Neutron-proton interaction in rare-earth nuclei: Role of tensor force

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We investigate the role of the tensor force in the description of doubly odd deformed nuclei within the framework of the particle-rotor model. We study the rare-earth nuclei ¹⁷⁴Lu, ¹⁸⁰Ta, ¹⁸²Ta, and ¹⁸⁸Re using a finite-range interaction, with and without tensor terms. Attention is focused on the lowest K=0 and K=1 bands, where the effects of the residual neutron-proton interaction are particularly evident. Comparison of the calculated results with experimental data evidences the importance of the tensor-force effects.

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I. INTRODUCTION

It has long been known that the spectroscopy of doubly odd deformed nuclei evidences phenomena which can be directly associated with the interaction between the unpaired neutron and proton. The two most important effects of this kind are known as Gallagher-Moszkowski (GM) splitting [1] and Newby (N) shift [2].

Recently, use of large multidetector γ -ray arrays with high analyzing power has largely extended the amount of experimental data on this class of nuclei. For instance, currently available data in the rare-earth region allow the empirical determination of 137 GM splittings and 36 N shifts for 25 doubly odd nuclei [3,4]. These are to be compared with the 50 GM splittings and 19 N shifts known in the mid 1970s, as reported in the extensive review article by Boisson et al. [5]. The new experimental data have also evidenced the occurrence of other phenomena which may provide further information on the neutron-proton (n-p) interaction. Of particular interest is the odd-even staggering in $K \neq 0$ bands, the most important mechanism responsible for it being the direct Coriolis coupling with one or more N-shifted K=0 bands [6]. Also to be mentioned is the signature inversion phenomenon, which in the last few years has attracted much attention, leading to several experimental and theoretical studies. Various attempts have been made to understand this phenomenon in term of different mechanisms, but its interpretation still remains an open question [7].

In a previous paper [8] (hereafter referred to as I) we have studied the doubly odd deformed nucleus ¹⁷⁶Lu by performing a complete Coriolis band-mixing calculation within the framework of the particle-rotor model. We focused attention on the lowest $K^{\pi} = 0^{-}$ and $K^{\pi} = 1^{+}$ bands, where the effects of the residual *n*-*p* interaction are particularly evident, producing a significant N shift in the former and an odd-even staggering in the latter. The aim of our study was in fact to investigate the role of the tensor force, which has long been a controversial matter. A detailed discussion of this point can be found in I. In that paper we have used for the n-p interaction both a finite-range force with a Gaussian radial shape and a zero-range interaction. To completely explore the role of the tensor force, we have performed two different calculations with the Gaussian potential, with and without the tensor terms, respectively. The main result of these calculations was that the tensor force is absolutely essential for the description of the above-mentioned bands in ¹⁷⁶Lu. In I we have also confirmed the relevance of exchange forces, which was already emphasized in Refs. [9,10].

Here, we extend our calculations to other doubly odd deformed nuclei in the rare-earth region, focusing attention on N-shifted and odd-even staggered bands in 174 Lu, 180 Ta, 182 Ta, and 188 Re (some preliminary results have already been reported in Refs. [11–13]). The main motivation for this work is to verify if the conclusions based on the study of 176 Lu are confirmed when considering a larger set of experimental data. In fact, the role of the tensor force in doubly odd deformed nuclei seems not yet completely recognized, as evidenced by the large number of recent calculations making use of zero-range forces (see, for example, Ref. [14] and references therein).

The outline of the paper is as follows. In Sec. II we give a brief description of the model and some details of our calculations. Our results are presented and compared with the experimental data in Sec. III. In Sec. IV we draw the conclusions of our study.

