Dynamical chiral bag model

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We study a dynamical chiral bag model, in which massless fermions are confined within an impenetrable but movable bag coupled to meson fields. The self-consistent motion of the bag is obtained by solving the equations of motion exactly assuming spherical symmetry. When the bag interacts with an external meson wave we find three different kinds of resonances: fermionic, geometric, and σ resonances. We discuss the phenomenological implications of our results.

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I. INTRODUCTION

The MIT bag model [1] and its chirally invariant versions, such as the chiral bag model [2,3] and the cloudy bag model [4], continue to be useful tools in the study of the physics of the nucleon and other baryons. They have also been used extensively in the discussion of various phenomena ranging from strange stars [5] to ultrarelativistic heavy-ion collisions [6], even though these often involve situations of high density/temperature where the applicability of the models is doubtful.

In most of the bag model studies so far, because of its simplicity, a static spherical bag is assumed. The few notable exceptions, which allowed for the possibility of a dynamical bag boundary, focused mainly on reproducing the correct phenomenological parity order of the low-lying states of the nucleon, although several approximations and modifications to the theory had to be employed. For example, Rebbi and DeGrand [7] studied a bosonic bag and quantized the full system with perturbation theory in the limit of small spherical oscillations. The authors in Refs. [8–10] considered a fermionic bag with a surface tension, as well as the volume energy, and quantized only the motion of the bag boundary in the adiabatic approximation. Nogami and Tomio [11] also quantized the motion of the boundary, but used the adiabatic approximation only for the mesons. Although these works gave a reasonable ordering of the low-lying states of the nucleon, the more fundamental question of whether it is consistent and feasible to use a dynamical bag to model hadrons was not addressed. That is the motivation of the present work.

In a previous paper [12] we proved that the original MIT bag model with massless quarks admits only one classical solution other than the static one, namely, a bag constantly expanding at the speed of light. We thus concluded that an additional field, such as the mesons in the chiral bag model, is needed to have a consistent and nontrivial dynamical bag model of hadrons. In this paper we implement a method that allows us to find the classical solutions of a spherically symmetric chiral bag for any motion of the bag radius. In particular we look for the self-consistent solution of the full theory without any approximation, in which the motion of the bag surface is determined by the conservation of the total energy. We can thus study the full nonlinear features of the model. We find that when the bag interacts with an external meson wave three different kinds of resonances occur: fermionic, geometric, and σ resonances.

In the present work, in order to obtain spherical symmetry, we consider hedgehog configurations in which the quarks are neither in a flavor eigenstate nor in a J eigenstate; they do not represent any known hadrons. However the occurence of the resonances we found does not depend on the flavor of the quarks, and since they are caused by spherical waves the angular momenta are not changed. Moreover, as we show in Sec. III and in the conclusion, they are not related to specific features of the hedgehog solution. Therefore, we conjecture that such resonances should occur also for more realistic solutions, i.e., with definite flavor and J.

This paper is organized as follows. We first show the method we use to solve the problem. We then discuss the resonances found with a driven bag motion. In the third section the problem of the self-consistent surface motion is addressed, and we discuss the results obtained with different incoming meson waves. We finally summarize our results and discuss their phenomenological implications. The Appendix provides more details about the method of solution.

II. METHOD OF SOLUTION

The Lagrangian of the system we study is [2]

$$\mathcal{L}=\frac{1}{2}\Big\{[i(\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi-(\partial_{\mu}\bar{\psi})\gamma^{\mu}\psi)-B]\theta_V(x) - \frac{1}{f_\pi}\bar{\psi}(\sigma+i\vec{\tau}\cdot\vec{\pi}\gamma_5)\psi\Delta_s+\partial_\mu\sigma\partial^\mu\sigma+\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}\Big\}, \tag{1}$$

where $\theta_V(x)$ is 1 inside the bag and 0 outside and

$$\frac{\partial\theta_V}{\partial x^\mu}=n_\mu\Delta_s, \tag{2}$$

Δ_s being the surface delta-function. From it we derive the following Euler-Lagrange equations of motion:

$$\gamma^\mu\partial_\mu\psi=0 \quad \text{inside the bag}, \tag{3}$$

$$i\gamma^\mu n_\mu\psi=\frac{1}{f_\pi}(\sigma+i\vec{\tau}\cdot\vec{\pi}\gamma_5)\psi \quad \text{on the bag surface}, \tag{4}$$

$$\partial_\mu \partial^\mu \sigma = -\frac{1}{2f_\pi}\bar{\psi}\psi\Delta_s, \tag{5}$$

$$\partial_\mu \partial^\mu \vec{\pi} = -\frac{1}{2f_\pi} i\bar{\psi}\gamma_5 \vec{\tau}\psi\Delta_s. \tag{6}$$

It is possible to look for spherically symmetric solutions of the above equations [2] by writing

$$\psi = \begin{pmatrix} g(t,r) \\ -i\vec{\sigma}\cdot\hat{r}f(t,r) \end{pmatrix} v, \tag{7}$$

$$\sigma = \sigma(t,r), \tag{8}$$

$$\vec{\pi} = \pi(t,r)\hat{r}, \tag{9}$$

where v includes the spin and isospin parts and can be written as

$$v = \frac{1}{2}(|\uparrow,d\rangle - |\downarrow,u\rangle). \tag{10}$$

The arrows indicate the spins while u and d the up and down flavors of the quarks, and v satisfies

$$(\vec{\sigma}+\vec{\tau})v = 0. \tag{11}$$

In Eqs. (7) and (11), $\vec{\sigma}$ are the three Pauli matrices and should not be confused with the field $\sigma(t,r)$. Equation (11) ensures that the right-hand side (RHS) of Eqs. (4) and (6) are spherically symmetric and causes $\vec{\pi}$ to be radially directed, as in Eq. (9). This kind of solutions is hence called hedgehog solutions [2,13].

For a static bag an analytic solution is known [2,13], which represents a stationary fermion field coupled at the surface of the bag to time-independent σ and π fields. Our goal is to find the hedgehog solution for any spherically symmetric motion of the bag's surface.

Substituting Eq. (7) for ψ in Eq. (3) we obtain

$$i\frac{\partial f}{\partial t} = \frac{\partial g}{\partial r}, \tag{12}$$

$$-i\frac{\partial g}{\partial t} = \frac{\partial f}{\partial r} + \frac{2}{r}f. \tag{13}$$

It is not difficult to verify [12] that the general solution of Eqs. (12), (13) has the form

$$g(t,r) = \frac{1}{r}[Q'(t-r) - Q'(t+r)], \tag{14}$$

$$f(t,r) = \frac{i}{r}\left\{Q'(t-r) + Q'(t+r) + \frac{1}{r}[Q(t-r) - Q(t+r)]\right\}, \tag{15}$$

where $Q(z)$ is an arbitrary function. In spherical coordinates Eqs. (5) and (6) become

$$\frac{\partial^2\sigma}{\partial t^2} - \frac{\partial^2\sigma}{\partial r^2} - \frac{2}{r}\frac{\partial\sigma}{\partial r} = -\frac{1}{2f_\pi}[g^*g - f^*f]\delta(R-r), \tag{16}$$

$$\frac{\partial^2\pi}{\partial t^2} - \frac{\partial^2\pi}{\partial r^2} - \frac{2}{r}\frac{\partial\pi}{\partial r} + \frac{2}{r^2}\pi = \frac{1}{2f_\pi}[g^*f + gf^*]\delta(R-r). \tag{17}$$

