} \tag{60}$$

Second, the matrix element of one-particle current contains now, contrary to Eq. (22), the magnetic form factors of quarks as well as the charge ones. Now the parametrization (the elementary-particle one following Ref. [4]) is of the form:

$$\begin{aligned}\langle \vec{p}, m | j^\mu(0) | \vec{p}\,', m' \rangle = {} & \bar{u}_{\vec{p}m} \gamma^\mu u_{\vec{p}\,'m'} F_1(Q^2) \\ & - \bar{u}_{\vec{p}m} \sigma^{\mu\nu} q_\nu u_{\vec{p}\,'m'} F_2(Q^2).\end{aligned} \tag{61}$$

Where $u_{\vec{p}m}$ is the Dirac bispinor and γ^μ the Dirac matrix,

$$\sigma^{\mu\nu} = \frac{1}{2}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu), \quad q_\nu = (p - p')_\nu .$$

Using multipole parametrization we can write the one-particle current matrix element in terms of Sachs form factors:

$$G_E(Q^2) = \tilde{F}_1(Q^2) + \frac{\kappa Q^2}{4M^2} \tilde{F}_2(Q^2),$$

$$G_M(Q^2) = \tilde{F}_1(Q^2) + \kappa \tilde{F}_2(Q^2),$$

$$F_1(Q^2) = e \tilde{F}_1(Q^2), \quad F_2(t) = \frac{\kappa}{2M} \tilde{F}_2(Q^2). \tag{62}$$

Here $G_{E,M}$ are the Sachs electric and magnetic form factors, respectively, e is the particle charge, and κ is the anomalous magnetic moment.

It is convenient to use the canonical parametrization of matrix elements [32]:

$$\begin{aligned}\langle \vec{p}, m | j_\mu(0) | \vec{p}\,', m' \rangle = {} & \sum_{m''} \langle m | D^j(p, p') | m'' \rangle \langle m'' | f_1(Q^2) K_\mu' \\ & + i\, f_2(Q^2) R_\mu | m' \rangle, \\ K_\mu' = (p + p')_\mu , \quad & R_\mu = \epsilon_{\mu\nu\lambda\rho} p^\nu p'^{\,\lambda} \Gamma^\rho(p').\end{aligned} \tag{63}$$

$\Gamma(p)$ is four-vector of spin:

$$\vec{\Gamma}(p) = M\,\vec{j} + \frac{\vec{p}(\vec{p}\cdot\vec{j})}{p_0 + M}, \quad \Gamma_0(p) = (\vec{p}\cdot\vec{j}).$$

The form factors $f_1(Q^2)$ and $f_2(Q^2)$ are the electric and magnetic form factors of particles. They are connected with Sachs form factors [48]:

$$\begin{aligned} f_1(Q^2) &= \frac{2M}{\sqrt{4M^2 + Q^2}} G_E(Q^2), \\ f_2(Q^2) &= -\frac{4}{M\sqrt{4M^2 + Q^2}} G_M(Q^2). \end{aligned} \tag{64}$$

Third, now the CG coefficients are of more complicated form. They are given by Eq. (8) with $J = S = l = 0$. Contrary to the previous simple case, now the CG coefficients contain the Wigner rotation matrices.

Finally, the free two-particle form factor for the system of two particles with spin $\frac{1}{2}$ and quantum numbers $J = S = l = 0$ is of the form (see also Ref. [19]):

$$\begin{aligned} g_0^{q\bar{q}}(s, Q^2, s') = {} & n_c \frac{(s + s' + Q^2) Q^2}{2\sqrt{(s - 4M^2)(s' - 4M^2)}} \\ & \times \frac{\theta(s, Q^2, s')}{[\lambda(s, -Q^2, s')]^{3/2}} \frac{1}{\sqrt{1 + Q^2/4M^2}} \\ & \times \Big\{ (s + s' + Q^2)[G_E^u(Q^2) + G_E^{\bar{d}}(Q^2)] \\ & \times \cos(\omega_1 + \omega_2) + \frac{1}{M} \xi(s, Q^2, s')(G_M^u(Q^2) \\ & + G_M^{\bar{d}}(Q^2)) \sin(\omega_1 + \omega_2) \Big\}. \end{aligned} \tag{65}$$