II. OUTLINE OF THE MODEL AND CALCULATIONS

As mentioned in the Introduction, our calculations are performed within the framework of the particle-rotor model, in which the unpaired neutron and proton are strongly coupled to an axially symmetric core and interact through a residual effective interaction. This is a well-known model and a detailed description can be found, for instance, in Refs. [5,15]. For the sake of completeness, we give in the following the most relevant formulas. The total Hamiltonian is written as

$$H = H_0 + H_{\rm RPC} + H_{\rm ppc} + V_{np} \,. \tag{1}$$

The term H_0 includes the rotational energy of the whole system, the deformed, axially symmetric field for the neutron and proton, and the one-body intrinsic contribution from the rotational degrees of freedom. It reads

$$H_0 = \frac{\hbar^2}{2\mathcal{J}} (\mathbf{I}^2 - I_3^2) + H_n + H_p + \frac{\hbar^2}{2\mathcal{J}} [(\mathbf{j}_n^2 - j_{n3}^2) + (\mathbf{j}_p^2 - j_{p3}^2)].$$
(2)

The two terms H_{RPC} and H_{ppc} in Eq. (1) stand for the Coriolis coupling and the coupling of particle degrees of freedom through the rotational motion, respectively. Their explicit expressions are

$$H_{\rm RPC} = -\frac{\hbar^2}{2\mathcal{J}} (I^+ J^- + I^- J^+)$$
 (3)

and

$$H_{\rm ppc} = \frac{\hbar^2}{2\mathcal{J}} (j_n^+ j_p^- + j_n^- j_p^+). \tag{4}$$

In Eqs. (2)–(4), \mathcal{J} is the moment of inertia of the core while I and $\mathbf{J} = \mathbf{j}_p + \mathbf{j}_n$ are the total and intrinsic angular momentum operators, respectively. I_3 and J_3 are their projections on the intrinsic symmetry axis and, owing to the axial symmetry, are represented by the same quantum number *K*.

The effective n-p interaction in Eq. (1) has the general form

$$V_{np} = V(r)[u_0 + u_1 \boldsymbol{\sigma}_p \cdot \boldsymbol{\sigma}_n + u_2 P_M + u_3 P_M \boldsymbol{\sigma}_p \cdot \boldsymbol{\sigma}_n + V_T S_{12} + V_{TM} P_M S_{12}],$$
(5)

where the notation is just the same as that adopted in Ref. [5].

As basis states we use the eigenfunctions of H_0 , which are written as a symmetrized product of rotational functions D_{MK}^I and intrinsic wave functions $|\nu_n \Omega_n\rangle |\nu_p \Omega_p\rangle$:

$$|\nu_{n}\Omega_{n}\nu_{p}\Omega_{p}IMK\rangle = \left(\frac{2I+1}{16\pi^{2}}\right)^{1/2} [D_{MK}^{I}|\nu_{n}\Omega_{n}\rangle|\nu_{p}\Omega_{p}\rangle$$
$$+ (-)^{I+K}D_{M-K}^{I}|\nu_{n}\overline{\Omega}_{n}\rangle|\nu_{p}\overline{\Omega}_{p}\rangle].$$
(6)

Here Ω is the the quantum number corresponding to j_3 and ν stands for all the additional quantum numbers necessary to completely specify the states. The state $|\nu\bar{\Omega}\rangle$ is the time-reversal partner of $|\nu\Omega\rangle$. The quantum number *K* has two possible values

$$K_{\pm} = ||\Omega_n| \pm |\Omega_p||, \tag{7}$$

corresponding to parallel or antiparallel coupling of Ω_n and Ω_p .

Each intrinsic state then gives rise to a rotational band, but the Hamiltonian (1) produces an admixture of different bands. The explicit expressions of the matrix elements of the total Hamiltonian can be found in Ref. [15], where it can be seen that while the Coriolis interaction has only $\Delta K = \pm 1$ matrix elements, the two terms V_{np} and H_{ppc} give rise to diagonal and nondiagonal contributions. In particular, the latter has diagonal matrix elements different from zero only for K=0 bands with $|\Omega_n| = |\Omega_p| = 1/2$. We would like to stress that all diagonal and nondiagonal terms of Hamiltonian (1) are explicitly taken into account in our calculations.

