Revisiting the Hugenholtz–Van Hove theorem in nuclear matter

P. Czerski
Institute of Nuclear Physics, PL-31-342 Kraków, Poland

A. De Pace and A. Molinari
Dipartimento di Fisica Teorica dell'Università di Torino and Istituto Nazionale di Fisica Nucleare, Sezione di Torino, via P.Giuria 1, I-10125 Torino, Italy

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An assessment of the magnitude of the rearrangement contribution to the Fermi energy and to the binding energy per particle is carried out in symmetric nuclear matter by extending the *G*-matrix framework. The restoration of the thermodynamic consistency or, equivalently, the fulfillment of the Hugenholtz–Van Hove theorem, is discussed.

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I. INTRODUCTION

The physics of nuclear matter beyond the Hartree-Fock (HF) mean field has been dealt with using a variety of techniques, such as the Bethe-Brueckner-Goldstone hole-line expansion [1], the variational approach based on correlated functions [2], and the Green's function Monte Carlo method [3], exploiting realistic nucleon-nucleon interactions, like the Bonn or the Urbana potentials, as input.

Basically, these studies focused on two observables: The binding energy per particle and the saturation density, experimentally extracted from the semi empirical mass formula and from the high energy nuclear electron scattering experiments. In addition, also the compression modulus, whose value can be inferred from the excitation energy of the breathing mode in nuclei, has been quite extensively explored.

As is well known, all these investigations indicate that the theory fails to account for the data. A remedy to this shortcoming is presently sought in the introduction of three-body forces [4] and/or in a covariant treatment of the nuclear many-body problem [5].

However, a successful theory of nuclear matter will be required not only to account for the data, but to fulfill as well general theorems, in particular the Hugenholtz–Van Hove (HVH) one [6]. Indeed, a violation of the latter would signal an inconsistency of the theory both at the global and at the single-particle level.

In the following we shall focus on the latter issue, which has received comparatively less attention. According to the HVH theorem, whose validity encompasses all the normal Fermi systems at zero temperature, at equilibrium the average energy per particle and the Fermi energy should coincide.

The HVH theorem holds because indeed a real (and not complex) energy can be assigned to the particles at the Fermi surface, as proved by Luttinger [7]. Actually, the concept of single-particle energy is still approximately tenable in the proximity of the Fermi surface, but not away from the latter as experimentally verified with exclusive inelastic electron scattering $(e,e'p)$ [8].

Accordingly, in this paper we investigate

(i) the contributions to the single-particle energy (in particular at the Fermi surface) beyond the mean field,

(ii) how they affect the total binding energy per particle,

(iii) whether they help to fulfill the HVH theorem in nuclear matter.

These items have been considered in the past by various authors. In nuclear matter starting from the old paper of Brueckner and Goldman [9] until the more recent work of Baldo *et al.* [10]. In finite nuclei, where the issues are even more delicate, the theme has been addressed in several investigations. To mention a few we recall those of Faessler *et al.* (see, for example, Ref. [11]) and of Meldner and Shakin [12]. A comprehensive review of the topic has been given by Hogdson [13].

From the above studies it appears that, in a given theoretical framework, the problem of fulfilling the HVH theorem is far from trivial, both in nuclear matter and in finite nuclei. This recognition is the basic motivation for the present work.

II. FORMALISM AND *G*-MATRIX RESULTS

We start from the *G*-matrix expression for the total energy of nuclear matter at zero temperature,

$$E=\sum_{\boldsymbol{k}_1}\frac{k_1^2}{2m}n(k_1,k_F)+\frac{1}{2}\sum_{\boldsymbol{k}_1\boldsymbol{k}_2}\langle \boldsymbol{k}_1\boldsymbol{k}_2|G|\boldsymbol{k}_1\boldsymbol{k}_2\rangle \times n(k_1,k_F)n(k_2,k_F), \tag{1}$$

