Calculation of $1/N_c$ corrections to the SU(3)-flavor Nambu–Jona-Lasino model

Hu Li, Qing Sun, and C. M. Shakin*
Department of Physics and Center for Nuclear Theory, Brooklyn College of the City University of New York, Brooklyn, New York 11210
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There is a very large body of literature in which the Nambu–Jona-Lasino (NJL) model has been applied in the study of mesons, baryons, and hadronic matter. One area of interest has been the calculation of corrections of order $1/N_c$ to various quantities calculated in the leading (Hartree) approximation. Of particular interest is the work of Dmitrašinović, Schulze, Tegen, and Lemmer [Ann. Phys. (N.Y.) **238**, 332 (1995)]. These authors have considered the SU(2)-flavor NJL model and identified a set of diagrams, whose calculation yields $1/N_c$ corrections, while at the same time maintaining the relations, such as the Goldberger-Treiman relation, that follow from the chiral symmetry of the theory. In the present work we extend the work of Dmitrašinović *et al.* to the case of SU(3)-flavor symmetry. In particular, we consider $1/N_c$ corrections to the quark vacuum condensates and to the "gap equation." While $1/N_c$ corrections to the pion decay constant are significant, the corrections to the condensates are found to be quite small, as was the case in the SU(2)-flavor analysis. The Pauli-Villars regularization procedure is thought to be particularly useful for such calculations, since that procedure is known to maintain the symmetries of the theory. As part of our analysis we extend the Pauli-Villars regularization procedure to the case in which a particular diagram contains quarks of different constituent mass. Such diagrams appear when we generalize the SU(2)-flavor analysis to SU(3). We also show that, if we use the "textbook" definition of the Pauli-Villars method, in which fictitious particles of large mass are added to the theory, the resulting formalism may not be used in the case of the NJL model. What is done in practice is that particular divergent (and nondivergent) integrals are regulated in a fashion that yields results that are quite similar to the results of the covariant regularization procedure that is used by many authors. For constituent quark mass values of the magnitude usually used in the case of the SU(3)-flavor NJL model, we find that the $1/N_c$ corrections in the calculation of the pion decay constant play a role in obtaining satisfactory values for the quark vacuum condensates and the strength of the 't Hooft interaction.

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I. INTRODUCTION

There have been a great number of applications of the Nambu–Jona-Lasinio (NJL) model [1], including recent work that studies quark matter at the very high densities appropriate for problems in astrophysics [2]. Useful reviews of earlier work may be found in Refs. [3–5]. One body of work of interest to us is the consideration of $1/N_c$ corrections to the simplest form of the theory [6–9]. Of particular interest is the work of Ref. [9], since the authors have shown how to calculate $1/N_c$ corrections while maintaining the relations that follow from the chiral symmetry of the theory. The analysis of Ref. [9] was made for the SU(2)-flavor version of the NJL model. One goal in this work is to extend the work of Ref. [9] to the SU(3)-flavor NJL model. To that end we consider the Lagrangian

$$\mathcal{L}=\bar{q}(i\partial\!\!\!/-m^0)q+\frac{G_s}{2}\sum_{a=0}^{8}[(\bar{q}\lambda^a q)^2+(\bar{q}i\gamma_5\lambda^a q)^2]+\frac{G_D}{2}\{\det[\bar{q}(1+\gamma_5)q]+\det[\bar{q}(1-\gamma_5)q]\}. \quad (1.1)$$

Here the λ^α are the Gell-Mann matrices with $\lambda^\alpha=\sqrt{2/3}\,\mathbf{1}$, where $\mathbf{1}$ is the unit matrix in the flavor space. The third term on the right-hand side of Eq. (1.1) is the 't Hooft interaction. Further, $m^0=\mathrm{diag}\,(m_u^0,m_d^0,m_s^0)$ is the quark (current) mass matrix.

Using Eq. (1.1) we may identify the quark self-energy for the SU(3)-flavor analysis as in Fig. 1. There, the first two terms define the Hartree approximation. (The Fock terms may be incorporated in an effective Hartree term after making a Fierz transformation [8,9].) The third term of Fig. 1 was considered in Ref. [9]. The wavy line denotes pseudoscalar and scalar mesons. In the SU(2)-flavor version of the NJL model these are just the π and σ mesons. For SU(3)-flavor we consider the π, K, and η_s pseudoscalar mesons and the σ, K_0^*, and a_0 scalar mesons. (Note that the $\eta(547)$ has been shown to be largely of flavor-octet structure [10].) The last term of the self-energy shown in Fig. 1 represents the contribution of the 't Hooft interaction [3–5].

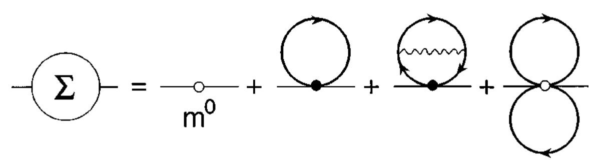

FIG. 1. The figure depicts the model of the quark self-energy considered in this work. The third diagram on the right-hand side represents a $1/N_c$ correction calculated in Ref. [9] for the SU(2), flavor version of the NJL model. Here we calculate this diagram for the SU(3)-flavor model. The wavy line denotes the exchanged meson of momentum q. The last term represents the 't Hooft interaction that plays an important role in the SU(3)-flavor analysis. [3–5].