As regards the single-particle Hamiltonians H_n and H_p , they have been generated by a standard Nilsson potential as

TABLE I. Values of the parameters β_2 and $\hbar^2/2\mathcal{J}$ (keV).

Nucleus	β_2	$\hbar^2/2\mathcal{J}$	
¹⁷⁴ Lu	0.331	12	
¹⁸⁰ Ta	0.278	14	
¹⁸² Ta	0.274	14	
¹⁸⁸ Re	0.226	18	

defined, for instance, in [16]. In our calculations for the four nuclei ¹⁷⁴Lu, ¹⁸⁰Ta, ¹⁸²Ta, and ¹⁸⁸Re the parameters μ and κ in this potential have been fixed by using the massdependent formulas of Ref. [17], while the harmonic oscillator parameter ν has been chosen according to the expression $\nu = m\omega/\hbar$ fm⁻². The deformation parameter β_2 has been deduced for each doubly odd nucleus from the even-even neighbor. In Table I, we report the values of β_2 together with those of the rotational parameter $\hbar^2/2\mathcal{J}$ used in our calculations. For each doubly odd nucleus this latter quantity has been deduced from low-lying bands with a pure rotational character. The adopted single-particle schemes for the neutron and proton are listed in Table II for the four considered nuclei, each scheme being essentially derived from the experimental spectra of the two neighboring odd-mass nuclei.

As regards the residual n-p interaction (5), we have used a finite-range force with a radial dependence V(r) of the Gaussian form

$$V(r) = \exp(-r^2/r_0^2).$$
 (8)

As mentioned in the Introduction, the main aim of this study is to assess the role of the tensor force. To this end, we have performed two different calculations with the Gaussian potential, with and without the tensor terms, respectively. In both cases, for all the parameters of the force we have adopted the values determined by Boisson *et al.* [5], the only exception being u_0 , which does not contribute either to the GM splttings or to the N shifts. These values, as well as that of u_0 , are reported in I, where the reasons of our choice are also discussed.

Before closing this section it is worth mentioning that we have also performed calculations making use of a δ interaction with a strength of the spin-spin parameter varied over a large range of values. We have found that no value of this strength gives a satisfactory description of the N shifts and odd-even staggerings in any of the considered nuclei, thus confirming our results for ¹⁷⁶Lu (see I). As a consequence, here we will not compare with experiment the results obtained by using a δ force, but make only some comments to point out the inadequacy of this force to explain the above-mentioned effects.

III. RESULTS AND COMPARISON WITH EXPERIMENT

We report here the results of our calculations for ¹⁷⁴Lu, ¹⁸⁰Ta, ¹⁸²Ta, and ¹⁸⁸Re, which are all well-deformed nuclei. The choice of these nuclei is motivated by the fact that enough experimental information relevant to our study is available for them. In fact, in the experimental spectra of