k_F being the Fermi momentum and the *G*-matrix obeying the equation

$$\langle \boldsymbol{k}_1\boldsymbol{k}_2|G|\boldsymbol{k}_1\boldsymbol{k}_2\rangle=\langle \boldsymbol{k}_1\boldsymbol{k}_2|V|\boldsymbol{k}_1\boldsymbol{k}_2\rangle-\sum_{\boldsymbol{k}_3\boldsymbol{k}_4}\frac{\langle \boldsymbol{k}_1\boldsymbol{k}_2|V|\boldsymbol{k}_3\boldsymbol{k}_4\rangle[1-n(k_3,k_F)][1-n(k_4,k_F)]\langle \boldsymbol{k}_3\boldsymbol{k}_4|G|\boldsymbol{k}_1\boldsymbol{k}_2\rangle}{e_G(k_3,k_F)+e_G(k_4,k_F)-e_G(k_1,k_F)-e_G(k_2,k_F)}, \tag{2}$$

where the summation includes the spin-isospin degrees of freedom and the matrix elements are meant to be antisymmetrized. We then solve Eq. (2) *self-consistently* with respect to the single-particle energies appearing in the denominator. These, to be defined below, are continuous functions of the energy across the Fermi surface. For the bare interaction V we use the Bonn potential [14].

Concerning self-consistency, we recall that in Landau's theory of Fermi liquids the energy of a single quasiparticle $e(k,k_F)$ is obtained according to the prescription

$$\frac{\delta E}{\delta n(k,k_F)} = e(k,k_F), \tag{3}$$

where $n(k,k_F)$ gives the quasiparticle number in the k state. The distribution function $n(k,k_F)$ fixes the density of the system and depends upon k_F, to be viewed as a parameter. When Eq. (3) is implemented in the G-matrix framework (but ignoring the dependence upon n of the G-matrix elements), it yields

$$e_G(k,k_F) = \frac{k^2}{2m} + \sum_{\boldsymbol{k}_2} \langle \boldsymbol{k}\boldsymbol{k}_2|G|\boldsymbol{k}\boldsymbol{k}_2\rangle n(k_2,k_F), \tag{4}$$

which we view as the energy of a particle of given momentum k.

In accord with the theory of Brueckner [15], which represent the leading term of the hole-line expansion and exactly incorporates the two-particle correlations, we solve then Eq. (2) seeking for self-consistency on the basis of Eq. (4). From a diagrammatic point of view this amounts to dress the particles and the *two-hole* lines of a ladder diagram with a *first order* self-energy containing a G-matrix interaction.

However, it has been known for a long time that, when computed at $k=k_F$, Eq. (4) violates the HVH theorem, in contrast with the HF theory (see the Appendix). This is indeed seen to occur in Fig. 1, where we display, as a function of $\rho = 2k_F^3/3\pi^2$, the results of our calculation of the binding energy per particle (1) and of the Fermi energy, as obtained from Eq. (4), carried out with the distribution $n(k,k_F) = \theta(k_F - k)$. It appears from the figure that our self-consistent G-matrix yields a good binding energy per particle (-16.1 MeV), but at the wrong density ($\rho = 0.25$ fm^{-3}, i.e., $k_F = 1.55$ fm^{-1}); these results are close to those recently obtained in Ref. [16], also in the Brueckner-Hartree-Fock (BHF) scheme, with the $v14$ Argonne potential and with a continuous auxiliary potential. In addition, a most substantial violation of the HVH theorem (of 17.2 MeV, the Fermi energy being $\epsilon_F = -33.3$ MeV) is seen to occur, *in quantitative* accord with the findings of Ref. [10].

Note that the above quoted ϵ_F corresponds to a BHF potential energy of about 80 MeV: Hence the rearrangement corresponds to a correction of $\cong 20\%$ of the latter.