For $r \neq R$, we notice that $\sigma(t,r)$ obeys the equation of a free s wave while $\pi(t,r)$, being the radial part of the vector $\vec{\pi}$, satisfies the equation of a free p wave. Hence we can look for a solution of the form

$$\sigma(t,r) = \sigma_{\rm in}(t,r)\,\theta(R-r) + \sigma_{\rm out}(t,r)[1-\theta(R-r)], \tag{18}$$

$$\pi(t,r) = \pi_{\rm in}(t,r)\,\theta(R-r) + \pi_{\rm out}(t,r)[1-\theta(R-r)], \tag{19}$$

where the fields inside and outside of the bag can be written accordingly as

$$\sigma_{\rm in}(t,r) = \frac{1}{r}[\Sigma_{\rm in}(t-r) - \Sigma_{\rm in}(t+r)] + \sigma_{0,{\rm in}}(r), \tag{20}$$

$$\sigma_{\rm out}(t,r) = \frac{1}{r}[\Sigma_{\rm out-}(t-r) - \Sigma_{\rm out+}(t+r)] + \sigma_{0,{\rm out}}(r), \tag{21}$$

$$\pi_{\rm in}(t,r) = \frac{1}{r}\left\{\Pi'_{\rm in}(t-r) + \Pi'_{\rm in}(t+r) + \frac{1}{r}[\Pi_{\rm in}(t-r) - \Pi_{\rm in}(t+r)]\right\} + \pi_{0,{\rm in}}(r), \tag{22}$$

$$\pi_{\rm out}(t,r) = \frac{1}{r}\left\{\Pi'_{\rm out-}(t-r) + \Pi'_{\rm out+}(t+r) + \frac{1}{r}[\Pi_{\rm out-}(t-r) - \Pi_{\rm out+}(t+r)]\right\} + \pi_{0,{\rm out}}(r), \tag{23}$$

where $\Sigma_{\rm out+}$, $\Sigma_{\rm out-}$, $\Sigma_{\rm in}$, $\Pi_{\rm in}$, $\Pi_{\rm out+}$, and $\Pi_{\rm out-}$ are arbitrary functions. Notice that $\Sigma_{\rm out+}$ and $\Sigma_{\rm out-}$ are in general different functions as are also $\Pi_{\rm out+}$ and $\Pi_{\rm out-}$. Here, the time-independent terms $\sigma_{0,{\rm in}}(r)$, $\sigma_{0,{\rm out}}(r)$, $\pi_{0,{\rm in}}(r)$, and $\pi_{0,{\rm out}}(r)$ are the static-bag solutions given by [2,13]

$$\sigma_{0,{\rm in}}(r) = g_0,$$

$$\sigma_{0,{\rm out}}(r) = g_0 + \alpha R_0^2\left(\frac{1}{R_0} - \frac{1}{r}\right),$$

$$\pi_{0,{\rm in}}(r) = -\frac{\beta}{3}r,$$

$$\pi_{0,{\rm out}}(r) = -\frac{\beta}{3}\frac{R_0^3}{r^2}.$$

Substituting Eqs. (18) and (19) in Eqs. (16) and (17) and requiring the continuity of $\sigma(t,r)$ and $\pi(t,r)$ at $r=R$, we finally obtain two relations that can be viewed as boundary conditions for the fields g, f, $\sigma_{\rm in}$, $\sigma_{\rm out}$, $\pi_{\rm in}$, and $\pi_{\rm out}$:

$$\dot{R}\left(\frac{\partial\sigma_{\rm in}}{\partial t}-\frac{\partial\sigma_{\rm out}}{\partial t}\right)+\left(\frac{\partial\sigma_{\rm in}}{\partial r}-\frac{\partial\sigma_{\rm out}}{\partial r}\right)=-\frac{1}{2f_\pi}(g^*g-f^*f) \quad \text{at} \quad r=R, \tag{24}$$

$$\dot{R}\left(\frac{\partial\pi_{\rm in}}{\partial t}-\frac{\partial\pi_{\rm out}}{\partial t}\right)+\left(\frac{\partial\pi_{\rm in}}{\partial r}-\frac{\partial\pi_{\rm out}}{\partial r}\right)=\frac{1}{2f_\pi}(g^*f+gf^*) \quad \text{at} \quad r=R. \tag{25}$$

From Eq. (4) we can express $\sigma(t,R)$ and $\pi(t,R)$ in terms of $g(t,R)$ and $f(t,R)$ (see the Appendix) and use Eqs. (24) and (25) to find $g(t,r)$, $f(t,r)$, i.e., $Q_{\rm re}(z)$ and $Q_{\rm im}(z)$ [see Eqs. (14), (15)] with the null-lines method [14,15].

From the point of view of the null-lines method the unknowns in Eqs. (24) and (25), as long as $|\dot{R}|\leqslant 1$ [15], are $Q_{\rm re}(t+R)$, $Q_{\rm im}(t+R)$, $\Sigma_{\rm in}(t+R)$, $\Pi_{\rm in}(t+R)$, $\Sigma_{\rm out-}(t-R)$, and $\Pi_{\rm out-}(t-R)$. Furthermore, the latter four are fixed once $Q_{\rm re}(t+R)$ and $Q_{\rm im}(t+R)$ are known, using

$$\Sigma_{\rm in}(t+R)=\Sigma_{\rm in}(t-R)+Rg_0-R\sigma(t,R), \tag{26}$$

$$\Sigma_{\rm out-}(t-R)=\Sigma_{\rm out+}(t+R)+\alpha R_0^2-R(g_0+\alpha R_0)+R\sigma(t,R), \tag{27}$$

$$\Pi'_{\rm in}(t+R)=\frac{1}{R}[\Pi_{\rm in}(t+R)-\Pi_{\rm in}(t-R)]+\Pi'_{\rm in}(t-R)+\frac{\beta}{3}R^2+R\pi(t,R), \tag{28}$$

$$\Pi'_{\rm out-}(t-R)=\frac{1}{R}[\Pi_{\rm out+}(t+R)-\Pi_{\rm out-}(t-R)]-\Pi'_{\rm out+}(t+R)+\frac{\beta}{3}\frac{R_0^3}{R}+R\pi(t,R). \tag{29}$$

Here for convenience we have not written explicitly the dependence on $Q_{\rm re}(t+R)$ and $Q_{\rm im}(t+R)$ which are hidden in $\sigma(t,R)$ and $\pi(t,R)$.

We still need to solve Eqs. (28) and (29). This can be done numerically either by simply replacing $\Pi'_{\rm in}(t+R)$ and $\Pi'_{\rm out}(t-R)$ with their finite-difference counterparts or by integrating between $z-dz$ and z, where $z=t+R$ and $z=t-R$, respectively, for the first and second equations, and by approximating all quantities other than $\Pi_{\rm in}(t+R)$ and $\Pi_{\rm out}(t-R)$ as being constant in this infinitesimal interval. The numerical results turn out to be slightly more accurate with the second method. To solve Eqs. (24) and (25) we used a fourth-order Runge-Kutta algorithm.