Nucleus	Configuration Proton	E	Configuration Neutron	Ε	Nucleus	Configuration Proton	Ε	Configuration Neutron	E
¹⁷⁴ Lu	$\frac{\frac{7}{2} + [404]}{\frac{1}{2} - [541]}$ $\frac{\frac{5}{2} + [402]}{\frac{9}{2} - [514]}$ $\frac{1}{2} + [411]$ $\frac{3}{2} - [532]$	0 208 369 437 449 903	$\frac{5}{2}^{-}[512]$ $\frac{7}{2}^{+}[633]$ $\frac{7}{2}^{-}[514]$ $\frac{1}{2}^{-}[521]$ $\frac{1}{2}^{-}[510]$ $\frac{5}{2}^{+}[642]$	0 389 400 431 1053 1207	¹⁸⁰ Ta	$\frac{\frac{7}{2}}{\frac{9}{2}} = \frac{1}{514}$ $\frac{\frac{5}{2}}{\frac{1}{2}} = \frac{1}{402}$ $\frac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{541}$	0 20 250 546 700	$\frac{9}{2} + [624]$ $\frac{7}{2} - [514]$ $\frac{1}{2} - [510]$ $\frac{5}{2} - [512]$ $\frac{1}{2} - [521]$ $\frac{3}{2} - [512]$ $\frac{7}{2} - [503]$ $\frac{7}{2} + [633]$ $\frac{5}{2} + [642]$	0 200 395 518 647 785 812 1506 2000
¹⁸² Ta	$\frac{\frac{7}{2}}{\frac{9}{2}} + [404]$ $\frac{9}{2} - [514]$ $\frac{5}{2} + [402]$ $\frac{1}{2} + [411]$	0 5 350 477	$\frac{\frac{1}{2}}{\frac{3}{2}} = [510]$ $\frac{\frac{3}{2}}{\frac{11}{2}} = [615]$ $\frac{\frac{9}{2}}{\frac{1}{2}} = [624]$ $\frac{7}{2} = [503]$ $\frac{7}{2} = [514]$	0 220 300 550 670 1100	¹⁸⁸ Re	$\frac{\frac{5}{2}}{\frac{9}{2}} + [402]$ $\frac{9}{2} - [514]$ $\frac{1}{2} + [411]$ $\frac{7}{2} + [404]$	0 176 625 748	$\frac{\frac{3}{2}}{\frac{1}{2}} = [512]$ $\frac{\frac{1}{2}}{\frac{1}{2}} = [510]$ $\frac{\frac{7}{2}}{\frac{1}{2}} = [503]$ $\frac{\frac{9}{2}}{\frac{5}{2}} = [503]$	0 161 310 340 640

TABLE II. Energies (in keV) of the proton and neutron single-particle states.

¹⁷⁴Lu, ¹⁸²Ta, and ¹⁸⁸Re K=0 bands, which are not strongly perturbed but have a sizable N shift, have been recognized, while in ¹⁸⁰Ta and ¹⁸⁸Re there are K=1 bands which exhibit a significant odd-even staggering. We have focused attention on those data which are directly related to the *n*-*p* interaction, so as to give a sound answer to the question regarding the role of the tensor term. In fact, aside from the residual *n*-*p* interaction, also the terms H_{ppc} and H_{RPC} of the Hamiltonian (1) may give a contribution to the odd-even shift (see Ref. [6]) and a complex interplay of all these interactions may occur in bands characterized by a strong admixture of different intrinsic wave functions. In these cases, even taking explicitly into account all contributions coming from Hamiltonian (1), it would be a very difficult task to disentangle the genuine effects of the *n*-*p* interaction.

In Figs. 1–3 the experimental spectra of the K=0 bands [4] are reported and compared with the calculated ones, which are obtained making use of the two different interactions mentioned in Sec. II. The $K^+ = 0^+ p_2^7 [404] n_2^7 [633]$ band of ¹⁷⁴Lu starting at 281 keV excitation energy is shown in Fig. 1. This band, recognized in several other odd-odd nuclei, has been observed in ¹⁷⁴Lu up to $I^{\pi} = 9^+$. We see that a very good agreement with experiment is obtained for the Gaussian force with tensor terms. This is not the case for the calculations performed with the pure central interaction. From Fig. 1 it appears that the two calculations yield almost the same level spacings for states with even and odd angular momenta separately. The main difference between the two spectra resides in the energy displacement of levels of odd I relative to those of even I, i.e., the N shift. In first-order perturbation theory the N shift is just defined as the matrix element $\langle \nu_p \Omega_p, \nu_n \Omega_n | V_{np} | \nu_p \Omega_p, \nu_n \Omega_n \rangle$. In Table III we report the values of this matrix element obtained with the central plus tensor force together with the individual contributions. For the above-mentioned K=0 band in ¹⁷⁴Lu we see that the magnitude of the matrix element is almost entirely determined by the tensor force, the central part giving only a 1 keV contribution. This result is in line with the early findings of Refs. [5,18], where the importance of the contribution of tensor force to the N shift for bands based on antiparallel or parallel intrinsic spins was evidenced. For the latter the effects of the tensor part are predominant, as expected from the study of Ref. [2], where an analysis in the asymptotic limit was performed. In this context, we should also mention that when using a δ force, as defined, for instance, in I, there is no reasonable way to reproduce the odd-even shift of this band. In fact, taking a negative value for the strength v_1 of the spin-spin term, the odd-even shift has the opposite sign as compared to the experimental one, its magnitude increasing almost linearly with v_1 . As an example, when using $v_1 = -0.9$ MeV, which is a reasonable value to reproduce the GM splitting in the rare-earth region [3,19], the matrix element $\langle \nu_p \Omega_p, \nu_n \overline{\Omega}_n | V_{np} | \nu_p \overline{\Omega}_p, \nu_n \Omega_n \rangle$ becomes 22 keV. The fact that a δ force produces a shift with the wrong sign while a pure central force gives a very small contribution, but with the right sign, makes evident the importance of the exchange forces.

