

## First observation of the intruder band in $^{108}\text{Cd}$

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The study of intruder and multiphonon configurations is of great interest in nuclear structure physics because it is related to single-particle and collective motion. The rarest of the stable cadmium isotopes was investigated with the powerful combination of two complementary experimental techniques:  $\gamma\gamma$  spectroscopy following the  $\beta$  decay of  $^{108}\text{In}$ , and population of excited states using the nonselective  $(\alpha, n)$  fusion-evaporation reaction. The intruder band in  $^{108}\text{Cd}$  up to its  $4^+$  member is established on the measurement of branching ratios of in-band and intraband decays supported by upper limits for the lifetimes using Doppler shift attenuation method. The heavily suppressed absolute  $E2$  transition strength out of the proposed intruder band indicates this band structure to be fairly pure. These findings are compared to neighboring Cd isotopes and related six quasiproton structures. An analysis of the multiphonon structures observed in  $^{108}\text{Cd}$  and heavier Cd isotopes shows that the even mass cadmium chain forms a transitional path between the vibrational U(5) and the gamma-unstable O(6) limit with  $^{108}\text{Cd}$  closer to the O(6) description.

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The stable cadmium isotopes represent an unique laboratory for the study of the nuclear many-body problem because two protons are missing for the  $Z=50$  shell closure, while the neutrons are situated near midshell with  $N\approx 60-70$ . The isotopes  $^{110-114}\text{Cd}$  are classical examples for a spherical vibrator close to the dynamical U(5) symmetry of the interacting boson model [1-4]. In the investigation of low lying collective states in these isotopes one systematically finds an additional  $0^+$  state at about the energy of the two-phonon triplet  $(4_1^+, 2_2^+, 0_2^+)$ , which seems to destroy the simple phonon concept even at low energy. These states have been investigated using various experimental techniques and theoretical models. Their strong population in the two-proton transfer reaction  $(^3\text{He}, n)$  [5] is generally assessed as proof of the  $2p-2h$  proton intruder structure [6,7]. The high energy that is necessary to excite two protons across the shell closure is compensated by the strongly attractive proton-neutron interaction. These intruder states are a common phenomenon in regions where one kind of nucleon is near semimagic shell closures and the other at midshell. These  $2p-2h$  excitations serve to explain the unexpected states at low energies in the chain of cadmium isotopes. In the even mass Sn isotopes fast  $E0$  transitions between the intruder  $0^+$  and the ground state were observed [8] pointing at a sizable deformation of the intruder configuration. In  $^{110-114}\text{Cd}$ , band structures based on the intruder  $0^+$  state were established up to the  $6_1^+$  member [9] in  $^{110}\text{Cd}$  even up to higher spin values [4,10]. In  $^{108}\text{Cd}$  the  $0_2^+$  and  $2_3^+$  excitations were suggested as intruder states but a clear signature, for example, a connecting transition establishing a bandlike structure, was missing [9].

We investigated  $^{108}\text{Cd}$  using a powerful combination of two different population mechanisms. Excited states were populated in the  $\beta$  decay of  $^{108}\text{In}$  and also in the nonselective  $^{105}\text{Pd}(\alpha, n)^{108}\text{Cd}$  fusion-evaporation reaction. While the  $\beta$  decay serves to populate predominantly positive parity states with spin values from 1 to  $3\hbar$  up to high energies, the compound nucleus reaction near the Coulomb barrier excites states up to  $(7-8)\hbar$  independently of the parity. Both experi-

mental techniques offer the possibility to determine accurate multipole mixing ratios of transitions and to assign spin values by the analysis of  $\gamma\gamma$  angular correlations. The angular momentum transfer in the  $(\alpha, xn)$  type of reaction prevents the excitation of dipole or quadrupole states at high energies, in that sense the two experimental methods are complementary. In a fusion-evaporation reaction it is possible to determine effective lifetimes using the Doppler shift attenuation method (DSAM). The  $\gamma$  rays emitted in flight by the recoiling nuclei during the deceleration process in the target or backing material are observed with Doppler-shifted energies under forward and backward angles. Lifetimes of the order of femtoseconds to picoseconds are accessible by this technique.

