Coherent pair state of the pion in the constituent quark model

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A coherent state of pions is introduced to the nonrelativistic quark model. The coherent pair approximation is employed for the pion field in order to maintain the spin-isospin symmetry. In this approximation the pion is localized in the momentum space, and the vertex form factor in the pion-quark interaction is derived from this localization. The nucleon masses and wave functions are calculated using this model, and our results are compared to those of the quark model with the one pion exchange potential. Similar result is obtained for the mass spectrum, but there exists a clear difference in the internal structure of nucleon resonances.

I. INTRODUCTION

The nonrelativistic constituent quark model (NRCQM) is one of the effective models of QCD. This model describes a baryon as a bound state of the constituent quarks in a phenomenological confinement potential.

Isgur *et al.* successfully applied the NRCQM to the study of baryon spectra [1,2]. They used the one gluon exchange potential (OGEP) with a phenomenological confinement potential in order to consider spin dependence in the effective quark-quark interaction. As suggested by De Rújula *et al.* [3], the OGEP is justified by asymptotic freedom.

On the other hand, the idea of meson contribution to baryon structure has been introduced [4], and recently extended to the quark model with the one meson exchange potential (OMEP) [5–7]. In this potential mesons belong to the pseudoscalar octet, and are closely related with the spontaneous breaking of chiral symmetry (SBχS). This model has also succeeded in reproducing the observed baryon spectra [8].

Many theoretical works have suggested that these mesons should be important degrees of freedom in the NRCQM. The OMEP seems to have desirable properties with respect to the spin-flavor dependence of the quark-quark interaction in contrast to the OGEP [5]. The dynamical role of mesons has been considered in order to deal with the long-standing problems in the NRCQM. For example, the mass difference between $\Lambda(1405)$ and $\Lambda(1520)$ can be produced by the coupling with the $\bar{K}N$ channel [9]. In a recent work, a positive parity state appears as a first excited state in the nucleon mass spectrum when the scalar-isoscalar excitation of a meson field is taken into account [10].

In many quark models, mesons are usually treated in perturbation theory. However, the perturbative method is not appropriate for mesons interacting strongly with quarks. As for the OMEP, only the one particle exchange is included. The static approximation is also used for the OMEP, while the dynamical effects of mesons such as a self-energy should not be ignored [11].

In order to understand the mesonic effects on the nucleon structure, the nonperturbative property of mesons should be considered in a effective model. In this paper we make use of the coherent state formalism to describe the pion field in the NRCQM, which is one of the nonperturbative representations of the pion field. Instead of using Glauber's coherent state [12], we apply the coherent pair approximation (CPA) [13] to our calculation. In this approximation the vacuum of the pion field is constructed from indefinite numbers of scalar-isoscalar pion pairs, and has definite quantum numbers for the orbital angular momentum and the isospin.

The projection method based on the hedgehog ansatz is known as another method of treating the pion field. In this method the quantum numbers associated with spin-isospin symmetry are projected out by semiclassical or adiabatic method from the meson field with the hedgehog structure. Reference [14] has shown that the projection with the hedgehog ansatz is a better approximation for the pion field in a ground state nucleon than the CPA [15–17]. However, this projection method has some limitation when baryon resonances are considered, in which the excitation energy of quarks in a baryon is on the same order of the fluctuation energy of the pion field [18–20]. It is difficult to improve this method for calculating the baryon spectrum as far as we rely on the spin-isospin projection. We tried to avoid this problem by choosing the eigenstates of spin-isospin symmetry for the pion field.

As argued by Isgur [21], the quark model should apply not only to baryons but also to mesons. However, it is not easy to make such a model that can deal with all hadrons systematically. At present, we approximately describe a baryon as a composite system of the constituent quarks and mesons, and focus our attention on the collective behavior of mesons in a baryon. Assuming that the meson structure is not affected by its collective motion, we treat the meson as a point particle in this paper.

We calculate the nucleon mass spectrum by taking account of the pion field in the CPA (the CPA model). We apply the CPA to the study of excited baryons, while this approximation has been used to study the ground state nucleon [15,16]. This application is considered as a good test ground to see the role of mesons in the nucleon structure. We employ a variational method to determine the momentum distribution of pion, which is similar to Tomonaga's intermedi-

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ate coupling approximation applied to the pion-nucleon and the polaron systems [22,23]. We also calculate the mass spectrum by using the NRCQM with the one pion exchange potential (the OPEP model), and compare the result with that of ours.