*Email address: casbc@cunyvm.cuny.edu

We may write the "gap equations" whose solution yields the constituent quark masses as,

$$m_u = m_u^0 - 2G_s\langle\overline{u}u\rangle - G_D\langle\overline{d}d\rangle\langle\overline{s}s\rangle, \tag{1.2}$$

$$m_d = m_d^0 - 2G_s\langle dd\rangle - G_D\langle\overline{u}u\rangle\langle\overline{s}s\rangle, \tag{1.3}$$

$$m_u = m_u^0 - 2G_s\langle\overline{s}s\rangle - G_D\langle\overline{u}u\rangle\langle\overline{d}d\rangle. \tag{1.4}$$

We define $\delta\langle\overline{u}u\rangle$ to be the correction due to the third term in Fig. 1, etc. Therefore, we write

$$\langle\overline{u}u\rangle = \langle\overline{u}u\rangle_0 + \delta\langle\overline{u}u\rangle, \tag{1.5}$$

$$\langle\overline{d}d\rangle = \langle\overline{d}d\rangle_0 + \delta\langle\overline{d}d\rangle, \tag{1.6}$$

and

$$\langle\overline{s}s\rangle = \langle\overline{s}s\rangle_0 + \delta\langle\overline{s}s\rangle. \tag{1.7}$$

(Since we will use $m_u^0 = m_d^0$, we have $\langle\overline{u}u\rangle = \langle\overline{d}d\rangle$.) In Eqs. (1.5)–(1.7) $\langle\overline{u}u\rangle_0$ is of order N_c and $\delta\langle\overline{u}u\rangle$ is of order 1, etc. Therefore, since G_S is of order $1/N_c$, $G_S\delta\langle\overline{u}u\rangle$ is a correction of order $1/N_c$. Further, G_D is of order $(1/N_c)^3$ [11], so that G_D $\langle\overline{d}d\rangle$ $\langle\overline{s}s\rangle$ is of order $1/N_c$. Note that $\delta\langle\overline{u}u\rangle$, $\delta\langle\overline{d}d\rangle$, and $\delta\langle\overline{s}s\rangle$ are quite small, so it does not matter whether we write G_D $\langle\overline{d}d\rangle_0$ $\langle\overline{s}s\rangle_0$ or G_D $\langle\overline{d}d\rangle$ $\langle\overline{s}s\rangle$ in Eq. (1.2). (The former choice is more consistent with $1/N_c$ counting procedures, however.)

The organization of our work is as follows. In Sec. II we describe the calculation of $\delta\langle\overline{u}u\rangle$, $\delta\langle\overline{d}d\rangle$, and $\delta\langle\overline{s}s\rangle$. The regularization of the various diagrams we calculate is discussed in the Appendixes. Our discussion of the Pauli-Villars regularization for the SU(3)-flavor case represents a generalization of the procedures introduced in Ref. [9] for the SU(2)-flavor analysis. However, in Sec. III we show that the Pauli-Villars procedure, as usually defined in quantum field theory, cannot be applied in a systematic fashion in the case of the NJL model. We also explain why the authors of Ref. [9] were able to obtain reasonable results in their version of the Pauli-Villars regularization procedure. In Sec. IV we make use of the method of Ref. [9] to implement the Pauli-Villars procedure and also use the standard covariant regularization procedure. We present some results of our numerical calculations in Sec. IV. Section V contains some further discussion and conclusions.

II. CALCULATION OF VACUUM CONDENSATES IN THE SU(3)-FLAVOR NJL MODEL

After passing to Euclidean space with $q_E^2 = -q^2$ we obtain the following result for $\delta\langle\overline{u}u\rangle$:

$$\begin{aligned}
\delta\langle\overline{u}u\rangle = &-\frac{3}{4}\left(\frac{N_c}{2\pi^2}\right)\int_0^{\tilde{\Lambda}} dq_E q_E^3 \frac{G_S}{1-G_S J_P(m_u,m_u,q_E)}\, iT_{PS}(m_u,m_u,q_E)\\
&-\frac{1}{2}\left(\frac{N_c}{2\pi^2}\right)\int_0^{\tilde{\Lambda}} dq_E q_E^3 \frac{G_S}{1-G_S J_P(m_u,m_s,q_E)}\, iT_{PS}(m_s,m_u,q_E)\\
&-\frac{1}{12}\left(\frac{N_c}{2\pi^2}\right)\int_0^{\tilde{\Lambda}} dq_E q_E^3 \frac{G_S}{1-G_S J_P^{88}(m_u,m_s,q_E)}\, iT_{PS}(m_u,m_u,q_E)\\
&+\frac{3}{4}\left(\frac{N_c}{2\pi^2}\right)\int_0^{\tilde{\Lambda}} dq_E q_E^3 \frac{G_S}{1-G_S J_S(m_u,m_u,q_E)}\, iT_S(m_u,m_u,q_E)\\
&+\frac{1}{2}\left(\frac{N_c}{2\pi^2}\right)\int_0^{\tilde{\Lambda}} dq_E q_E^3 \frac{G_S}{1-G_S J_S(m_u,m_u,q_E)}\, iT_S(m_s,m_u,q_E)\\
&+\frac{1}{4}\left(\frac{N_c}{2\pi^2}\right)\int_0^{\tilde{\Lambda}} dq_E q_E^3 \frac{G_E}{1-G_S J_S(m_u,m_u,q_E)}\, iT_S(m_u,m_u,q_E).
\end{aligned} \tag{2.1}$$

The contributions in Eq. (2.1) are for the sequence of mesons π, K, η_s, a_0, K_0^*, and σ. Note that the pseudoscalar meson contribution enters with the opposite sign to that of the scalar mesons. [We may make contact with Ref. [9] by noting relations of the form

$$\frac{G_S}{1-G_S J_P(m_u,m_u,q_E)} = \frac{g_\pi^2(q_E^2)}{q_E^2+m_\pi^2}. \tag{2.2}$$

The approximation $g_\pi^2(q_E^2) \approx g_\pi^2(0)$ is often used.]

