Nuclear $\bar{K}$ bound states in light nuclei

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The possible existence of deeply bound nuclear $\bar{K}$ states is investigated theoretically for few-body systems. The nuclear ground states of a K^- in ^{3}He, ^{4}He, and ^{8}Be are predicted to be discrete states with binding energies of 108, 86, and 113 MeV and widths of 20, 34, and 38 MeV, respectively. The smallness of the widths arises from their energy-level locations below the $\Sigma\pi$ emission threshold. It is found that a substantial contraction of the surrounding nucleus is induced due to the strong attraction of the $I=0$ $\bar{K}N$ pair, thus forming an unusually dense nuclear medium. Formation of the $T=0$ $K^-\otimes{}^3\mathrm{He}+\bar{K}^0\otimes{}^3\mathrm{H}$ state in the ^{4}He(stopped K^-, n) reaction is proposed, with a calculated branching ratio of about 2%.

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I. INTRODUCTION

One of the most important, yet unsolved, problems in hadron physics is how the hadron masses and interactions change in the nuclear medium (see, for instance, Refs. [1–3]). In the strangeness sector, the possibility of a dropping K^- mass in nuclei has been theoretically asserted [4–7]. This problem is connected to an exciting issue of whether strangeness condensation in nuclear matter could occur. Despite the importance of studying in-medium hadron masses, no clear experimental method for deducing in-medium scalar masses has yet been established. It was pointed out by the present authors [8] that the invariant-mass spectroscopy, when applied to hadrons embedded in nuclei, provides information not on the scalar mass itself but only on the energy state (in a scalar + vector potential), which suffers from collisional shifts and broadening in nuclei. Recently, a new type of "in-medium hadron-mass spectroscopy" has been carried out. The method involves producing deeply bound states of a hadron from which the hadron-nucleus potential, and subsequently the in-medium hadron mass, can be deduced. The first successful example was the observation of narrow $1s$ and $2p$ states of a π^- in ^{207}Pb [9–12] and ^{205}Pb [13], which had been predicted to be produced by using suitable "pion-transfer" nuclear reactions [14,15]. The effective mass shift of the π^- in this nucleus was deduced to be around +26 MeV [10,12,13,16], from which a partial restoration of chiral symmetry breaking was indicated [17,18]. This type of nuclear spectroscopy is being extended to search for η and ω bound states in light nuclei [19]. In this paper we consider the possible existence of discrete nuclear bound states of a $\bar{K}$ and how to populate such exotic nuclear states.

Recently, definite information about the strong-interaction level shifts of the kaonic hydrogen atom was obtained from the experiment KpX at KEK [20,21], which indicates a "repulsive"-type shift of $(-323\pm63\pm11)+i(407\pm208\pm100)$ eV for the $1s$ orbit. For heavier nuclei, Batty *et al.* [22,23] reanalyzed all of the existing data of K^- atoms, including a density-dependent term for the K^-N scattering length, and deduced an optical potential with a strongly attractive real part ($V_0^{\mathrm{atom}}=-183$ MeV) and also a strongly absorptive imaginary part ($W_0^{\mathrm{atom}}=-70$ MeV). The reason for such a highly attractive potential, despite the fact that the strong-interaction shifts appear to be negative (of "repulsive" type), comes from the assertion that the $\Lambda(1405)$ state is not an "elementary particle," but the bound state of $\bar{K}+N$. From such a potential one expects deeply bound *nuclear* states in heavier nuclei, but their widths are estimated to be on the order of 100 MeV or more if their potential parameters are strictly applied, and thus such nuclear states may not be identified as discrete states. On the other hand, a number of narrow deeply bound *atomic* states are expected in all nuclei, as studied theoretically by Friedman and Gal [24]. Such atomic states are, however, not very sensitive to the inner behavior of the asserted strong-interaction potential. Certainly, direct knowledge about the potential should be obtained from $\bar{K}$ *nuclear* bound states, if they exist as discrete states. In the following sections, therefore, we seek possible narrow discrete nuclear states by examining the $\bar{K}N$ interaction and treating few-body systems of relatively simple structure. Short versions of the present study have already been published [25,26].

II. FORMALISM

A. $\bar{K}N$ interaction

First, we construct phenomenologically a quantitative $\bar{K}N$ interaction model that is as simple as possible using free $\bar{K}N$ scattering data [27], the KpX data of kaonic hydrogen [20,21] and the binding energy and width of $\Lambda(1405)$, which can be regarded as an isospin $I=0$ bound state of $\bar{K}+N$. Then, the $I=0$ and $I=1$ $\bar{K}N$ interactions are found to be

$$v_{\bar{K}N}^{I}(r)=v_D^I\exp[-(r/0.66\ \ \mathrm{fm})^2], \tag{1}$$

$$v_{\bar{K}N,\pi\Sigma}^{I}(r)=v_{C_1}^I\exp[-(r/0.66\ \ \mathrm{fm})^2], \tag{2}$$

$$v_{\bar{K}N,\pi\Lambda}^{I}(r)=v_{C_2}^I\exp[-(r/0.66\ \ \mathrm{fm})^2] \tag{3}$$

with

$$v_D^{I=0} = -436 \text{ MeV}, \quad v_{C_1}^{I=0} = -412 \text{ MeV},$$

$$v_{C_2}^{I=0} = \text{none}, \tag{4}$$

$$v_D^{I=1} = -62 \text{ MeV}, \quad v_{C_1}^{I=1} = -285 \text{ MeV},$$

$$v_{C_2}^{I=1} = -285 \text{ MeV}, \tag{5}$$

where $v^I_{\pi\Sigma}(r) = v^I_{\pi\Lambda}(r) = 0$ is taken to simply reduce the number of parameters. The $I=0$ interaction produces an unstable bound state of $\Lambda(1405)$ with $E_{\bar{K}N} = -29.5$ MeV (-27 MeV) from the $I=0$ threshold (from the $K^- + p$ threshold) and $\Gamma = 40$ MeV. The interaction gives a scattering length of $a^{I=0} = -1.76 + i0.46$ fm, which can be compared to Martin's empirical value $a^{I=0} = (-1.70 \pm 0.07) + i(0.68 \pm 0.04)$ fm [27]. It is noted that the data for $\Lambda(1405)$ and the scattering length enable us to determine both the strength and the range of the interaction. The $I=1$ interaction, having no bound state, gives a scattering length of $a^{I=1} = 0.37 + i0.60$ fm, reproducing Martin's $I=1$ value. The $K^- p$ scattering length is calculated from both the $I=0$ and $I=1$ interactions to be

