d^* dibaryon in the extended quark-delocalization, color-screening model

Jialun Ping
Department of Physics, Nanjing Normal University, Nanjing 210097, People's Republic of China
and Center for Theoretical Physics, Nanjing University, Nanjing 210093, People's Republic of China

Hourong Pang
Department of Physics, Nanjing University, Nanjing 210093, People's Republic of China

Fan Wang
Center for Theoretical Physics and Department of Physics, Nanjing University, Nanjing 210093, People's Republic of China

T. Goldman
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 6 December 2000; revised manuscript received 8 November 2001; published 26 March 2002)

The quark-delocalization, color-screening model, extended by inclusion of a one-pion-exchange (OPE) tail, is applied to the study of the deuteron and the d^* dibaryon. The results show that the properties of the deuteron (an extended object) are well reproduced, greatly improving the agreement with experimental data as compared to our previous study (without OPE). At the same time, the mass and decay width of the d^* (a compact object) are, as expected, not altered significantly.

DOI: 10.1103/PhysRevC.65.044003 PACS number(s): 12.39.−x, 14.20.Pt, 13.75.Cs

I. INTRODUCTION

Quantum chromodynamics (QCD) is believed to be the fundamental theory of the strong interactions. High-energy phenomena can be described very well by using its fundamental degrees of freedom, quarks and gluons. However, the direct use of QCD for low-energy hadronic interactions, for example, the nucleon-nucleon (NN) interaction, is still impossible because of the nonperturbative complications of QCD. Quark models are, therefore, a useful phenomenological tool. Which of the models or which effective degrees of freedom represent the physics of low-energy QCD better remains an open question.

The traditional meson-exchange model [1–5] describes the NN scattering data quantitatively very well, where the effective degrees of freedom are nucleons and mesons. The intermediate- and long-range parts of the NN interaction are attributed to two-pion-exchange part, usually parametrized in terms of a σ meson, and one-pion-exchange (OPE) part, respectively. The short-range part is either parametrized by a repulsive core or regularized by means of vertex form factors. Such parametrizations are difficult to extend to the study of new phenomena, such as multiquark systems.

In light of its success in describing the properties of hadrons, the constituent quark model (CQM) [6], where the effective degrees of freedom are constituent quarks and gluons, has been extended to the study of the NN interaction. The short-range repulsive core is successfully reproduced by a combination of the quark Pauli exclusion principle and the color hyperfine interaction. On the other hand, the intermediate- and long-range part of the NN interaction cannot be accounted for in the CQM. Meson exchange has to be invoked again; this leads to ''hybrid'' models [7–9]. However, the quantitative agreement with experimental data is not as good as for traditional meson-exchange models.

Recently a new approach, the Goldstone-boson-exchange (GBE) model [10], where the effective degrees of freedom are constituent quarks and Goldstone bosons, has appeared. It appears to give a rather good description of the baryon spectrum and has also been applied to the NN interaction [11].

Another quark-model approach, which is closer in spirit to the original CQM, the quark-delocalization, color-screening model (QDCSM) [12], has been developed with the aim of understanding the well-known similarities between nuclear and molecular forces despite the obvious energy and length scale differences. The model has been applied to baryon-baryon interactions [12–16] and dibaryons [17–19]. Quantitative agreement with the experimental data on NN and YN (hyperon) scattering has been obtained [16].

Although the intermediate-range attraction of the NN interaction is reproduced in the model, the long-range tail is missing, similar to the case of the CQM. For example, the deuteron, a highly extended object, is not well reproduced [20]. An increased value for the color-screening parameter in the QDCSM can generate enough attraction to bind the deuteron, but the radius and D-wave mixing thus obtained are both too small. Also, the attraction in NN scattering is a little bit too strong. From these effects, from the parallel to the CQM and from consideration of the coordinate space behavior of the adiabatic NN potential obtained in the model [16], we concluded that the long-range part of the NN interaction was what was missing. Additionally, the most convincing result of spontaneously broken chiral symmetry is the small mass of the pion and its coupling to the nucleon. Pion exchange between nucleons is also uniquely well established by partial wave analyses of NN scattering data [21].