Thus, in the spirit of Landau's theory, we carry out more thoroughly the variation (3) of the energy versus the distribution $n(k,k_F)$ writing

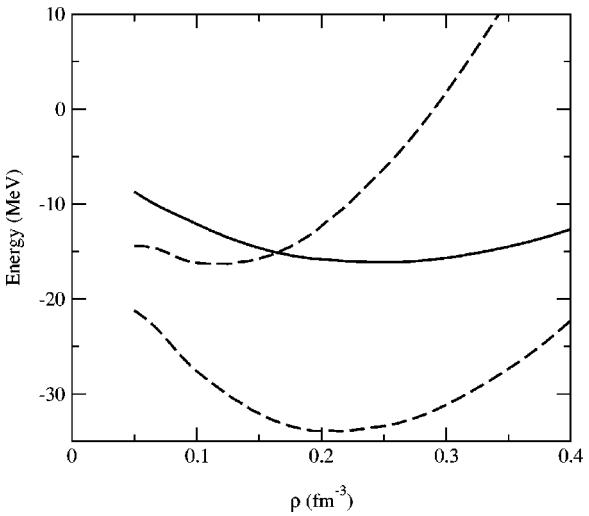

FIG. 1. Symmetric nuclear matter binding energy per particle (solid line) and Fermi energy (dashed) as a function of the density. The lower and upper dashed lines refer to Fermi energies without and with rearrangement contribution, respectively, whereas the solid line has no rearrangement.

$$e(k,k_F) = e_G(k,k_F) + \frac{1}{2}\sum_{\boldsymbol{k}_1\boldsymbol{k}_2} n(k_1,k_F)n(k_2,k_F) \times \frac{\delta}{\delta n(k,k_F)}\langle \boldsymbol{k}_1\boldsymbol{k}_2|G|\boldsymbol{k}_1\boldsymbol{k}_2\rangle, \tag{5}$$

the last term on the right-hand side being often referred to as the *rearrangement* contribution. This in principle should be computed by making the variation on the right-hand side of Eq. (2) with respect to $n(k_3,k_F)$ and $n(k_4,k_F)$ and also to the $n(k,k_F)$ entering into the single-particle energies of the denominator. These variations represent corrections to the Pauli operator and to the energies of the initial and intermediate states, respectively, resulting from the removal of one particle.

For a discussion of the associated diagrams and for the convergence of the hole-line expansion in general we refer the reader to Refs. [10,16–18], where the impact of the hole-hole ladder diagrams and of the off-shell T matrix in connection with the HVH theorem is explored.

Here we aim instead to compute the rearrangement contribution to the single-particle energy at (or in the proximity of) the Fermi surface by using the following procedure in carrying out the functional derivative, that is

$$e(k\approx k_F,k_F) = e_G(k,k_F) + \frac{1}{2}\sum_{\boldsymbol{k}_1\boldsymbol{k}_2} n(k_1,k_F)n(k_2,k_F) \times \frac{\partial}{\partial k_F}\langle \boldsymbol{k}_1\boldsymbol{k}_2|G|\boldsymbol{k}_1\boldsymbol{k}_2\rangle \frac{\partial k_F}{\partial \rho}\frac{\delta \rho}{\delta n(k,k_F)}, \tag{6}$$

where $\rho \equiv \rho(k_F)$ is the density of the system. Of course, if the (large) volume V enclosing the nuclear matter is kept fixed, then

$$\frac{\delta\rho}{\delta n(k,k_F)} = \frac{1}{V}\frac{\delta}{\delta n(k,k_F)}\sum_{\boldsymbol{k}_1} n(k_1,k_F)$$

$$= \frac{1}{V} 4 \frac{V}{(2\pi)^3}\int d\boldsymbol{k}_1 \frac{\delta n(k_1,k_F)}{\delta n(k,k_F)} = \frac{1}{V}. \qquad (7)$$

In the Appendix we shall show that the above prescription for the functional derivative of the energy with respect to the distribution, worked out at constant volume, just yields the Fermi energy, whereas, when the number of particles is kept fixed, is proportional to the pressure. The latter, of course, should vanish for a system at equilibrium.