This article is organized as follows. In Sec. II our Hamiltonian for the CPA model is presented, and the nucleon state in the CPA model is constructed in Sec. III. The calculation with the CPA model is explained in Sec. IV. A brief review of the OPEP model are given in Sec. V. Our results obtained by the CPA model are exhibited in Sec. VI, and are compared with those by the OPEP model. A brief summary is given in the last section.

II. MODEL HAMILTONIAN

We describe the nucleon as a bound state of the three constituent quarks surrounded by the coherent state of pions. Our model Hamiltonian is written as

$$H = H_0 + H_\pi + H_{\pi q}, \tag{1}$$

where H_0 describes the relative motion of the three quarks in the center of mass (c.m.) frame, H_π the pion kinetic energy, and $H_{\pi q}$ the pion-quark interaction. We apply the nonrelativistic kinematics for the constituent quarks, and we write H_0 as

$$H_0 = \sum_{i=1}^{3} \frac{\mathbf{p}_i^2}{2m_q} - T_g + \sum_{i<j}^{3} V(r_{ij}), \tag{2}$$

where $\mathbf{p}_i$ is the momentum of the ith quark, m_q the constituent quark mass, $r_{ij} = |\mathbf{r}_{ij}| = |\mathbf{r}_i - \mathbf{r}_j|$ and $V(r_{ij})$ the phenomenological confinement potential. T_g is the c.m. kinetic energy of the three quarks. The mass difference between the u and d quarks is not considered in this work. As for the confinement potential $V(r_{ij})$, although the linear form is often used for heavy-light quark systems [24,25], we choose the quadratic potential for simplicity:

$$V(r_{ij}) = \frac{1}{6} m_q \omega^2 r_{ij}^2, \tag{3}$$

where the parameter ω determines the excitation energy of this three-quark system and its spatial distribution.

The standard form of H_π is employed in this model

$$H_\pi = \int_0^\infty dk k^2 \omega_k \sum_{l_\pi \mu \nu} a_{\mu\nu}^{l_\pi\dagger}(k) a_{\mu\nu}^{l_\pi}(k), \tag{4}$$

where $\omega_k = \sqrt{k^2 + m_\pi^2}$ with the pion mass m_π and $a_{\mu\nu}^{l_\pi}(k)$ is the isovector annihilation operator with the orbital angular momentum (l_π, μ), the isospin component ν, and the momentum $k = |\mathbf{k}|$. The annihilation operator in the spherical representation

$$a_{\mu\nu}^{l_\pi}(k) = i^{l_\pi} \int d\hat{k} Y_{l_\pi \mu}^*(\hat{k}) a_\nu(\mathbf{k}) \tag{5}$$

satisfies the commutation relation

$$[a_{\mu\nu}^{l_\pi}(k), a_{\mu'\nu'}^{l'_\pi\dagger}(k')] = \delta_{l_\pi l'_\pi} \delta_{\mu\mu'} \delta_{\nu\nu'} \frac{1}{k^2} \delta(k-k'). \tag{6}$$

The pion-quark interaction $H_{\pi q}$ takes the nonrelativistic form of the pseudoscalar (PS) coupling

$$H_{\pi q} = \sum_{j=1}^{3} H_{\pi q}^j$$

$$= \sum_{j=1}^{3} \int \frac{d^3k}{\sqrt{2\omega_k(2\pi)^3}} \frac{g}{2m_q} [i\boldsymbol{\sigma}^j \cdot \mathbf{k}\boldsymbol{\tau}^j \cdot \mathbf{a}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}_j} + \text{H.c.}], \tag{7}$$

where g is the pion-quark coupling constant, and $\boldsymbol{\sigma}^j$ and $\boldsymbol{\tau}^j$ are the spin and isospin operators for the jth quark, respectively. The pseudovector coupling is also possible for the pion-quark interaction. Since this coupling is equivalent to the PS coupling in the low energy region, we use the PS coupling for simplicity.

III. BASIS STATE FOR NUCLEON

We construct the basis state for the representation of baryons by making a direct product of a three-quark state and a coherent state of pions. The three-quark state is given by a proper combination of four parts

$$|3q\rangle = |[\psi \otimes \chi]^{j_q} \zeta^{t_q} C\rangle, \tag{8}$$

where ψ, χ, ζ, and C stand for the orbital, spin, flavor, and color parts, respectively. The total angular momentum is denoted by j_q and the total isospin by t_q.

We use the eigenstate of H_0 for the orbital part ψ. The orbital part becomes a product of two independent harmonic oscillators when we use the Jacobi coordinates. The energy eigenvalue of H_0 is $E_0 = (N_s + 3)\hbar\omega$, where $N_s = 0, 1, 2, \ldots$.