Here

$$\xi(s, Q^2, s') = \sqrt{s s' Q^2 - M^2 \lambda(s, -Q^2, s')},$$

n_c is the number of quark colors, and ω_1 and ω_2 are the Wigner rotation parameters:

$$\omega_1 = \arctan \frac{\xi(s, Q^2, s')}{M[(\sqrt{s} + \sqrt{s'})^2 + Q^2] + \sqrt{s s'}(\sqrt{s} + \sqrt{s'})},$$

$$\omega_2 = \arctan\frac{\alpha(s,s')\xi(s,Q^2,s')}{M(s+s'+Q^2)\alpha(s,s')+\sqrt{ss'}(4M^2+Q^2)}, \tag{66}$$

with $\alpha(s,s')=2M+\sqrt{s}+\sqrt{s'}$, and $G^{u,\bar{d}}_{E,M}(Q^2)$ are Sachs form factors for quarks. The θ function in Eq. (65) is the same as that in Eq. (29).

An interesting effect follows from Eq. (65): due to the relativistic Wigner spin rotation effect the pion charge form factor contains the contribution of quark magnetic form factors. The pion charge form factor can be calculated using Eq. (48), with Eq. (65) for the free two-particle form factor:

$$F_\pi(Q^2)=\int d\sqrt{s}d\sqrt{s'}\,\varphi(s)g_0^{q\bar{q}}(s,Q^2,s')\varphi(s'). \tag{67}$$

B. The lepton decay constant of pions

The lepton decay constant f_π is defined by the electroweak-current matrix element [15]:

$$\langle 0|j_\mu(0)|p_\pi\rangle = i f_\pi p_{\pi_\mu}\frac{1}{(2\pi)^{3/2}}. \tag{68}$$

p_π is the four-momentum of the meson. Let us decompose the l.h.s. of Eq. (68) in the basis (7). Using the explicit form of the meson wave function (10) one can obtain for Eq. (68)

$$\int \frac{N_c}{N_{\mathrm{CG}}} d\sqrt{s}\langle 0|j_\mu(0)|\vec{p}_\pi,\sqrt{s}\rangle\varphi(s) = i f_\pi p_{\pi_\mu}\frac{1}{(2\pi)^{3/2}}. \tag{69}$$

As in Sec. II [Eq. (41)] one can divide the integrand in Eq. (69) into two parts: the covariant part (smooth ordinary function) and the invariant part:

$$\frac{N_c}{N_{\mathrm{CG}}}\langle 0|j_\mu(0)|\vec{p}_\pi,\sqrt{s}\rangle = iG(s)B_\mu(s)\frac{1}{(2\pi)^{3/2}}. \tag{70}$$

The invariant form factor $G(s)$ is a generalized function. In the same way as in calculating Eq. (46) of the preceding section, we now obtain the lepton decay constant of pion in the form

$$\int d\sqrt{s}G(s)\varphi(s) = f_\pi. \tag{71}$$

In general, the form factor $G(s)$ can be calculated in the frame of the standard model for electroweak interactions. However, in this paper we limit ourselves to a four-fermion interaction. We take for $G(s)$ the form factor that parametrizes the decay of free two-quark system:

$$\langle 0|j_\mu^{(0)}(0)|\vec{P},\sqrt{s}\rangle = iG_0(s)P_\mu\frac{1}{(2\pi)^{3/2}}. \tag{72}$$

The explicit form (72) is written by analogy to Eq. (25), not taking into account the current conservation law. The form (72) is quite similar to Eq. (68) but instead of the constant f_π the form factor depending on invariant variables is written. To calculate $G_0(s)$ let us decompose Eq. (72) in the one-particle basis (6). Now we obtain for Eq. (72):

$$\begin{aligned} iG_0(s)P_\mu\frac{1}{(2\pi)^{3/2}} = \sum_{m_1,m_2,i_c}\int\frac{d\vec{p}_1}{2p_{10}}\frac{d\vec{p}_2}{2p_{20}} &\times\langle 0|j^{(0)}_{\mu i_c}|\vec{p}_1,m_1;\vec{p}_2,m_2\rangle \\ &\times\langle\vec{p}_1,m_1;\vec{p}_2,m_2|\vec{P},\sqrt{s}\rangle. \end{aligned} \tag{73}$$