To proceed further we now define

$$A(k_F) = \frac{1}{2N}\sum_{\boldsymbol{k}_1\boldsymbol{k}_2} n(k_1,k_F) n(k_2,k_F)$$

$$\times \frac{\partial}{\partial k_F}\langle \boldsymbol{k}_1,\boldsymbol{k}_2|G|\boldsymbol{k}_1,\boldsymbol{k}_2\rangle, \qquad (8)$$

which, together with Eq. (7), allows us to recast Eq. (6) as follows:

$$e(k\approx k_F,k_F) = e_G(k,k_F) + \rho\frac{\partial k_F}{\partial\rho}A(k_F)$$

$$= e_G(k,k_F) + \left(\frac{\partial}{\partial k_F}\ln\rho\right)^{-1} A(k_F). \qquad (9)$$

The above equation, where the G matrix and the rearrangement contributions to the single-particle (or, better, quasiparticle) energy are neatly separated, yields the correct Fermi energy $e(k_F,k_F)\equiv\epsilon_F$, being rigorously valid at the Fermi surface.

We should now compute the impact of the rearrangement on the binding energy per particle. In this connection, we are aware that the BHF formula,

$$\frac{E_G}{N} = \frac{1}{2N}\sum_{\boldsymbol{k}} n(k,k_F)\left[\frac{k^2}{2m} + e_G(k,k_F)\right], \qquad (10)$$

linking the total and the single particle energies, no longer holds when the rearrangement is included. Furthermore, calculations of the nucleon momentum distribution including correlations among nucleons beyond the BHF ones indicate that the corrections to the latter tend to be more pronounced near the Fermi surface [19], where the Landau theory applies. Hence, inspired by the Landau theory, we heuristically assume Eq. (9) to be applicable as well in the proximity of the Fermi surface, where the concept of quasiparticle is tenable. In the conclusions the foundation and the limits of this assumption will be addressed. Here we write, as an extension of Eq. (10),

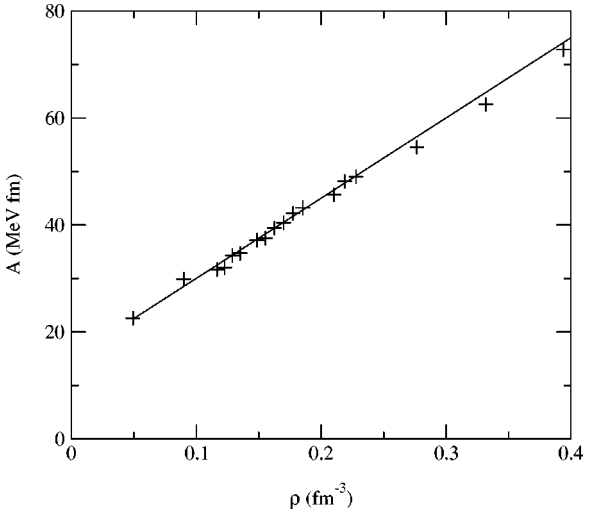

FIG. 2. The quantity $A(k_F)$, defined in the formula (8) of the text versus the density $\rho = 2k_F^3/2\pi^2$. The crosses show the calculated values, whereas the solid line corresponds to the linear fit $A = 150\rho + 15$ MeV fm. We do not display $A(k_F)$ for $\rho < 0.05$ fm^{-3} because here its numerical evaluation becomes quite inaccurate.

$$\frac{E}{N} = \frac{E_G}{N} + A(k_F)\left(\frac{\partial}{\partial k_F}\ln\rho\right)^{-1}\frac{1}{2N}\sum_{k>k_{\rm QP}} n(k,k_F)$$

$$= \frac{E_G}{N} + A(k_F)\left(\frac{\partial}{\partial k_F}\ln\rho\right)^{-1}\frac{1}{\pi^2\rho}\int_{k_{\rm QP}}^{k_F} dk\, k^2$$

$$= \frac{E_G}{N} + A(k_F)\left(\frac{\partial}{\partial k_F}\ln\rho\right)^{-1}\frac{1}{2}\left[1-\left(\frac{k_{\rm QP}}{k_F}\right)^3\right]. \qquad (11)$$

In the above the summation is meant to be carried out in a restricted region near the Fermi surface, namely in the range of momenta $k_{\rm QP}\leqslant k\leqslant k_F$. In the derivation of Eq. (11), the θ-function distribution is used to be consistent with the G-matrix calculation. Obviously the momentum $k_{\rm QP}$ should be viewed as a parameter: If it will turn out to be close to k_F, then the omission of the k dependence in the rearrangement contribution to the single-particle energy (9) should be expected to have not too serious consequences.