The spin part χ has either the total spin 3/2 or 1/2, and is the irreducible representation of the spin $SU(2)$ and the permutation group S_3 simultaneously. The symmetry of the flavor part ζ is the isospin $SU(2)$ because we deal with the nonstrange baryons in this work. Thus the flavor part is constructed in the same way as the spin part. The color part C is the singlet representation of the color $SU(3)$. The three-quark state $|3q\rangle$ is totally antisymmetric under the exchange of quarks.

Each $N_s\hbar\omega$ level is degenerate. In the case of $J = T = 1/2$, for example, the degeneracy is as follows. There is only one nucleon state for the $0\hbar\omega$ level. The first excited level $1\hbar\omega$ is twofold degenerate, i.e., two kinds of the p-wave excitation. The second excited level $2\hbar\omega$ is fourfold degenerate, i.e., the one-node excitation, the d-wave excitation, and two kinds of the p-wave excitation. These three-quark states are sequentially labeled by the subscript i: $|3q;i\rangle$ $(i = 1, 2, 3 \cdots)$.

The pion field in the CPA is described as the coherent pair state (CPS), which is the simultaneous eigenstate of the orbital angular momentum and the isospin. The CPS is first introduced by Bolsterli [13], and is applied to studying the

ground state nucleon by Goeke *et al.* [15]. Here we summarize the properties of the CPS briefly. See Refs. [13,15,16] for details. Following Bolsterli's definition [26], we introduce the operator $b_{\mu\nu}^{l_\pi}$ instead of using $a_{\mu\nu}^{l_\pi}(k)$:

$$b_{\mu\nu}^{l_\pi} = \int_0^\infty dk\, k^2 \xi(k) a_{\mu\nu}^{l_\pi}(k), \qquad (9)$$

with the momentum distribution function $\xi(k)$. The function $\xi(k)$ is real, and it is normalized as

$$\int_0^\infty dk\, k^2 \xi(k)^2 = 1. \qquad (10)$$

We determine $\xi(k)$ variationally in the following calculation.

The ground state of the CPS $|\text{c.p.0};\xi\rangle$, i.e., the 0-unpaired pion state, is defined as

$$|\text{c.p.0};\xi\rangle = \sum_{n=0} \frac{f_n(x)}{(2n)!} (b^{l_\pi\dagger} \cdot b^{l_\pi\dagger})^n |0\rangle, \qquad (11)$$

satisfying the eigenvalue equation

$$b^{l_\pi} \cdot b^{l_\pi} |\text{c.p.0};\xi\rangle = \sum_{\mu,\nu} (-1)^\mu b_{\mu\nu}^{l_\pi} \cdot b_{-\mu,-\nu}^{l_\pi} |\text{c.p.0};\xi\rangle$$

$$= x |\text{c.p.0};\xi\rangle, \qquad (12)$$

where x is the coherence parameter and $b^{l_\pi} \cdot b^{l_\pi}$ is scalar-isoscalar combination for the pion pair. Owing to the coherent property Eq. (12), the function $f_n(x)$ satisfies the recursion relation

$$f_{n+1}(x) = \frac{x(2n+1)}{(2L+1)(2T+1)+2n} f_n(x). \qquad (13)$$

We determine $f_0(x)$ so that $|\text{c.p.0};\xi\rangle$ is normalized. The one-unpaired pion state of the CPS is defined as

$$|\text{c.p.1};\xi;l_\pi\mu,\nu\rangle = \frac{1}{\mathcal{N}(x)} (-)^{\mu+\nu} b_{\mu\nu}^{l_\pi} |\text{c.p.0};\xi\rangle$$

$$= \frac{1}{\mathcal{N}(x)} \sum_{n=0} \frac{f_{n+1}(x)}{(2n+1)!} (b^{l_\pi\dagger} \cdot b^{l_\pi\dagger})^n b_{\mu\nu}^{l_\pi\dagger} |0\rangle, \qquad (14)$$

where $\mathcal{N}(x)$ is the normalization factor.

