PHYSICAL REVIEW C, VOLUME 65, 041303(R)

Wobbling motion in atomic nuclei with positive-γ shapes

Masayuki Matsuzaki*
Department of Physics, Fukuoka University of Education, Munakata, Fukuoka 811-4192, Japan

Yoshifumi R. Shimizu†
Department of Physics, Graduate School of Sciences, Kyushu University, Fukuoka 812-8581, Japan

Kenichi Matsuyanagi‡
Department of Physics, Graduate School of Science, Kyoto University, Kyoto 606-8502, Japan

(Received 1 January 2002; published 20 March 2002)

The three moments of inertia associated with the wobbling mode built on the superdeformed states in ^{163}Lu are investigated by means of the cranked shell model plus random phase approximation to the configuration with an aligned quasiparticle. The result indicates that it is crucial to take into account the direct contribution to the moments of inertia from the aligned quasiparticle so as to realize $\mathcal{J}_x > \mathcal{J}_y$ in positive-γ shapes. Quenching of the pairing gap cooperates with the alignment effect. The peculiarity of the recently observed ^{163}Lu data is discussed by calculating not only the electromagnetic properties but also the excitation spectra.

DOI: 10.1103/PhysRevC.65.041303 PACS number(s): 21.10.Re, 21.60.Jz, 23.20.Lv, 27.70.+q

Rotation is one of the specific collective motions in finite many-body systems. Most of the nuclear rotational spectra can be understood as the outcome of one-dimensional (1D) rotations of axially symmetric nuclei. Two representative models—the moment of inertia of the irrotational fluid, $\mathcal{J}^{\mathrm{irr}}$, and that of the rigid rotor, $\mathcal{J}^{\mathrm{rig}}$, both specified by an appropriate axially symmetric deformation parameter β—could not reproduce the experimental ones given by $\mathcal{J}^{\mathrm{exp}}$; $\mathcal{J}^{\mathrm{irr}} < \mathcal{J}^{\mathrm{exp}} < \mathcal{J}^{\mathrm{rig}}$. From a microscopic viewpoint, the moment of inertia can be calculated as the response of the many-body system to an externally forced rotation—the cranking model [1]. This reproduces $\mathcal{J}^{\mathrm{exp}}$ well by taking into account the pairing correlation. Triaxial nuclei can rotate about their three principal axes and the three corresponding moments of inertia depend on their shapes in general. In spite of a lot of theoretical studies, their shape (in particular the triaxiality parameter γ) dependence has not been understood well because of the lack of decisive experimental data. Recently, some evidences of 3D rotations have been observed, such as the shears bands and the so-called chiral-twin bands [2]. In addition to these fully 3D motions, from the general argument of symmetry breaking, there must be a low-lying collective mode associated with the symmetry reduction from a 1D rotating axially symmetric mean field to a 3D rotating triaxial one. This is called the wobbling mode. Notice that the collective mode associated with the "phase transition" from an axially symmetric to a triaxial mean field in the nonrotating case is the well-known γ vibration. Therefore, the wobbling mode can be said to be produced by an interplay of triaxiality and rotation. The wobbling mode is described as a small amplitude fluctuation of the rotational axis away from the principal axis with the largest moment of inertia. Bohr and Mottelson first discussed this mode [3]. Mikhailov and Janssen [4] and Marshalek [5] described this mode in terms of the random phase approximation (RPA) in the rotating frame. In these works it was shown that at $\gamma = 0$ this mode turns into the odd-spin members of the γ vibrational band while at $\gamma = 60°$ or $-120°$ it becomes the precession mode built on the top of the high-K isomeric states [6]. Here we note that, according to the direction of the rotational axis relative to the three principal axes of the shape, γ runs from $-120°$ to $60°$.