This paper reports a study of some effects of adding OPE to the QDCSM. This is carried out with the inclusion of a short-distance coordinate space cutoff in order to avoid, or at least, to minimize double counting of the intermediate to short-range part of meson exchange already accounted for in the model by delocalization of the quark wave functions. We first recalculate the deuteron, then apply this same addition to the d^* dibaryon. Another important application is that to NN scattering, but this is left to future work. Section II gives a brief description of the model Hamiltonian, wave functions, and calculation method. The results and a discussion are presented in Sec. III.

II. MODEL HAMILTONIAN, WAVE FUNCTIONS, AND CALCULATION METHOD

The details of the QDCSM can be found in Refs. [12,17] and the resonating-group calculation method (RGM) has been presented in Refs. [20,22]. Here we present only the model Hamiltonian, wave functions, and the necessary equations used in the current calculation.

The Hamiltonian for the three-quark system is the same as the usual potential model. For the six-quark system, it is assumed to be

$$H_6=\sum_{i=1}^{6}\left(m_i+\frac{p_i^2}{2m_i}\right)-T_{CM}+\sum_{i<j=1}^{6}(V_{ij}^c+V_{ij}^G+V_{ij}^\pi),$$

$$V_{ij}^G=\alpha_s\frac{\vec{\lambda}_i\cdot\vec{\lambda}_j}{4}\left[\frac{1}{r}-\frac{\pi\delta(\vec{r})}{m_im_j}\left(1+\frac{2}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j\right)+\frac{1}{4m_im_j}\left(\frac{3(\vec{\sigma}_i\cdot\vec{r})(\vec{\sigma}_j\cdot\vec{r})}{r^5}-\frac{\vec{\sigma}_i\cdot\vec{\sigma}_j}{r^3}\right)\right],$$

$$V_{ij}^\pi=\theta(r-r_0)f_{qq\pi}^2\vec{\tau}_i\cdot\vec{\tau}_j\frac{1}{r}e^{-\mu_\pi r}\times\left[\frac{1}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j+\left(\frac{3(\vec{\sigma}_i\cdot\vec{r})(\vec{\sigma}_j\cdot\vec{r})}{r^2}-\vec{\sigma}_i\cdot\vec{\sigma}_j\right)\left(\frac{1}{(\mu_\pi r)^2}+\frac{1}{\mu_\pi r}+\frac{1}{3}\right)\right],$$

$$V_{ij}^c=-a_c\vec{\lambda}_i\cdot\vec{\lambda}_j\begin{cases}r^2 & \text{if } i,j \text{ occur in the same baryon orbit,}\\ \dfrac{1-e^{-\mu r^2}}{\mu} & \text{if } i,j \text{ occur in different baryon orbits,}\end{cases}$$

$$\theta(r-r_0)=\begin{cases}0, & r<r_0,\\ 1 & \text{otherwise,}\end{cases} \tag{1}$$

where all the symbols have their usual meaning, and the confinement potential V_{ij}^c has been discussed in Refs. [19,20].

The OPE takes the usual Yukawa form except for a short-range cutoff. The quark-pion coupling constant $f_{qq\pi}$ can be obtained from the nucleon-pion coupling constant $f_{NN\pi}$ by using the equivalence of the quark and nucleon pictures of the NN interaction. If the separation between two nucleons is large, then their interaction energy can be well described by a Yukawa potential; the quark description at the same separation should lead to the same potential. For example, for the (IS)=(01) NN channel, (with the nucleon taken as a point particle,) the Yukawa potential at separation R, is

$$V_{NN}^{\pi N}=-f_{NN\pi}^2\frac{1}{R}e^{-\mu_\pi R}. \tag{2}$$

Taking the nucleon as a $3q$ system, for sufficiently large separation R we can express the potential between the two $3q$ systems as

$$V_{NN}^{\pi q}=\langle[N(123)N(456)]^{IS}|9V_{14}^\pi|[N(123)N(456)]^{IS}\rangle$$
$$=-\frac{25}{9}f_{qq\pi}^2\frac{1}{R}e^{-\mu_\pi R}e^{\mu_\pi^2 b^2/2}, \tag{3}$$

since all of the exchange terms tend to zero at large R. The quantity b (~ 0.6 fm) is the size parameter characterizing the quark wave functions in a nucleon; see Refs. [19,20] and below. From the above expression, it is clear that the classic symmetry relation, $f_{qq\pi}=\frac{3}{5}f_{NN\pi}$, holds except for a small correction (since $\mu_\pi b\sim 0.4$) due to the finite size of the nucleon in the quark description.