In the calculation of Eq. (2.1) we have defined

$$\begin{aligned}
T_P(m_1,m_2,q_E) = {}&[m_2 I(m_2,m_2,0) + (m_2-m_1) I(m_1,m_2,q_E)\\
&-(-m_2 q_E^2 - m_2^3 + m_1 m_2^2) K(m_1,m_2,q_E)],
\end{aligned} \tag{2.3}$$

so that

$$T_P(m,m,q_E)=m[I(m,m,0)+q_E^2K(m,m,q_E^2)]. \quad (2.4)$$

Also,

$$T_S(m_1,m_2,q_E)=[m_2I(m_2,m_2,0)+(m_1+m_2)I(m_1,m_2,q_E) + (m_2^3+2m_1m_2^2+m_2m_1^2 + m_2q_E^2)K(m_1,m_2,q_E)], \quad (2.5)$$

and

$$T_S(m,m,q_E)=m[I(m,m,0)+2I(m,m,q_E) + (4m^2+q_E^2)K(m,m,q_E^2)]. \quad (2.6)$$

The evaluation of the functions $J_P(m_1,m_2,q_E)$, $J_S(m_1,m_2,q_E)$, $I(m_1,m_2,q_E)$, and $K(m_1,m_2,q_E)$ is discussed in the Appendixes. In Eq. (2.1) we have used

$$J_p^{ss}(m_u,m_s,q_E)=\tfrac{1}{6}[J_P(m_u,m_u,q_E)+J_P(m_d,m_d,q_E) + 4J_P(m_s,m_s,q_E)] \quad (2.7)$$

$$=\tfrac{1}{3}[J_P(m_u,m_u,q_E)+2J_P(m_s,m_z,q_E)], \quad (2.8)$$

where a factor of $\frac{1}{2}$ is included to remove a factor of 2 in the definition of J_P that arises from the flavor trace.

The result for $\delta\langle\bar{s}s\rangle$ is

$$\delta\langle\bar{s}s\rangle=-\left(\frac{N_c}{2\pi^2}\right)\int_0^{\tilde{\Lambda}}dq_E q_E^3\frac{G_S}{1-G_SJ_P(m_u,m_s,q_E)}\,iT_{PS}(m_u,m_s,q_E) - \frac{1}{3}\left(\frac{N_c}{2\pi^2}\right)\int_0^{\tilde{\Lambda}}dq_E q_E^3\frac{G_S}{1-G_SJ_P^{88}(m_u,m_s,q_E)}\,iT_{PS}(m_s,m_s,q_E) + \left(\frac{N_c}{2\pi^2}\right)\int_0^{\tilde{\Lambda}}dq_E q_E^3\frac{G_S}{1-G_SJ_P(m_s,m_u,q_E)}\,iT_S(m_u,m_s,q_E). \quad (2.9)$$

Here, the sequence of contributions are those due to K, η_s, and K_0^* mesons.

III. THE PAULI-VILLARS REGULARIZATION PROCEDURE

In a quantum field theory the Pauli-Villars regularization procedure introduces fictitious large mass particles, which may have real or imaginary coupling constants. The standard discussion of this procedure in the case of QED vacuum polarization diagrams may be found in the textbook of Itzkson and Zuber [12], for example. If we follow that procedure for the vacuum polarization diagrams of the NJL model, we would have

$$J_S^{PV}(m,m,q_E^2)=J_S(m,m,q_E^2)+J_S(M_1,M_1,q_E^2) - 2J_S(M_2,M_2,q_E^2), \quad (3.1)$$

with $M_1^2=m^2+2\Lambda^2$ and $M_1^2=m^2+\Lambda^2$. (The unequal mass case is discussed in the Appendixes.)

In Ref. [9] the value of Λ is fixed by calculating the pion decay constant. There is an additional parameter $\tilde{\Lambda}$ that regulates the integral over the momentum of the meson in the third term of Fig. 1, and which also appears in the calculation of f_π. For definiteness let us consider $\tilde{\Lambda}=\Lambda=0.80$ GeV, and up and down quark masses of 364 MeV. We then have $M_1=1.19$ GeV and $M_2=0.879$ GeV. Note that M_2 is not much greater than $m_\sigma=2m_u=0.728$ GeV. That suggests that the approximation

$$\frac{G_S}{1-G_SJ_S(p^2)}\approx\frac{g_\sigma^2}{p^2+m_\sigma^2} \quad (3.2)$$

is inadequate. Alternatively, we may argue that the fictitious particles introduced in the Pauli-Villars procedure of mass $M_1^2=m^2+2\Lambda^2$ and $M_2^2=m^2+\Lambda^2$ have masses that are too close to the mass values of the physical mesons for the procedure to work well. To be more precise, we have calculated $J_S^{PV}(m,m,q_E^2)$ and $J_P^{PV}(m,m,q_E^2)$ with $m=0.364$ GeV and $\Lambda=0.80$ GeV. We find that $J_S^{PV}(m,m,0)$ is greater than $J_P^{PV}(m,m,0)$. That means that there are scalar bound states of negative energy, since $G_S^{-1}=J_P^{PV}(m,m,0)$ in the chiral limit $(m_\pi=0)$. This problem may be avoided if we do not regulate the vacuum polarization functions as in Eq. (3.1), but only regulate the functions $F(m^2)$ and $I(m,m,q_E)$ that appear in our expressions for $J_g(m,m,q_E^2)$ and $J_P(m,m,q_E^2)$. In a similar fashion, we also regulate the convergent integral $K(m,m,q_E)$ [9].