Recently, electromagnetic (EM) properties of the second triaxial superdeformed (TSD2) band in ^{163}Lu were reported and it was concluded that the TSD2 is a wobbling band excited on the previously known yrast TSD1 band, on the basis of comparisons to a particle-rotor model (PRM) calculation [7,8]. In conventional PRM calculations an irrotational moment of inertia,

$$\mathcal{J}_k^{\mathrm{irr}} = \frac{4}{3}\mathcal{J}_0 \sin^2\left(\gamma + \frac{2}{3}\pi k\right), \tag{1}$$

where $k = 1$–3 denote the x, y, and z principal axes, is assumed. The magnitude $\mathcal{J}_0$ is treated as an adjustable parameter although it can be identified as $\mathcal{J}_0 = 3B_2\beta^2$, where B_2 is the inertia parameter in the Bohr Hamiltonian [9]. This reduces to $\mathcal{J}^{\mathrm{irr}}$ in the first paragraph by substituting $\gamma = 0$ and $k = 1$, and satisfies such a required property that collective rotations about the symmetry axes are forbidden. Since $\mathcal{J}_y^{\mathrm{irr}}$ is largest for $0 < \gamma < 60°$ and the main rotation occurs about the axis of the largest inertia, the PRM with $\mathcal{J}_k^{\mathrm{irr}}$ cannot describe the positive-γ rotation, that is, the rotation about the shortest axis (x axis). Then in Refs. [7,8] the so-called γ-reversed moment of inertia [10], $\mathcal{J}_k^{\mathrm{rev}}$, defined by inverting the sign of γ in Eq. (1), was adopted. Although this reproduced the measured EM properties well, this does not satisfy the required property mentioned above and its physical implications are not very clear. In this Rapid Communication, therefore, we study the moments of inertia associated with the wobbling motion excited on the positive-γ states by

*Email address: matsuza@fukuoka-edu.ac.jp
†Email address: yrsh2scp@mbox.nc.kyushu-u.ac.jp
‡Email address: ken@ruby.scphys.kyoto-u.ac.jp

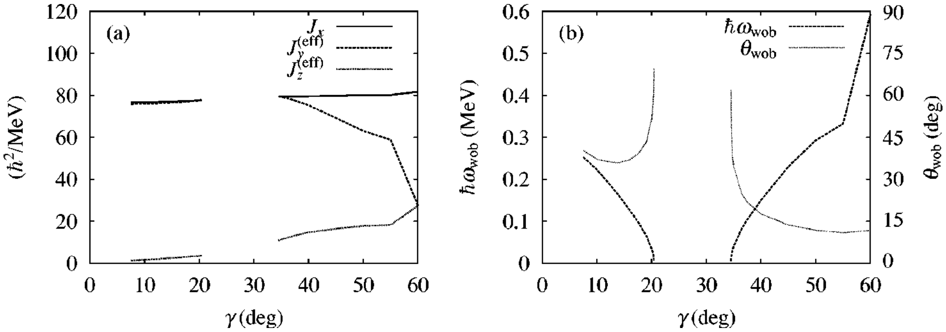

FIG. 1. (a) Moments of inertia and (b) wobbling frequency (left scale) and wobbling angle (right) in the five quasiparticle state in ^{147}Gd calculated as functions of γ at $\hbar\omega_{\rm rot}=0.3$ MeV. The dip around $\gamma=55°$ stems from a weak fragmentation of collectivity. Note that the present method of calculation does not apply to $\gamma\simeq 0$.

means of the cranked shell model plus RPA. This framework does not divide the system into a valence particle and a rotor, and therefore, can calculate the three moments of inertia of the whole system microscopically. We believe that this is the first step toward understanding the fully 3D nuclear rotations.

We have developed a computer code for the RPA to excitation modes built on configurations with arbitrary number of aligned quasiparticles (QPs). In this paper, we present the results for the 4–6 QP configurations in Gd isotopes and the 1 QP one in ^{163}Lu. In particular, this is the first RPA calculation for the rotating odd-A configurations, to our knowledge. Note that this approach is different from the conventional particle-vibration coupling calculations where the RPA itself is performed for the even-even "core" configurations. Since the details of the formulation have already been given in Refs. [11,12], here we describe only the outline. The QP states were obtained by diagonalizing the cranked triaxial Nilsson plus BCS Hamiltonian at each rotational frequency $\omega_{\rm rot}$ by adjusting chemical potentials to give correct average particle numbers. The doubly stretched $\mathbf{l}^2$ and $\mathbf{l}\cdot\mathbf{s}$ potentials were adopted, and their strengths were taken from Ref. [13]. The RPA calculation was performed by adopting the pairing plus doubly stretched Q-Q interaction. The existence of aligned QPs is taken into account by exchanging the definitions of the QP creation and annihilation operators in an appropriate manner. Actual calculations were done in five major shells ($N_n^{(\rm osc)}=3$–7 and $N_p^{(\rm osc)}=2$–6) by using the dispersion equation [5],