Using the three-quark state (8) and the CPS's (11) and (14), we can write the basis states for nucleons as

$$[|3q;i\rangle \otimes |\text{c.p.0};\xi^i\rangle]^{JT}, [|3q;i\rangle \otimes |\text{c.p.1};\xi^i;l_\pi\rangle]^{JT}, \quad (15)$$

where J and T mean the total angular momentum and the total isospin of a baryon, respectively. Note that we attach the label i to $\xi(k)$ because we consider the dependence of the CPS on the three-quark state $|3q;i\rangle$. The jth nucleon resonance is written as

$$|N_j\rangle = \sum_i (\alpha_{ji}[|3q;i\rangle \otimes |\text{c.p.0};\xi^i\rangle]^{(1/2)\,(1/2)}$$

$$+ \beta_{ji}[|3q;i\rangle \otimes |\text{c.p.1};\xi^i;l_\pi\rangle]^{(1/2)\,(1/2)}), \qquad (16)$$

where the mixing coefficients satisfy $\Sigma_i(\alpha_{ji}^2 + \beta_{ji}^2) = 1$. The ground state nucleon is labeled by $j=0$.

IV. VARIATIONAL METHOD

Here we comment on our four assumptions made in our calculation. The first assumption is for the model space in our calculation. Because we are interested in the low-lying nucleon states, we truncate the three-quark states at $2\hbar\omega$. The same reason will hold for our second assumption, i.e., we neglect the higher partial waves of the pion. We consider the pion with $l_\pi = 1$ in the following calculation. The third assumption is for the unpaired pion state in the CPA. We take account of the 0- and one-unpaired pion states defined in Eqs. (12) and (14). This is sufficient to see the nonperturbative effects, although the CPS with larger number of unpaired pions should be included for better approximation [14,15]. The last assumption is that the c.m. of three quarks is always at rest when each quark interacts with the pion. The c.m. energy of the πN system, for example, amounts to about 25% of the excitation energy 500 MeV of the Roper resonance, which should be taken into account for precise estimation of the mass spectrum. We expect, however, that the inclusion of this contribution will not change our conclusions in this paper drastically. Thus we do not consider the c.m. correction here for simplicity.

Now we construct the nucleon states with $(J,T) = (1/2,1/2)$. The 0-unpaired pion state $|\text{c.p.0};\xi^i\rangle$ has quantum numbers $(l_\pi,t_\pi) = (0,0)$, so that only the three-quark state with $(j_q,t_q) = (1/2,1/2)$ is combined with $|\text{c.p.0};\xi^i\rangle$. On the other hand, because the one-unpaired pion state $|\text{c.p.1};\xi^i;l_\pi\rangle$ has quantum numbers $(l_\pi,t_\pi) = (1,1)$, not only the three-quark state with $(j_q,t_q) = (1/2,1/2)$ but also the state with $(j_q,t_q) = (3/2,3/2)$ are combined with $|\text{c.p.1};\xi^i;l_\pi\rangle$. In this case we numerically checked that other possible quantum numbers for the three-quark states, for example, $(j_q,t_q) = (1/2,3/2)$, etc., can be neglected in our calculation.

In order to determine the momentum distribution functions $\xi^i(k)$ and the coherence parameters x_i, we minimize the expectation value of H for the ground state nucleon variationally. Considering the normalization condition for $\xi^i(k)$, we take the variation with respect to $\xi^i(k)$,

$$\delta\left(\langle N_0|H|N_0\rangle - \sum_i c_i \int_0^\infty dk\, k^2 \xi^i(k)^2 \right) = 0, \qquad (17)$$

where the Lagrange multipliers c_i are introduced. Then we obtain the explicit forms of $\xi^i(k)$ in terms of α_{0i}, β_{0i}, c_i, and x_i.

The numerical calculation is performed iteratively as follows. First we prepare the initial values of α_{0i} and β_{0i} for the fixed values of x_i and write the explicit forms of $\xi^i(k)$. The constants c_i are determined so as to normalize $\xi^i(k)$.

Next we calculate the matrix elements of H by using $\xi^i(k)$, and reevaluate the mixing coefficients α_{0i} and β_{0i} by diagonalization. This procedure is continued until these values converge. The values of x_i are chosen to give the minimum energy of the ground state nucleon.

V. CONSTITUENT QUARK MODEL WITH OPEP

In this section, we briefly review the NRCQM with the one pion exchange potential (OPEP). We intend to compare the result of this conventional model (the OPEP model) with that of our CPA model. The Hamiltonian is now

$$H = H_0 + \sum_{i<j}^{3} H_{ij}^{OPEP}, \tag{18}$$

where H_0 is the same as that of Eq. (2). The OPEP has the form [11]

$$H_{ij}^{OPEP} = \frac{g^2}{4\pi} \frac{1}{12m_q^2} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \left[S_\pi(\mathbf{r}_{ij}) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + T_\pi(\mathbf{r}_{ij}) \right.$$
$$\left. \times \left(\frac{3\boldsymbol{\sigma}_i \cdot \mathbf{r}_{ij} \boldsymbol{\sigma}_j \cdot \mathbf{r}_{ij}}{r_{ij}^2} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right) \right]. \tag{19}$$