IV. RESULTS OF NUMERICAL CALCULATIONS

Here we consider the solutions of Eqs. (1.2)–(1.4). We chose values of $m_u=m_d$ and m_s. We then chose a value for t such that $\tilde{\Lambda}^2=t\Lambda^2$, where $\tilde{\Lambda}$ is the cutoff for the meson momentum seen in Eqs. (2.1) and (2.9). We then find Λ such that the value of the pion decay constant $f_\pi=0.0924$ GeV. (For this work we neglect the strangeness content of the pion and use the formula for f_π given in Ref. [9].) We then calculate the condensates $\langle\bar{u}u\rangle_0=\langle\bar{d}d\rangle_0$ and $\langle\bar{s}s\rangle_0$. We calcu-

TABLE I. Results for the Pauli-Villars regularization procedure. Here we have determined the value of Λ using the procedure of Ref. [9] with $\tilde{\Lambda}^2 = t\Lambda^2$. We use $m_u = m_d = 0.364$ GeV and $m_u = 0.522$ GeV, and calculate $-\langle \bar{u}u \rangle_0^{1/3}$, $-\langle \bar{s}s \rangle_0^{1/3}$ and the corresponding values of G_S and G_D. We also present results for the corrected condensate values $-\langle \bar{u}u \rangle^{1/3}$ and $-\langle \bar{s}s \rangle^{1/3}$ (see Sec. II).

t	Λ [GeV]	$\tilde{\Lambda}$ [GeV]	f_π^0 [GeV]	G_S [GeV^{-2}]	G_D [GeV^{-5}]	$-\langle \bar{u}u \rangle_0^{1/3}$ [GeV]	$-\langle \bar{s}s \rangle_0^{1/3}$ [GeV]	$-\langle \bar{u}u \rangle^{1/3}$ [GeV]	$-\langle \bar{s}s \rangle^{1/3}$ [GeV]	$\frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle}$
1.0	0.816	0.816	0.109	7.650	−221.5	0.2621	0.2750	0.2652	0.2699	1.054
0.9	0.799	0.758	0.108	8.221	−234.1	0.2510	0.2690	0.2597	0.2648	1.061
0.8	0.781	0.699	0.106	8.864	−245.4	0.2526	0.2640	0.2541	0.2597	1.068
0.7	0.763	0.638	0.105	9.595	−254.4	0.2477	0.2584	0.2486	0.2546	1.074
0.6	0.745	0.577	0.103	10.43	−259.0	0.2428	0.2528	0.2431	0.2494	1.080
0.5	0.727	0.514	0.101	11.38	−256.4	0.2379	0.2471	0.2379	0.2449	1.085
0.4	0.710	0.449	0.0996	12.48	−242.4	0.2329	0.2414	0.2327	0.2394	1.089
0.3	0.692	0.379	0.0978	13.75	−210.7	0.2279	0.2357	0.2276	0.2344	1.091
0.2	0.675	0.302	0.0961	15.24	−152.2	0.2230	0.2300	0.2227	0.2293	1.092
0.1	0.658	0.208	0.0942	16.98	−53.2	0.2181	0.2245	0.2179	0.2242	1.090
0.0	0.693	0.000	0.0924	19.06	106.6	0.2132	0.2189	0.2132	0.2189	1.083

late G_S and G_D after putting $m_u^0 = 0.0055$ GeV and $m_u^0 = 0.132$ GeV. Once we obtain Λ, $\tilde{\Lambda}$, and G_S we calculate $\delta\langle \bar{u}u \rangle = \delta\langle \bar{d}d \rangle$ and $\delta\langle \bar{s}s \rangle$. The values shown in Table I were calculated for $m_u = m_d = 0.364$ GeV and $m_s = 0.522$ GeV using the Pauli-Villars regularization as defined in Ref. [9]. The values chosen for m_u and m_s are those used in the extensive calculations reported in Ref. [4]. From the table we see that $\langle \bar{u}u \rangle^{1/3}$ and $\langle \bar{s}s \rangle^{1/3}$ differ from $\langle \bar{u}u \rangle_0^{1/3}$ and $\langle \bar{s}s \rangle_0^{1/3}$ by about 1–2%. There is about a 3–5% difference between $\langle \bar{u}u \rangle_0$ and $\langle \bar{u}u \rangle$ or $\langle \bar{s}s \rangle_0$ and $\langle \bar{s}s \rangle$. The value of the condensate is somewhat uncertain with $\langle \bar{u}u \rangle \approx -(0.250 \pm 0.025 \text{ GeV})^3$. For most values of Λ and $\tilde{\Lambda}$ listed in Table I the condensates take on acceptable values, with the values for $t=0$ and $t=0.1$ being somewhat too small.