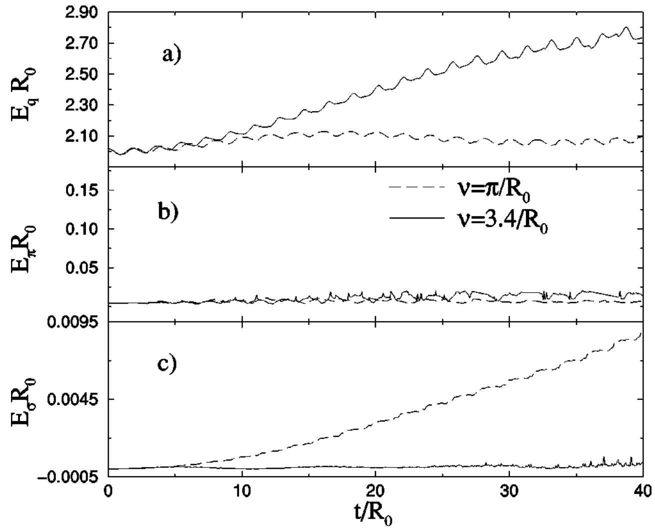

FIG. 1. Time evolution of the energy for driven surface oscillations at the $\nu=\pi/R_0$ (dashed lines) and $\nu=E_2-E_1$ (solid lines) resonances with $\epsilon=0.01R_0$. (a) Energy of the fermion field. (b) Energy of the π-field. (c) Energy of the σ-field.

III. RESONANCES WITH A DRIVEN BAG MOTION

With the method discussed in the previous section we first computed the solution for a static bag and then for a slowly moving one. We verified that the norm of the fermion field is conserved and that our numerical method is accurate up to the second derivative of $Q(z)$ for a bag of initial radius $R_0=1$ fm and $f_\pi=1$ fm^{-1}. With these parameters the static chiral bag is similar to the MIT bag, with an almost constant $\sigma(r)$ and a very small $\pi(r)$. All the results presented below are obtained with such values of R_0 and f_π, which is a representative set of parameters for showing the qualitative features of a dynamical chiral bag model.

Since we are particularly interested in the behavior of the fields under the effect of the motion of the boundary, we first study the chiral bag with an imposed surface motion. Subjecting the bag boundary to a sinusoidal motion, $R(t)=R_0+\epsilon[\cos(\nu t)-1]$, we found three different kinds of resonances (i) the *fermionic* resonances, which are excited when the oscillation frequencies are close to the difference between two static-bag eigenenergies, $\nu\simeq E_n-E_k$, (ii) the *geometric* σ resonances, for $\nu\simeq n\pi/R_0$, and (iii) the *parametric* σ resonances, for $\nu\simeq(2n+1)\pi/(2R_0)$, where n is an integer.

The origin of the fermionic resonances at $\nu\simeq E_n-E_k$ is similar to those found for a Schrödinger particle in an oscillating cavity [16]. The difference here is that the fermions cannot really be excited to the upper static-cavity level because the upper level is associated with different static pion fields which cannot be produced by the boundary motion. However, since with our choice of parameters $\sigma(r)$ and $\pi(r)$ change little for different static solutions, the system still gets excited for oscillation frequencies close to the static energy gaps. The smaller f_π is, these resonance frequencies deviate more from E_n-E_k. As an example, we show for $\nu=3.4/R_0\simeq E_2-E_1$ the time dependence of the energies of the fermion and the meson fields in Figs. 1(a), 1(b), and 1(c), respectively. It is interesting to note that neither the σ nor the

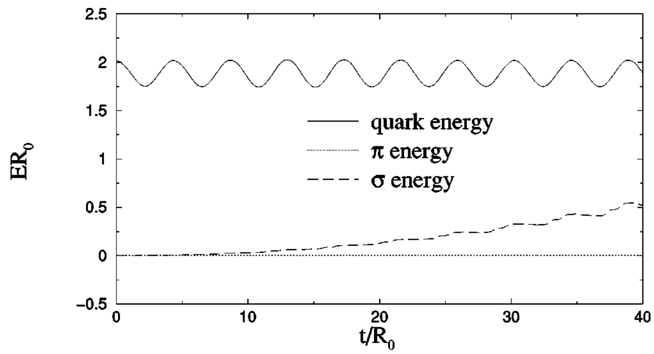

FIG. 2. Time evolution of the energy for driven surface oscillations at the $\nu=\pi/(2R_{\rm av})$ resonance with $\epsilon=0.08R_0$ ($R_{\rm av}=R_0+\epsilon$).

π fields gain considerable energy.

For $\nu=n\pi/R_0$ we found resonances involving the σ field. As can be seen in Fig. 1 the energy of the σ field increases remarkably, while the energies of the fermion and the π field change little. These resonances may be considered *geometric*, since the resonance frequencies are related to the time it takes for the wave components of $\sigma(t,r)$, i.e., $\Sigma_{\rm in}(t-r)$ and $\Sigma_{\rm in}(t+r)$, to travel from the boundary of the bag to its center and back again. It has been shown [17] that *p waves* in an oscillating spherical cavity also manifest resonances at $\nu=n\pi/R_0$, and so it is somewhat surprising that here the energies of the fermion and π fields are little affected. The strongly nonlinear interaction at the bag boundary seems to damp out the resonant evolution. The energy of the fermions actually shows some resonant behavior, but this is probably mainly due to the fact that the driving frequency is close to E_2-E_1.

The third kind of resonances we found is a peculiar feature of the system under analysis. As we can see from Fig. 2, it involves mainly the σ field. Note that $\Sigma_{\rm in}(z)$ (Fig. 3) has an almost periodic dependence and the period is about half that of the oscillating bag. In other words the bag surface, oscillating at frequencies $\nu=(2n+1)\pi/(2R_0)$, acts as a source for a σ field with frequencies $(2n+1)\pi/R_0$. Such frequencies are obviously resonant with the cavity so that the σ field is resonantly enhanced. The occurence of this kind of resonances can be understood in the following way. Since the fermion field is not excited we can approximate its value at the bag boundary with its static-bag expression

$$g[t,R(t)]=N\exp(-iEt)j_0[ER(t)], \tag{30}$$

$$f[t,R(t)]=-N\exp(-iEt)j_1[ER(t)]. \tag{31}$$

From Eqs. (A3)–(A6) we obtain after some manipulations

$$\pi(t,R)=f_\pi\frac{f_{\rm Re}^2(t,R)+f_{\rm Im}^2(t,R)-g_{\rm Re}^2(t,R)-g_{\rm Im}^2(t,R)}{f_{\rm Re}^2(t,R)+f_{\rm Im}^2(t,R)+g_{\rm Re}^2(t,R)+g_{\rm Im}^2(t,R)}, \tag{32}$$

$$\sigma(t,R)=-2f_\pi\frac{f_{\rm Re}(t,R)g_{\rm Re}(t,R)+f_{\rm Im}(t,R)g_{\rm Im}(t,R)}{f_{\rm Re}^2(t,R)+f_{\rm Im}^2(t,R)+g_{\rm Re}^2(t,R)+g_{\rm Im}^2(t,R)}. \tag{33}$$