$$a_{K^- p} = \frac{1}{2}(a^{I=0} + a^{I=1}) = -0.70 + i0.53 \text{ fm}, \tag{6}$$

and is in good agreement with the data obtained by the KpX measurement [20,21],

$$a_{K^- p} = (-0.78 \pm 0.15 \pm 0.03) + i(0.49 \pm 0.25 \pm 0.12) \text{ fm}. \tag{7}$$

In order to see the properties of a $\bar{K}N$ effective interaction, we calculate the t matrix, which is given by

$$t = v + v\frac{\hat{O}}{E_{\bar{K}N} - \hat{T}} t, \tag{8}$$

where $E_{\bar{K}N}$ and $\hat{T}$ are the internal energy and the kinetic energy operator of the $\bar{K}N$ system, respectively. The operator $\hat{O}$ is either 1 for $\bar{K}N$ in free space or Q_N (the Pauli exclusion operator for the nucleon) for $\bar{K}N$ in a nuclear medium. The t matrix elements are related to s-wave scattering amplitudes (f^I) as

$$t^I(k) = -\frac{1}{2\pi^2}\frac{\hbar^2}{2\mu} f^I(k), \tag{9}$$

where μ and k are the reduced mass and relative momentum (which becomes imaginary for negative $E_{\bar{K}N}$) of $\bar{K}N$, respectively. Figure 1 shows the behavior of the scattering amplitudes in free space and in a nuclear medium with a Fermi momentum of $k_F = 1.2$ fm^{-1} as a function of $E_{\bar{K}N}$, where the c.m. momentum is taken to be zero. Just below the threshold corresponding to the atomic states, the $I=0$ t matrix changes from repulsion in free space to attraction in the medium due to the dissolution of $\Lambda(1405)$ [5,28]. At $E_{\bar{K}N} = 0$ the scattering amplitude (f^I) is equal to the scattering length (a^I). When $E_{\bar{K}N}$ falls below the $\Sigma + \pi$ threshold, the imaginary part of $f^{I=0}$ vanishes and gives no contribution to the decay of a very deeply bound state in a nucleus, if it exists. In usual treatments the t matrices are used to obtain the $\bar{K}$-nucleus optical potential in the form of "$t\rho$" [23] with a nuclear density of $\rho(r)$. We, however, use g matrices hereafter instead of the in-medium t matrices to take into account the binding effects more properly; a real k value for $\bar{K}$ moving in nuclear matter is compatible with any negative value of $E_{\bar{K}N}$ in the g matrices, but is not in the t matrices due to the on-energy-shell constraint $(\hbar^2/2\mu)k^2 = E_{\bar{K}N}$.

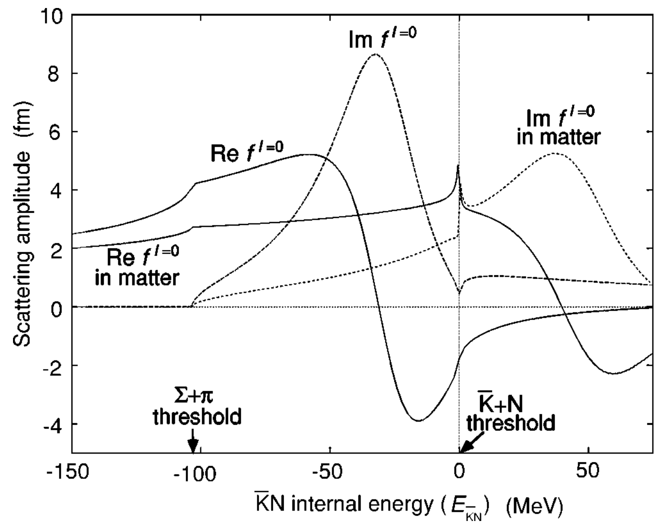

FIG. 1. Calculated scattering amplitudes ($f^{I=0}$) of $\bar{K}N$ versus $E_{\bar{K}N}$ in free space and in nuclear matter.

B. g-matrix method

The binding of $\bar{K}$ in nuclei is calculated within the framework of the Brueckner-Hartree-Fock (BHF) theory. Here, the $\bar{K}N$ g matrix in a nuclear medium is defined by

$$g = v + v\frac{Q_N}{E_{\mathrm{st}} - Q_N \hat{T} Q_N} g. \tag{10}$$

The binding effects of $\bar{K}$ and N are properly taken into account through the starting energy (E_{st}) which is a quantity independent of k. We employ the "$Q_N \hat{T} Q_N$" prescription for intermediate states in the g-matrix equation [29,30].

