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Dynamical moment of inertia and quadrupole vibrations in rotating nuclei

R. G. Nazmitdinov
Max-Planck-Institut für Physik komplexer Systeme, D-01187 Dresden, Germany
and Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, RU-141980 Dubna, Russia

D. Almehed
Department of Physics, UMIST, P.O. Box 88, Manchester M60 1QD, United Kingdom
and Institut für Kern- und Hadronenphysik, FZ Rossendorf, D-01314 Dresden, Germany

F. Dönau
Institut für Kern- und Hadronenphysik, FZ Rossendorf, D-01314 Dresden, Germany
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The contribution of quantum shape fluctuations to inertial properties of rotating nuclei has been analyzed within the self-consistent one-dimensional cranking oscillator model. It is numerically proven that for even-even nuclei the dynamical moment of inertia calculated in a mean field approximation in the rotating frame is equivalent to the Thouless-Valatin moment of inertia. If the contribution of the quantum fluctuations to the total energy is taken into account, the dynamical moment of inertia differs from the Thouless-Valatin value.

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A description of rotational states is one of the oldest, yet not fully solved, problem in nuclear structure physics. While various microscopic models based on the cranking approach [1,2] describe reasonably well the kinematical moment of inertia $\mathcal{J}^{(1)}=-(dE/d\Omega)/\Omega$ for a finite angular frequency Ω, there is still a systematic deviation of the dynamical moment of inertia $\mathcal{J}^{(2)}=-d^2E/d\Omega^2$ (E is the total energy in the rotating frame) from the experimental data at high spins [3]. Since the moments of inertia are the benchmarks for microscopic models of collective motion in nuclei, the understanding of the source of the discrepancy becomes a challenge for a many-body theory of finite Fermi systems.

The description of the moments of inertia can be improved if quantum oscillations around the mean solution will be incorporated as it was suggested by Thouless and Valatin [4]. In fact, using the cranking plus random phase approximation (CM+RPA) approach the effects of pairing correlations onto the moments of inertia were considered in Ref. [5] (see also references therein). For the case of pairing and quadrupole vibrations such calculations in a restricted configuration space (only three shells have been included) were performed only once [6]. For a time new attempts to study those effects were postponed, since there was no practical recipe to calculate the RPA correlation energy for realistic cases where a large configuration space must be considered. On the other hand, the development of microscopic cranking approaches which start with effective nucleon-nucleon interactions treated within the self-consistent Hartree-Fock (Bogoliubov) method (see, for a review, Ref. [7]) can renew the interest in this problem. Moreover, using the integral representation method developed recently in [8], the total energy E can be calculated in the RPA order [1,9] with a high accuracy and with minimal numerical efforts. It is quite desirable to clarify under which conditions the basic principles formulated few decades ago could be valid. For example, the problem of the quantization of the angular momentum within the CM+RPA is still controversial (see the discussions in Refs. [10,11]) and needs a dedicated study. Further, in literature (see, for instance, Ref. [3]) it is stated that the dynamical moment of inertia calculated in the rotating frame should be equivalent to the Thouless-Valatin moment of inertia [4]. A practical check of the validity of this statement requires that the mean field and the RPA equations are solved self-consistently.

In the present paper, we will calculate the total energy including the mean field energy and the RPA correlation energy in a model that is fully self-consistent and analytically solvable. Our CM+RPA approach is based on the self-consistent mean field of the rotating triaxial oscillator. This model provides a relatively simple but still realistic frame to calculate the Thouless-Valatin moment of inertia and the desired contributions of shape oscillations without the usual restrictions of the configuration space. Notice that the contribution of the pairing vibrations to the correlation energy besides the one from the shape vibrations is also important (see Refs. [5,12,13] and references there). However, there are some open problems with the choice of the self-consistent pairing interaction. Therefore, the combined effect of both types of vibrations is beyond the scope of the present investigation and we leave this problem for the future. We focus our analysis upon the dynamical moment of inertia $\mathcal{J}^{(2)}$ that implies also information on the kinematical moment of inertia due to the obvious relation $\mathcal{J}^{(2)}=\mathcal{J}^{(1)}+\Omega d\mathcal{J}^{(1)}/d\Omega$. We will show that the dynamical moment of inertia calculated in the rotating frame is different from the Thouless-Valatin moment of inertia if the contribution of the quantum fluctuations is taken into account.