In these calculations the value of $f_\pi = 0.0924$ GeV is fixed. The value listed as $f_\pi^{(0)}$ is the value calculated for f_π in leading order, without the $1/N_c$ correction. For $t=1$ we see about an 18% correction arising from the $1/N_c$ correction term calculated for f_π.

The value of G_D used in Ref. [4] is -239.1 GeV^{-5} and that used in Ref. [3] is -185 GeV^{-5}. In our work on the properties of the η mesons we found that -180 GeV^{-5} $< G_D < -220$ GeV^{-5} gave good values for the masses, decay constants, and mixing angles of these mesons [10]. That suggests that the range $0.80 \leqslant t \leqslant 1.00$ leads to the best overall agreement with the values of f_π, G_D, and the quark vacuum condensates in the case we choose to work with $m_u = 0.364$ GeV and $m_s = 0.522$ GeV.

In Table II we present the results for the case of covariant regularization. We make use of the fact that it was shown in Ref. [9] that quite similar results are obtained for the covariant regularization scheme if $\Lambda_{\mathrm{COV}}^2 = 2 \ln 2 \Lambda_{PV}^2$, where Λ_{COV} is the regulator for the covariant scheme. Again, we define t such that $\tilde{\Lambda}_{\mathrm{COV}} = \sqrt{t}\Lambda_{\mathrm{COV}}$. In this manner we obtain the results shown in Table II. The results for $0.4 \leqslant t \leqslant 1.0$ are satisfactory, given the range of G_D that provides good results for the properties of light mesons [3–5,10].

TABLE II. Results for the covariant regularization procedure calculated with $\Lambda_{\mathrm{COV}}^2 = 2 \ln 2 \Lambda_{\mathrm{PV}}^2$ [9]. See the caption to Table I. (The value of $\Lambda_{\mathrm{COV}} = 0.90$ GeV was used in Ref. [4].)

t	Λ [GeV]	$\tilde{\Lambda}$ [GeV]	f_π^0 [GeV]	G_S [GeV^{-2}]	G_D [GeV^{-5}]	$-\langle \bar{u}u \rangle_0^{1/3}$ [GeV]	$-\langle \bar{s}s \rangle_0^{1/3}$ [GeV]	$-\langle \bar{u}u \rangle^{1/3}$ [GeV]	$-\langle \bar{s}s \rangle^{1/3}$ [GeV]	$\frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle}$
1.0	0.816	0.816	0.109	7.752	−216.4	0.2618	0.2744	0.2413	0.2593	1.241
0.9	0.799	0.758	0.108	8.335	−227.3	0.2571	0.2690	0.2404	0.2568	1.218
0.8	0.781	0.699	0.106	8.998	−236.6	0.2523	0.2634	0.2389	0.2538	1.198
0.7	0.763	0.638	0.105	9.753	−242.5	0.2474	0.2578	0.2369	0.2507	1.180
0.6	0.745	0.577	0.103	10.62	−243.1	0.2425	0.2522	0.2344	0.2466	1.163
0.5	0.727	0.514	0.101	11.61	−234.7	0.2376	0.2465	0.2316	0.2425	1.148
0.4	0.710	0.449	0.0996	12.76	−212.6	0.2326	0.2408	0.2285	0.2382	1.133
0.3	0.692	0.379	0.0978	14.10	−169.2	0.2276	0.2351	0.2250	0.2336	1.119
0.2	0.675	0.302	0.0961	15.67	−93.73	0.2227	0.2294	0.2212	0.2287	1.105
0.1	0.658	0.208	0.0942	17.54	30.48	0.2177	0.2238	0.2172	0.2236	1.091
0.0	0.693	0.000	0.0924	19.46	226.2	0.2129	0.2183	0.2128	0.2183	1.078

V. DISCUSSION

The advantage of the formalism presented in Ref. [9] is that the summed diagrams represent an approximation that preserves the relations that follow from the chiral symmetry of the Lagrangian. One of the goals of our work has been to extend the calculations of Ref. [9] to the case of SU(3)-flavor symmetry. We have found that the small values of $\delta\langle\bar{u}u\rangle$ found in that work are also found in our analysis.

The analysis of the SU(3) model may be made using different procedures. We were interested in investigating the effect of changing the value of t for the values $m_u = m_d = 0.364$ GeV and $m_s = 0.522$ GeV that were used in Ref. [4]. In that work the covariant regularization scheme was used with $\Lambda_{\rm COV} = 0.90$ GeV, $G_S = 9.80\ {\rm GeV}^{-2}$, $G_D = -239.1\ {\rm GeV}^{-5}$, $m_u^0 = 0.0055$ GeV, and $m_s^0 = 0.132$ GeV. In the covariant analysis we have

$$[f_\pi^{(0)}]^2 = \frac{3}{4\pi^2} m_u^2 \left[\ln(1+x) - \frac{x}{1+x} \right], \tag{5.1}$$

where $x = \Lambda_{\rm COV}^2/m_u^2$. Using the parameters of Ref. [4] we obtain $f_\pi^{(0)} = 0.105$ GeV which is consistent with the results given in Tables I and II. Therefore, we conclude that the results of Ref. [4], calculated with $t=0$, would be improved if the $1/N_c$ correction to f_π were included in the analysis. Use of $t=0.7$ or 0.8 would improve the value obtained for f_π and only give rise to small corrections to the condensate values of $\langle\bar{u}u\rangle_0 = \langle\bar{d}d\rangle_0 = -(0.248\ {\rm GeV})^3$ and $\langle\bar{s}s\rangle_0 = -(0.258\ {\rm GeV})^3$ obtained in Ref. [4].