For atomic states we take $E_{\mathrm{st}} = E_{K^-} + E_N \approx E_N \approx -20$ MeV and calculate g matrices with $k_{\mathrm{F}} = 1.36$ fm^{-1}. In this case it is convenient for practical use to approximate the obtained g matrices with zero-range effective interactions having the same volume-integral values. The resultant interactions are

$$g^I \approx g_0^I \quad \delta(\vec{r}), \tag{11}$$

$$g_0^{I=0} = -2175 - i849 \ \text{MeV fm}^3, \tag{12}$$

$$g_0^{I=1} = -323 - i225 \ \text{MeV fm}^3. \tag{13}$$

The central depth of the K^--nucleus optical potential for atomic states is now obtained as

$$V_0^{\text{atom}} + iW_0^{\text{atom}} = \frac{1}{4}(g_0^{I=0} + 3g_0^{I=1})\rho_0, \tag{14}$$

where ρ_0 is the normal density of the nucleus. For heavy nuclei of isospin $T=0$ we get

$$V_0^{\text{atom}} = -134 \ \text{MeV and} \ W_0^{\text{atom}} = -65 \ \text{MeV}, \tag{15}$$

which is considerably deeper than those obtained in previous theoretical studies [31–34,28]. Although this potential is compatible with a deep potential of $V_0^{\text{atom}} = -183$ MeV and $W_0^{\text{atom}} = -70$ MeV determined phenomenologically from atomic data by Batty *et al.* [22,23], it should be noticed that atomic states are not sensitive to the interior of the optical potential. In fact, a relatively shallow optical potential of $V_0^{\text{atom}} = -55$ MeV and $W_0^{\text{atom}} = -60$ MeV has recently been recommended from the analysis of the same atomic data by Gal *et al.* [35].

The optical potential for deeply bound nuclear states must be different from that of Eq. (15). For $E_{\text{st}} = -110$ MeV, for example, the g matrices change to

$$g_0^{I=0} = -1704 - i\ 0 \ \text{MeV fm}^3, \tag{16}$$

$$g_0^{I=1} = -363 - i87 \ \text{MeV fm}^3, \tag{17}$$

and the optical potential is obtained to be

$$V_0^{\text{nucl}} = -119 \ \text{MeV and} \ W_0^{\text{nucl}} = -11 \ \text{MeV} \tag{18}$$

with the normal density (ρ_0). The small imaginary part is due to closing the $\Sigma + \pi$ channel. This strongly attractive potential is consistent with a substantial reduction of the K^- mass in the nuclear medium, predicted by a chiral SU(3) theory of Waas *et al.* [5] and supported experimentally by a large subthreshold K^- production in heavy-ion reactions by Laue *et al.* [36]. The existence of discrete bound states with small decay widths, which we have derived here, was assumed by Kishimoto [37] in a proposal to search for deeply bound K^- nuclear states in medium-mass nuclei by using (in-flight K^-,N) reactions.

It was recently theoretically shown by Schaffner-Bielich *et al.* [38] and by Ramos *et al.* [39] that the optical potential obtained from a similar procedure is rather shallow, on the order of -40 MeV, in contradiction to Eq. (18). The essential difference comes from the treatment of self-consistency in intermediate states of the $\bar{K}$ propagation. While they introduced a self-consistent spectrum in the intermediate states, we do not use any spectrum, but apply the $Q_N \hat{T} Q_N$ prescription to $\bar{K}$ similarly to N or Λ. In the case of N, $Q_N \hat{T} Q_N$ gives a better description of the intermediate spectrum in the g-matrix equation [40,29]. Also, in the case of Λ the energy correction due to the potential insertion to the intermediate states is largely canceled out with the rearrangement energy of the nuclear medium [41,42]. The rearrangement effect of the medium would become much more important for the case of $\bar{K}$ in relation to "contraction," which is discussed below. Thus, since many-body theory for $\bar{K}$ has not yet been established, we should say that there remains some ambiguity as to whether the optical potential is deep or shallow for $\bar{K}$ embedded inside heavy nuclei. It is, however, noted that the results of Refs. [38,39] cannot be directly applied to the atomic case because of the large mismatch of E_{st}, and the result of Eq. (15) holds as being reasonable.

III. $\bar{K}$ IN ^{3}He AND ^{4}He

A. *g*-matrix calculation

Now let us investigate nuclear $\bar{K}$ bound states in ^{3}He and ^{4}He. An advantage is that few-body systems have less ambiguity in the many-body treatment. The $\bar{K}N$ effective interactions, i.e., g matrices of Eq. (10), have the following weights in possible $\bar{K}$ nuclear states:

$$3\left(\frac{1}{2}g^{I=0} + \frac{1}{2}g^{I=1}\right) \quad \text{for} \ {}^3_K\text{H}(T=0), \tag{19}$$

$$3\left(\frac{1}{6}g^{I=0} + \frac{5}{6}g^{I=1}\right) \quad \text{for} \ {}^3_K\text{H}(T=1), \tag{20}$$

$$4\left(\frac{1}{4}g^{I=0} + \frac{3}{4}g^{I=1}\right) \quad \text{for} \ {}^4_K\text{H}(T=1/2), \tag{21}$$

where we employ the conventional nomenclature to express composite nuclei, namely, ${}^3_K\text{H}(T=0,1)$ are the states of $K^- \otimes {}^3\text{He} + \bar{K}^0 \otimes {}^3\text{H}$ $(T=0,1)$ and ${}^4_K\text{H}(T=1/2)$ is $K^- \otimes {}^4\text{He}$ $(T=1/2)$. It is to be noted that the ${}^3_K\text{H}(T=0)$ state acquires the largest weight (3/2) of the more attractive interaction $g^{I=0}$ among the three states.

The nucleon density distributions of the core nuclei ($A = 3,4$) are given after eliminating the c.m. motion in a harmonic-oscillator model $[\hbar\omega = (\hbar^2/M_N)\beta]$ as follows:

$$\rho(r) = A\left(\frac{A}{A-1}\frac{\beta}{\pi}\right)^{3/2}\exp\left(-\frac{A}{A-1}\beta r^2\right), \tag{22}$$

where the parameter β is related to the rms nuclear radius (R_{core}) as

$$R_{\text{core}} = \sqrt{\frac{3(A-1)}{2A\beta}}. \tag{23}$$

The g matrices are calculated in a local density approximation with an average Fermi momentum

$$\bar{k}_F = \bar{k}_F^{\text{free}}\sqrt{\frac{\beta}{\beta^{\text{free}}}}, \tag{24}$$

where $(\overline{k_F^{\rm free}},\ \beta^{\rm free})$ is taken to be (1.1 fm^{-1},0.39 fm^{-2}) for the ^{3}He core and (1.3 fm^{-1},0.52 fm^{-2}) for the ^{4}He core. These values of $(\overline{k_F^{\rm free}},\ \beta^{\rm free})$ were used in the BHF calculation of the hypernuclei, $^4_\Lambda$He and $^5_\Lambda$He [43]. In the case of $^4_\Lambda$He the Λ separation energy for the 0^+ state was obtained to be 2.18 MeV, which is in good agreement with recent results, 2.28 MeV by a Gaussian-basis variational method [44] and 2.32 MeV by a Faddeev-Yakubovsky method [45].