One can see that the dependence on $\exp(-iEt)$ cancels out in our approximation and the whole expressions become periodic functions with period $T=2\pi/\nu$. The Fourier expansion of such functions involves all multiple frequencies of ν and, in the case of $\nu=(2n+1)\pi/(2R_0)$, its even multiples are also integral multiples of π/R_0. It is then evident that the expressions for $\sigma(R)$ and $\pi(R)$ contain terms in resonance with the cavity. However, it is surprising that for oscillation amplitudes $\epsilon>0.005R_0$ the frequency 2ν for $\sigma(R)$ becomes the dominating one even before the first bag oscillation is completed. In Fig. 3 we can see clearly how the amplitude of $\Sigma_{\rm in}(z)$ increases with each bag oscillation. Although the expression for $\pi(R)$ also contains terms with frequency being integral multiples of π/R_0, we observe no resonant behavior for the π field, which is consistent with the previous observation that this field is not excited for $\nu=n\pi/R_0$.

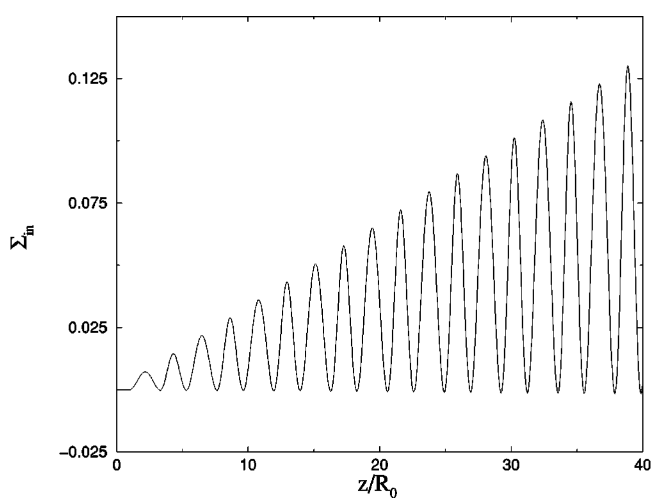

FIG. 3. The function $\Sigma_{\rm in}(z)$ for driven surface oscillations at the $\nu=\pi/(2R_{\rm av})$ resonance with $\epsilon=0.08R_0$. The function clearly contains a periodic contribution with $T\simeq 2\,R_0$.

IV. SELF-CONSISTENT SURFACE MOTION

A. Equation for the radius and nonresonant interaction

Our main interest in this work is to study the behavior of the fields and the bag's surface when perturbed from their static-bag states by, for example, an incoming pion wave. To this end we need to find the self-consistent dynamics of the bag surface and fields.

We notice that the velocity of points on the bag surface does not appear in the Lagrangian, Eq. (1), and hence we cannot derive from it an equation of motion for the bag radius [7,8]. The motion of the bag, however, is constrained by the conservation of the total energy. Let us consider the energy-momentum tensor

$$T^{\mu\nu}=-g^{\mu\nu}\mathcal{L}+\frac{i}{2}[\bar\psi\gamma^\mu\partial^\nu\psi-\partial^\nu\bar\psi\gamma^\mu\psi]\theta_V$$
$$+\partial^\mu\sigma\partial^\nu\sigma+\partial^\mu\vec\pi\cdot\partial^\nu\vec\pi. \tag{34}$$

The conservation of energy and momentum requires $\partial_\mu T^{\mu\nu}=0$, from which, by using the equations of motion Eqs. (3)–(6) and after some algebra, we derive

$$Bn^\nu=\frac{1}{2f_\pi}\partial^\nu[\bar{\psi}(\sigma+i\vec{\tau}\cdot\vec{\pi}\gamma_5)\psi] \quad \text{on the surface.} \tag{35}$$

For spherically symmetric solutions $n^\nu\equiv(\dot{R},\hat{r})$, and the above equation can be written as

$$B\dot{R}=\frac{1}{2f_\pi}\left\{\frac{\partial}{\partial t}[\bar{\psi}(\sigma+i\vec{\tau}\cdot\hat{r}\pi\gamma_5)\psi]\right\}_{r=R}, \tag{36}$$

$$B=\frac{1}{2f_\pi}\left\{\frac{\partial}{\partial r}[\bar{\psi}(\sigma+i\vec{\tau}\cdot\hat{r}\pi\gamma_5)\psi]\right\}_{r=R}. \tag{37}$$

In the static case Eq. (36) is identically satisfied because $\dot{R}=0$ and $\bar{\psi}(\sigma+i\vec{\tau}\cdot\vec{\pi}\gamma_5)\psi$ is time independent, and we could use Eq. (37) to derive B. However, the RHS of Eq. (37) is an ambiguous expression, because it involves the derivatives of σ and π at the boundary which are discontinuous. To overcome this difficulty we use the fact that $T^{\mu\nu}$ can also be written as [2]

$$T^{\mu\nu}=T^{\mu\nu}_{\rm in}\theta_V+T^{\mu\nu}_{\rm out}(1-\theta_V), \tag{38}$$

because the surface term is zero along the trajectories of motion. Since $\partial_\mu T^{\mu\nu}_{\rm in}=0$ and $\partial_\mu T^{\mu\nu}_{\rm out}=0$, the conservation condition for energy and momentum becomes

$$n_\mu T^{\mu\nu}_{\rm in}=n_\mu T^{\mu\nu}_{\rm out} \quad \text{on the surface.} \tag{39}$$

Again using the equations of motion we obtain

$$\begin{aligned}n_\mu T^{\mu\nu}_{\rm in}-n_\mu T^{\mu\nu}_{\rm out}={}&n^\nu[B-D(t)]+\frac{1}{2f_\pi}\partial^\nu[\bar{\psi}(\sigma_{\rm av}\\&+i\vec{\tau}\cdot\vec{\pi}_{\rm av}\gamma_5)\psi]-n_\mu\partial^\mu\sigma_{\rm in}\partial^\nu\sigma_{\rm out}\\&+n_\mu\partial^\mu\sigma_{\rm out}\partial^\nu\sigma_{\rm in}-n_\mu\partial^\mu\pi_{\rm in}\partial^\nu\pi_{\rm out}\\&+n_\mu\partial^\mu\pi_{\rm out}\partial^\nu\pi_{\rm in}=0,\end{aligned} \tag{40}$$

with

$$\sigma_{\rm av}\equiv\frac{1}{2}(\sigma_{\rm in}+\sigma_{\rm out}), \tag{41}$$

$$\pi_{\rm av}\equiv\frac{1}{2}(\pi_{\rm in}+\pi_{\rm out}), \tag{42}$$

$$D(t)\equiv\frac{1}{2}[(\partial_\rho\sigma_{\rm in})^2+(\partial_\rho\vec{\pi}_{\rm in})^2-(\partial_\rho\sigma_{\rm out})^2-(\partial_\rho\vec{\pi}_{\rm out})^2], \tag{43}$$

and all the functions in the above expressions are evaluated at the surface of the bag.