The mean field part of the many-body Hamiltonian (Routhian) in the rotating frame is given by

$$H=\sum_{i=1}^{N}(h_0-\Omega l_x)_i=H_0-\Omega L_x, \tag{1}$$

where the single-particle triaxial harmonic oscillator Hamiltonian h_0 is aligned along its principal axes and reads

$$h_0=\frac{\vec{p}^2}{2m}+\frac{m}{2}(\omega_x^2x^2+\omega_y^2y^2+\omega_z^2z^2). \tag{2}$$

The eigenmodes and the total energy of the mean field Hamiltonian Eq. (1) are well known [14–16],[1]

$$\omega_\pm^2=\frac{1}{2}\{\omega_y^2+\omega_z^2+2\Omega^2\pm[(\omega_y^2-\omega_z^2)^2+8\Omega^2(\omega_y^2+\omega_z^2)]^{1/2}\}, \tag{3}$$

$$E_{MF}=\omega_x\Sigma_x+\omega_+\Sigma_++\omega_-\Sigma_-. \tag{4}$$

Here, $\Sigma_k=\langle\Sigma_j^N(n_k+1/2)_j\rangle$ and $n_k=a_k^\dagger a_k$ $(k=x,+,-)$ where $a_k^\dagger$, a_k are the oscillator quanta operators. The lowest levels are filled from the bottom, which give the ground state energy in the rotating frame. The Pauli principle is taken into account such that two particles occupy one level. The minimization of the total energy Eq. (4) with respect to all three frequencies, subject to the volume conservation condition $\omega_x\omega_y\omega_z=\omega_0^3$, yields the self-consistent condition [17,18] for a finite rotational frequency

$$\omega_x^2\langle x^2\rangle=\omega_y^2\langle y^2\rangle=\omega_z^2\langle z^2\rangle. \tag{5}$$

It should be pointed out that the condition Eq. (5) provides generally the absolute minima in comparison with the local minima obtained from the condition of the *isotropic velocity distribution* [15,16]

$$\omega_x\Sigma_x=\omega_+\Sigma_+=\omega_-\Sigma_- \tag{6}$$

at large rotational frequency. Since all shells are mixed, we go beyond the approximation used in Ref. [6] (for a cranking harmonic oscillator, see also Ref. [19]).

To analyze the contribution of the quadrupole shape oscillations we add to the mean field Hamiltonian Eq. (1) the self-consistent interaction resulting from small angular rotations around the $x-$, $y-$, $z-$ axes and small variations of the two intrinsic shape parameters ε and γ [20]. Consequently, the total Hamiltonian can be expressed as

$$H_{\mathrm{RPA}}=H_0-\Omega L_x-\frac{\kappa}{2}\sum_{\mu=-2}^{2}Q_\mu^\dagger Q_\mu=\tilde{H}-\Omega L_x. \tag{7}$$

Here, the quadrupole operators $Q_\mu=r^2\overline{Y_{2\mu}}$ are expressed in terms of the doubly stretched coordinates $\overline{q_i}=(\omega_i/\omega_0)q_i$, $(q_i=x,y,z)$. The effective quadrupole interaction restores the rotational invariance of the Hamiltonian H_0 such that now $[\tilde{H},L_i]=0$ $(i=x,y,z)$ in the RPA order. The self-consistency condition Eq. (5) fixes the quadrupole strength $\kappa=(4\pi/5)(m\omega_0^2/\langle r^2\rangle)$, where $\langle r^2\rangle=\langle\overline{x^2}+\overline{y^2}+\overline{z^2}\rangle$. We remind that the self-consistent residual interaction does not affect the equilibrium deformation obtained from the minimization procedure.