The optical potential between $\bar{K}$ and the core nucleus is constructed by folding g matrices of finite range without any approximation such as Eq. (11) in the case of few-body systems. The bound-state energy ($E_{\bar{K}}$) is obtained by solving the $\bar{K}$-core relative motion

$$\left[-\frac{\hbar^2}{2\mu_{\bar{K}A}}\frac{d^2}{dr^2}+V_{\bar{K}A}(r)\right]u_{\bar{K}}(r)=E_{\bar{K}}u_{\bar{K}}(r), \qquad (25)$$

with

$$V_{\bar{K}A}(r)=\int g(\vec{r}-\vec{r}\,')\rho(\vec{r}\,')d\vec{r}\,' \qquad (26)$$

repeatedly in such a way that $E_{\rm st}$, g, and $E_{\bar{K}}$ become to be self-consistent at a given β.

By using the $\beta^{\rm free}$'s of ^{3}He and ^{4}He we obtain the following results: $E_{\bar{K}}=-76$ MeV with $\Gamma=82$ MeV for ^3_KH($T=0$) and $E_{\bar{K}}=-69$ MeV with $\Gamma=66$ MeV for ^4_KH. The bindings are fairly large, as expected, but the widths are also broad because the bound states are located above the $\Sigma+\pi$ threshold.

B. Nuclear contraction effect

Here, a question arises: do the sizes of the core nuclei remain unchanged when the $\bar{K}$ interacts with the core by several tens of MeV binding? Due to the strong $\bar{K}N$ attraction, the bound $\bar{K}$ would act as a *contractor* to combine the nucleons closer. Suppose a zero-size limit of the core nucleus, ^{3}He, namely, the $\bar{K}\otimes{}^3$He system is considered as a "pseudo-$\Lambda(1405)$" with $M_N\rightarrow 3M_N$ and $v^{I=0}\rightarrow\frac{3}{2}v^{I=0}$. The change in $v^{I=0}\rightarrow\frac{3}{2}v^{I=0}$ increases the binding energy from 27 to 142 MeV, and the change in $M_N\rightarrow 3M_N$ brings an additional binding of 47 MeV. Thus, the fictitious "pseudo-$\Lambda(1405)$" becomes a bound state of $E_{\bar{K}\text{``}N\text{''}}=-189$ MeV with no decay width.

Of course, such a shrinkage effect induced by $\bar{K}$ must be counterbalanced by hard incompressibility of the core nucleus. The internal energy of the core nucleus is given by

$$E_{\rm core}(\beta)=(A-1)\frac{3}{4}\frac{\hbar^2}{M_N}\beta+\frac{A(A-1)}{2}\times\sum_j v_j^{(0)}\left(1+\frac{2\gamma_j}{\beta}\right)^{-3/2} \qquad (27)$$

with an effective NN interaction of Gaussian type

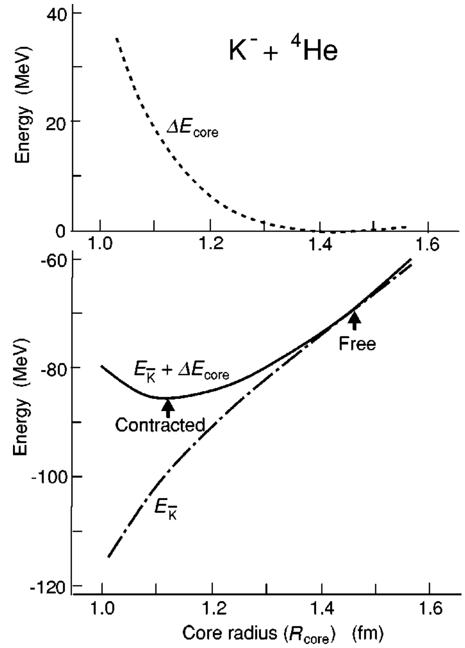

FIG. 2. $\bar{K}$-nucleus energy and incompressibility versus core-radius contraction. The incompressibility is given by $\Delta E_{\rm core}=E_{\rm core}(R_{\rm core})-E_{\rm core}(R^{\rm free}_{\rm core})$. The *contraction* increases the $\bar{K}$-nucleus binding energy by 17 MeV.

$$g_{NN}=\sum_j v_j^{(0)}\exp(-\gamma_j r^2). \qquad (28)$$

By taking into account the incompressibility with Hasegawa-Nagata's effective NN interaction [46], we minimize the sum of the energy $[E_{\bar{K}}(\beta)]$ of the bound $\bar{K}$ and the internal energy $[E_{\rm core}(\beta)]$ of the core nucleus with respect to the core-size parameter (β). Figure 2 illustrates the contraction effect of the core nucleus in the case of $K^-+\ ^4$He.

The final results after a size-optimization are presented in Table I, which shows a substantial lowering of the bound-state energies due to shrinkage of the core nuclear radii. The optimized rms radii of the core nuclei are 60 and 74 % of the free one (1.61 fm) for ^3_KH($T=0$) and ^3_KH($T=1$), respectively, and 76% of the free one (1.47 fm) for ^4_KH($T=1/2$). The $\bar{K}$ in the light nuclei creates by itself compressed surroundings and provides some information about the proper-

TABLE I. Summary of the present calculations for the energies ($E_{\bar{K}}$), widths (Γ), and rms radii of the core nuclei ($R^{\bar{K}}_{\rm core}$) of the $\bar{K}$ nuclear states. The energies are measured from the respective K^-+ nucleus threshold.