Equation (40) is well defined, and since the spatial part of n^ν in the case of spherical symmetry is $\hat{r}$, it can be used to calculate the bag constant B as

$$\begin{aligned}B={}&D(t)+\frac{1}{2f_\pi}\frac{\partial}{\partial r}[\bar{\psi}(\sigma_{\rm av}+i\vec{\tau}\cdot\vec{\pi}_{\rm av}\gamma_5)\psi]-n_\mu\partial^\mu\sigma_{\rm in}\frac{\partial\sigma_{\rm out}}{\partial r}\\&+n_\mu\partial^\mu\sigma_{\rm out}\frac{\partial\sigma_{\rm in}}{\partial r}-n_\mu\partial^\mu\pi_{\rm in}\frac{\partial\pi_{\rm out}}{\partial r}+n_\mu\partial^\mu\pi_{\rm out}\frac{\partial\pi_{\rm in}}{\partial r},\end{aligned} \tag{44}$$

where the static-bag solution has to be used. With the static hedgehog solution the terms involving products of fields inside and outside the bag actually cancel out each other. Such a value of B ensures that the continuity equation for the linear momentum is satisfied. Choosing $\nu=0$ from Eq. (40) we can derive an equation for $\dot{R}$:

$$\begin{aligned}\dot{R}={}&\frac{-1}{B-D(t)}\left\{\frac{1}{2f_\pi}\frac{\partial}{\partial t}[\bar{\psi}(\sigma_{\rm av}+i\vec{\tau}\cdot\vec{\pi}_{\rm av}\gamma_5)\psi]-\frac{\partial\sigma_{\rm in}}{\partial r}\frac{\partial\sigma_{\rm out}}{\partial t}\right.\\&\left.+\frac{\partial\sigma_{\rm out}}{\partial r}\frac{\partial\sigma_{\rm in}}{\partial t}-\frac{\partial\pi_{\rm in}}{\partial r}\frac{\partial\pi_{\rm out}}{\partial t}+\frac{\partial\pi_{\rm out}}{\partial r}\frac{\partial\pi_{\rm in}}{\partial t}\right\}.\end{aligned} \tag{45}$$

We use the above expression to compute the motion of the bag's surface, which conserves the total energy and momentum.

It is important to notice that for spherically symmetric solutions the total linear momentum is conserved regardless of the value of B, because the associated current is radial and the vector sum always gives a zero total momentum. This guarantees the conservation of the total momentum also for a nonstatic bag surface, because in such a case the RHS of Eq. (44) is not constant and hence the equation is not satisfied.

The first question we want to address is whether the static hedgehog solution is stable with respect to a small perturbation or it is just a special field configuration permitted only with a static boundary. If the static hedgehog models a hadron state one would like it to be little affected by a small nonresonant perturbation. We therefore considered an incoming wave packet incident on the bag, and we computed the evolution of the system. We used both a π field and a σ field as the incoming packet, with the following form:

$$\left.\begin{aligned}\sigma_{\rm out}(t_0,r)\\ \pi_{\rm out}(t_0,r)\end{aligned}\right\}=\begin{aligned}&A[e^{-\beta(r-R_0)}-1]^3\sin[\nu(t_0+r)]e^{-\alpha(r-R_0)}\\&\qquad\qquad r>R_0,\end{aligned} \tag{46}$$

which, at $r=R_0$, is zero up to the third derivative, in order to avoid discontinuities at the instant of the collision. The motion of the bag surface depends on the bag constant B. With $R_0=1$ fm and $f_\pi=1$ fm^{-1}, from Eq. (44) we have $B\simeq0.16$ fm^{-4}. However with such a small value of B our numerical implementation allowed us to obtain accurate solutions only for short times. We use $B=1$ fm^{-4} to demonstrate the qualitative features of the system even if it is an unrealistically high value, and we have verified that using smaller bag constants do not change the qualitative features of the system.

In Fig. 4 we show the energy of the fields inside and outside the bag versus time for small A and ν. In all the computations we used $\beta=1/R_0$. Both for a π wave and a σ wave the bag is hardly changed, and after the interaction the

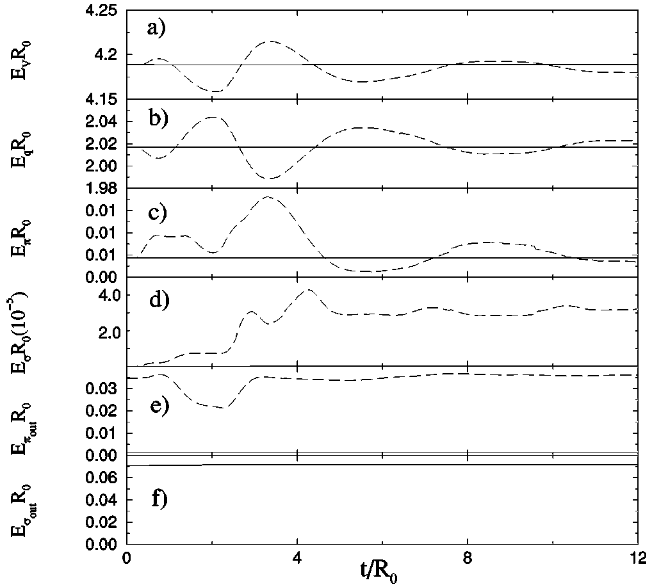

FIG. 4. Time evolution of the energy in the case of a collision with a wavepacket with $\nu = 1/R_0$, $\alpha = 0.2/R_0$, π wave with $A = 0.05/R_0$ (dashed line) or σ wave with $A = 0.1/R_0$ (solid line) (see Eq. 46): (a) volume energy, (b) fermion energy, (c) π-field energy inside the bag, (d) σ-field energy inside the bag, (e) π-field energy outside the bag, and (f) σ-field energy outside the bag.

velocity of the surface goes back to zero gradually as expected. Part of the pion field is reflected back at the surface of the bag, while the other part after penetrating the bag goes back out towards infinity. It is interesting to observe that a σ wave has almost no effect at all on the bag and nearly does not enter it. We verified that the static hedgehog solution remains little affected also for larger A and higher frequencies ν, thus validating its use as a stationary state of a hadron.

B. Resonances

We next consider whether the resonances found in the case of a driven bag motion still occur for the self-consistent motion caused by an incoming wave of appropriate frequency. This is a nontrivial question because the nonlinear relation between the fields and the motion of the boundary, as expressed in Eq. (45), might in principle destroy any phase coherence on which a resonance is built up. We hence performed our computation with incoming π waves given by Eq. (46) with $\alpha = 0$ and $\nu = n\pi/R_0$, $\nu = (E_n - E_k)$, and $\nu = (2n+1)\pi/(2R_0)$. Again due to numerical limitations we had to use small values of A.