Thus, the expression (11), beyond the standard G-matrix contribution (namely E_G/N), explicitly embodies the rearrangement one as well, but the latter, and this is important, is reduced by the factor $[1-(k_{\rm QP}/k_F)^3]/2$ with respect to Eq. (9).

III. RESULTS FOR THE REARRANGEMENT ENERGY

In order to compute the rearrangement contribution, the quantity $A(k_F)$ is needed: It has been numerically evaluated and it is displayed as a function of ρ in Fig. 2. It appears that $A(k_F)$ is always positive, growing linearly with ρ, and substantial. Actually, it turns out to be well approximated by the expression $A(k_F) = (150\rho + 15)$ MeV fm.

Since the above findings on $A(k_F)$ have been obtained with a θ-function distribution, for consistency we evaluate

$[\partial(\ln\rho)/\partial k_F]^{-1}$ with the θ function as well. An elementary calculation yields

$$\left(\frac{\partial}{\partial k_F}\ln\rho\right)^{-1}=\frac{k_F}{3}. \tag{12}$$

Hence, from Eq. (9) it follows that the rearrangement contribution to the Fermi energy grows as k_F^4, in accord with Ref. [20], where it was conjectured to grow as k_F^5 and found to vary as k_F^4 in a schematic estimate.

In Fig. 1, we display the Fermi energy including the rearrangement, obtained using Eq. (12), versus the density $\rho=2k_F^3/3\pi^2$.

We note

(i) the occurrence, in correspondence of the minimum of the binding energy, of a *positive* rearrangement contribution of about 25 MeV, larger than the value of 17 MeV obtained in Ref. [10] in a perturbative scheme. Since in Ref. [10] only the leading diagrams, beyond the BHF ones, were kept, one could view the difference as an estimate of the contribution of the higher order terms in the hole-line expansion. However, this estimate should not be taken at face value, since in Ref. [10] the Paris potential was employed, whereas we use the Bonn one;

(ii) notably, our rearrangement contribution stays remarkably constant to the left of our saturation density $\rho\approx 0.25$ fm^{-3} (say, in the range $0.20\leqslant\rho\leqslant 0.25$ fm^{-3}), in accord with the old finding of Thouless [20];

(iii) while in Ref. [10] the obtained rearrangement contribution was just enough to restore the thermodynamic consistency, in our case we ''overcure'' the HVH violation: Indeed, we predict the equality between the Fermi energy and the binding energy to occur at $\rho\approx 0.17$ fm^{-3}, i.e., not at equilibrium. However, in Ref. [10] no change was assumed to occur in the ground state energy due to rearrangement.

Here, as discussed above, we schematically estimate the magnitude of this change by resorting to formula (11) and searching for a value of the momentum $k_{\rm QP}$ such to restore the validity of the HVH theorem. This turns out to be fulfilled when $k_{\rm QP}=0.09k_F$, as it appears from Fig. 3, where we display, in correspondence to this value, the binding and Fermi energies, both including rearrangement, versus the density.

We see in the figure that the minimum of the binding energy, while reduced, as expected, to a value (-13.15 MeV) higher than the BHF prediction, occurs at a saturation density $\rho_{\rm min}\cong 0.19$ fm^{-3} ($k_F=1.41$ fm^{-1}), in closer contact with the experimental value $\rho_{\rm exp}=0.17$ fm^{-3}.