After introducing generator coordinates in which to expand the relative motion wave function and including the wave function for the center-of-mass motion,[1] the ansatz for the two-cluster wave function used in the RGM can be written as

$$\Psi_{6q}=\mathcal{A}\sum_k\sum_{i=1}^{n}\sum_{L_k=0,2}C_{k,i,L_k}\int\frac{d\Omega_{S_i}}{\sqrt{4\pi}}\prod_{\alpha=1}^{3}\psi_\alpha(\vec{S}_i,\epsilon)$$
$$\times\prod_{\beta=4}^{6}\psi_\beta(-\vec{S}_i,\epsilon)\{[\eta_{I_{1k}S_{1k}}(B_{1k})$$
$$\times\eta_{I_{2k}S_{2k}}(B_{2k})]^{IS_k}Y^{L_k}(\hat{\vec{S}}_i)\}^J[\chi_c(B_1)\chi_c(B_2)]^{[\sigma]}, \tag{4}$$

[1]For details, see Refs. [20,22].

TABLE I. Model parameters and calculated results for deuteron and d^*.

	$r_0=0.6$ fm		$r_0=1.0$ fm		Without OPE	
	Deuteron	d^*	Deuteron	d^*	Deuteron	d^*
m (MeV)	313		313		313	
b (fm)	0.6010		0.6021		0.6034	
a_c (MeV fm^{-2})	25.40		25.02		25.13	
α_s	1.573		1.550		1.543	
μ (fm^{-2})	0.75		0.95		1.50	
Mass (MeV)	1876	2186	1876	2165	1876	2116
$\sqrt{\langle r^2\rangle}$ (fm)	2.1	1.3	1.9	1.3	1.5	1.2
P_D	5.2%		4.5%		0.2%	
Decay width (MeV)		7.92		5.76		4.02

where k is the channel index and $S_i, i=1,\ldots,n$ are the generating coordinates. For example, for the deuteron, we have $k=1,\ldots,5$, corresponding to the channels NN $S=1$ $L=0$, $\Delta\Delta$ $S=1$ $L=0$, $\Delta\Delta$ $S=3$ $L=2$, NN $S=1$ $L=2$, and $\Delta\Delta$ $S=1$ $L=2$. Also,

$$\psi_\alpha(\vec{S}_i,\epsilon)=[\phi_\alpha(\vec{S}_i)+\epsilon\phi_\alpha(-\vec{S}_i)]/N(\epsilon),$$

$$\psi_\beta(-\vec{S}_i,\epsilon)=[\phi_\beta(-\vec{S}_i)+\epsilon\phi_\beta(\vec{S}_i)]/N(\epsilon),$$

$$N(\epsilon)=\sqrt{1+\epsilon^2+2\epsilon e^{-S_i^2/4b^2}}.$$

$$\phi_\alpha(\vec{S}_i)=\left(\frac{1}{\pi b^2}\right)^{3/4}\exp\left(-\frac{1}{2b^2}(\vec{r}_\alpha-\vec{S}_i/2)^2\right)$$

$$\phi_\beta(-\vec{S}_i)=\left(\frac{1}{\pi b^2}\right)^{3/4}\exp\left(-\frac{1}{2b^2}(\vec{r}_\beta+\vec{S}_i/2)^2\right). \quad (5)$$

are the delocalized single-particle wave functions used in QDCSM. The delocalization parameter, ϵ, is determined by the six-quark dynamics.

With the above ansatz, Eq. (4), the RGM equation becomes an algebraic eigenvalue equation,

$$\sum_{j,k,L_k} C_{j,k,L_k} H_{i,j}^{k',L'_{k'},k,L_k}=E\sum_j C_{j,k,L_k} N_{i,j}^{k',L'_{k'}}, \quad (6)$$

where $N_{i,j}^{k',L'_{k'}}, H_{i,j}^{k,L_k,k',L'_{k'}}$ are the [Eq. (4)] wave function overlaps and Hamiltonian matrix elements (without the summation over L'), respectively. By solving the generalized eigenvalue problem, we obtain the energies of the six-quark systems and their corresponding wave functions.