With the powerful combination of two complementary methods for low-spin spectroscopy we were able to establish the intruder band structure up to the  $4_1^+$  state including lower limits for the absolute transition strengths of interband and intraband decays. So the systematics of intruder bands in the even mass cadmium chain could be extended to the rarest isotope  $^{108}\text{Cd}$ . Comparing our findings to the neighboring nuclei  $^{110}\text{Cd}$  and  $^{112}\text{Cd}$  it is found that the chain of Cd isotopes forms a transitional path between the vibrational U(5) and gamma-unstable O(6) dynamical symmetry of the interacting boson model (IBM) [11]. This last conclusion gained importance due to the recently discussed E(5) symmetry [13] describing the physics at the critical point of the U(5)-O(6) phase transition analytically.

Former investigations of  $^{108}\text{Cd}$  were performed with  $\beta$  decay techniques [14,15], investigations using a  $(p, 2n)$  reaction [9], and heavy ion induced high-spin experiments [16]. Our experiments were done at the FN TANDEM accelerator facility of the University of Cologne. Excited states of  $^{108}\text{Cd}$  were populated in the fusion-evaporation reaction  $^{105}\text{Pd}(\alpha, n)^{108}\text{Cd}$  at a beam energy of 13.75 MeV. The  $\gamma$  singles and  $\gamma\gamma$  coincidences were observed by means of the OSIRIS cube spectrometer, which was equipped with six escape-suppression shielded high purity germanium detectors positioned at the faces of the cubic target chamber. Un-

shielded detectors at three corners of the cube offered important additional angles for the  $\gamma\gamma$  angular correlation analysis. This setup together with a fourth unshielded detector at one corner of the cube served to study excited states of  $^{108}\text{Cd}$  following the  $\beta$  decay of the  $7^+$  ground state and a  $2^+$  low-spin isomer in  $^{108}\text{In}$ . The radioactive  $^{108}\text{In}$  was produced with the reaction  $^{108}\text{Cd}(p,n)^{108}\text{In}$  at 13.5 MeV beam energy. We applied a cyclic procedure of activation with the beam on target for 1 s followed by 1 s of measuring  $\gamma\gamma$  coincidences and  $\gamma$  singles off-beam. This off-beam data taking technique results in very clean spectra with low background.

In an off-line sorting process, the coincidence time information was used to subtract random coincidences. The energies of coincident events were sorted into  $8k \times 8k$   $\gamma\gamma$  matrices. A matrix including the coincidences between all detector pairs was used to analyze the level and branching scheme of  $^{108}\text{Cd}$ . According to the angular relations between the detector pairs and the beam axis, additional matrices were sorted providing the intensities for the angular correlation analysis. A detailed description of the  $\gamma\gamma$  angular correlation method following the  $\beta$  decay is given in [17]. A review of the analysis of angular correlations involving states oriented in a fusion-evaporation reaction for the discussed experimental setup can be found in [18]. We follow the theory and sign conventions of Krane, Steffen, and Wheeler [19]. We used the method of angular correlation for spin assignments as well as for the determination of multipole mixing ratios  $\delta$ . In the case of decays with  $\Delta J=0,1$  and  $\pi_i\pi_f=1$ , the square of the mixing ratio equals  $\delta^2=I_\gamma(E2)/I_\gamma(M1)$  and provides important information on nuclear structure.

The reaction kinematics of our  $(\alpha,n)$  experiment determines the velocity of the recoiling nuclei to be  $v/c=0.19\%$ . The stopping time averages 500 fs.  $\gamma$  rays emitted from recoiling nuclei in flight were detected to be Doppler shifted under  $45^\circ$  forward and backward angles. The program DSTOP [20,21] was used to simulate the stopping process of the recoiling  $^{108}\text{Cd}$  nuclei, according to our target consisting of  $1\text{ mg/cm}^2$   $^{105}\text{Pd}$  on a  $0.5\text{ mg/cm}^2$  gold backing. With this knowledge of the velocity-time relation, information on the level lifetime can be extracted from a line shape analysis of Doppler-shifted transitions. In our analyses the feeding was assumed to be prompt. Using DSAM this way potentially results in a systematic overestimation of the lifetime. Due to the neglect of a finite feeding time, the determined lifetimes only serve as upper limits.