Concerning the rearrangement effect, at $k_F=1.41$ fm^{-1} we obtain a positive contribution of ≈ 20 MeV for the Fermi energy and of ≈ 1.8 MeV for the binding energy per particle. These values might be compared with those obtained long ago by Brueckner *et al.* [21], namely about 12 and 1.5 MeV for the Fermi and mean energies, respectively, and with the recent ones of Ref. [18], namely about 12 and 1 MeV, respectively. It should, however, be remarked that these values have been obtained with simple, as compared to the Bonn potential, interactions.

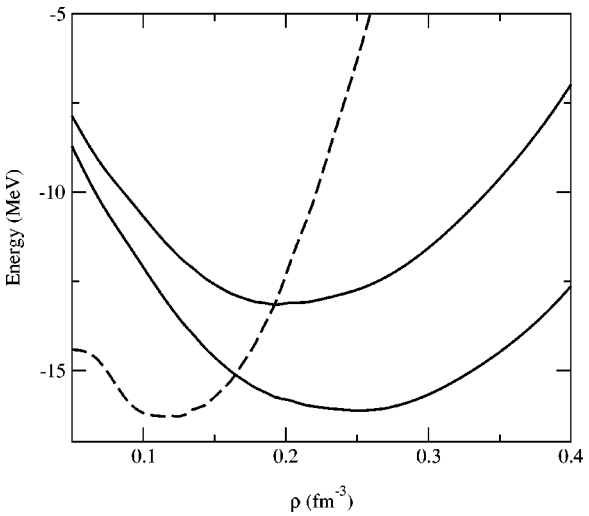

FIG. 3. Symmetric nuclear matter binding energy per particle (solid lines) and Fermi energy (dashed) as a function of the density. The lower and upper solid lines refer to binding energies without and with rearrangement contribution, respectively, whereas the dashed line includes rearrangement.

Finally, the compression modulus predicted in our framework, via a polynomial interpolation of the binding energy, turns out to be ≈ 150 MeV, significantly larger than the one obtained in a pure BHF scheme (≈ 120 MeV), as expected owing to the rapid k_F dependence of the rearrangement term, but still somewhat lower than the experimental value.

IV. CONCLUSIONS

In these concluding remarks we seek for some justification of our empirical procedure in computing the rearrangement contribution to the binding energy per particle.

We start by recalling the link [see Eq. (10)] between the single-particle and mean energies, which holds only for strict mean field theories, like HF and BHF. Should these schemes be valid, then, as it is well known [13], the separation and the single-particle energies would coincide, as it is (nearly) true in atoms (Koopman's theorem, see Ref. [13]).

In nuclei, of course, HF (which respects HVH) is not applicable and BHF, as shown by many calculations, while not unrealistic, fails to fulfill the HVH theorem. Indeed, the amount of the failure measures the impossibility of describing the system in terms of independent constituents. Actually, beyond the mean field framework the only remaining link between the mean and single-particle energies is the one expressed by the HVH theorem itself.

Our simple approach is based on the premise that the contribution to the system's mean energy arising from the proximity of the Fermi surface can still be simply related to the energy of individual entities. We identify the latter with Landau's quasiparticles and not with the BHF particles, since, as previously mentioned, the BHF framework fails to account for the appreciable depletion of the single-particle orbits induced by the strong short-range repulsion among the nucleons, a depletion occurring mainly, although not only, near the Fermi surface. Indeed, it is established that the re-

pulsion much affects both the momentum distribution and the rearrangement energy [20].

Accordingly, we use the quasiparticle approximation for the propagator $G(\boldsymbol{k},\omega)$ in the expression [22]

$$\frac{E}{N}=\frac{1}{\rho}\lim_{\eta\to 0^+}\int\frac{d\boldsymbol{k}}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi i}e^{i\omega\eta}\times\left[\frac{k^2}{2m}+\frac{1}{2}\Sigma^*(\boldsymbol{k},\omega)\right]\mathrm{Tr}\,G(\boldsymbol{k},\omega), \tag{13}$$

which yields the binding energy per particle. The quasiparticle approximation for $G(\boldsymbol{k},\omega)$ is best grasped by starting from the canonical spectral representation,

$$G(\boldsymbol{k},\omega)=\int_{\epsilon_F}^{\infty}d\omega'\frac{S_{\mathrm{p}}(\boldsymbol{k},\omega')}{\omega-\omega'+i\eta}+\int_{-\infty}^{\epsilon_F}d\omega'\frac{S_{\mathrm{h}}(\boldsymbol{k},\omega')}{\omega-\omega'-i\eta}. \tag{14}$$