In Fig. 5 we plot the energy of the fields inside the bag vs time for $\nu = \pi/R_0$ and in Fig. 6 for $\nu = E_2 - E_1$. We can see that the resonant behavior is still present. From the point of view of the energy gained by the bag the two resonances merge and appear as a broad resonance peaked at $\nu = E_2 - E_1$. With a closer look, however, one can observe two different physical phenomena. For ν close to π/R_0 the σ field is excited and the bag expands while the fermion field gets only a small contribution from the second static hedgehog state. For ν close to $E_2 - E_1$ the σ field is slightly out of

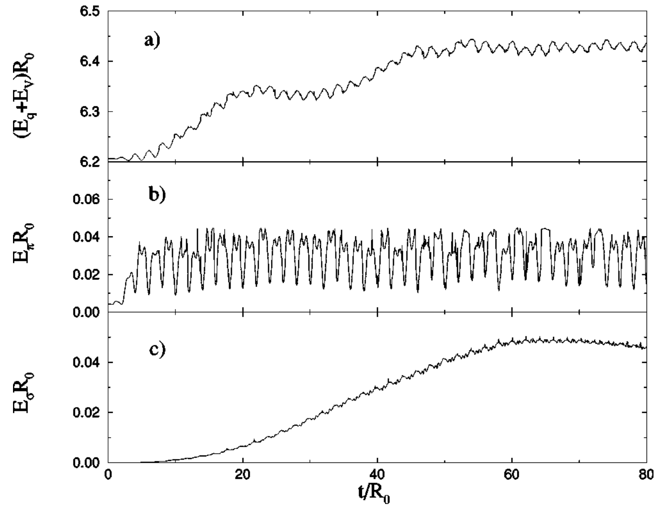

FIG. 5. Time evolution of the energy for a resonance with an incoming $\nu = \pi/R_0$, $A = 0.005/R_0$, $\alpha = 0$ π wave [see Eq. (46)], showing (a) fermion energy plus volume energy, (b) π field energy inside the bag, and (c) σ-field energy inside the bag.

resonance while the fermion field tends to be excited towards the second static hedgehog state and the volume of the bag decreases.

In the case of a wave with $\nu = \pi/(2R_0)$ we also have a clear resonance that involves the σ field (Fig. 7). At this frequency the fermion field is completely out of resonance. Overall the bag's energy increases not only as the pion energy but also in the form of volume energy due to the expansion of the bag. The excitation mechanism for the σ field is the same as explained in the previous section by means of Eqs. (32) and (33).

It is very interesting to notice that the expansion of the bag is related to the excitation of the σ field and not directly to the fermions. The increase of energy due to a larger bag radius is the classical counterpart of the breathing modes proposed by other authors [7–11] to explain certain radial excitations of the baryons such as the Roper resonance. These authors propose that such resonances are excitations of

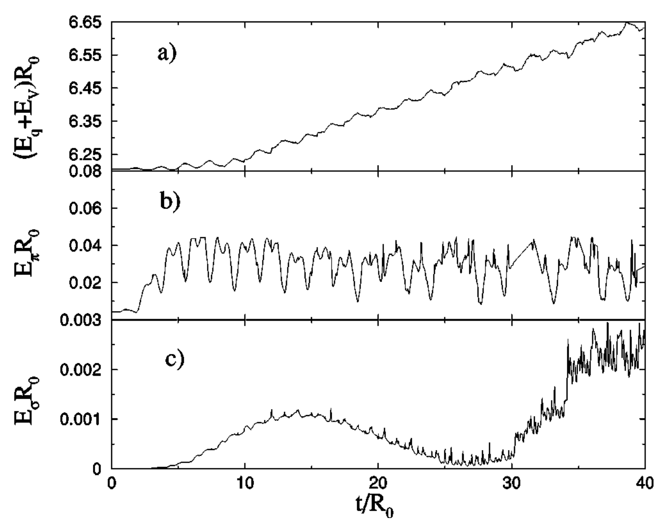

FIG. 6. Same as Fig. 6, but for $\nu = E_2 - E_1$.

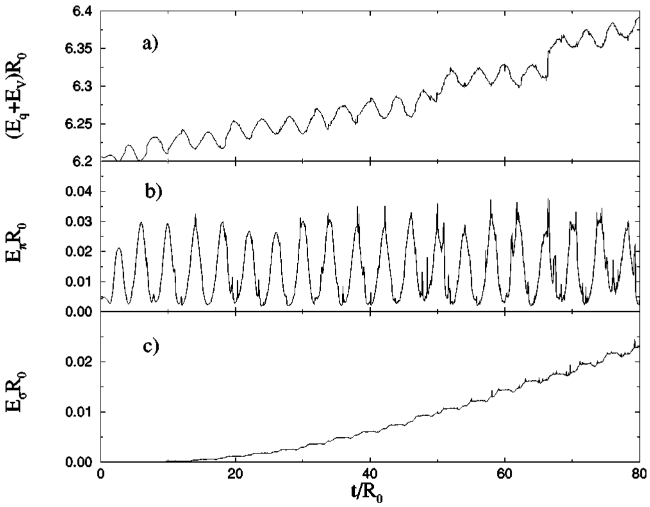

FIG. 7. Same as Fig. 6, but for $\nu=\pi/(2R_0)$ and $A=0.02/R_0$.

the collective degrees of freedom of the bag, represented in the models they considered by the surface coordinates, while the quarks essentially remain in the ground state. In the chiral bag model, in addition to the bag's radius, the pions too describe collective degrees of freedom, and hence our results strongly support the above scenario.

For larger odd multiples of $\pi/(2R_0)$ the resonant behavior is much attenuated. We believe that this is due to the fact that at higher frequencies and with a self-consistent bag motion an approximation as the one shown in Eqs. (30) and (31) is no longer acceptable. The perturbation still appears to be resonant for the σ field, but its energy increases very slowly.

Another remarkable property of the chiral bag is that it shows a realistic behavior in a scattering process. We have already mentioned the dynamics for the scattering with a nonresonant pion wave packet, but it is interesting to examine how the bag releases its energy after being excited by a resonant wave packet. In Fig. 8 it can be seen how, after the incoming wave packet has been scattered away, the bag remains in its excited state for some time before starting to slowly release the energy gained, hence looking similar to a stable particle. Also in this case we can see that the π wave inside the bag is not excited and its temporary increase in energy is simply due to the part of the incoming wave that enters the bag before being reflected away.

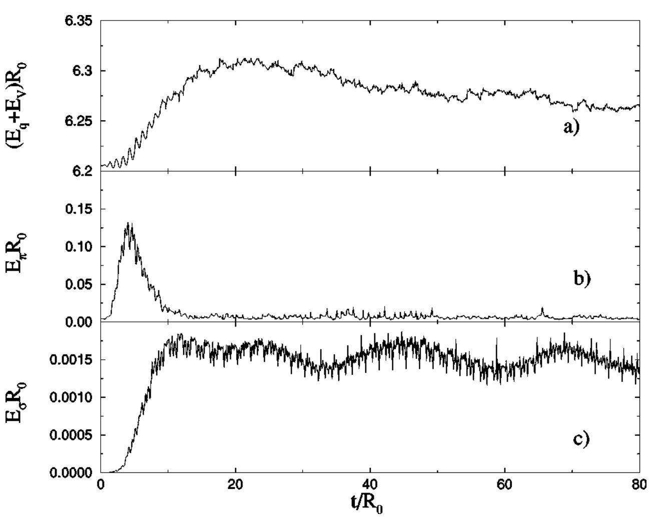

FIG. 8. Same as Fig. 6, but for $\nu=6.55/R_0\simeq E_3-E_1$, $A=0.005/R_0$, and $\alpha=0.2/R_0$.