As mentioned before, the  $0_2^+$  excitation at 1720 keV and the  $2_3^+$  state at 2163 keV were suggested to belong to the proton  $2p$ - $2h$  intruder configuration, but a connecting transition pointing to a bandlike structure was missing in the past. In our  $\beta$  decay experiment we observed this decay for the first time in  $^{108}\text{Cd}$  and extended the band up to the  $4^+$  member. In the following we briefly discuss the intruder band.

The 1720 keV state is the second  $0^+$  state in  $^{108}\text{Cd}$  and was proposed as the band head of the intruder configuration, solely based on considerations involving systematics in 1992 [9]. The only depopulating transition of this excitation is the

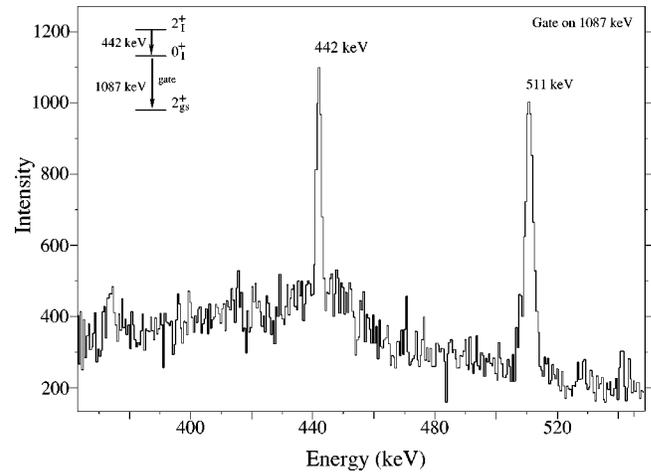


FIG. 1. Part of a coincidence spectrum obtained by gating on the  $0_1^+ \rightarrow 2_1^+$  transition. The peak at 442 keV is the  $2_1^+ \rightarrow 0_1^+$  decay within the newly established intruder band. Its  $\gamma$  intensity equals 0.3% of the strongest branch. The observation of this weak branch was only possible in the spectra detected off-beam following the  $\beta$  decay of  $^{108}\text{In}$ .

decay to the  $2_1^+$  state. We observed this transition with an unshifted line shape, therefore the lifetime of this state was not accessible to DSAM.

For the 2163 keV level, two depopulating transitions were known earlier: the decays to the ground state and the first excited  $2^+$  state with branching ratios of 6% and 100%, respectively. We were able to extend the decay scheme of this state by the transition to the  $0_2^+$  excitation. This was possible only due to the very strong population (7.1%) of the  $2_3^+$  state in the  $\beta$  decay of the  $2^+$  isomer of  $^{108}\text{In}$ . We determined the 442 keV transition  $2_3^+ \rightarrow 0_2^+$  to have a branching ratio of 0.3% compared to the transition to the first  $2^+$  state, and its spectroscopy was possible only from our coincidence data recorded off-beam following the  $\beta$  decay. Such weak branching ratios were determined in the coincidence spectra obtained by gating on all depopulating transitions of the involved final states. The resulting intensities were corrected for coincidence efficiency. In Fig. 1 we show a part of a gated spectrum containing all coincidences to the  $0_2^+ \rightarrow 2_1^+$  transition. The in-band 442 keV transition depopulating this level is marked. In our spectra from the  $(\alpha,n)$  measurement, we observed Doppler shifts of the strongest decay under  $45^\circ$  forward and backward angles allowing a line shape analysis. The extracted upper limit for the lifetime equals 483(195) fs. The relatively large error is mainly caused by a contamination in the peak. With this lifetime information it was possible to extract a lower limit for the transition strengths. The results are summarized in Fig. 2 and Table I.