It amounts to set for the hole spectral function the expression

$$S_{\mathrm{h}}^{\mathrm{QP}}(\boldsymbol{k},\omega)=\frac{1}{\pi}\frac{Z^2(k)|W(k)|}{[\omega-E(k)]^2+[Z(k)W(k)]^2}, \tag{15}$$

where $W(k)$ is the imaginary part of the quasiparticle self-energy and $Z(k)$ the so-called quasiparticle strength.

In our crude and empirical model, to avoid the introduction of further parameters beyond k_{QP}, we have set $Z(k)=1$ and $W(k)=0$. Of course, the former should be lower than 1, but not too much, in order not to spoil the concept of quasiparticle, and the latter should be small for the same reason. Thus our quasiparticle propagator differs from the BHF one only in the location of the pole, which is moved by the rearrangement contribution.

Finally, we find it gratifying that the only parameter of our approach, namely k_{QP}, confines the quasiparticle existence to a quite narrow domain close to the Fermi surface. Also satisfying is our result that the rearrangement affects the Fermi energy an order of magnitude more than the mean energy, a finding on which a general consensus exists.

APPENDIX

We comment here on the formula

$$\left.\frac{\delta E[n]}{\delta n(k,k_F)}\right|_{k\cong k_F}=\frac{\partial E}{\partial k_F}\frac{\partial k_F}{\partial\rho}\frac{\delta\rho}{\delta n(k,k_F)}, \tag{A1}$$

where

$$\frac{\delta\rho}{\delta n(k,k_F)}=\frac{\delta}{\delta n(k,k_F)}\left[\frac{1}{V}\sum_{\boldsymbol{k}_1}n(k_1,k_F)\right], \tag{A2}$$

for the functional derivative of the energy with respect to the distribution function. The sum over spin and isospin is understood.

We first consider the volume V constant. In this case

$$\frac{\delta\rho}{\delta n(k,k_F)}=\frac{1}{V}\sum_{\boldsymbol{k}_1}\frac{\delta n(k_1,k_F)}{\delta n(k,k_F)}=\frac{1}{V}. \tag{A3}$$

Hence

$$\left.\frac{\delta E[n]}{\delta n(k,k_F)}\right|_{k\cong k_F}=\frac{\partial E}{\partial k_F}\frac{\partial k_F}{\partial(N/V)}\frac{1}{V}=\frac{\partial E}{\partial k_F}\frac{\partial k_F}{\partial N}=\frac{\partial E}{\partial N}=\mu=\epsilon_F. \tag{A4}$$

In the case of a free Fermi gas, Eq. (A4) yields indeed

$$\left.\frac{\delta E[n]}{\delta n(k,k_F)}\right|_{k\cong k_F}=\frac{\partial E}{\partial k_F}\frac{\pi^2}{2k_F^2}\frac{1}{V}=\frac{\pi^2}{2k_F^2}\frac{\partial}{\partial k_F}\left(\frac{N}{V}\frac{3}{5}\frac{k_F^2}{2m}\right)=\frac{k_F^2}{2m}\equiv\epsilon_F^{\mathrm{FG}}. \tag{A5}$$

On the other hand, when N is constant (but V varies),

$$\delta N=\sum_{\boldsymbol{k}}\frac{\delta N}{\delta n(k,k_F)}\delta n(k,k_F)=\sum_{\boldsymbol{k}}\delta n(k,k_F)=0, \tag{A6}$$

since

$$\frac{\delta N}{\delta n(k,k_F)}=\frac{\delta}{\delta n(k,k_F)}\sum_{\boldsymbol{k}_1}n(k_1,k_F)=1. \tag{A7}$$