Since the chiral bag is generally used to describe baryons, we have also performed the previous calculations with three quarks inside the bag in order to verify that our findings hold in this case too. Not having quantized the theory, we have to consider already in the Lagrangian three distinct fermion fields. Moreover, the coupling with the pions causes the solution to differ from the one-quark bag. In fact, while each quark has to satisfy the same equations (3) and (4), the equations for the pions change because the RHS of Eqs. (5) and (6) must be multiplied by a factor 3. Such a difference produces different eigenvalues for the static cavity solution and yields the following equation for the motion of the bag's surface

$$\dot{R}=\frac{-1}{B-D(t)}\left\{\frac{3}{2f_\pi}\frac{\partial}{\partial t}[\overline{\psi}(\sigma_{\rm av}+i\vec{\tau}\cdot\vec{\pi}_{\rm av}\gamma_5)\psi]-\frac{\partial\sigma_{\rm in}}{\partial r}\frac{\partial\sigma_{\rm out}}{\partial t}+\frac{\partial\sigma_{\rm out}}{\partial r}\frac{\partial\sigma_{\rm in}}{\partial t}-\frac{\partial\pi_{\rm in}}{\partial r}\frac{\partial\pi_{\rm out}}{\partial t}+\frac{\partial\pi_{\rm out}}{\partial r}\frac{\partial\pi_{\rm in}}{\partial t}\right\}. \quad (47)$$

We have verified that the features found with only one quark remain with three quarks.

For smaller values of B or bigger amplitudes A we have been able to obtain fairly accurate solutions for short times ($t<10$ fm/c) and, for resonant perturbations, we observe a much stronger and faster excitation process, which could probably model realistic energy levels.

V. SIMILARITIES WITH THE SOLITON

We point out here the very interesting similarities between the hedgehog solution and the soliton solution of a nonlinear field theory. We can define a classical soliton as any spatially confined and nondispersive solution of a classical field theory [18]. In order to have soliton solutions it is necessary to have some nonlinear couplings among the fields. The MIT bag model with only fermions inside the bag does not have nonlinear couplings. In fact, as we proved in a previous paper [12], it admits baglike solutions, but these are unstable with respect to perturbations of the bag surface. In the chiral bag model the quark-pion coupling, although it is linear, introduces a nonlinear self-coupling for the fermion field through the boundary conditions, as apparent from Eqs. (24), (25). We have seen that the hedgehog solution is indeed stable with respect to perturbations of the bag surface.

For a boson field the various nonlinear couplings can be characterized by a dimensionless coupling constant g. If $g=0$, the theory is linear and there is no soliton solution. However if g, however small, is different from zero, the theory admits soliton solutions. In the limit $g\rightarrow 0$ the soliton solution grows to infinity. This is remarkably similar to what happens in the dynamical chiral bag model. The dimensionless coupling constant is in this case γ/f_π, with γ an arbitrary constant with dimension of L^{-1}. If we set $\gamma/f_\pi=0$

already in the Lagrangian, we have the MIT bag model, and no stable, spatially confined solution exists. If we take the limit $\gamma/f_\pi \rightarrow 0$ ($f_\pi \rightarrow \infty$), we still have stable hedgehog solutions, but the field σ goes to infinity, so that $\sigma/f_\pi \rightarrow 1$.

Such similarities seem more than a coincidence, especially if we consider the bag models as simplifications of more general models. In fact it has been shown [19] that a chiral model, similar to the Skyrme Lagrangian, can automatically produce baglike solutions. From this point of view one may put some features of the baglike solutions already in the Lagrangian, thus obtaining a bag model. The fact that the MIT bag model does not admit a stable baglike solution may be viewed as due to an oversimplification, having completely neglected the quark-pion interaction, while in the chiral bag model such interaction is maintained at least at the surface of the bag.

VI. CONCLUSION

We have shown that the chiral bag described by the Lagrangian Eq. (1) is stable in the sense that a baglike solution exists even if the static bag is perturbed either by arbitrary radial motions of its boundary or by its interaction with a meson wave. This is in contrast with the purely fermionic MIT bag which has been proved to be unstable [12].

Computing the solution for a bag perturbed by a nonresonant meson wave, we found that it remains close to the static hedgehog solution and that, after the incoming wave is scattered away, it returns to the static hedgehog. Such a result validates the use of the static hedgehog as a stationary state of a hadron.

We also examined the existence of resonant perturbations, and we discovered three kinds of resonances which occur when the bag interacts with an incoming π wave. When the frequency of the incoming wave is close to an energy gap $\nu \simeq E_n - E_k$, the fermion field is in resonance. The σ field is also excited since ν is close to an integral multiple of π/R_0. The fermion field tends to go to an upper static hedgehog level, but these resonances are not simply transitions from a lower hedgehog state to an upper one, because in that case there would be only a static meson field in the final state, while here we have remarkable energy contribution from a nonstatic σ field. At $\nu = n\pi/R_0$ the fermion field is little excited while the σ field is in resonance and the bag expands. The third type of resonance occurs when ν equals odd multiples of $\pi/(2R_0)$, which is a consequence of the linear boundary condition Eq. (4). In this case the fermion field is not excited and the increase in energy comes from the σ field and the increase in the volume energy associated with the expansion of the bag. The occurence of a resonance at $\nu = \pi/(2R_0)$, which is much smaller than the energy gap between the first and second static hedgehog states, gives support to the description of the Roper resonance as a radial excitation of the collective degrees of freedom. In particular the expansion of the bag without fermion excitation is the classical analog of the breathing modes proposed by other authors [7–11]. Our results, however, show that the expansion of the bag is strictly related to the excitation of the σ field and in this sense warn against the use of the adiabatic approximation.

Compared to previous studies of dynamical bag models, our approach has two main advantages: we solve the equations of motion without approximations and we show the dynamics of the resonances. In the present work we have considered hedgehog configurations in which the quarks are neither in a flavor eigenstate nor in a J eigenstate; they do not represent any known hadrons. However, the occurence of the resonances we found does not depend on the flavor of the quarks, and since they are caused by spherical waves the angular momenta are not changed. Moreover they are not related to specific features of the hedgehog solution. In fact the $\nu = E_n - E_k$ resonance is a general feature of discrete-level systems. The resonances at $\nu = n\pi/R_0$ are geometrical resonances related to the fact that the mesons in this model are massless, and the ones at $\nu = (2n+1)\pi/(2R_0)$ are related to the linear boundary condition Eq. (4). Therefore, we conjecture that such resonances should occur also for more realistic solutions, i.e., with definite flavor and J.