The fourth  $4^+$  state (2739 keV) and its transitions to the  $4_1^+$  and  $2_1^+$  states were known earlier. In the  $(\alpha,n)$  experiment we observed two new depopulating decays of this level containing the important transition to the  $2_3^+$  state with a branch of only 0.9%. Again we were able to determine an upper limit for the lifetime. The result of the DSAM analysis yields  $\tau \leq 530(129)$  fs. The resulting lower limits for the  $E2$  excitation strengths are presented in Fig. 2 and Table I.

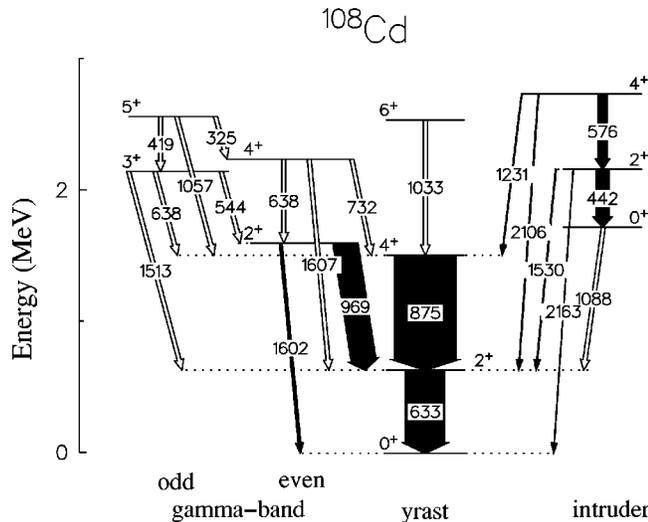


FIG. 2. Prominent band structures observed in  $^{108}\text{Cd}$ . The level scheme comprises the ground band, the even and odd part of the quasigamma band, and the newly established intruder band. The width of the black arrows indicates absolute  $E2$  transition strength. We stress that the results for the intruder band only can serve as lower limits due to the mentioned feeding problematic in the lifetime determination using DSAM. Transition strengths for the yrast band and the second  $2^+$  state were taken from [22].

With the powerful combination of two different experimental techniques, it was possible to establish the intruder band up to the  $4^+$  member by measuring the tiny branching ratios of the in-band transitions. The upper limits of the lifetimes led to the determination of lower limits for the absolute transition strengths, which prove the collectivity of the transitions in the intruder band. In Fig. 2 we present the observed bandlike structures in  $^{108}\text{Cd}$  comprising the ground band, the even and odd part of the quasigamma band, and the new intruder band. The widths of the black arrows indicate the  $E2$  transition strengths. For the ground band and the second

TABLE I. Experimental results for the states forming the intruder band in  $^{108}\text{Cd}$ . We give the transition energy,  $\gamma$ -intensity ratio, multipole mixing ratio, and a lower limit for the  $E2$  transition strengths, obtained from effective DSAM lifetimes. In the last column we compare our findings to the absolute transition strengths in  $^{110}\text{Cd}$  [4]. The branching of the  $2_1^+$  state was determined from our  $\beta$  decay data (marked by bold print), all other results stem from the  $(\alpha, n)$  measurement.

Decay	$^{108}\text{Cd}$				$^{110}\text{Cd}$
	$E_\gamma$ (keV)	Branch (%)	$\delta$	$B(E2)$ (Wu)	$B(E2)$ (Wu)
$2_1^+ \rightarrow 0_1^+$	442.0 <sup>a</sup>	<b>0.3(1)</b>	$E2$	$\geq 9_{-5}^{+12}$	$23_{-18}^{+27}$
$4_1^+ \rightarrow 2_1^+$	575.9 <sup>a</sup>	0.9(1)	$E2$	$\geq 6_{-2}^{+4}$	$109_{-53}^{+62}$
$2_2^+ \rightarrow 2_1^+$	1529.8	<b>100(8)</b>	0.27(4)	$\geq 0.4_{-0.2}^{+0.3}$	$0.16_{-0.09}^{+0.12}$
$2_2^+ \rightarrow 0_1^+$	2162.8	<b>5.6(7)</b>	$E2$	$\geq 0.06(3)$	0.30(10)
$4_2^+ \rightarrow 4_1^+$	1230.4	100(9)	0.16(8)	$\geq 0.39(10)$	$2_{-1}^{+4}$
$4_2^+ \rightarrow 2_1^+$	2105.6	14(2)	$E2$	$\geq 0.14(4)$	$0.20_{-0.10}^{+0.27}$

<sup>a</sup>Decays observed for the first time.