Using the transformation [16] from p_i,q_i variables to the oscillator quanta $a_k^\dagger,a_k$ of the rotating oscillator, all matrix elements are calculated analytically. We solve the RPA equation of motion for the generalized coordinates $\mathcal{X}_\lambda$ and momenta $\mathcal{P}_\lambda$,

$$[H_{\mathrm{RPA}},\mathcal{X}_\lambda]=-i\omega_\lambda\mathcal{P}_\lambda,\quad [H_{\mathrm{RPA}},\mathcal{P}_\lambda]=i\omega_\lambda\mathcal{X}_\lambda, \tag{8}$$
$$[\mathcal{X}_\lambda,\mathcal{P}_\lambda]=i\delta_{\lambda,\lambda'},$$

where ω_λ are the RPA eigenfrequencies in the rotating frame and the associated phonon operators are $O_\lambda=(\mathcal{X}_\lambda-i\mathcal{P}_\lambda)/\sqrt{2}$. Here $\mathcal{X}_\lambda=\Sigma_sX_s^\lambda\hat{f}_s$, $\mathcal{P}_\lambda=i\Sigma_sP_s^\lambda\hat{g}_s$ are bilinear combinations of the quanta $a_k^\dagger,a_k$ such that $\langle[\hat{f}_s,\hat{g}_{s'}]\rangle=V_s\delta_{s,s'}$, where quantities V_s are proportional to different combinations of Σ_i $(i=x,+,-)$. Further, $\langle\cdots\rangle$ means the averaging over mean field states. Since the mean field violates the rotational invariance, among the RPA eigenfrequencies there exist two spurious solutions. One solution with zero frequency is associated with the rotation around the x axes, since $[H,L_x]=0$. The other "spurious" solution at $\omega\equiv\Omega$ corresponds to a collective rotation, since $[H,L_\pm]=[H,L_y\pm iL_z]=\mp\Omega L_\pm$ [21]. The Hamiltonian Eq. (7) possesses the signature symmetry, i.e., $[R_x,H_{\mathrm{RPA}}]=0$ $(R_x=e^{-i\pi\hat{L}_x})$, such that it decomposes into positive and negative signature terms

$$H_{\mathrm{RPA}}=H(+)+H(-) \tag{9}$$

that can be separately diagonalized [21–23]. The negative signature Hamiltonian contains the rotational mode and the vibrational mode describing the wobbling motion [22,24]. We focus on the positive signature Hamiltonian. It contains the zero-frequency mode defined by

$$[H(+),\phi_x]=\frac{-iL_x}{\mathcal{J}_{TV}},\quad [\phi_x,L_x]=i \tag{10}$$

and allows one to determine the Thouless-Valatin moment of inertia $\mathcal{J}_{TV}$ [25]. Here, the angular momentum operator $L_x=\Sigma_sl_s^x\hat{f}_s$ and the canonically conjugated angle $\phi_x=i\Sigma_s\phi_s^x\hat{g}_s$ are expressed via $\hat{f}_s$ and $\hat{g}_s$, which obey the condition $R_x\hat{d}_sR_x^{-1}=\hat{d}_s$ $(\hat{d}_s=\hat{f}_s$ or $\hat{g}_s)$. Solving Eqs. (10) for the Hamiltonian $H(+)$,

$$H(+)=\sum_{k=x,+,-}\sum_j\omega_k(a_k^\dagger a_k+1/2)_j -\frac{\kappa}{2}(Q_0^2+Q_1^{(+)2}+Q_2^{(+)2}), \tag{11}$$

where

$$Q_0=\sqrt{\frac{5}{16\pi}}(2\overline{z^2}-\overline{x^2}-\overline{y^2})=\sqrt{\frac{5}{16\pi}}\sum_s q_s^0\hat{f}_s, \tag{12}$$

[1]To simplify our notation the unit $\hbar$ entering the angular momentum and the frequencies is suppressed.

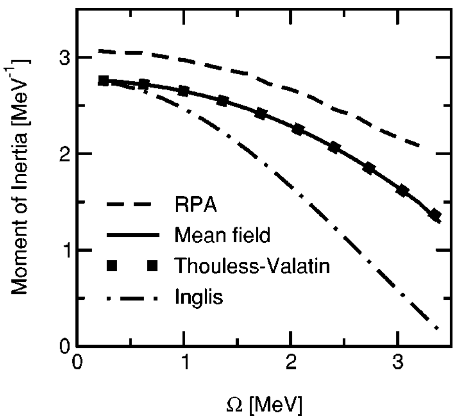

FIG. 1. Moments of inertia for $N=Z=10$ system as a function of the rotational frequency Ω. The definitions of different moments of inertia are given in the text.