Where $i_c=1,2,3$ and the sum over i_c is the sum over the colors. The CG coefficients are known [Eq. (8)]. The current matrix element in the basis (6) can be written in the standard way in terms of the lepton decay current matrix element [15]:

$$\begin{aligned} &\langle 0|j_\mu^{(0)}|\vec{p}_1,m_1;\vec{p}_2,m_2\rangle \\ &= \frac{1}{(2\pi)^3}\bar{v}(\vec{p}_2,m_2)\gamma_\mu(1+\gamma^5)u(\vec{p}_1,m_1). \end{aligned} \tag{74}$$

We integrate in Eq. (73) in the coordinate frame with $\vec{P}=\vec{0}$. Finally, we obtain

$$G_0(s) = \frac{n_c}{2\sqrt{2}\pi P_0}(p_0+M)\left[1-\frac{k^2}{(p_0+M)^2}\right], \tag{75}$$

$$p_0=\sqrt{k^2+M^2}.$$

Substituting Eq. (75) in Eq. (71) we obtain the result, which has the following form if written in invariant variables:

$$f_\pi = \frac{2M\,n_c}{2\sqrt{2}\pi}\int d\sqrt{s}\frac{1}{\sqrt{s}}\varphi(s). \tag{76}$$

Let us notice that Eq. (76) coincides with that obtained in the frame of light-front dynamics [15]. However, although all forms of RHD are unitary equivalent [12], nevertheless after the physical approximations are made in more complicated cases the results, e.g., for form factors, can be different. This is possibly due to the fact that the unitary operators connecting different forms of RHD are interaction dependent [12] and so the RHD forms realize one and the same approximation in different ways. Let us note that the nonrelativistic limit of Eq. (76) gives the standard form in terms of coordinate space wave function at zero value.

C. The results of calculations

To calculate the electroweak structure of pions using Eqs. (67), (65), (76), and (11) the following meson wave functions were utilized:

(1) A Gaussian or harmonic oscillator (HO) wave function

$$u(k) = N_{\mathrm{HO}}\exp(-k^2/2b^2). \tag{77}$$

(2) A power-law (PL) wave function

$$u(k)=N_{\mathrm{PL}}(k^2/b^2+1)^{-n},\quad n=2,\quad 3. \tag{78}$$

(3) The wave function with linear confinement from Ref. [49]:

$$u(r)=N_T\exp(-\alpha r^{3/2}-\beta r),\quad \alpha=\frac{2}{3}\sqrt{M\,a},$$

$$\beta=\frac{M}{2}b, \tag{79}$$

where a and b are parameters of linear and Coulomb parts of the potential, respectively.

In Ref. [19] in the calculation of pion electromagnetic structure we assumed the quarks to be pointlike. The results of Ref. [19] can be considered as preliminary results. However, one has to take into account the structure of constituent quarks [50], in particular, the anomalous magnetic moment. As anomalous magnetic moments are connected with the finite size of quark, one has to take into account the explicit form of quark form factors entering Eq. (65) and the pion charge form factor (67). As in Ref. [18], let us use the following forms for quark form factors:

$$G_E^q(Q^2)=e_q f(Q^2),$$
$$G_M^q(Q^2)=(e_q+\kappa_q)f(Q^2). \tag{80}$$

Here e_q is the quark charge and κ_q the quark anomalous magnetic moment (in natural units). To obtain the explicit form of the function $f(Q^2)$ let us consider the asymptotics of pion charge form factor as $Q^2\to\infty, M\to 0$.