The partial width for d^* decay into the NN D-wave state is obtained by using ''Fermi's Golden Rule,''

$$\Gamma=\frac{1}{7}\sum_{M_{J_i},M_{J_f}}\frac{1}{(2\pi)^2}\int p^2 dp d\Omega \quad \delta(E_f-E_i)|M|^2$$

$$=\frac{1}{7}\sum_{M_{J_i},M_{J_f}}\frac{1}{32\pi^2}m_{d^*}\sqrt{m_{d^*}^2-4m_N^2}\int |M|^2 d\Omega, \quad (7)$$

where M_{J_i} and M_{J_f} are the spin projections of the initial and final states. The nonrelativistic transition matrix element, M, includes the effect of the relative motion wave function between the two final state nucleons,

$$M=\langle d^*|H_I|[\Psi_{N_1}\Psi_{N_2}]^{IS}\chi(\vec{R})\rangle, \quad (8)$$

where $\vec{R}$ is the relative motion coordinate of the two clusters of quarks (nucleons) and $\chi(\vec{R})$ is the wave function of the relative motion. The interaction Hamiltonian, H_I, is comprised of the tensor parts of OGE and OPE.

III. RESULTS AND DISCUSSION

Our model parameters are given in Table I. They have been fixed by matching baryon properties, except for the color-screening parameter [μ in the confining potential in Eq. (1)] which has been determined by matching the mass of the deuteron. We have examined the values 0.6 fm and 1.0 fm for the short-range cutoff of OPE. For each cutoff, the model parameters were readjusted to best match the baryon properties. In all cases, the contribution of OPE to the baryon mass is not large (about 10 MeV or less) because of the short-range cutoff; correspondingly, the changes from the original QDCSM parameters are also quite small. Obviously the mass shift is model dependent, as the mass of the baryon comes mainly from OPE in the GBE model [10], and there is no net contribution from OPE in the hybrid model of Fujiwara [9].

We have pointed out in the introduction that which model represents the physics of low-energy QCD better remains an open question. Many different pictures of baryons—the quark-gluon coupling of Isgur [23], the quark-meson coupling of Glozman and Riska [10], and the intermediate models, namely, the Manohar-Georgi quark-gluon-meson coupling model [24] and its variants—all give good (although not perfect) baryon spectra. We found such a ''Cheshire Cat'' principle [25] also appears in the baryon-baryon interaction [26]. Since the QDCSM reduces to the Isgur model in the case of individual baryons, and since the ground state spectra are negligibly affected by addition of the cutoff pion-quark coupling *à la* Manohar-Georgi, it is clear that the QDCSM, with or without the pion, reproduces the quality of the Isgur

TABLE II. Deuteron in the soft cutoff model calculation.

	$\Lambda=4.2$ (fm^{-1})	$\Lambda=2.0$ (fm^{-1})	$\Lambda=0.8$ (fm^{-1})
m (MeV)	313	313	313
b (fm)	0.613	0.611	0.606
a_c (MeV fm^{-2})	13.90	19.65	23.77
α_s	1.057	1.285	1.473
μ (fm^{-2})	12.5	2.8	1.75
Mass (MeV)	1876	1876	1876
$\sqrt{\langle r^2\rangle}$ (fm)	1.52	1.5	1.51
P_D	5.98%	5.2%	2.4%

model agreement with the baryon spectrum. While this is not perfect, and may perhaps be improved in our approach, by considering such things as pion breathing modes for the Roper, for example, this is not the focus of our work here. We rely on the wide acceptance of the Isgur model as assurance that the QDCSM does acceptably well for the baryon spectrum, since it explicitly follows that model for the ground state baryons.

For comparison, the results of our earlier calculation [20] without OPE is also included in Table I. Clearly, the "deuteron" obtained there is not the physically correct one, due to its small size and negligible *D*-wave mixing, although it has the correct binding energy. By adding OPE with a cutoff, our results are significantly improved; the deuteron is now reproduced with either cutoff scale. The results can be made even better by finetuning of the model parameters.

We did not do such a finetuning. However, we did carry out a numerical calculation, with Yukawa smearing of the short-range part of the OPE interaction, to check our model results for sensitivity to the sharp cutoff used above:

$$V_{ij}^{\pi}=\frac{g_8^2}{4\pi}\frac{\mu_\pi^2}{12m_u^2}\frac{\Lambda^2}{\Lambda^2-\mu_\pi^2}\vec{\tau}_i\cdot\vec{\tau}_j\left(\frac{1}{r}e^{-\mu_\pi r}-\frac{\Lambda^2}{\mu_\pi^2}\frac{e^{-\Lambda r}}{r}\right)\times\left[\frac{1}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j+\left(\frac{3(\vec{\sigma}_i\cdot\vec{r})(\vec{\sigma}_j\cdot\vec{r})}{r^2}-\vec{\sigma}_i\cdot\vec{\sigma}_j\right)\times\left(\frac{1}{(\mu_\pi r)^2}+\frac{1}{\mu_\pi r}+\frac{1}{3}\right)\right]. \tag{9}$$