APPENDIX A

In this appendix we discuss the application of the Pauli-Villars procedure to regulate divergent integrals. As discussed in Ref. [9], convergent integrals are regulated by the same procedure. In Ref. [9] the following integrals are defined:

$$I(m^2,q) = \int \frac{d^4p}{(2\pi)^4} \frac{1}{[p^2-m^2][(p+q)^2-m^2]}, \tag{A1}$$

$$K(m^2,q) = \int \frac{d^4p}{(2\pi)^4} \frac{1}{[p^2-m^2]^2[(p+q)^2-m^2]}, \tag{A2}$$

$$F(m^2) = (4\pi)^2 i \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2-m^2}. \tag{A3}$$

The regulated integrals are formed by writing [9]

$$I^{PV}(q) = I(m^2,q) + I(M_1^2,q) - 2I(M_2^2,q) \tag{A4}$$

and

$$K^{PV}(q) = K(m^2,q) + K(M_1^2,q) - 2K(M_2^2,q), \tag{A5}$$

with $M_1^2 = m^2 + 2\Lambda^2$ and $M_2^2 = m^2 + \Lambda^2$.
Similarly, we define

$$F^{PV}(m^2) = F(m^2) + F(M_1^2) - 2F(M_2^2). \tag{A6}$$

One obtains

$$F^{PV}(m^2) = m^2[(1+2x)\ln(1+2x) - (1+x)\ln(1+x)], \tag{A7}$$

where $x = \Lambda^2/m^2$. We also have

$$I(m^2,q) = \frac{i}{(4\pi)^2}[2\ln(1+x) - \ln(1+2x) + \{\Delta(y)\}^{PV}] \tag{A8}$$

and

$$q^2 K(m^2,q) = \frac{i}{(4\pi)^2} \left\{ \frac{y}{y+1}[2-\Delta(y)] \right\}^{PV} \tag{A9}$$

with

$$\Delta(y) = 2(1 - \sqrt{1+1/y}\,\ln[\sqrt{y} + \sqrt{1+y}]) \tag{A10}$$

and $y = -q^2/4m^2 > 0$. The regularization procedure is defined by the relation [9]

$$\{f(y)\}^{PV} = f(y) + f\left(\frac{y}{1+2x}\right) - 2f\left(\frac{y}{1+x}\right). \tag{A11}$$

We now turn to the integrals needed for our SU(3)-flavor analysis. Consider the vacuum polarization integral ($q^2 < 0$),

$$J_S(m_1,m_2,q^2) = -2N_c i\, {\rm Tr} \int \frac{d^4k}{(2\pi)^4} \frac{i}{\not{q} + \not{k} - m_1} \frac{i}{\not{k} - m_2}. \tag{A12}$$

We define the regulated function in a field theoretic analysis to be

$$\{J_E(m_1,m_2,q^2)\}^{PV} = J_E(m_1,m_2,q^2) + J_S(M_1,M_1,q^2) - 2J_E(M_2,M_2,q^2) \tag{A13}$$

where, in this case,

$$M_1^2 = m_1 m_2 + 2\Lambda^2 \tag{A14}$$

and

$$M_2^2 = \frac{m_1^2 + m_2^2}{2} + \Lambda^2. \tag{A15}$$

A regulated form of

$$J_P(m_1,m_2,q^2) = -2N_c i\, {\rm Tr} \int \frac{d^4k}{(2\pi)^4} \left[\gamma_5 \frac{1}{\not{q} + \not{k} - m_1} \gamma_5 \frac{1}{\not{k} - m_2} \right] \tag{A16}$$

is defined in analogy to the definition given in Eqs. (A13)–(A15).

As described in the text, Eq. (A13) cannot be used for the NJL model. As discussed previously, the regulated form used here is

$$J_S^{PV}(m_1,m_2,q_E^2) = 4n_C\left[\frac{F^{PV}(m_1^2)}{(4\pi)^2}+\frac{F^{PV}(m_2^2)}{(4\pi)^2} + (q_E^2+m_1^2+m_2^2+2m_1m_2)iI^{PV}(m_1,m_2,q_E)\right], \tag{A17}$$

with a similar definition of $J_P^{PV}(m_1,m_2,q_E^2)$.

We next turn to the regularization of $T_s(m_1,m_2,q)$ of Eq. (2.5). For the moment, we work in Minkowski space where $q^2=-q_E^2$. Rather than work with Eq. (2.5), it is best to return to the integral that appears when evaluating the third diagram in Fig. 1. We consider

$$T_E(m_1,m_2,q)=\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\frac{1}{[\not p+\not q-m_1]}\frac{1}{[\not p-m_2]^2} \tag{A18}$$

and define the field-theoretic regularization

$$\{T_S(m_1,m_2,q)\}^{PV}=\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}\left[\frac{1}{\not p+\not q-m_1}\frac{1}{[\not p-m_2]^2} +\alpha\frac{1}{\not p+\not q-\hat M_1}\frac{1}{[\not p-\hat M_1]^2} +\beta\frac{1}{\not p+\not q-\hat M_2}\frac{1}{[\not p-\hat M_2]^2}\right], \tag{A19}$$

with $\alpha=1$ and $\beta=-2$. We also find that we may choose

$$\hat M_1=\left(\frac{2m_2+m_1}{3}\right)+2\hat\Lambda \tag{A20}$$

and

$$\hat M_2=\left(\frac{2m_2+m_1}{3}\right)+\hat\Lambda, \tag{A21}$$

so that we may regulate $T_P(m_1,m_2,q_E)$ by writing

$$\{T_P(m_1,m_2,q_E)\}^{PV}=T_P(m_1,m_2,q_E)+T_P(\hat M_1,\hat M_1,q_E) -2T_P(\hat M_2,\hat M_2,q_E). \tag{A22}$$