Thus the vanishing of δN does not imply the vanishing of $\delta N/\delta n(k,k_F)$. Hence

$$\left.\frac{\delta E[n]}{\delta n(k,k_F)}\right|_{k\cong k_F}=\frac{\partial E}{\partial k_F}\frac{\partial k_F}{\partial\rho}\frac{1}{V}-\frac{\partial E}{\partial k_F}\frac{\partial k_F}{\partial\rho}\frac{\rho}{V}\frac{\delta V}{\delta n(k,k_F)}, \tag{A8}$$

the second term on the right-hand side actually not contributing because V and $n(k,k_F)$ are varying independently. Accordingly,

$$\left.\frac{\delta E[n]}{\delta n(k,k_F)}\right|_{k\cong k_F}=\frac{\partial E}{\partial k_F}\frac{1}{N}\frac{\partial k_F}{\partial(1/V)}\frac{1}{V}=-\frac{1}{\rho}\frac{\partial E}{\partial V}=\frac{P}{\rho}, \tag{A9}$$

which vanishes at equilibrium because so does the pressure P.

Curiously, from Eq. (A9) for a perfect Fermi gas it follows that

$$\left.\frac{\delta E[n]}{\delta n(k,k_F)}\right|_{k\cong k_F}=\frac{2}{5}\epsilon_F=\epsilon_F-\frac{3}{5}\epsilon_F, \tag{A10}$$

showing that a nonconfined Fermi gas satisfies the HVH theorem, i.e., it reaches equilibrium, at zero density.

On the other hand, for a translationally invariant Fermi system, with a generic interaction $V(q)$, one has from Eq. (A9)

$$\left.\frac{\delta E[n]}{\delta n(k,k_F)}\right|_{k\equiv k_F}=\frac{k_F}{3}\frac{\partial(E/N)}{\partial k_F}=0. \tag{A11}$$

In the HF approximation [22],

$$\begin{aligned}\frac{E}{N}&=\frac{3}{5}\epsilon_F^{\mathrm{FG}}+\frac{1}{2}\rho V(0)-\int\frac{d\boldsymbol{k}}{(2\pi)^3}\theta(k_F-k)\\&\quad\times\int\frac{d\boldsymbol{k}'}{(2\pi)^3}\theta(k_F-k')V(|\boldsymbol{k}-\boldsymbol{k}'|)\\&=\frac{3}{5}\epsilon_F^{\mathrm{FG}}+\frac{1}{2}\rho V(0)-\frac{1}{(2\pi)^2}\int_0^{2k_F}dq\,q^2V(q)\\&\quad\times\left[1-\frac{3}{2}\frac{q}{2k_F}+\frac{1}{2}\left(\frac{q}{2k_F}\right)^3\right]\end{aligned} \tag{A12}$$

and it is an easy matter to carry out the derivative in Eq. (A11), getting

$$\begin{aligned}\frac{\partial(E/N)}{\partial k_F}&=-\frac{E}{N}+\epsilon_F^{\mathrm{FG}}+\rho V(0)-\frac{1}{(2\pi)^2}\\&\quad\times\int_0^{2k_F}dq\,q^2V(q)\left(1-\frac{q}{2k_F}\right).\end{aligned} \tag{A13}$$

Now, in HF the Fermi energy reads

$$\begin{aligned}\epsilon_F^{\mathrm{HF}}&=\epsilon_F^{\mathrm{FG}}+\rho V(0)-\int\frac{d\boldsymbol{k}'}{(2\pi)^3}\theta(k_F-k')V(|\boldsymbol{k}_F-\boldsymbol{k}'|)=\epsilon_F^{\mathrm{FG}}\\&\quad+\rho V(0)\\&\quad-\frac{1}{(2\pi)^2}\int_0^{2k_F}dq\,q^2V(q)\left(1-\frac{q}{2k_F}\right).\end{aligned} \tag{A14}$$

Hence, by comparing with Eq. (A13), one sees that the HVH theorem is fulfilled in the HF approximation no matter what the interaction is, providing the latter is independent of the density.

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