$$(\hbar\omega)^2=(\hbar\omega_{\rm rot})^2\frac{[\mathcal{J}_x-\mathcal{J}_y^{(\rm eff)}(\omega)][\mathcal{J}_x-\mathcal{J}_z^{(\rm eff)}(\omega)]}{\mathcal{J}_y^{(\rm eff)}(\omega)\mathcal{J}_z^{(\rm eff)}(\omega)}, \quad (2)$$

obtained by decoupling the Nambu-Goldstone mode analytically assuming $\gamma\neq 0$. This equation is independent of the strengths of the interaction. Not only the collective wobbling mode ($\omega=\omega_{\rm wob}$) but also many noncollective modes are obtained from this equation. The effective inertia $\mathcal{J}_{y,z}^{(\rm eff)}(\omega)=J_{y,z}^{(\rm PA)}(\omega)/\Omega_{y,z}(\omega)$, defined in the principal-axis (PA) frame (their concrete expressions were given in Ref. [12]), depend on the eigenmode while the kinematical $\mathcal{J}_x=\langle J_x\rangle/\omega_{\rm rot}$, where the expectation value taken with respect to the whole system is common to all the modes. It should be noted that Eq. (2) coincides with the original expression for $\omega_{\rm wob}$ [3] if $\mathcal{J}_x$ and $\mathcal{J}_{y,z}^{(\rm eff)}(\omega)$ are replaced with constant moments of inertia.

In the following, we present some numerical results. Here the parameters ϵ_2 (alternative to β), γ, Δ_n, and Δ_p were chosen so as to reproduce the available experimental data, and kept constant as functions of $\omega_{\rm rot}$. We have confirmed that qualitative features of the result are robust and the details of the parameter dependence will be given in a separate publication [14]. It is nontrivial to obtain the wobbling solution in the RPA for positive-γ nuclei and the QP alignment is indispensable for its appearance. In order to show this, we first discuss a theoretical calculation for a precession mode that might be built on top of the $I^\pi=49/2^+$ isomeric state in ^{147}Gd, where the whole angular momentum is built up by the alignment of the five QPs, $[(\pi h_{11/2})^2(\nu h_{9/2},f_{7/2})^2]_{18^+}$ in ^{146}Gd plus $[\nu i_{13/2}]_{13/2^+}$, so that a $\gamma=60°$ shape (axially symmetric about the x axis) is realized. This state is obtained by cranking with $\hbar\omega_{\rm rot}=0.3$ MeV. We chose $\epsilon_2=0.19$ and $\Delta_n=\Delta_p=0.6$ MeV, and reproduced the observed static quadrupole moment and the g factor [15,16]. In order to see the behavior of the three moments of inertia, we calculated the wobbling mode by changing the parameter γ from 60°. The result is presented in Fig. 1(a). Although at a first glance their γ dependence resembles that of the rigid rotor,

$$\mathcal{J}_k^{\rm rig}=\frac{16\pi}{15}B_2\left(1-\sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma+\frac{2}{3}\pi k\right)\right), \quad (3)$$