In Figs. 2 and 3 we compare the experimental and calculated spectra for two lowest K=0 bands in ¹⁸²Ta and ¹⁸⁸Re, respectively. The first is the $K^+=0^ p\frac{7}{2}[404]n\frac{7}{2}[503]$ band with a bandhead energy at 558 keV while the second is the $K^+=0^+$ $p\frac{9}{2}[514]n\frac{9}{2}[505]$ band starting at 208 keV. Unfortunately, only low-spin members of these two bands have been identified. However, as it was the case for ¹⁷⁴Lu, an excellent agreement is obtained in both cases when a Gaussian force with tensor terms is used. From Figs. 2 and 3 we see that the right level order is obtained while the discrepan-



FIG. 1. Experimental and calculated spectra of the lowest $K^{\pi} = 0^+$ band in ¹⁷⁴Lu. The theoretical spectra have been obtained by using (a) a central plus tensor force with a Gaussian radial shape and (b) a Gaussian central force.

cies in the energies are less than 20 keV for all states. As regards the calculations with a pure central force, the situation is quite similar to that discussed for 174 Lu. In fact, the disagreement between the experimental data and the results of these calculations resides in the relative displacement in the energy of the states with odd and even angular momenta, which, as mentioned before, is essentially due to the *n*-*p* interaction. In Table III we report the values of the relevant matrix elements. We see again that the pure central force gives only a small contribution, the magnitude of these matrix elements being essentially determined by the tensor force.

To conclude this discussion, we have shown that the tensor force is quite able to explain the odd-even shift in K = 0 bands of ¹⁷⁴Lu, ¹⁸²Ta, and ¹⁸⁸Re. We now compare in Table IV the empirical GM splittings [3] for these three nuclei with those derived from the calculated bandhead energies. In this table we only report the results obtained using the Gaussian plus tensor force. This is to show how this force, parametrized as in Ref. [5], leads to a very good de-



FIG. 2. Same as Fig. 1, but for the lowest $K^{\pi} = 0^{-}$ band in ¹⁸²Ta.

scription of the three nuclei ¹⁷⁴Lu, ¹⁸²Ta, and ¹⁸⁸Re on the whole. In fact, we see from Table IV that not only is the sign correctly reproduced in all cases, also the quantitative agreement is quite satisfactory. The discrepancy between the calculated and experimental values is always less than 50 keV, except in one case where it turns out to be 101 keV. In this case, however, the empirical GM splitting is affected by an error of 57 keV.

Let us now come to the odd-even staggering effect. In Fig. 4 the experimental [20,21] ratio [E(I) - E(I-1)]/2I for the $K^{\pi} = 1^+ p_2^{-1} [404] n_2^{-1} [624]$ ground-state band in ¹⁸⁰Ta is plotted vs *I* and compared with the calculated ones. We see that this band exhibits a rather large odd-even staggering, the even *I* states being energetically favored with respect to the odd ones. This behavior is satisfactorily reproduced by the



FIG. 3. Same as Fig. 1, but for the lowest $K^{\pi} = 0^+$ band in ¹⁸⁸Re.