The first term of Eq. (19) is the spin-spin interaction which generates the mass difference between N and Δ, for example. The spatial part $S_\pi(\mathbf{r}_{ij})$ of this interaction is explicitly written as

$$S_\pi(\mathbf{r}_{ij}) = \frac{2}{\pi} m_\pi^2 \int_0^\infty dq \frac{q^2}{q^2 + m_\pi^2} j_0(q r_{ij}) \tilde{U}(\mathbf{q}) - 4\pi U(\mathbf{r}_{ij}), \tag{20}$$

where $\tilde{U}(\mathbf{q})$ is the Fourier transformation of $U(\mathbf{r}_{ij})$. When the pion and the quark are considered as point particles $U(\mathbf{r}_{ij}) = \delta^3(\mathbf{r}_{ij})$. This singular function is usually regularized properly to take account of the structures of these effective particles. Because the spin-spin interaction generally has a large effect on the baryon masses, the spectrum in this model depends on the choice of this regularization function. For example, the first excited positive parity state moves down if the special form is used for the regularization function [5]. Here we employ the simple form [11]

$$U(\mathbf{r}_{ij}) = \left(\frac{\kappa}{\pi} \right)^{3/2} e^{-\kappa r_{ij}^2}, \tag{21}$$

where the range parameter κ is introduced.

The second term of Eq. (19) is the tensor interaction. The spatial part is

$$T_\pi(\mathbf{r}_{ij}) = \frac{2}{\pi} \int_0^\infty dq \frac{q^4}{q^2 + m_\pi^2} j_2(q r_{ij}) \tilde{U}(q). \tag{22}$$

The tensor force generates the mass splitting between $N(1535)$ and $N(1650)$ and has large effects on the internal structures of these states.

TABLE I. Parameter sets in the CPA model and the OPEP model.

	ω	g	κ
CPA	380 MeV	5.10	
OPEP	550 MeV	1.75	5.0 fm^{-2}

Note that in the OPEP model the nucleons are purely expressed by the three-quark states $|3q;i\rangle$, while in the CPA model the coherent pair states are included. We consider those states with $(j_q, t_q) = (1/2, 1/2)$ up to the $2\hbar\omega$ energy level in this model.

VI. RESULTS

We calculate the nucleon mass spectrum and the ground state energy of Δ by using the CPA model. The parameters are determined as follows. The experimentally observed value is used for the pion mass $m_\pi = 140$ MeV. The constituent quark mass m_q is fixed at 300 MeV. The strength ω of the confinement potential and the pion-quark coupling constant g are chosen so that the energy differences between the ground state nucleon and the negative parity states agree with the experimental data [8].

We also calculate the mass spectrum by using the OPEP model, and the result is compared with that in the CPA model. Common values are used for the m_q and m_π in these models, and the other parameters (ω, g, and κ) are determined in the same way as the CPA model.

A. Energy levels of negative parity states and Δ

Both in the CPA model and in the OPEP model, we can find the parameter sets which reproduce the observed excitation energies of negative parity states. The splitting of these states is due to the tensor force generated by the pion-quark interaction. The numerical values of parameters are summarized in Table I. The obtained mass spectra are displayed in Fig. 1.

The N-Δ mass difference becomes about 120 MeV in the CPA model and about 80 MeV in the OPEP model. The result for this quantity is somewhat improved in the CPA model, even though both values are smaller than the observed value ~ 290 MeV.

Except for the values of model-dependent parameters, the qualitatively similar results are obtained by the two different models. As far as the pion and its interaction with quarks are considered in the NRCQM, it is difficult to reproduce the masses of negative parity nucleons and the N-Δ mass difference simultaneously. The complete description of the nucleon spectrum is not realized even if the nonperturbative coherent property is introduced to the pion field.

Let us discuss the difference between the two models by considering the matrix elements of $H_{\pi q}$. In the CPA model, the matrix element between $|c.p.0;\xi^\alpha\rangle$ and $|c.p.1;\xi^\beta;1\mu,\nu\rangle$ becomes
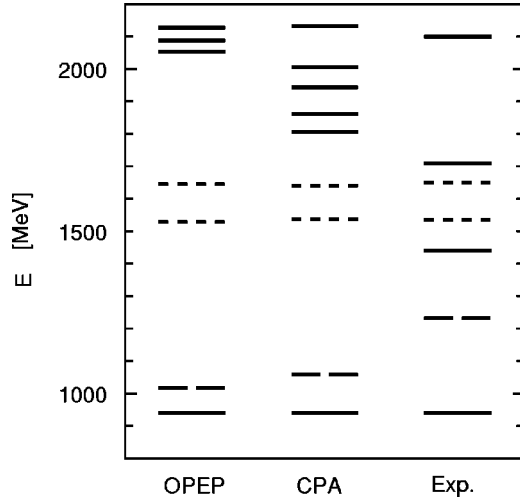
FIG. 1. Excitation spectra of the nucleon. The solid and short-dashed lines show the masses of positive parity and negative parity states, respectively. The ground state energy of Δ is shown by the dashed line for each case.