the physical content of $\mathcal{J}_x$ changes with γ; the fraction of the collective contribution decreases as γ increases and reaches 0 at $\gamma=60°$. Accordingly, it can be conjectured that the γ dependence of the "rotor" contribution is approximately irrotational and the QP contribution is superimposed on top of the former by aligning its angular momentum to the x axis. Our previous calculation [12,17] for a negative-γ nucleus, ^{182}Os, also supports this and consequently it is thought that the wobbling mode can appear relatively easily in superfluid negative-γ nuclei. To see if this conjecture is meaningful, starting from ^{146}Gd we add the $i_{13/2}$ quasineutrons sequentially. The result shows that $\mathcal{J}_x$ increases as the number of aligned QPs increases. Since the increase in $\mathcal{J}_{y,z}^{(\rm eff)}$ is rather moderate, the increase in $\mathcal{J}_x$ leads to that of the wobbling frequency $\omega_{\rm wob}$. Thus, the change from $\mathcal{J}_x<\mathcal{J}_y$ in $\mathcal{J}_k^{\rm irr}$ to $\mathcal{J}_x>\mathcal{J}_y$ in $\mathcal{J}_k^{\rm rev}$ may be related qualitatively to the increase in $\mathcal{J}_x$ stemming from the alignment that is not accounted for in the PRM, considering the fact that the alignment of particle states leads to $\gamma>0$.

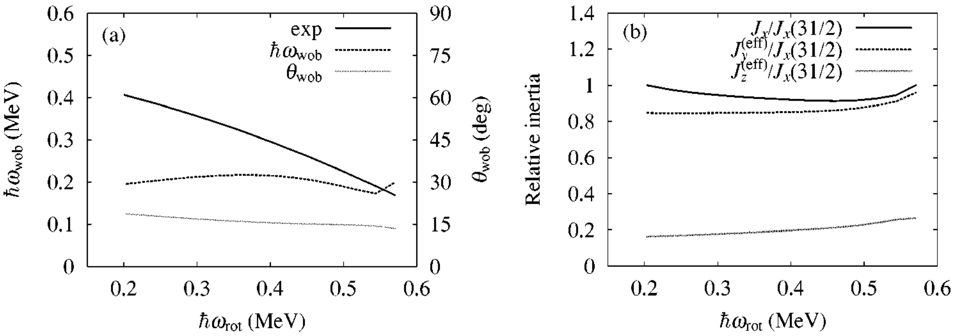

FIG. 2. (a) Wobbling frequency (left scale) and wobbling angle (right) and (b) moments of inertia in the TSD2 band in ^{163}Lu as functions of $\hbar\omega_{\rm rot}$. Here the latter were given by normalized to $\mathcal{J}_x(31/2)=99.2\hbar^2$/MeV. The proton BC crossing occurs at $\hbar\omega_{\rm rot}\gtrsim 0.55$ MeV in the calculation. Experimental values were calculated from the energy levels in Refs. [7,8].

At $\gamma\sim 30°$, where $\mathcal{J}_y$ reaches its maximum as in the irrotational model, we could not obtain a wobbling solution. In Fig. 1(b), $\hbar\omega_{\rm wob}$ and the wobbling angle

$$\theta_{\rm wob}=\tan^{-1}\frac{\sqrt{|J_y^{\rm (PA)}(\omega_{\rm wob})|^2+|J_z^{\rm (PA)}(\omega_{\rm wob})|^2}}{\langle J_x\rangle} \qquad (4)$$

are graphed. This shows that $\omega_{\rm wob}$ becomes imaginary and $\theta_{\rm wob}$ blows up in this region. Comparing Figs. 1(a) and 1(b), it may be inferred that the wobbling motion excitation on a mean field rotating about the x axis becomes unstable at $\gamma\sim 30°$ due to $\mathcal{J}_x<\mathcal{J}_y^{\rm (eff)}$, and that a tilted-axis rotation would be realized. Putting this unstable region in between, the solution in the larger-γ side is like a precession of an axially symmetric body about the x axis, whereas that in the smaller-γ side is like a γ vibration around an axially symmetric shape about the z axis.