$2^+$  state, the data were taken from the literature [22]. We stress that the  $B(E2)$  results for the intruder band only can serve as lower limits due to the feeding problem occurring in DSAM. Nevertheless it becomes clear that we found a band structure based on the  $0_2^+$  state.

The decays out of the band are very weak. Indeed these transitions are partly hindered by two orders of magnitude implying a fairly high purity of the intruder configuration. This is a rather remarkable fact, because the  $2_1^+$  and  $2_2^+$  states can mix as well as the  $4_1^+$  and  $4_2^+$  excitations. This can easily be understood in the U(5)-O(6) intruder extension of the IBM [6,7] developed by Lehmann and co-workers to describe intruder states as a manifestation of O(6) symmetry embedded within the U(5) vibrational structure of the “regular” multiphonon excitations. The mutual underlying O(5) group structure causes the  $d$ -boson seniority  $\tau$  to be a good quantum number in both dynamical symmetries, and only states with identical  $d$ -boson seniorities can mix in the framework of the mentioned theoretical approach [6]. The  $2_1^+$  and  $2_2^+$  states have  $\tau=1$  and the  $4_1^+$  and  $4_2^+$  have  $\tau=2$  but they are largely separated in energy. We conclude that the mixing between the vibrational phonon structure and the intruder states is weak, with a certain deviation. The lower limit of the  $B(M1; 2_3^+ \rightarrow 2_1^+)$  strength equals  $0.03(1)\mu_N^2$  indicating a possibly weak proton-neutron collective, so-called “mixed-symmetry,” contribution in the wave function. In the framework of the proton-neutron version of the interacting boson model [23], states with partial proton-neutron anti-symmetries in the wave function are called mixed-symmetry states and the isovector quadrupole excitation in the valence shell  $2_{ms}^+$  is predicted as the lowest lying state of this class [11]. Its signature is a strong  $M1$  decay to the proton-neutron symmetric  $2_1^+$  state and a weakly collective  $E2$  transition to the ground state. This state also has  $d$ -boson seniority  $\tau=1$  and, therefore, can mix with the intruder  $2_2^+$  excitation. Additionally the  $2_{ms}^+$  state is usually found at about 2 MeV excitation energy, which is fairly close to the energy of the  $2_2^+$  excitation. This mixing would explain the rather strong  $M1$  decay connecting the  $2_2^+$  and  $2_1^+$  state. We stress that also in  $^{112}\text{Cd}$ , intruder and mixed-symmetry  $2^+$  states seem to mix [24]. We will deepen the comparison of  $^{108}\text{Cd}$  and neighboring isotopes in the following.

In Fig. 3 we compare the multiphonon and intruder excitations of  $^{108-112}\text{Cd}$ . The comparison of the two- and three-phonon states is very instructive and offers an insight into nuclear structure effects. The chain of cadmium isotopes forms a transitional path between the U(5) and the O(6) dynamical symmetry of the interacting boson model. The geometric equivalents are the harmonic vibrator and the gamma-unstable rotor model of Wilets and Jean [25], respectively. A striking difference in the excitation spectra predicted within these limits is the positions of the excited  $0^+$  states. In the U(5) limit the coupling of two  $Q$ -phonons results in the well-known  $4_1^+, 2_2^+, 0_2^+$  triplet [1]. In the O(6) dynamical symmetry a two-phonon  $0^+$  state cannot exist and, therefore, the first excited  $0^+$  state belongs to the three-phonon quintuplet of states [11]. Looking at Fig. 3 the lowest excited (non-intruder)  $0^+$  state clearly belongs to the two-phonon triplet in