State	$E_{\bar{K}}$ (MeV)	Γ (MeV)	$R^{\bar{K}}_{\rm core}$ (fm)	$R^{\bar{K}}_{\rm core}/R^{\rm free}_{\rm core}$
^3_KH($T=0$)	-108	20	0.97	0.60
^3_KH($T=1$)	-21	95	1.20	0.74
^4_KH($T=1/2$)	-86	34	1.12	0.76

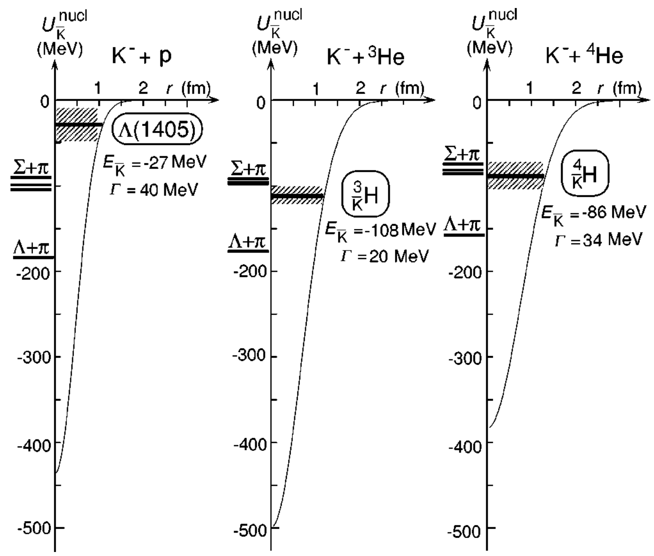

FIG. 3. Calculated $\bar{K}$-N and $\bar{K}$-nucleus potentials and the bound levels [$\Lambda(1405)$, ${}^{3}_{K}$H and ${}^{4}_{K}$H] for the K^--p, -^{3}He, and -^{4}He systems, respectively. The nuclear contraction effects are taken into account.

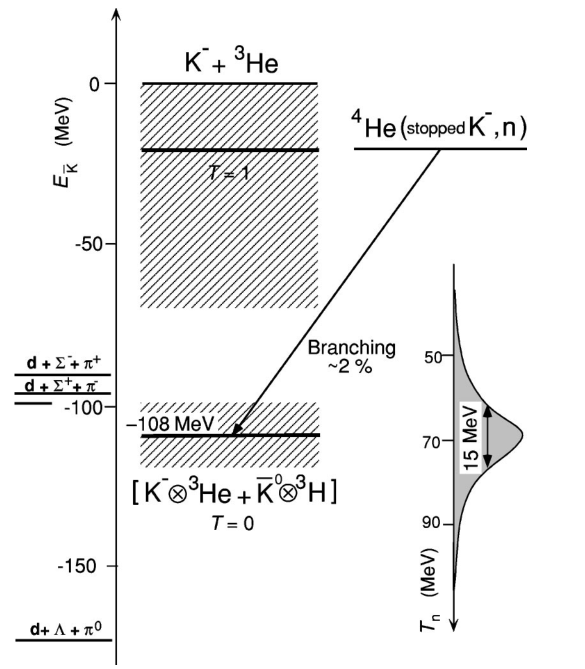

FIG. 4. Energy level diagram of the $K^- + {}^3$He system. The $T=0$ state can be excited and signaled with a discrete neutron emission from stopped K^- on ^{4}He. The neutron spectral intensity (branching ratio) calculated in Sec. V is also shown.

ties of the hadron in a dense nuclear medium, which is currently an important key issue [5].

C. Deeply bound $\bar{K}$ nuclear states

Figure 3 depicts the bound states of ${}^{3}_{K}$H and of ${}^{4}_{K}$H, which are extensions of the basic $K^- + p$ system with $\Lambda(1405)$ as its bound state, together with the optimized $\bar{K}$-nucleus potentials $U_{\bar{K}}$ of the $K^- + {}^3$He and $K^- + {}^4$He systems. The predicted bound states may be called a *strange tribaryon* and a *strange tetrabaryon*, respectively.

The nuclear bound state of $K^- \otimes {}^4$He has a binding energy of 86 MeV and a width of 34 MeV. Since this state is slightly below the $\Sigma + \pi$ threshold, and is open only for the $\Lambda + \pi$ channel, the width becomes fairly narrow. Although such a deeply bound narrow-width K^- state in ^{4}He was predicted by Staronski and Wycech [47], the contraction mechanism of its appearance given here is quite different from theirs.

More interesting is the system composed of $K^- \otimes {}^3$He $+ \bar{K}^0 \otimes {}^3$H shown in Fig. 4. The deeply bound $\bar{K}$ nuclear state with a much narrower width appears in this system of isospin $T=0$. Its energy level lies by 108 MeV below the ^{3}He + K^- threshold, i.e., by 13 MeV below the $\Sigma + \pi$ threshold, and the level width is 20 MeV which is about 20% of the binding energy. On the contrary, the other isospin $T=1$ state has a very large width of 95 MeV with only 21 MeV binding energy. The different features of these two states can be well understood by counting the contribution of the $I=0$ and $I=1$ $\bar{K}N$ interactions to these states. In the $T=0$ state the $I=0$ interaction has a weight of 3/2, which is three times larger than that in the $T=1$ state. This provides the $T=0$ state with stronger attraction and deeper binding, and thus the $T=0$ state lies below the $\Sigma + \pi$ threshold and cannot decay by the major $I=0$ component to the open $\Lambda + \pi$ channel due to the isospin selection. The width originates exclusively from the $I=1$ $\bar{K}N$ interaction, of which the weight in the $T=0$ state is only half of that in the deep $K^- \otimes {}^4$He state. Thus, the deeply bound ${}^{3}_{K}$H($T=0$) state has a markedly narrow width.