There is a similar relation for $\{T_S(m_1,m_2,q_E)\}^{PV}$.
We would like to have a relation between $\hat\Lambda$ of Eqs. (A20) and (A21) and the parameter Λ defined in Eqs. (A14) and (A15). We may write

$$M_2^2=\frac{m_1^2+m_2^2}{2}+\Lambda^2 \tag{A23}$$

$$=\hat M_2^2 \tag{A24}$$

$$=\left[\left(\frac{2m_2+m_1}{3}\right)+\hat\Lambda\right]^2, \tag{A25}$$

or use $M_1^2=\hat M_1^2$. The two values of $\hat\Lambda$ obtained in this fashion are rather close.

As discussed previously, the field-theoretic definition of the Pauli-Villars procedure is not applicable. Therefore, rather than use Eq. (A22), we use the results given in Eqs. (2.3) and (2.5) and write

$$T_P^{PV}(m_1,m_2,q_E) =[m_2I^{PV}(m_1,m_2,0)+(m_2-m_1)I^{PV}(m_1,m_2,q_E) +(m_2q_E^2+m_2^3-m_1m_2^2)K^{PV}(m_1,m_2,q_E)], \tag{A26}$$

with a similar definition of T_S^{PV} (m_1,m_2,q_E). [See Eq. (2.5).]

In Appendix B we provide the expressions we have obtained for $J_P(m_1,m_2,q_E)$, $J_S(m_1,m_2,q_E)$, $I(m_1,m_2,q_E)$, and $K(m_1,m_2,q_E)$.

APPENDIX B

We define

$$J_S(m_1,m_2,q^2)=-2N_ci\,\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}\frac{i}{\not q+\not k-m_1+i\varepsilon} \times\frac{i}{\not k-m_2+i\varepsilon} \tag{B1}$$

and find, with $p_E^2=-p^2$,

$$J_S(m_1,m_2,q_E^2)=4n_C\left[\frac{F(m_1^2)}{(4\pi)^2}+\frac{F(m_2^2)}{(4\pi)^2}+(p_E^2+m_1^2+m_2^2 +2m_1m_2)iI(m_1,m_2,q_E)\right]. \tag{B2}$$

We also define

$$J_P(m_1,m_2,q^2)=-2N_ci\,\mathrm{Tr}\int\frac{d^4k}{(2\pi)^4}i\gamma_5\left(\frac{i}{\not q+\not k-m_1+i\varepsilon}\right) \times i\gamma_5\left(\frac{i}{\not k-m_2+i\varepsilon}\right) \tag{B3}$$

and obtain

$$J_P(m_1,m_2,q_E^2)=4N_c\left[\frac{F(m_1^2)}{(4\pi)^2}+\frac{F(m_2^2)}{(4\pi)^2}+(q_E^2+m_1^2+m_2^2-2m_1m_2)iI(m_1,m_2,q_E)\right]. \tag{B4}$$

We define

$$K(m_1^2,m_2^2,q_E)=\int\frac{d^4p}{(2\pi)^4}\frac{1}{[(p+q)^2-m_1^2]}\frac{1}{[p^2-m_2^2]^2} \tag{B5}$$

and find

$$K(m_1^2,m_2^2,q_E)=\frac{-i}{(4\pi)^2}\left\{\frac{q_E^2+m_2^2-m_1^2}{q_E^2}\frac{1}{\sqrt{A^2-4m_1^2m_2^2}}\times\ln\left|\frac{A^2+\sqrt{A^4-4m_1^2m_2^2}}{2m_1m_2}\right|+\frac{1}{q_E^2}\ln\left(\frac{m_1}{m_2}\right)\right\}, \tag{B6}$$

with $A^2=q_E^2+m_1^2+m_2^2$. For $m_1=m_2$, we have

$$K(m,m,q_E)=\frac{i}{(4\pi)^2}\left(-\frac{1}{2}\right)\frac{1}{m^2\sqrt{y(1+y)}}\ln|\sqrt{y}+\sqrt{1+y}|, \tag{B7}$$

with $y=q_E^2/4m^2$.

We now write

$$K^{PV}(m_1^2,m_2^2,q_E)=K(m_1^2,m_2^2,q_E)+K(m_1^2+2\Lambda^2,m_2^2+2\Lambda^2,q_E)-2K(m_1^2+\Lambda^2,m_2^2+\Lambda^2,q_E), \tag{B8}$$

which is a generalization of the method used in Ref. [9] to regulate $K(m^2,q_E)$. [See Eqs. (A5) and (A9).]