TABLE III. Values of the matrix elements $\langle \nu_p \Omega_p, \nu_n \overline{\Omega}_n | V_{np} | \nu_p \overline{\Omega}_p, \nu_n \Omega_n \rangle$ (in keV) obtained using a central plus tensor force with a Gaussian radial shape. $\langle C \rangle$ and $\langle T \rangle$ represent the contributions of the central and tensor force, respectively.

	Config				
Nucleus	Proton	Neutron	$\langle C \rangle$	$\langle T \rangle$	$\langle V_{np} \rangle$
¹⁷⁴ Lu	$\frac{7}{2}$ + [404]	$\frac{7}{2}$ + [633]	-1	-26	-27
¹⁸² Ta	$\frac{7}{2}$ + [404]	$\frac{7}{2}$ [503]	3	19	22
¹⁸⁸ Re	$\frac{9}{2}$ [514]	$\frac{9}{2}$ [505]	-4	-60	-64
¹⁸⁸ Re	$\frac{5}{2}^{+}$ [402]	$\frac{5}{2}$ [503]	0	47	47

calculation including the tensor force while with the pure central Gaussian force the staggering is almost nonexistent. It should be pointed out that the staggering in this band may be traced to direct Coriolis coupling with the K^{π} $=0^+ p \frac{7}{2} [404] n \frac{7}{2} [633]$ band. In fact, in both our calculations we have found that the wave functions of the states of the $K^{\pi} = 1^{+}$ band contain significant components (up to 30%) of states of the above $K^{\pi} = 0^+$ band. The fact that only the calculation including the tensor terms gives the right staggering implies therefore that only this force is able to produce a sizable N shift for this $K^{\pi} = 0^+$ band. Since this band has not been recognized in ¹⁸⁰Ta, a direct comparison is not possible at the present time. It should be noted, however, that the $K^{\pi} = 0^{+}$ band in ¹⁷⁴Lu (see Fig. 1) has just the same intrinsic n-p configuration. As we have already pointed out above, only with the Gaussian plus tensor force the energy of the even I states of this band is decreased so as to bring the calculated spectrum in good agreement with the experimental one.

In Fig. 5 the experimental staggering observed in the K^{π}

TABLE IV. Experimental and calculated GM splittings (keV).

	Config	guration				
Nucleus	Proton	Neutron	K	ζ^{π}	Expt.	Calc.
¹⁷⁴ Lu	$\frac{7}{2}[404]$	$\frac{5}{2}[512]$	1 -	6-	-114.9 ± 1.3	-124
	$\frac{7}{2}[404]$	$\frac{7}{2}[633]$	0^+	7+	-64.2 ± 10.0	-82
	$\frac{7}{2}[404]$	$\frac{1}{2}[521]$	3-	4-	79.5 ± 5.0	89
	$\frac{1}{2}[541]$	$\frac{5}{2}[512]$	2^{+}	3+	-149.8	-123
	$\frac{5}{2}[402]$	$\frac{5}{2}[512]$	0^{-}	5^{-}	130.2 ± 2.1	155
	$\frac{9}{2}[514]$	$\frac{5}{2}[512]$	2^{+}	7+	135.3 ± 3.1	109
¹⁸² Ta	$\frac{7}{2}[404]$	$\frac{1}{2}[510]$	3 -	4^{-}	-92.6 ± 8.3	-118
	$\frac{7}{2}[404]$	$\frac{3}{2}[512]$	2^{-}	5^{-}	128.0 ± 0.7	142
	$\frac{7}{2}[404]$	$\frac{7}{2}$ [503]	0^{-}	7-	-122.0 ± 3.1	-143
	$\frac{9}{2}[514]$	$\frac{1}{2}[510]$	4+	5+	147.6 ± 0.6	105
	$\frac{9}{2}[514]$	$\frac{3}{2}[512]$	3+	6^+	-111.1 ± 12.4	-123
	$\frac{9}{2}[514]$	$\frac{11}{2}[615]$	1 -	10^{-}	147.7 ± 0.6	122
	$\frac{5}{2}[402]$	$\frac{1}{2}[510]$	2^{-}	3-	301.2	296
¹⁸⁸ Re	$\frac{5}{2}[402]$	$\frac{3}{2}[512]$	1^{-}	4^{-}	-138.7 ± 1.4	-117
	$\frac{5}{2}[402]$	$\frac{1}{2}[510]$	2^{-}	3-	93.5 ± 1.1	100
	$\frac{5}{2}[402]$	$\frac{7}{2}[503]$	1^{-}	6^{-}	209.4 ± 0.2	205
	$\frac{1}{2}$ [411]	$\frac{3}{2}[512]$	1^{-}	2^{-}	197.2 ± 57.3	96