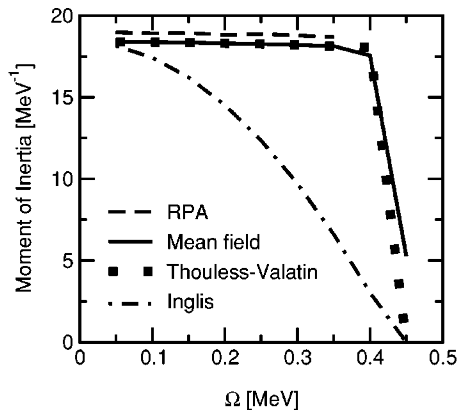

FIG. 2. As in Fig. 1 for $N=Z=32$ system.

$$Q_1^{(+)}=\sqrt{\frac{15}{4\pi}}\overline{yz}=\sqrt{\frac{15}{4\pi}}i\sum_s q_s^1\hat{g}_s, \tag{13}$$

$$Q_2^{(+)}=\sqrt{\frac{15}{16\pi}}(\overline{x^2}-\overline{y^2})=\sqrt{\frac{15}{16\pi}}\sum_s q_s^2\hat{f}_s, \tag{14}$$

we obtain the expression for the Thouless-Valatin moment of inertia

$$\mathcal{J}_{TV}=\mathcal{J}_I+\frac{2S_{x0}S_{x2}S_{02}-S_{x0}^2\left(S_{22}-\frac{1}{\kappa_2}\right)-S_{x2}^2\left(S_{00}-\frac{1}{\kappa_0}\right)}{\left(S_{00}-\frac{1}{\kappa_0}\right)\left(S_{22}-\frac{1}{\kappa_2}\right)-S_{02}^2}. \tag{15}$$

Here, the term $\mathcal{J}_I$ corresponds to the Inglis moment of inertia

$$\mathcal{J}_I=\sum_s \frac{(l_s^x)^2V_s}{E_s}. \tag{16}$$

The second term in Eq. (15) is a contribution of the quadrupole residual interaction in the cranking model. In the cranking harmonic oscillator it consists of terms that have the following structure,

$$S_{xm}=\sum_s \frac{l_s^x q_s^m V_s}{E_s},\quad S_{nm}=\sum_s \frac{q_s^n q_s^m V_s}{E_s},\quad n,m=0,2, \tag{17}$$

where E_s are the energies of particle-hole excitations: $E_1=2\omega_+$, $E_2=2\omega_-$, $E_3=2\omega_x$, $E_4=\omega_+ + \omega_-$, and $E_5=\omega_+ - \omega_-$. We also introduced the following notations: $\kappa_0=(5/16\pi)\kappa$ and $\kappa_2=(15/16\pi)\kappa$.

The above results are the starting point for our numerical analysis. It should be noted that a general discussion about the RPA corrections to the cranking model has been presented in Ref. [21]. The total energy in Ref. [21] is a sum of the mean field energy E_{SCC} defined in the *laboratory frame* and the RPA correlation energy defined in the *rotating frame*. This inconsistency was already mentioned in Ref. [10]. Further, an equivalence is claimed between the Thouless-Valatin moment of inertia defined by Eq. (10) and the quantity $g_{33}=(\partial^2E_{SCC}/\partial I^2)_0$ [21]. However, the angular momentum is not *a good quantum number* in the cranking model (see, e.g., [1]). In addition, the quantization condition for the angular momentum in the CM+RPA approach depends on the definition of the total energy [10,11]. Therefore, a question arises about the validity of this equivalence.

We recall that all calculated quantities, i.e., the mean field energy, the quasiparticle (particle-hole) excitations, the RPA eigenfrequencies, are functions of the rotational frequency Ω that is the only free parameter. We stress that we shall follow the option to study the rotational properties as the Thouless-Valatin moment and the dynamical moment of inertia in terms of this parameter and avoid making any transformation to the laboratory system. In fact, this analysis is consistent with the experimental definition of the dynamical moment of inertia $\mathcal{J}^{(2)}=dI/d\Omega\approx 4/\Delta E_\gamma$ (see, for example, Ref. [19]). Here, $\Delta\Omega=\Delta E_\gamma/2$, where ΔE_γ is the difference between two consecutive γ transitions, and E_γ is the γ-transition energy between two neighboring states that differ on two units of the angular momentum.