In the short-range term, a δ function form has been smeared to Yukawa form with a parameter Λ, which plays an important role in the GBE model. For each value of the constant, Λ, we refit the values of b, α_s, and a_c by reproducing the $N-\Delta$ mass difference, the nucleon mass, and by satisfying the stability condition for a baryon. It should be noted that the OPE interaction given in Eq. (9) contributes to $N-\Delta$ mass splitting with the opposite sign to that of the sharp cutoff. The color-screening parameter, μ, is again determined by matching the mass of deuteron. Our results are shown in Table II. From the table, we can see that it is more difficult to reproduce the deuteron in our model when the short distance part is smoothly suppressed. Although the binding energy and even the *D*-wave mixing, can be reproduced by adjusting the color-screening parameter, the root-mean-square radius of the deuteron is always too small. Other calculations have found similar results: For example, Zhang *et al.* [27] also obtained a small rms radius and Fujiwara *et al.* [9] used a c_δ to reduce the contact term contribution in order to obtain an acceptable deuteron and good *NN* phase shifts.

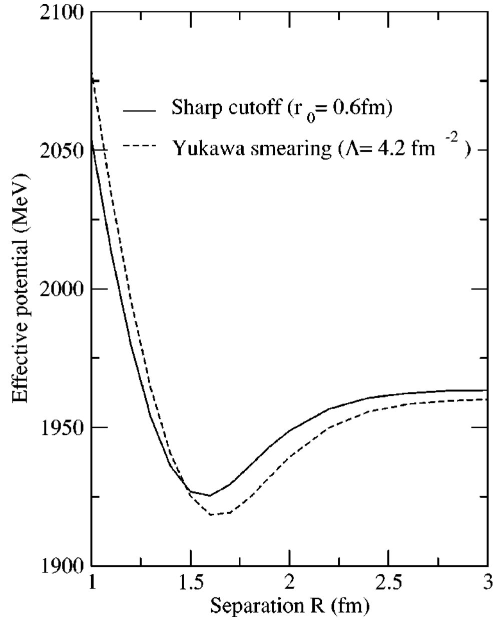

FIG. 1. Effective intercluster potential for deuteron with two OPE cutoffs. The sharp cutoff case is shown by the solid curve, the Yukawa-smeared cutoff is given by the dashed curve. Parameter values are as described in the text.

We suspect that these results are due to the fact that, in our model approach, the short-range repulsion is driven by a combination of the effects of the quark Pauli principle and the color magnetic interaction due to gluon exchange. If we include the short-range OPE interaction further, the short-range repulsion is overestimated. In order to reproduce the deuteron binding energy, the intermediate-range attraction must then be artificially enhanced. Figure 1 shows the adiabatic potentials of the deuteron channel with the sharp cutoff and the Yukawa smearing OPE, respectively. The deuteron is a delicate system; it is sensitive to fine details of the effective *NN* interaction. It would, therefore, be interesting to determine whether the GBE model, with Goldstone boson exchange only, can fully reproduce the deuteron.

We conclude that the QDCSM, after extension to include OPE with a short-range cutoff, fits to the deuteron quantitatively well. (Our preliminary results also show an improvement in the fit to the *NN* phase shifts; that result will be reported fully elsewhere.) This confirms our expectation that the quark delocalization and color-screening mechanism describes the short- and intermediate-range *NN* interaction well and provides an alternative to the short-range phenomenology and the intermediate-range σ or two pion exchange used in conventional meson-exchange models [2].

The case of the d^*, however, is quite different. The OPE changes the mass of the d^* by only a few percent and increases the *NN* decay width by less than 4 MeV. We expected these results because of the high degree of compactness of the d^*, as seen from its rms size, and the short-range cutoff of OPE. The combination of these two effects significantly reduces the contribution to the d^* available from OPE.

In summary, we find that the deuteron can be well described in the extended QDCSM. The quark delocalization and color-screening mechanism can account for the short-range repulsive core and most of the intermediate-range attraction, while the missing long-range tail can be economically incorporated by OPE with a short-range cutoff. With new parameters thus fixed, the properties of the d^* dibaryon are minimally affected.