In order to confirm the above conclusion, the accuracy of the BHF treatment with $Q_N \hat{T} Q_N$ is checked by using a variational method, named ATMS [48], for a $ppnK^-$ system interacting with a test $\bar{K}N$ interaction of $v_{\bar{K}N} = -(500$ MeV $+ i\, w_0)\exp[-(r/0.66\ \mathrm{fm})^2]$ and a suitable NN interaction [49]. The ATMS results of $(E_{\bar{K}}, \Gamma)$ are $(-107, 24)$, $(-106, 48)$, and $(-104, 72)$ MeV for $w_0 = 20$, 40, and 60 MeV, respectively, whereas the corresponding BHF results are $(-109, 29)$, $(-108, 60)$, and $(-105, 91)$ MeV. This agreement demonstrates that the BHF treatment used in this paper works well for the few-body cases.

It should be mentioned that the widths discussed so far do not contain contributions from the nonpionic decay modes $\bar{K}NN \to \Sigma N/\Lambda N$. Since the calculated imaginary strength ($W_0^{\mathrm{atom}} = -65$ MeV) agrees with the phenomenological one from K^- atom data (-60–-70 MeV [23,35]), the contribution of the processes which are not taken into account in the present calculation cannot be very large. From the ^{4}He bubble chamber data the nonpionic branching ratio for K^- absorption at rest is known to be $16.5 \pm 2.6\%$ [50]. So we try to estimate these contributions to the widths of ${}^{3}_{K}$H and ${}^{4}_{K}$H. About 17% of the phenomenological imaginary strength of -60–-70 MeV [35], i.e., ~ -11 MeV, could be attributed to the $\bar{K}NN \to \Sigma N/\Lambda N$ decays. Here, we make a rough assumption that the imaginary strength of $W_0^{\mathrm{nonpion}} \sim$

-11 MeV is also applicable to the deeply bound $\bar{K}$ nuclear states in light nuclei. When this imaginary strength is added to that of $V_{\bar{K}\alpha}$ of Eq. (26), the widths of ${}^{3}_{K}$H and ${}^{4}_{K}$H increase by ~ 12 MeV, which can be regarded as the widths of $\bar{K}NN \to \Sigma N/\Lambda N$. Thus, we obtain a crude estimate of the nonpionic decay contribution

$$\Gamma^{\text{nonpion}}_{\bar{K}NN} \approx 12 \ \ \text{MeV}. \tag{29}$$

The width of ${}^{3}_{K}$H remains still narrower than that of $\Lambda(1405)$ even when this nonpionic decay width is included.

IV. $\bar{K}$ IN ^{8}Be

The ^{9}Be nucleus is known to have a structure of well-developed α clusters. Let us consider the case when a K^- is injected into ^{9} Be, which may lead to a K^- bound system with ^{8}Be. The ^{8}Be nucleus itself is unbound, but the resonance states of the α-α molecular type are known as its excited states. The kaon plays a drastic role in the K^--^{8}Be system, as shown below. We solve the three-body system $\alpha\alpha\bar{K}$ variationally by the ATMS method. The Hamiltonian is given by

$$H = -\sum_{(ij)} \frac{\hbar^2}{2\mu_{ij}} \frac{1}{r_{ij}^2} \frac{\partial}{\partial r_{ij}} r_{ij}^2 \frac{\partial}{\partial r_{ij}} - \sum_{(ijk)} \frac{\hbar^2}{M_k} \frac{r_{jk}^2 + r_{ki}^2 - r_{ij}^2}{2 r_{jk} r_{ki}} \frac{\partial}{\partial r_{jk}} \frac{\partial}{\partial r_{ki}} + \sum_{(ij)} V_{ij}, \tag{30}$$

where $(ij)=(12)$, (23), (31) and $(ijk)=$cyclic permutations of (123). V_{ij} and μ_{ij} are the folding potentials and the reduced masses. A variational wave function is set to be

$$\Psi = \prod_{(ij)} f_{ij}(r_{ij}), \tag{31}$$

and f_{ij}'s are determined by solving Euler-Lagrange's equation

$$\int [f_{jk}(r_{jk}) f_{ki}(r_{ki}) \{H - \lambda\} \Psi(r_{12}, r_{23}, r_{31})]_{\xi_k = r} d\vec{\rho}_k = 0, \tag{32}$$

consistently for all (ij) pairs, where $\vec{\xi}_k = \vec{r}_i - \vec{r}_j$ and $\vec{\rho}_k = \vec{r}_k - (M_i \vec{r}_i + M_j \vec{r}_j)/(M_i + M_j)$. We assign 1, 2, 3 to α, α, $\bar{K}$. It is noted that $f_{12}(r)$ with r being the $\alpha\alpha$ distance is not the relative wave function of two α's, since $|f_{12}|^2$ has no meaning of the probability density. The $\alpha\alpha$ relative wave function in the $\alpha\alpha\bar{K}$ system is obtained by an "off-shell" transformation [48]

$$u_{\alpha\alpha}(r)/r = [S_{12}(r)]^{1/2} f_{12}(r), \tag{33}$$

where

$$S_{12}(r) = \int [f_{23}^2(r_{23}) f_{31}^2(r_{31})]_{\xi_3 = r} d\vec{\rho}_3. \tag{34}$$

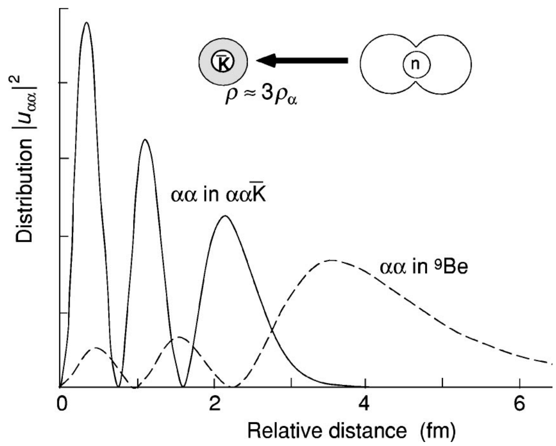

FIG. 5. Distribution ($|u_{\alpha\alpha}(r)|^2$) of the $\alpha\alpha$ relative motion in the $\alpha\alpha\bar{K}$ system, which is compared to that in ^{9}Be. The kaon behaves as a *contractor*, and makes the central nucleon-density about three times as high as that of the α particle.