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APPENDIX

For a spherical bag n_μ can be written as $(\dot{R}, -\hat{r})$ and, as shown for example in Ref. [12], Eq. (4) in radial coordinates becomes

$$i\dot{R}g(t,R) - f(t,R) = \frac{1}{f_\pi}[\sigma(t,R)g(t,R) - \pi(t,R)f(t,R)], \tag{A1}$$

$$-i\dot{R}f(t,R) - g(t,R) = \frac{1}{f_\pi}[\sigma(t,R)f(t,R) + \pi(t,R)g(t,R)]. \tag{A2}$$

Equating separately the real and imaginary parts of the two equations we obtain the following four equations:

$$-\dot{R}g_{\rm im}(t,R) - f_{\rm re}(t,R) = \frac{1}{f_\pi}[\sigma(t,R)g_{\rm re}(t,R) - \pi(t,R)f_{\rm re}(t,R)], \tag{A3}$$

$$\dot{R}f_{\rm im}(t,R) - g_{\rm re}(t,R) = \frac{1}{f_\pi}[\sigma(t,R)f_{\rm re}(t,R) + \pi(t,R)g_{\rm re}(t,R)], \tag{A4}$$

$$\dot{R}g_{\rm re}(t,R) - f_{\rm im}(t,R) = \frac{1}{f_\pi}[\sigma(t,R)g_{\rm im}(t,R) - \pi(t,R)f_{\rm im}(t,R)], \tag{A5}$$

$$-\dot{R}f_{\rm re}(t,R)-g_{\rm im}(t,R)=\frac{1}{f_\pi}[\sigma(t,R)f_{\rm im}(t,R)+\pi(t,R)g_{\rm im}(t,R)]. \quad \text{(A6)}$$

At this point we have four equations and four unknowns, i.e., $Q_{\rm re}(t+R)$, $Q_{\rm im}(t+R)$, $\sigma(t,R)$, and $\pi(t,R)$. This fact seems to make our requirement, that $\sigma(t,r)$ and $\pi(t,r)$ be continuous at $r=R$, redundant hence making the whole problem inconsistent. In fact if we could derive from Eqs. (A3)–(A6) all the four functions mentioned above, then Eqs. (16) and (17) would require $\sigma_{\rm out}$ and $\pi_{\rm out}$ to be singular at $r=R$, and even this would not guarantee the existence of a solution in general. However, it turns out that Eqs. (A3)–(A6) are not independent and we need to impose some condition on the functions in order to have a unique solution. We have verified this in two ways, as explained below.

Solving Eqs. (A3)–(A6) for $\sigma(t,R)$ and $\pi(t,R)$ we obtain two different expressions for each function

$$\sigma(t,R)=f_\pi\{-2f_{\rm im}(t,R)g_{\rm im}(t,R)+[g_{\rm re}(t,R)g_{\rm im}(t,R)-f_{\rm re}(t,R)f_{\rm im}(t,R)]\dot{R}\}/[g_{\rm im}^2(t,R)+f_{\rm im}^2(t,R)], \quad \text{(A7)}$$

$$\sigma(t,R)=f_\pi\{-2f_{\rm re}(t,R)g_{\rm re}(t,R)-[g_{\rm re}(t,R)g_{\rm im}(t,R)-f_{\rm re}(t,R)f_{\rm im}(t,R)]\dot{R}\}/[g_{\rm re}^2(t,R)+f_{\rm re}^2(t,R)], \quad \text{(A8)}$$

$$\pi(t,R)=f_\pi\{f_{\rm im}^2(t,R)-g_{\rm im}^2(t,R)-[f_{\rm re}(t,R)g_{\rm im}(t,R)+g_{\rm re}(t,R)f_{\rm im}(t,R)]\dot{R}\}/[g_{\rm im}^2(t,R)+f_{\rm im}^2(t,R)], \quad \text{(A9)}$$

$$\pi(t,R)=f_\pi\{f_{\rm re}^2(t,R)-g_{\rm re}^2(t,R)+[f_{\rm re}(t,R)g_{\rm im}(t,R)+g_{\rm re}(t,R)f_{\rm im}(t,R)]\dot{R}\}/[g_{\rm re}^2(t,R)+f_{\rm re}^2(t,R)]. \quad \text{(A10)}$$

Equating Eq. (A7) with Eq. (A8) and Eq. (A9) with Eq. (A10) we obtain two nonlinear equations for the real and imaginary parts of f and g,

$$\{2f_{\rm im}(t,R)g_{\rm im}(t,R)-[g_{\rm re}(t,R)g_{\rm im}(t,R)-f_{\rm re}(t,R)f_{\rm im}(t,R)]\dot{R}\}[g_{\rm re}^2(t,R)+f_{\rm re}^2(t,R)]$$
$$=\{2f_{\rm re}(t,R)g_{\rm re}(t,R)+[g_{\rm re}(t,R)g_{\rm im}(t,R)-f_{\rm re}(t,R)f_{\rm im}(t,R)]\dot{R}\}[g_{\rm im}^2(t,R)+f_{\rm im}^2(t,R)], \quad \text{(A11)}$$

$$\{f_{\rm im}^2(t,R)-g_{\rm im}^2(t,R)-[f_{\rm re}(t,R)g_{\rm im}(t,R)+g_{\rm re}(t,R)f_{\rm im}(t,R)]\dot{R}\}[g_{\rm re}^2(t,R)+f_{\rm re}^2(t,R)]$$
$$=\{f_{\rm re}^2(t,R)-g_{\rm re}^2(t,R)+[f_{\rm re}(t,R)g_{\rm im}(t,R)+g_{\rm re}(t,R)f_{\rm im}(t,R)]\dot{R}\}[g_{\rm im}^2(t,R)+f_{\rm im}^2(t,R)]. \quad \text{(A12)}$$

The problem is evidently extremely difficult to handle analytically, and so we used a numerical approach. Substituting Q' with a finite incremental ratio, Eqs. (A11) and (A12) become two nonlinear algebraic equations for $Q_{\rm re}(t+R)$ and $Q_{\rm im}(t+R)$. Solving numerically the algebraic equations, we have found that a whole region exists, in the $Q_{\rm re}$-$Q_{\rm im}$ plane around $Q(t+R-dz)$, in which the algebraic equations are satisfied, thus indicating that the solution is not unique. Since this is not a rigorous proof, we need to cross check our finding by imposing the continuity of $\sigma(t,r)$ and $\pi(t,r)$ on the surface of the bag and by verifying whether the solutions, found without using Eqs. (A11) and (A12), satisfy all four Eqs. (A3)–(A6).

In order to do this we have solved numerically Eqs. (24) and (25) for $Q_{\rm re}(t+R)$ and $Q_{\rm im}(t+R)$, as discussed in the main part of the paper, with $\sigma(t,R)$ and $\pi(t,R)$ given by anyone of the above expressions or a combination of them. We thereby verified that Eqs. (A3)–(A6) are automatically satisfied.

In the case of $\dot{R}=0$ we can analytically prove that Eqs. (A3)–(A6) are not independent. In fact, looking for solutions of the form $g_{\rm re}=P(t)g(r)$, $f_{\rm re}=P(t)f(r)$, $g_{\rm im}=S(t)g(r)$, $f_{\rm im}=S(t)f(r)$, we easily verify that Eqs. (A11), (A12) are automatically satisfied leaving $P(t)$ and $S(t)$ undetermined. It is reasonable to think that as $\dot{R}$ slowly departs from zero the four equations still remain not independent, though we are not able to provide a rigorous proof for it.

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