$$\langle \text{c.p.0}; \xi^\alpha | H_{\pi q} | \text{c.p.1}; \xi^\beta; 1\mu, \nu \rangle$$

$$= \sum_j \int \frac{d^3 k}{\sqrt{2\omega_k (2\pi)^3}} \frac{g}{2m_q} \boldsymbol{\sigma}^j \cdot \mathbf{k} \tau_\nu^{j\dagger} Y_{1\mu}(\hat{k})$$

$$\times [\xi^\beta(k) F^+(x_\alpha, x_\beta) e^{i\mathbf{k}\cdot\mathbf{r}_j} + \xi^\alpha(k) F^-(x_\alpha, x_\beta) e^{-i\mathbf{k}\cdot\mathbf{r}_j}],$$
$$(23)$$

where $F^+(x_\alpha, x_\beta)$ and $F^-(x_\alpha, x_\beta)$ are given by

$$F^+(x_\alpha, x_\beta) = \frac{f_1(x_\beta)}{\mathcal{N}(x_\beta)} \sum_{n=0}^\infty \frac{f_n(x_\alpha)}{(2n)!} x_\beta^n s_{\alpha\beta}^{2n},$$

$$F^-(x_\alpha, x_\beta) = \frac{f_1(x_\alpha)}{\mathcal{N}(x_\beta)} \sum_{n=0}^\infty \frac{f_{n+1}(x_\beta)}{(2n+1)!} x_\alpha^n s_{\alpha\beta}^{2n+1}, \quad (24)$$

and $s_{\alpha\beta}$ is defined as

$$s_{\alpha\beta} = \int_0^\infty dk k^2 \xi^\alpha(k) \xi^\beta(k). \quad (25)$$

We make an effective interaction which is active only for the model space composed of the three-quark and the 0-unpaired pion states

$$H_{\text{eff}}^{\text{CPA}} = \langle \text{c.p.0}; \xi^\alpha | H_{\pi q} | \text{c.p.1}; \xi^\gamma \rangle \frac{1}{E - \langle \text{c.p.1}; \xi^\gamma | H_\pi | \text{c.p.1}; \xi^\gamma \rangle}$$

$$\times \langle \text{c.p.1}; \xi^\gamma | H_{\pi q} | \text{c.p.1}; \xi^\beta \rangle. \quad (26)$$

In the OPEP model, the matrix element between $|0\rangle$ (the normal vacuum for the pion field) and $|\mathbf{q}, \nu\rangle = a_\nu^\dagger(\mathbf{q})|0\rangle$ is

$$\langle 0 | H_{\pi q}^j | \mathbf{q}, \nu \rangle = \frac{1}{\sqrt{2\omega_q (2\pi)^3}} \frac{g}{2m_q} i\boldsymbol{\sigma}^j \cdot \mathbf{q} e^{i\mathbf{q}\cdot\mathbf{r}_j} \tau_\nu^{j\dagger} \sqrt{\tilde{U}(q)},$$
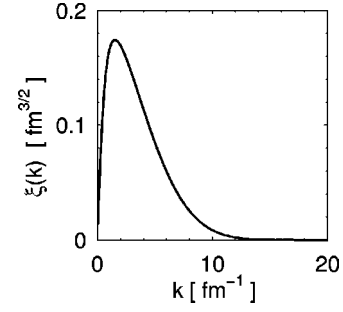$$(27)$$

FIG. 2. Momentum distribution function $\xi^0(k)$ of the pion field for the three-quark ground state with $(j_q, t_q) = (1/2, 1/2)$.

where $\mathbf{q}$ is the momentum of exchanged pion and $\tilde{U}(q)$ the form factor introduced in Sec. V. By using this matrix element, we can write the OPEP as

$$H^{\text{OPEP}} = \sum_{i<j} \left(\sum_\nu \int d^3 q \langle 0 | H_{\pi q}^i | \mathbf{q}, \nu \rangle \right.$$

$$\left. \times \frac{1}{-\omega_q} \langle \mathbf{q}, \nu | H_{\pi q}^j | 0 \rangle + (i \leftrightarrow j) \right). \quad (28)$$

Note that the closure is assumed for the intermediate baryons in this process.