FIG. 4. Experimental and calculated odd-even staggering of the $K^{\pi} = 1^+$ ground-state band in ¹⁸⁰Ta. Solid circles correspond to experimental data. The theoretical results are represented by open circles (Gaussian central plus tensor force) and squares (Gaussian central force).

 $=1^{-}p_{2}^{5}[402]n_{2}^{3}[512]$ ground-state band of ¹⁸⁸Re [4] is compared with the calculated ones. Although this band does not exhibit a large odd-even staggering, we see that only with the Gaussian plus tensor force the calculated behavior reproduces the experimental one. Also in this case the wave functions of the states of the $K^{\pi} = 1^{-}$ band obtained from both calculations contain appreciable components (5%-10%) of states with $K^{\pi} = 0^{-} p \frac{5}{2} [402] n \frac{5}{2} [503]$. The difference in the two calculations may therefore be traced back to the different values of the matrix element $\langle p\frac{5}{2}[402]n\frac{5}{2}[503]|V_{np}|p\frac{5}{2}[402]n\frac{5}{2}[503]\rangle$. In fact, as can be seen from Table III, only the matrix element of the Gaussian plus tensor force turns out to be adequate to produce a sizable N shift for the $K^{\pi} = 0^{-}$ band. Unfortunately, this band has not been observed in ¹⁸⁸Re or in other rare-earth nuclei.

IV. CONCLUDING REMARKS

In this paper, we have presented the results of a study of the well deformed doubly odd nuclei ¹⁷⁴Lu, ¹⁸⁰Ta, ¹⁸²Ta, and ¹⁸⁸Re within the framework of the particle-rotor model.



FIG. 5. Same as Fig. 4, but for the $K^{\pi} = 1^{-}$ ground-state band in ¹⁸⁸Re.

We have performed complete Coriolis band-mixing calculations, focusing attention on the lowest K=0 and K=1bands, where the effects of the neutron-proton interaction are particularly evident. As regards this interaction, we have used in our calculations a Gaussian force with and without tensor terms adopting in both cases the values of the parameters originally determined by Boisson *et al.* [5].

This study represents an extension of our previous work on ¹⁷⁶Lu aimed at obtaining further insight into the role of the tensor force in the description of doubly odd deformed nuclei. The results presented here, as those reported in I, show that the tensor force is an essential ingredient, if not unique in some cases, to explain the odd-even shift in K= 0 bands. Also the staggering in $K \neq 0$ bands, whose basic mechanism is the Coriolis mixing with N-shifted K=0bands, is very well accounted for by our calculations with tensor terms.

It should be mentioned that in more complex situations,

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where there is a strong band mixing, other mechanisms may play a significant role and only the right interplay between them can explain the observed effects. This is, for instance, the case of the so called doubly decoupled bands: namely, of bands based on high-j unique-parity shell-model states. Also in this context, however, the tensor force should not be neglected, as is done in most of the existing studies to date. The use of an incomplete neutron-proton interaction may in fact lead to an incorrect evaluation of the weight of other mechanisms.

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