Now we turn to the TSD bands in ^{163}Lu. We chose $\epsilon_2=0.43$, $\gamma=20°$, and $\Delta_n=\Delta_p=0.3$ MeV, and obtained transition quadrupole moments $Q_t=10.9$–11.3 e b for $\hbar\omega_{\rm rot}=0.20$–$0.57$ MeV in accordance with the data, $Q_t=10.7\pm 0.7$ e b [18]. We have obtained for the first time (aside from the theoretical simulation above) the wobbling solution in the RPA for positive-γ nuclei. Here it should be stressed that the inclusion of the five major shells and the alignment effect of the proton $i_{13/2}$ quasiparticle is essential for obtaining this result. In Fig. 2(a) the measured excitation energy of the TSD2 band relative to that of the TSD1 and the calculated $\hbar\omega_{\rm wob}$ are shown. The most peculiar point in the experimental data is that $\omega_{\rm wob}$ decreases as a function of $\omega_{\rm rot}$. If $\omega_{\rm rot}$-independent moments of inertia such as the irrotational ones are adopted, $\omega_{\rm wob}$ increases linearly with $\omega_{\rm rot}$, see the comment below Eq. (2). The wobbling frequency is sensitive to the difference among the three moments of inertia, and the ratios $\mathcal{J}_y^{\rm (eff)}/\mathcal{J}_x$ and $\mathcal{J}_z^{\rm (eff)}/\mathcal{J}_x$ actually determine $\omega_{\rm wob}$. For example, the γ-reversed moments of inertia give $\mathcal{J}_y^{\rm rev}/\mathcal{J}_x^{\rm rev}=0.43$ and $\mathcal{J}_z^{\rm rev}/\mathcal{J}_x^{\rm rev}=0.12$ for $\gamma=20°$ leading to $\omega_{\rm wob}\simeq 3\omega_{\rm rot}$, which is quite different from the experimental data. In contrast, as shown in Fig. 2(b), the three moments of inertia calculated microscopically depend on $\omega_{\rm rot}$ even when the shape parameters are fixed, and the resultant $\omega_{\rm wob}$ can either increase or decrease in general. In the present case of ^{163}Lu in Fig. 2, $\mathcal{J}_x-\mathcal{J}_y^{\rm (eff)}$ mainly determines the $\omega_{\rm rot}$ dependence. Its decrease is a consequence of that of $\mathcal{J}_x$; the partial contribution to $\mathcal{J}_x$ from the proton $i_{13/2}$, $i_x/\omega_{\rm rot}$, decreases as $\omega_{\rm rot}$ increases since this orbital is already fully aligned and therefore the aligned angular momentum i_x is approximately constant. Thus, our result for $\omega_{\rm wob}$ stays almost constant against $\omega_{\rm rot}$, and even decreases slightly at higher frequencies approaching the experimentally observed one. This clearly shows that microscopic calculation of the three moments of inertia is crucial to understand the $\omega_{\rm rot}$ dependence of $\omega_{\rm wob}$ in ^{163}Lu. Let us compare this result with that for ^{147}Gd above. In ^{147}Gd, $\mathcal{J}_y^{\rm (eff)}/\mathcal{J}_x\simeq 1$, $\mathcal{J}_z^{\rm (eff)}\sim 0$, and $|Q_1^{(-)}/Q_2^{(-)}|\ll 1$ at $\gamma\lesssim 20°$. The last quantity measures the rotational K-mixing. This indicates that this solution is essentially similar to the γ vibration in an axially symmetric nucleus as mentioned above. In contrast, the result that $\mathcal{J}_y^{\rm (eff)}/\mathcal{J}_x=0.90$, $\mathcal{J}_z^{\rm (eff)}/\mathcal{J}_x=0.19$, and $|Q_1^{(-)}/Q_2^{(-)}|=0.78$ for ^{163}Lu at $\hbar\omega_{\rm rot}=0.3$ MeV, for example, indicates that this solution is more like a wobbling motion of a triaxial body. The wobbling angle shown in Fig. 2(a) is 19°–13° for the calculated range. It is evident that the present small-amplitude approximation holds better at high spins. We confirmed that this wobbling solution disappeared as γ decreased. Another feature distinct from the γ vibration is that the present solution exists even at $\Delta_n=\Delta_p=0$, whereas it is well known that the pairing field is indispensable for the existence of low-lying shape vibrations. This is related to such a tendency that the moments of inertia approach the rigid ones, $\mathcal{J}_x>\mathcal{J}_y$ for $\gamma>0$, as the pairing gap decreases even without aligned QPs.