To obtain the asymptotic behavior let us first make the asymptotic estimation of the integrals in Eq. (67) in the point like quark approximation [$f(Q^2)=1, \kappa=0$ in Eq. (80)]. Omitting the details of calculation (given in Ref. [51]) we write the final result for the asymptotics in the form:

$$F_\pi(Q^2)\sim Q^{-2}. \tag{81}$$

The asymptotics does not depend on the actual form of the wave function and coincides with that obtained in QCD. The actual form we obtain, e.g., for Eq. (77) is

$$F_\pi(Q^2)\sim 32\sqrt{2}\frac{[\Gamma(\frac{5}{4})]^2}{\sqrt{\pi}}\,\frac{b^2}{Q^2}. \tag{82}$$

It is worth to compare the form (82) with the detailed QCD result [52]:

$$F_\pi(Q^2)=\frac{8\pi\alpha_s f_\pi^2}{Q^2}. \tag{83}$$

If $\alpha_s/\pi\sim 0.1$, then Eqs. (82) and (83) coincide at $b\sim 0.1$. So the asymptotics (81) is quite realistic.

In the case of non-point-like quarks we obtain another asymptotics because the form factor depends upon the momentum transfer. It is known that QCD gives logarithmic

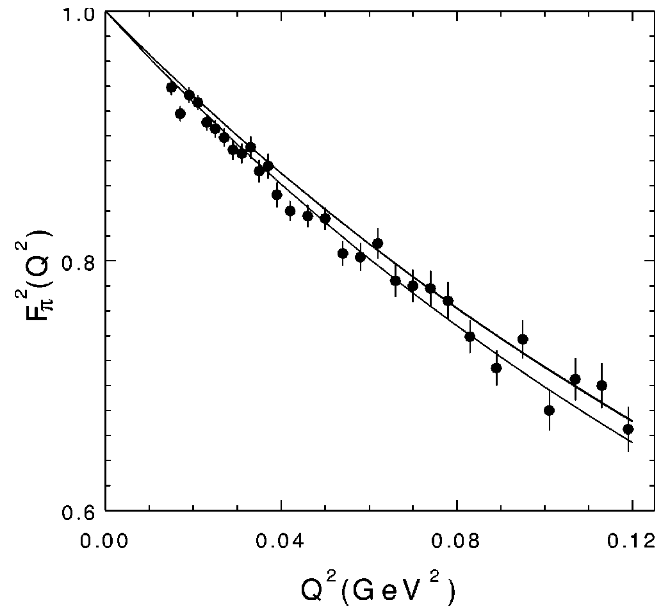

FIG. 1. The square of the pion form factor at small values of momentum transfers for different models.

corrections to relation (81). To agree with this QCD-corrected asymptotics we can, for example, choose the following form for $f(Q^2)$:

$$f(Q^2)=\frac{1}{1+\ln(1+\langle r_q^2\rangle Q^2/6)}. \tag{84}$$

Here $\langle r_q^2\rangle$ is the MSR of the constituent quark, which can be considered as the model parameter. Let us fix it (as in Ref. [18]) to be $\langle r_q^2\rangle\simeq 0.3/M^2$.

For the constituent quark mass in pions we use the value that is usually used in the calculations in RHD: $M=0.25$ GeV. The quark anomalous magnetic moments can be taken from Ref. [50]: $\kappa_u=0.029, \kappa_d=-0.059$.

We choose the parameters b in Eqs. (77) and (78) and a in Eq. (79) in such a way as to fit the pion MSR: $\langle r_\pi^2\rangle=(0.432\pm 0.016)$ fm^2 [53]. We choose this way to fix the model parameters because the pion MSR is defined by the form factor at small values of Q^2, that is, the range where potential models work well.

The fit of the pion MSR gives the following parameters of the wave functions: in the model (77) $b=0.2784$ GeV; model (78) at $n=2$, $b=0.3394$ GeV; model (78) at $n=3$, $b=0.5150$ GeV; model (79) $b=(4/3)\alpha_s$, $\alpha_s=0.59$ at the light meson mass scale, $a=0.0567$ GeV^2. The results of the calculation are presented on Figs. 1 and 2.