We can now impose an orthogonality condition [51] on this $u_{\alpha\alpha}$ function by taking account of the Pauli principle among eight nucleons. Then, Eq. (32) for $\alpha\alpha$ is extended to be

$$\left[-\frac{\hbar^2}{2\mu_{\alpha\alpha}} \frac{d^2}{dr^2} + V_{\alpha\alpha}(r) + U^{K}_{\alpha\alpha}(r) \right] u_{\alpha\alpha}(r) + \Lambda \sum_n \langle u_{\mathrm{F}}^{(n)} | u_{\alpha\alpha} \rangle u_{\mathrm{F}}^{(n)}(r) = E u_{\alpha\alpha}(r), \tag{35}$$

where $V_{\alpha\alpha}$ is the folding potential (we use one given in Ref. [52]) and $U^{K}_{\alpha\alpha}$ is the $\bar{K}$-mediated potential [48] which is calculated with f_{23} and f_{31}. The Pauli-forbidden states,

$$u_F^{(1)}(r) = 2(8\beta^3/\pi)^{1/4} r \ \exp(-\beta r^2),$$

$$u_F^{(2)}(r) = 2(18\beta^3/\pi)^{1/4} \left\{ r - \frac{4}{3}\beta r^3 \right\} \exp(-\beta r^2), \tag{36}$$

are excluded with a large positive factor ($\Lambda = 10^8$ MeV) from the relative wave function ($u_{\alpha\alpha}$).

The minimum energy of the $\alpha\alpha\bar{K}$ system with respect to β is obtained at a small α-cluster size corresponding to the harmonic-oscillator strength $\hbar\omega = (\hbar^2/M_N)\beta = 30$ MeV, which is greatly shrunk from the size of the α cluster in ^{9}Be corresponding to $\hbar\omega \approx (\hbar^2/M_N)\beta^{\text{free}} = 22$ MeV. It is noted that the folding potential ($V_{\bar{K}\alpha}$) and the forbidden states are given in a consistent way at $\hbar\omega = 30$ MeV to obtain the final results of the $\alpha\alpha\bar{K}$ system. The self-consistent folding potential obtained for $\bar{K}\alpha$ is well simulated as

$$V_{\bar{K}\alpha}(r) = \{(-285 - i22) \ \ \text{MeV}\} \exp[-(r/1.18 \ \text{fm})^2], \tag{37}$$

which is a little shallower than the corresponding one shown in Fig. 3.

The behavior of the $\alpha\alpha$ relative motion is shown in Fig. 5. When the neutron in the ^{9}Be nucleus is replaced by a K^-, it strikingly increases the binding energy to 113 MeV, which exceeds the $\Sigma+\pi$ threshold. The width is 38 MeV, decaying to the $\Lambda+\pi$ channel. Most interesting is the structure change of the system; the kaon attracts all nucleons within its interaction range and builds a high-density system at "ultralow temperature," i.e., almost the $(0s)^4\ (0p)^4$ nucleon configuration. The structure drastically changes from a loose "cluster" to a compact "shell" as is indicated in Fig. 5. The central nucleon density of the system attains almost three times that of the α particle, i.e., $\sim$ five times the normal density (ρ_0). The formation of deeply bound $\bar{K}$ nuclear states in Be and other nuclei would provide a means to obtain high-density nuclear matter at low temperatures and a new way to investigate the properties of hadrons in a cold and dense nuclear medium. It would also be an interesting problem to know how such high-density matter behaves after the kaon has died as $\bar{K}N\rightarrow\Lambda\pi$ or $\bar{K}NN\rightarrow\Sigma N/\Lambda N$.

V. ^{4}He (STOPPED K^-, n) REACTION

The ${}^3_K$H$(T=0)$ bound state may be formed and identified in K^- absorption at rest on ^{4}He. The K^- in an atomic orbit falls into the deeply bound nuclear orbit of ${}^3_K$H$(T=0)$, while emitting a neutron, which helps to form the core nucleus with one less neutron, and simultaneously serves as a spectator of the formed state. The neutron energy is related to the $\bar{K}$ binding energy as

$$T_n=\frac{M_{\bar{K}\mathrm{H}}}{M_{\bar{K}\mathrm{H}}+M_n}Q, \quad Q=-E_{\bar{K}}-20.6\ \ \mathrm{MeV}, \tag{38}$$

where $M_{\bar{K}\mathrm{H}}$ is the mass of ${}^3_K$H. For $E_{\bar{K}}=-108$ MeV we expect $T_n=68$ MeV, as shown in Fig. 4.