The regulated form of $I(m_1,m_2,0)$ for $m_1\neq m_2$ is

$$I^{PV}(m_1,m_2,0)=\frac{i}{(4\pi)^2}\left\{\frac{1}{2}\left[-\ln\frac{(m_1^2+2\lambda^2)(m_2^2+2\Lambda^2)}{m_1^2m_2^2}+2\ln\frac{(m_1^2+\Lambda^2)(m_2^2+\Lambda^2)}{m_1^2m_2^2}\right]-[f(m_1^2,m_2^2)+f(m_1^2+2\Lambda^2,m_2^2+2\Lambda^2)-2f(m_1^2+\Lambda^2,m_2^2+\Lambda^2)]\right\}, \tag{B9}$$

where

$$f(m_1^2,m_2^2)=\frac{m_1^2+m_2^2}{2}\left\{\frac{1}{|m_1^2-m_2^2|}\ln\left[\frac{m_1^2+m_2^2-|m_1^2-m_2^2|}{m_1^2+m_2^2+|m_1^2-m_2^2|}\right]\right\}, \tag{B10}$$

We also have

$$I(m_1,m_2,q_E)=-\frac{i}{(4\pi)^2}\int_0^1dx\{\ln[b(m_1^2,m_2^2,q_E)]+\ln[b(m_1^2+2\Lambda^2,m_2^2+2\Lambda^2,q_E)]-2\ln[b(m_1^2+\Lambda^2,m_2^2+\Lambda^2,q_E)]\}, \tag{B11}$$

where

$$b(m_1^2,m_2^2,q_E)=q_E^2x(1-x)+m_2^2x+m_1^2(1-x). \tag{B12}$$

Completion of the integral yields

$$I(m_1,m_2,q_E)=-\frac{i}{(4\pi)^2}\left\{\frac{q_E^2+m_1^2-m_2^2}{2q_E^2}\ln m_2^2-2+\frac{q_E^2+m_2^2-m_1^2}{2q_E^2}\ln m_1^2+\frac{\sqrt{A^2-4m_1^2m_2^2}}{2q_E^2}\ln\left[\frac{A^2+\sqrt{A^2-4m_1^2m_2^2}}{A^2-\sqrt{A^4-4m_1^2m_2^2}}\right]\right\}, \tag{B13}$$

where $A^2=q_E^2+m_1^2+m_2^2$.

In Eq. (B13) we may replace $\ln m_1^2$ and $\ln m_2^2$ by $\ln(m_1^2/M^2)$ and $\ln(m_2^2/M^2)$ where M is an arbitrary constant of the dimension of a mass. The result for I^{PV} (m_1,m_2,q_E) does not depend upon the value of M.

The regularization procedure used in Ref. [9] requires only a regularization of the integrals $F(m^2)$, $I(m_1,m_2,q_E)$ and $K(m_1,m_2,q_E)$ using the "Pauli-Villars method." For example,

$$\{F(m^2)\}^{PV}=F(m^2)+F(m^2+2\Lambda^2)-2F(m^2+\Lambda^2), \tag{B14}$$

etc. The essential point is to avoid the use of our Eq. (A13), since that leads to the problems discussed in the text.

APPENDIX C

In this appendix we provide expressions for $I(m_1,m_2,q_E)$, $K(m_1,m_2,q_E)$, and $F(m)$ using the covariant regularization procedure, which is based upon the introduction of the factor $\theta(\Lambda_{\rm COV}^2-p_E^2)$ in the Euclidean-space integrals over the values of p_E. [See Eqs. (A1)–(A3).] [The Feymann method for combining the energy denominators in Eqs. (A1)–(A3) is used.] We find

$$I(m_1,m_2,q)=\frac{1}{16\pi^2}i\int_0^1dx\left[\frac{b(q^2,x)}{b(q^2,x)+\Lambda_{\rm COV}^2}-1-\ln\left(\frac{b(q^2,x)}{b(q^2,x)+\Lambda_{\rm COV}^2}\right)\right] \tag{C1}$$

with

$$b(q^2,x)=q^2(1-x)x+m_2^2x+m_1^2(1-x). \quad \text{(C2)}$$

Further,

$$K(m_1,m_2,q)=-\frac{1}{16\pi^2}i\int_0^1 dx\,\frac{\Lambda_{\mathrm{COV}}^4 x}{b(q^2,x)[b(q^2,x)+\Lambda_{\mathrm{COV}}^2]^2}. \quad \text{(C3)}$$

We also note that [9]

$$F_{\mathrm{COV}}(m^2)=m^2[x-\ln(1-x)], \quad \text{(C4)}$$

where $x=\Lambda_{\mathrm{COV}}^2/m^2$. In the equal mass case ($m_1=m_2=m$), one finds

$$I(m,m,0)=\frac{1}{(4\pi)^2}i\left[\ln(x+1)-\frac{x}{x+1}\right], \quad \text{(C5)}$$

while, for $m_1\neq m_2$,

$$I(m_1,m_2,0)=\frac{i}{(4\pi)^2}\left\{\frac{1}{2}\ln\left[\left(\frac{m_1^2+\Lambda_{\mathrm{COV}}^2}{m_1^2}\right)\left(\frac{m_2^2+\Lambda_{\mathrm{COV}}^2}{m_2^2}\right)\right] -\frac{m_1^2+m_2^2}{2(m_1^2-m_2^2)}\ln\left[\left(\frac{m_2^2+\Lambda_{\mathrm{COV}}^2}{m_2^2}\right)\times\left(\frac{m_1^2}{m_1^2+\Lambda_{\mathrm{COV}}^2}\right)\right]\right\}. \quad \text{(C6)}$$

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