The square of the pion form factor at small values of momentum transfers for different models (77)–(79) is presented on Fig. 1. Results of calculation in the models (77), (78) at $n=3$ and (79) coincide very closely.

The calculations of product $Q^2F_\pi(Q^2)$ at high momentum transfers for different models (77)–(79) are presented on Fig. 2. The legend is following: 1, harmonic oscillator wave function (77); 2, power-law wave function (78) at $n=2$; 3,

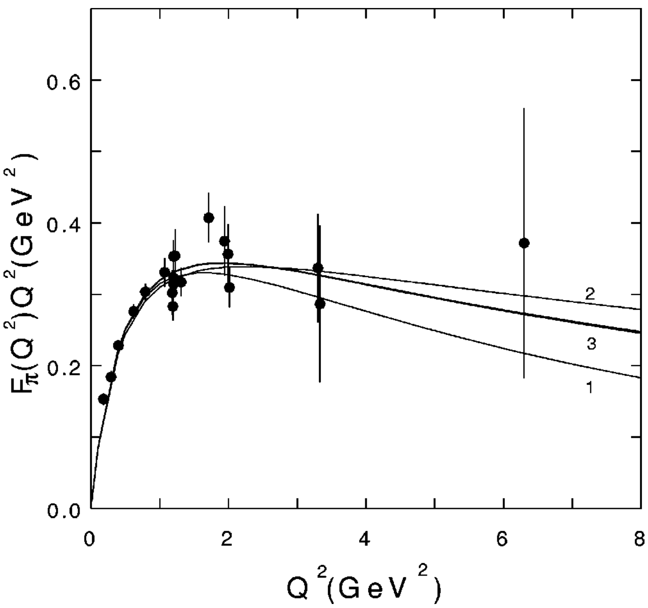

FIG. 2. Electromagnetic form factor, $Q^2F_\pi(Q^2)$, at high momentum transfers.

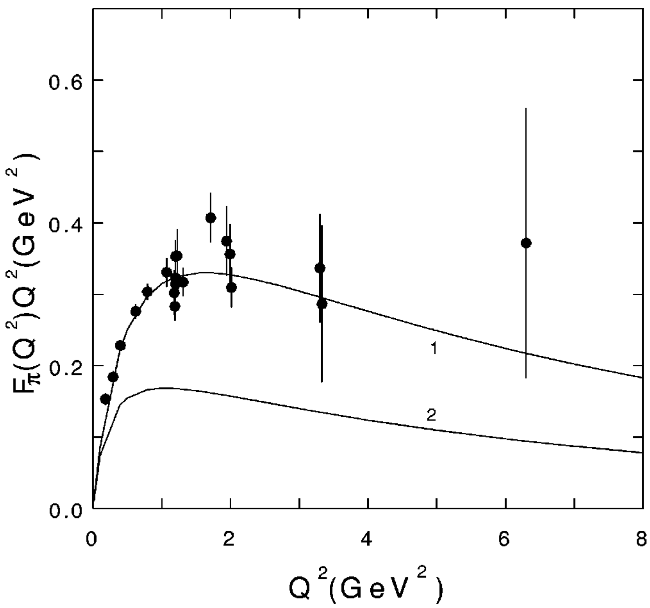

FIG. 3. $Q^2F(Q^2)$ for MIA (1) and for IA (2). Results of the calculation with the wave function (77).Parameters are the same as in Fig. 1.

power-law wave function (78) at $n=3$ and wave function from model with linear confinement (79) (these curves coincide very closely).

All the models for the interaction (77), (78), and (79) give a good description of the existing experimental data.[3] The dependence of the results on the actual model is much less pronounced that in the case of pointlike quarks [19].

The lepton decay constants calculated following Eq. (76) with different wave functions have the following values: $f_\pi=0.1210$ GeV in the model (77); $f_\pi=0.1327$ GeV in the model (78) with $n=2$; $f_\pi=0.1282$ GeV in the model (78) with $n=3$; and $f_\pi=0.1290$ GeV in the model (79). Let us emphasize that we have used no fitting parameters to calculate the lepton decay constant. Nevertheless, the obtained values are very close to the experimental value: $f_{\pi\,\mathrm{expt}}=0.1317\pm0.0002$ GeV [54].