We first comment on the vertex form factor appeared in the matrix elements (23) and (27). The vertex form factor is necessary for the low energy effective model in order to suppress the high momentum contribution of pion. The form factor also suppresses the coupling with highly excited baryons and higher partial waves of pion included implicitly in the intermediate states. Many kinds of explanation are given for the origin of the form factor, e.g., due to the finite size of the pion and quark. Thus in the OPEP model we must supply the form factor $\tilde{U}(q)$ to the pion-quark interaction with an additional parameter κ.

In the CPA model, we consider only the p-wave pion. The momentum distribution function $\xi^\alpha(k)$ in the matrix element Eq. (23), which is displayed in Fig. 2, plays the role of a vertex form factor such as $\tilde{U}(q)$ in the OPEP model. The coherent property accounts for the origin of the form factor: the coherent state of pions is localized, and high momentum contribution of pion is cut off. The form factor in the CPA model does not include any free parameters, and is self-consistently determined by the variation.

Secondly, we notice that the values of g in the two models are different. In the OPEP model all the continuum states of the plane wave pion (except for the high momentum components) contribute to the interaction. On the other hand, in the CPA model the pion in the coherent pair state always has the localized distribution specified by $\xi^i(k)$, and continuum states of pion is excluded from this model. Because of this constraint on the pion state, the matrix elements of $H_{\text{eff}}^{\text{CPA}}$ become smaller than those of H^{OPEP}. The large value of g is required in the CPA model in order to reproduce the observed spectrum, while a smaller value of g is found in the OPEP model.

TABLE II. Mixing coefficients of the negative parity states. Masses calculated for each state are displayed in the second column. Both states include the spatial part with the p-wave excitation of one quark (thus the parity is negative), and the flavor part with mixed symmetry ($t_q = 1/2$). These states are distinguished by the total intrinsic spin s_q ($s_q = 1/2, 3/2$). Both states have total spin $j_q = 1/2$.

CPA	mass [MeV]	$[\lvert 3q; s_q=3/2\rangle \otimes \lvert \text{c.p.0}\rangle]^{(1/2)\,(1/2)}$	$[\lvert 3q; s_q=1/2\rangle \otimes \lvert \text{c.p.0}\rangle]^{(1/2)\,(1/2)}$
	1537	0.682	0.652
	1641	0.690	−0.720
OPEP	mass [MeV]	$\lvert 3q; s_q=3/2\rangle$	$\lvert 3q; s_q=1/2\rangle$
	1529	0.399	0.917
	1646	0.917	−0.399

Here we comment on our assumption about the partial wave of pion in the intermediate state. As demonstrated by our calculation, we obtain the qualitatively similar results in the two models for the masses of negative parity nucleons and the N-Δ mass difference. The use of the large value of g compensates for neglecting all partial waves except the p wave in the CPA model. We conclude that the contribution to the nucleon masses is dominated by the p-wave pion in the NRCQM including the pionic degree of freedom.

B. Mixings coefficients

Table II shows the mixing coefficients for the negative parity excited states. In both models these two resonances are mostly due to the p-wave excitation of quarks in the nucleon, and are also mixtures of the two basis states with spin 1/2 and 3/2 for the three-quark part. The structure of these states in the CPA model is found to be different from that in the OPEP model.

Let us consider the mixture of the two basis states, which is generated by the tensor force appearing in H^{OPEP} and in the effective interaction $H^{\text{CPA}}_{\text{eff}}$. There are three matrix elements of the Hamiltonian H with respect to these basis states: two diagonal elements $\langle H\rangle_{1/2}$, $\langle H\rangle_{3/2}$ and an off-diagonal element $\langle H\rangle_{1/2,3/2}$. The mixing coefficients are solely determined by the ratio $(\langle H\rangle_{3/2} - \langle H\rangle_{1/2}) : \langle H\rangle_{1/2,3/2}$. Calculating the matrix elements, we obtain 0.1 : 1 when we use the Hamiltonian $H = H_\pi + H^{\text{CPA}}_{\text{eff}}$ in the CPA model, and 1.9:1 in the OPEP model with $H = H^{\text{OPEP}}$. The difference in this ratio between the two models can be explained if we consider the intermediate baryons included in the effective interactions (26) and (28). The mixing coefficients of negative parity nucleons are sensitive to the model space for baryons considered in the effective model.