ACKNOWLEDGMENTS

This research was supported by the National Science Foundation of China, the Fok Yingdung Educational Fund, the Natural Science Foundation of Jiangsu Province, and the U.S. Department of Energy under Contract No. W-7405-ENG-36.

[1] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987), and references therein.

[2] R. Machleidt, Adv. Nucl. Phys. **19**, 1 (1988); Nucl. Phys. **A689**, 11c (2001), and references therein.

[3] R. Vinh Mau, C. Semay, B. Loiseau, and M. Lacombe, Phys. Rev. Lett. **67**, 1392 (1991).

[4] D. Hadjimichef, J. Haidenbauer, and G. Krein, nucl-th/0010044.

[5] V. G. J. Stoks, R. A. M. Klomp, M. C. M. Rentmeester, and J. J. de Swart, Phys. Rev. C **48**, 792 (1993).

[6] For a list of references, see A. Le Yaouanc, L. I. Olive, O. Pene, and J. C. Raynal, *Hadron Transitions in the Quark Model* (Gordon and Breach, New York, 1998).

[7] F. Fernandez, A. Valcarce, U. Straub, and A. Faessler, J. Phys. G **19**, 2013 (1993); A. Valcarce, A. Buchmann, F. Fernandez, and A. Faessler, Phys. Rev. C **50**, 2246 (1994); **51**, 1480 (1995).

[8] S. Takeuchi, K. Shimizu, and K. Yazaki, Nucl. Phys. **A504**, 777 (1989).

[9] Y. Fujiwara, C. Nakamoto, and Y. Suzuki, Phys. Rev. Lett. **76**, 2242 (1996); Phys. Rev. C **54**, 2180 (1996).

[10] L. Ya. Glozman and D. O. Riska, Phys. Rep. **268**, 263 (1996); Phys. Lett. B **381**, 311 (1996).

[11] Fl. Stancu, S. Pepin, and L. Ya. Glozman, Phys. Rev. D **57**, 4393 (1998); Phys. Rev. C **60**, 055207 (1999).

[12] F. Wang, G. H. Wu, L. J. Teng, and T. Goldman, Phys. Rev. Lett. **69**, 2901 (1992).

[13] G. H. Wu, L. J. Teng, J. L. Ping, F. Wang, and T. Goldman, Phys. Rev. C **53**, 1161 (1996).

[14] G. H. Wu, L. J. Teng, J. L. Ping, F. Wang, and T. Goldman, Mod. Phys. Lett. A **10**, 1895 (1995).

[15] J. L. Ping, F. Wang, and T. Goldman, Nucl. Phys. **A657**, 95 (1999).

[16] G. H. Wu, J. L. Ping, L. J. Teng, F. Wang, and T. Goldman, Nucl. Phys. **A673**, 279 (2000).

[17] F. Wang, J. L. Ping, G. H. Wu, L. J. Teng, and T. Goldman, Phys. Rev. C **51**, 3411 (1995).

[18] T. Goldman, K. Maltman, G. J. Stephenson, Jr., J. L. Ping, and F. Wang, Mod. Phys. Lett. A **13**, 59 (1998).

[19] J. L. Ping, F. Wang, and T. Goldman, Phys. Rev. C **62**, 054007 (2000).

[20] J. L. Ping, F. Wang, and T. Goldman, Nucl. Phys. **A688**, 871 (2001).

[21] R. Timmermans, in *Properties and Interactions of Hyperons*, edited by B. F. Gibson *et al.* (World Scientific, Singapore, 1994), p. 179.

[22] A. J. Buchmann, Y. Yamauchi, and A. Faessler, Nucl. Phys. **A496**, 621 (1989).

[23] N. Isgur, Phys. Rev. D **62**, 054026 (2000); **62**, 014025 (2000), and references therein. See also, R. Jaffe, hep-ph/0001123.

[24] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).

[25] V. Vento, Nucl. Phys. **A689**, 223c (2001).

[26] H. R. Pang, J. L. Ping, F. Wang, and T. Goldman, Phys. Rev. C **65**, 014003 (2002).

[27] Zong-ye Zhang, Amand Faessler, U. Straub, and L. Ya. Glozman, Nucl. Phys. **A578**, 573 (1994).