Now let us compare the numerical results for the pion form factor obtained in MIA (67) with that of the traditional IA. Let us choose for the comparison, for example, the null component of the current.

To obtain the pion form factor in IA we proceed in the same way as while obtaining Eq. (51) of the preceding section. Now, however,

(1) the decomposition (15) of the IA matrix current element over the state set (6) is realized following Eq. (60),

(2) the parametrization of the one-particle matrix element is given by Eqs. (63) and (64) [instead of Eq. (22)],

(3) the CG coefficient (8) in Eq. (50) is for pion quantum numbers.

Acting in the same way as Eq. (51) was obtained and using the null component of the current matrix element, we can write the pion form factor in IA in the following form:

$$F_\pi(Q^2)=\frac{M_\pi}{4}\frac{\sqrt{2(2\,M_\pi^2+Q^2)}}{4\,M_\pi^2+Q^2}\frac{n_c}{\sqrt{1+Q^2/4M^2}}$$
$$\times\int\sqrt{\frac{s}{s'}}\frac{d\sqrt{s}d\sqrt{s'}}{\sqrt{(s-4\,M^2)(s'-4\,M^2)}}$$
$$\times\frac{(s+s'+Q^2)^3Q^2}{[\lambda(s,-Q^2,s')]^{3/2}}\frac{1}{(s\,s')^{1/4}}\frac{1}{\sqrt{s'}(s+Q^2)}$$
$$\times\varphi(s)\varphi(s')\Big\{(s+s'+Q^2)[G_E^u(Q^2)$$
$$+G_E^{\bar d}(Q^2)]\cos(\omega_1+\omega_2)+\frac{1}{M}\xi(s,Q^2,s')$$
$$\times[G_M^u(Q^2)+G_M^{\bar d}(Q^2)]\sin(\omega_1+\omega_2)\Big\}. \quad (85)$$

Here $M_\pi=139.568\pm0.001$ MeV [54] is the mass of a pion. The normalization condition $F_\pi(0)=1$ is satisfied for the form factor (85) if the wave functions (11) satisfy Eq. (13).

To compare the numerical results given by Eqs. (67) and (65) with that given by Eq. (85) let us calculate the pion form factor using the wave function (77) with the parameters of the calculations presented in Figs. 1 and 2. The results are shown in Fig. 3. The results obtained with the use of the parametrization (48) and (65) differ essentially from that obtained without such a parametrization (85). The form factor calculated in our approach describes the existing experimental data adequately.

[3]The JLab new results [55] are discussed in connection with our approach in Ref. [56].

Let us emphasize once again that the form factor obtained in MIA does not depend on the choice of coordinate frame. This is an important advantage of our relativistic MIA.

V. CONCLUSION

Let us summarize the results.

(1) A new approach to the electromagnetic properties of two-particle composite systems is developed. The approach is based on IF RHD.

(2) The main feature of this approach is the new method of construction of the matrix element of the electroweak current operator. The electroweak current matrix element satisfies the relativistic covariance conditions and in the case of the electromagnetic current also satisfies the conservation law automatically.

(3) The method of the construction of the current operator matrix element consists of the extraction of the invariant part—the reduced matrix element on the Lorentz group (form factor)—and the covariant part defining the transformation properties of the current. The form factors contain all the dynamical information about transition. The properties of the system as well as the approximations used are formulated in terms of form factors, which in general have to be considered as generalized functions.

(4) The approach makes it possible to formulate relativistic impulse approximation [modified impulse approximation (MIA)] in such a way that the Lorentz covariance of the current is ensured. In the electromagnetic case the current conservation law is ensured, too.

(5) The results of the calculations are unambiguous: they do not depend on the choice of the coordinate frame and on the choice of ''good'' components of the current as it takes place in the standard form of light-front dynamics.

(6) The effectiveness of the approach is demonstrated by the calculation of the electroweak structure of the pion. Our approach gives good results for the pion electromagnetic form factor in the whole range of momentum transfers available for experiments at present time.

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