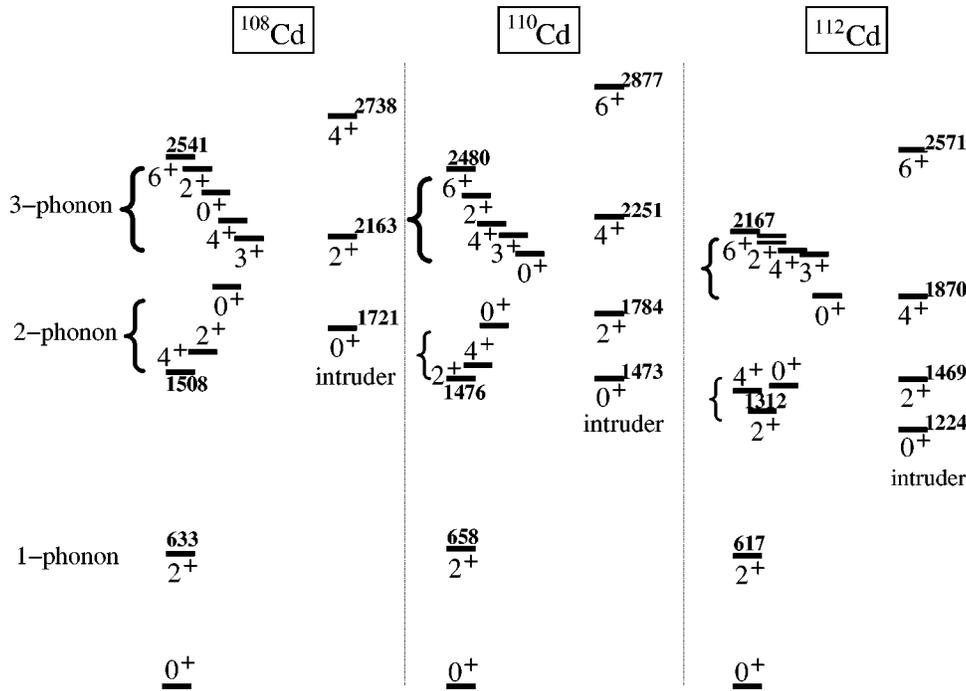


FIG. 3. Comparison of multiphonon and intruder structures in  $^{108-112}\text{Cd}$ . For the classification of the isotopes under the aspect of the underlying dynamical symmetry within the IBM it is very instructive to track the positions of the excited  $0^+$  states. As discussed in the text we stress that the chain of cadmium isotopes obviously lies on a transitional path between the dynamical U(5) and O(6) symmetry limits. We note that the energies and level spacings within the intruder bands (particle-hole configuration) change fairly rapidly compared to the collective multiphonon configurations.

$^{112}\text{Cd}$ , starts to separate in  $^{110}\text{Cd}$ , and lies even closer to the three-phonon quintuplet in  $^{108}\text{Cd}$ , while the next higher excited  $0^+$  state leaves the three-phonon multiplet possibly to become the ground state of the  $\sigma = N - 2$  subset in the O(6) dynamical symmetry [11]. Unfortunately the hindrance of the E2 decay rate could not be determined. That a transition to a O(6)-like structure is found is rather unexpected, and certainly would need confirmation, for instance, by measuring the absolute  $B(E2)$  rate of the  $0^+$  state.

With decreasing neutron number the energy of the intruder band head as well as the level spacings within the intruder band change rapidly while the position of the collective structures remains rather constant. The observed tendencies can be understood by comparing the proton  $2p-2h$  intruder states to an equivalent structure. The energetical increase of the intruder band head corresponds to the decreasing quadrupole interaction strength between protons and neutrons. For the intruder configuration a proton  $2p-2h$  structure (four quasiprotons) is assumed. Together with the two proton holes of  $Z=48$  the intruder band has a  $2p-4h$  configuration in the valence space (six quasiprotons). The proton-neutron interaction for this configuration has to be comparable to the  $p-n$  interaction in the ground band of the respective ruthenium isotope (Ru,  $Z=50-6$ ) [12]. The most useful terms to compare are the following  $4^+/2^+$  energy ratios [1,27]:

$$\text{ground band Ru, } R_{4/2}^{gb} = \frac{E(4_1^+)}{E(2_1^+)},$$

$$\text{intruder band Cd, } R_{4/2}^I = \frac{E(4_1^+) - E(0_1^+)}{E(2_1^+) - E(0_1^+)}.$$