The magnitudes and the relative sign of the mixing coefficients manifest the internal structures of $N(1535)$ and $N(1650)$. It is well known that the OPEP model fails to describe their configuration mixing, from the analyses of πN and ηN reactions [27]. The CPA model does not improve this shortcoming of the OPEP model. As explained in Ref. [28], the meson-quark coupling is not sufficient to explain

TABLE III. Mixing coefficients of $N(940)$. The quarks are always in the $0\hbar\omega$ state in this table. $\lvert 3q; N\rangle$ stands for the three-quark state with $(j_q, t_q) = (1/2, 1/2)$, and $\lvert 3q; \Delta\rangle$ with $(j_q, t_q) = (3/2, 3/2)$.

CPA	$[\lvert 3q;N\rangle \otimes \lvert \text{c.p.0}\rangle]^{(1/2)\,(1/2)}$	$[\lvert 3q;N\rangle \otimes \lvert \text{c.p.1}\rangle]^{(1/2)\,(1/2)}$	$[\lvert 3q;\Delta\rangle \otimes \lvert \text{c.p.1}\rangle]^{(1/2)\,(1/2)}$
	0.860	−0.293	−0.334
OPEP	$\lvert 3q;N\rangle$		
	0.998		

these reactions, and we need something else for the quark-quark interaction.

The structures of $N(940)$ and $\Delta(1232)$ are shown in Tables III and IV, respectively. In both models, $N(940)$ is dominated by the three-quark state in the lowest energy. This is also the case for $\Delta(1232)$, and the d-wave excitation of quarks is hardly found in this state [29–31]. We note that $N(940)$ and $\Delta(1232)$ in the CPA model have the three-quark component with $(j_q, t_q) = (3/2, 3/2)$ and $(j_q, t_q) = (1/2, 1/2)$, respectively. This is because the CPA model includes the one-unpaired pion state in the description of baryons.

C. First excited state of positive parity nucleon

The first excited state of the nucleon is called the Roper resonance $N(1440)$. The excitation energy of this resonance is hard to reproduce in the constituent quark model without introducing strong short-range attractive force among the quarks [5–7]. Several recent works have suggested that the positive parity state can be interpreted as a scalar-isoscalar excitation of some extra degrees of freedom [10,32]. The first positive parity excitation in our calculation does not correspond to the observed Roper resonance but to another state with higher mass. The coherent state of pions in the CPA does not produce such a strong short-range force as suggested by Refs. [5–7]. As for the spectrum of positive parity nucleons, the CPA model is similar to the OPEP model.

However, the internal structure for the positive parity excitation is remarkably different between the two models. In the OPEP model, the first positive parity excitation is mainly due to the nodal excitation of quarks (about 97%) which has the symmetric spatial part. On the other hand, in the CPA model, the components of this excited state are classified as follows: the three quarks in the lowest energy with the one-unpaired pion (about 47%), and the nodal excitation of quarks with the 0-unpaired pion (about 38%). This nodal

TABLE IV. Mixing coefficients of $\Delta(1232)$. The notation is the same as in Table III.

CPA	$[\lvert 3q;\Delta\rangle \otimes \lvert \text{c.p.0}\rangle]^{(3/2)\,(3/2)}$	$[\lvert 3q;N\rangle \otimes \lvert \text{c.p.1}\rangle]^{(3/2)\,(3/2)}$
	0.898	−0.389
OPEP	$\lvert 3q;\Delta\rangle$	
	0.9998	

excitation has the mixed symmetric spatial part in contrast to the symmetric excitation in the OPEP model.

This result shows that the three-quark state accompanied by the one-unpaired pion constitutes an additional positive parity state in the nucleon spectrum. This is the new aspect of our study taking account of the explicit role of pion in the NRCQM. However, there is no Roper-like excitation below the negative parity nucleons because the additional state (pion mode excitation in the nucleon) has an excitation energy on the order of $2\hbar\omega$.

VII. SUMMARY

We have introduced the coherent state of pions to the NRCQM in the CPA. The coherent property of the pion does not change the prediction of the OPEP model drastically. As for the nucleon mass spectrum, the CPA model gives quali-tatively similar results in comparison with the OPEP model. However, the difference between the two models is clearly seen in the pion-quark interaction and the internal structure of baryons.

In view of the SBχS, the pion is also an important con-stituent of the nucleon as well as the constituent quarks. Be-cause the mass spectrum generated only by the excitation of quarks is not satisfactory for explaining the observed nucleon spectrum, the excitation of extra degrees of freedom, such as the pion in our work, should be considered in the analyses of the nucleon mass spectrum.

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