To take into account shell effects, we consider two systems with number of particles $A=20$, 64 $(N=Z)$. For $\Omega=0$ MeV, the global minimum occurs for a prolate shape and for a near oblate triaxial shape for $A=20$ and 64, respectively [18]. If we trace the configurations that characterize the ground states, with increasing rotational frequency both systems become oblate. At this point the moment of inertia vanishes, since there is no a kinetic energy associated with such a rotation.

In order to compare various moments of inertia, i.e., the Thouless-Valatin, Eq. (15), the Inglis, Eq. (16), and $\mathcal{J}_{MF}^{(2)}=-d^2E_{MF}/d\Omega^2$ with $\mathcal{J}_{RPA}^{(2)}=-d^2E_{RPA}/d\Omega^2$, we calculate the RPA correlation energy $E_{corr}^{RPA}=\frac{1}{2}(\Sigma_\lambda\omega_\lambda-\Sigma_s E_s)$ that includes the positive and negative signature contributions.

In Figs. 1 and 2 the results of calculations for different moments of inertia are presented for $A=20$ and 64, respectively. To our knowledge this is the first numerical demonstration of the equivalence between the dynamical moment of inertia $\mathcal{J}_{MF}^{(2)}$ calculated in the mean field approximation and the Thouless-Valatin moment of inertia $\mathcal{J}_{TV}$ calculated in the RPA. For the both systems the Inglis moment of inertia

$\mathcal{J}_I$ is smaller than the $\mathcal{J}_{TV}$ and $\mathcal{J}^{(2)}_{MF}$ and has a different rotational dependence.

While the Inglis moment of inertia characterizes the collective properties of noninteracting fermions, the dynamical moment of inertia reflects the changes in the rotating self-consistent mean field due to an internucleon interaction. As it was pointed out in Ref. [26], the volume conservation condition, used as a constraint in the mean field calculations, can be interpreted as a Hartree approximation applied to an interaction that involves the sum of one-body, two-body, etc., forces. The sharp drop in all moments of inertia in Fig. 2 is caused by the onset of the oblate shape where the collective rotation does not exist. For $A=64$ the onset of the oblate deformation occurs at a smaller rotational frequency in contrast to the one for the system $A=20$.

The dynamical moment of inertia $\mathcal{J}^{(2)}_{RPA}$ is larger than the Thouless-Valatin moment of inertia. However, from our calculations it follows that the contribution of the RPA ground state correlations decreases with an increase of the number of particles. The difference between the $\mathcal{J}^{(2)}_{RPA}$ and the $\mathcal{J}_{TV}$ is due to the following reason. The Inglis moment of inertia is smaller than the Thouless-Valatin (or $\mathcal{J}^{(2)}_{MF}$) value, since the $\mathcal{J}_{TV}$ contains the effect of the residual particle-hole interaction. On the other hand, the Thouless-Valatin moment of inertia manifests the rotational dependence of the residual interaction. Thus, we may speculate that inclusion of the phonon interaction could help us to reproduce the behavior of the $\mathcal{J}^{(2)}_{RPA}$ that characterizes the rotational dependence of the phonon-phonon interaction.

In summary, using the self-consistent cranking harmonic oscillator model, we have numerically proved the equivalence of the mean field dynamical moment of inertia calculated *in the rotating frame* to the Thouless-Valatin moment of inertia calculated in the CM+RPA approach. Our result is a consequence of the self-consistent condition Eq. (5) that minimizes the expectation value of the mean field Hamiltonian, Eq. (1). This condition is equivalent to the stability condition of collective modes in the RPA [27], i.e., ω_λ to be real, and has been used to calculate different moments of inertia. The rotational dependence of both the dynamical moments of inertia, $\mathcal{J}^{(2)}_{MF}$ and $\mathcal{J}^{(2)}_{RPA}$, is similar, however, the $\mathcal{J}^{(2)}_{RPA}$ is larger than the $\mathcal{J}^{(2)}_{MF}$ due to the contribution of the ground state correlations. This difference between the moments of inertia is less important for heavy systems.

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