For the corresponding ruthenium and cadmium isotones these energy ratios are compared in Fig. 4. Obviously, the

tendency of increasing  $R_{4/2}$  with increasing neutron number in the Ru isotopes can also be found for the intruder bands of the corresponding Cd isotones. In fact, the two curves only differ by an offset. We stress that it would be naive to expect exact equality for the  $R_{4/2}$  values, because the proton-neutron interaction is just one important part of the complex system of interactions forming the excitation spectrum of an atomic nucleus. Nevertheless the similar qualitative behavior is remarkable.

At the end we would like to stress that the multiphonon description presented in “The enigma of  $^{114}\text{Cd}$ ” [26] (see also [27]) cannot hold for the observations in  $^{108}\text{Cd}$ . In Ref. [26], states that perfectly fit into the intruder systematics and that are well described with the U(5)-O(6) interacting boson intruder mixing model [6,7] were also interpreted to belong

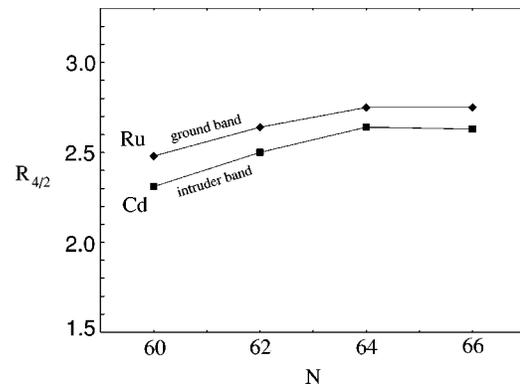


FIG. 4. Comparison of the six quasiproton  $2p-4h$  intruder bands in Cd (proton  $2p-2h$  configuration plus  $2h$  from  $Z=48$ ) with the ground band of the corresponding ruthenium isotones. We stress that Ru with  $Z=50-6$  also has six proton holes in the valence space. Indeed in Cd and Ru, one observes impressively similar tendencies for the level spacings.

to phonon multiplets using bizarre anharmonicities to reproduce their excitation energies. From our findings the assumption of a three-phonon nature of the  $2_3^+$  state can be ruled out, because all  $E2$  transitions to two-phonon states are small or nonexistent. The example of  $^{114}\text{Cd}$  nevertheless showed the importance of disentangling multiphonon and intruder structures, and prompted many studies on this subject.

Using a powerful combination of two complementary experimental techniques we established the intruder band in  $^{108}\text{Cd}$  up to the  $4_1^+$  member based on lower limits for the absolute  $E2$  transition strengths of interband and intraband decays. Excited states of  $^{108}\text{Cd}$  were populated in the fusion-evaporation reaction  $^{105}\text{Pd}(\alpha, n)^{108}\text{Cd}$  and also following the  $\beta$  decay of the  $7^+$  ground state and  $2^+$  low-spin isomer of  $^{108}\text{In}$ . The strong population of the  $2_3^+$  state in the  $\beta$  decay and the resulting off-beam  $\gamma\gamma$  spectroscopy enabled us to measure the tiny branching ratio of 0.3(1)% for its transition

to the  $0^+$  intruder state establishing a bandlike structure. In the  $(\alpha, n)$  reaction we finally could extend the intruder band up to the  $4^+$  member and determined effective lifetimes for the  $2_1^+$  and  $4_1^+$  excitations from a DSA line shape analysis. From the heavily suppressed transition strengths for the decays out of the band, we conclude that the mixing between multiphonon and intruder structures is weak in  $^{108}\text{Cd}$ . We successfully compared our findings to neighboring even mass Cd isotopes and related six quasiproton structures (ground band of the corresponding Ru isotope).

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