The branching ratio to the state is calculated as follows. The K^- in an atomic orbit $[\Psi_{nlm}(\vec{r}_K)=R^{\mathrm{atom}}_{nl}(r_K)Y_{lm}(\Omega_K)]$ interacts with a neutron and ^{3}He $(=h)$ inside the target ^{4}He, where the absorption from $l=1$ with $n\geqslant 2$ covers about 80% of the total absorption as is known from a cascade calculation. When the K^- is absorbed to form the ${}^3_K$H$(T=0)$ state, the neutron is excited from the initial bound state $[\Phi_i(\vec{r}_n)=R_n(r_n)Y_{00}(\Omega_n)]$ to a continuum state (a nuclear Auger process). The effective K^-n and K^-h interactions are approximately taken as

$$V_{Kn}(\vec{r}_{Kn})=v_n^{(0)}\delta\left(\vec{r}_K-\frac{3}{4}\vec{r}_n\right),$$

$$V_{Kh}(\vec{r}_{Kh})=v_h^{(0)}\delta\left(\vec{r}_K+\frac{1}{4}\vec{r}_n\right), \tag{39}$$

of which the strengths are estimated from Eqs. (12) and (13) to be

$$v_n^{(0)}=\mathrm{Re}\left\{-\frac{1}{4}g_0^{I=0}+\frac{3}{4}g_0^{I=1}\right\}=302\ \ \mathrm{MeV\ fm}^3,$$

$$v_h^{(0)}=\mathrm{Re}\left\{\frac{3}{4}g_0^{I=0}+\frac{3}{4}g_0^{I=1}\right\}=-1874\ \ \mathrm{MeV\ fm}^3. \tag{40}$$

The partial decay rate for the formation of ${}^3_K$H is given by

$$\Gamma=\frac{\sqrt{2\mu_n}\ \mu_n}{(2\pi)^2\ \hbar^3}\sqrt{Q}\ \ |\tau_n+\tau_h|^2|C_{\mathrm{core}}|^2 \tag{41}$$

with

$$\tau_n=(-)^l\left(\frac{4}{3}\right)^3 v_n^{(0)}\int_0^\infty r_K^2 dr_K \tilde{j}_l\left(\frac{4k_n r_K}{3+(m_K/M_N)}\right)\times R_K^{\mathrm{nucl}}\left(\frac{4}{3}r_K\right)R_n\left(\frac{4}{3}r_K\right)R^{\mathrm{atom}}_{nl}(r_K), \tag{42}$$

$$\tau_h=4^3 v_h^{(0)}R_K^{\mathrm{nucl}}(0)\int_0^\infty r_K^2 dr_K\tilde{j}_l(4k_n r_K)R_n(4r_K)R^{\mathrm{atom}}_{nl}(r_K), \tag{43}$$

where μ_n, $\tilde{j}_l$, and k_n are the reduced mass, a distorted wave function, and the wave number of the emitted neutron, respectively. The coefficient $(|C_{\mathrm{core}}|)$ is the overlap between the core-nucleus wave function in the initial bound state and the shrunk one in the final bound state, and is calculated to be 0.83 by using $R^{\bar{K}}_{\mathrm{core}}/R^{\mathrm{free}}_{\mathrm{core}}$ given in Table I. The result obtained for the atomic $2p$ absorption is

$$\Gamma=1.2\ \ \mathrm{eV}. \tag{44}$$

Since the K^-h interaction is much stronger than the K^-n interaction, the τ_h term dominates and τ_n contributes only 0.1 eV. The branching ratio for the formation of the ${}^3_K$H$(T=0)$ state is expected to be at least 2%, which is a value evaluated by using the total-width data of $\Gamma_{\mathrm{tot}}=55\pm34$ eV [53–55], since the experimental data is known as an anomalous shift far bigger than theoretical estimations [56–58].

VI. CONCLUDING REMARKS

In summary, we predict the possible existence of nuclear $\bar{K}$ bound states with narrow widths in ^{3}He, ^{4}He, and ^{8}Be. They are the ground states of the $K^-+{}^3$He, $K^-+{}^4$He, and $K^-+{}^8$Be systems with binding energies of 108, 86, and 113 MeV and widths of 20, 34, and 38 MeV, respectively. The most interesting one is ${}^3_K$H$(T=0)$, i.e., the $K^-\otimes{}^3\mathrm{He}+\bar{K}^0\otimes{}^3\mathrm{H}$ $(T=0)$ state. The dominant $I=0$ $\bar{K}N$ interaction causes a large attraction which lowers the $T=0$ level to below the $\Sigma+\pi$ threshold, and therefore causes no pionic-decay width. It should, however, be noted that these widths do not contain contributions from the $\bar{K}NN$ nonpionic decay modes, which could be about 12 MeV. In the deeply bound states the core nuclei are largely compressed due to the strong attraction of a $\bar{K}$, which plays a unique role as a *contractor* to bind nucleons more tightly, and accommodates a nucleus of much higher density.

An experimental method is proposed to populate the

${}^{3}_{\bar{K}}\mathrm{H}(T=0)$ state, by measuring a discrete neutron component of about 70 MeV from the ${}^{4}\mathrm{He}$ (stopped K^-,n) reaction. The branching ratio to the relevant state is estimated to be about 2% of the total absorption, which makes the experiment feasible. An experimental proposal to search for the narrow peak in the neutron spectra was presented by Iwasaki *et al.* [59], which takes into account the possible neutron background from Σ, Λ, and other decay processes, and has been approved at KEK.

If confirmed experimentally, such ''bound-kaon nuclear spectroscopy'' would create a new paradigm in strangeness nuclear physics. Many important impacts are foreseen. (i) Very deep discrete states of $\bar{K}$ nuclear systems are formed with binding energies $B_{\bar{K}} \sim 100$ MeV. They are highly excited nuclear resonance states with excitation energies of $E_{\mathrm{ex}} \sim 400$ MeV. (ii) High-density cold nuclear matter would be formed around K^-, which could provide information concerning a modification of the $\bar{K}N$ interaction in the nuclear medium and a transition from the hadronic phase to a quark phase. Theoretical studies concerning such a dense and cold nuclear phase are called for. (iii) Empirical information on the possibility for kaon condensation and strange matter would be obtained. (iv) Nuclear dynamics under extreme conditions (nuclear compression, K^- ball, etc.) could be studied. For this purpose further experimental methods using (K^-,π^-) reactions through Λ^* formations to produce $\bar{K}$ bound states in exotic proton-rich nuclear systems are investigated [60,61].

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