A significant point of the data in Refs. [7,8] is that the interband EM transition rates connecting the states I (TSD2) to $I-1$ (TSD1) were precisely measured. In Fig. 3, we compare our numerical results with the measured ones in a form similar to those in Refs. [7,8]. Calculated values for I (TSD2) $\rightleftharpoons I+1$ (TSD1) are also included in order to show the staggering behavior characteristic to this kind of transitions [12]. Figure 3(a) presents the relative $B(E2)$. The data indicate huge collectivity of the interband $B(E2)$, such as 170 Weisskopf unit. Although the present RPA solution is extremely collective, $|c_{n=\rm wob}|\simeq 0.9$ in the sum rule (Eq. (4.30) in Ref. [12]), in comparison to usual low-lying vibrations, the calculation accounts for 1/2–1/3 of the measured strength. Figure

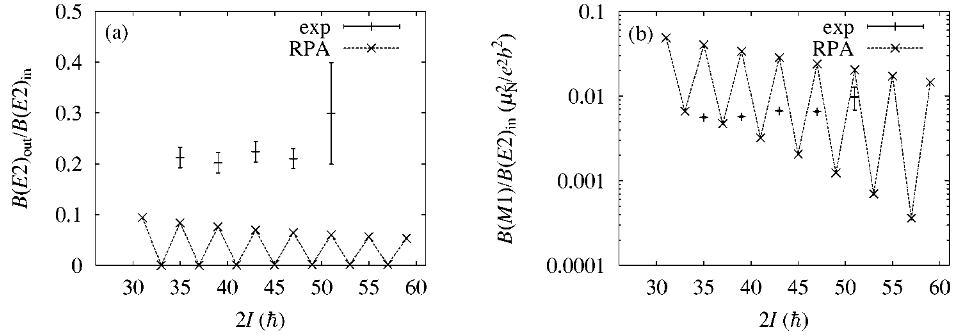

FIG. 3. Interband transition rates for I (TSD2) $\rightarrow I\pm 1$ (TSD1) transitions as functions of $2\times$ spin I, (a) $E2$ and (b) $M1$. They are divided by the in-band $E2(I\rightarrow I-2)$ transition rates. Experimental values were taken from Ref. [8]. Noting that the states $I+1$ (TSD1) are slightly higher in energy than I (TSD2) at $I>51/2$ and $B(T_\lambda;I\rightarrow I+1)\simeq B(T_\lambda;I+1\rightarrow I)$ at high spins, we plotted those for $I\rightarrow I+1$ at the places with the abscissae $I+1$ in order to show clearly their characteristic staggering behavior.

3(b) graphs $B(M1)/B(E2)_{\rm in}$. The smallness of $B(M1)$ also reflects collectivity, that is, the coherence with respect to the $E2$ operator, indirectly. Having confirmed the insensitivity to $g_s^{\rm (eff)}$, we adopted $0.6g_s^{\rm (free)}$ conforming to Ref. [8] and calculated $B(M1)$. The result is similar to that of the PRM. We confirmed that the sign of the $E2/M1$ mixing ratios was correct.

To summarize, we have performed, for the first time, the RPA calculation in the rotating frame to the triaxial superdeformed odd-A nucleus ^{163}Lu and discussed the physical conditions for the appearance of the wobbling solution in the RPA. We have confirmed that the proton $i_{13/2}$ alignment is indispensable for the appearance of the wobbling mode in this nucleus. The appearance of the wobbling mode requires $\mathcal{J}_x>\mathcal{J}_y^{\rm (eff)}(\neq\mathcal{J}_z^{\rm (eff)})$, but the moments of inertia of the even-even core exhibit irrotational-like γ dependence and, therefore, cannot fulfill this condition for positive-γ shapes. Consequently, the alignment effect that increases $\mathcal{J}_x$ is necessary. Quenching of the pairing correlation also cooperates with the alignment effect for making the γ dependence rigidlike.

We acknowledge useful discussions with I. Hamamoto and H. Madokoro. This work was supported in part by the Grant-in-Aid for scientific research from the Japan Ministry of Education, Science and Culture (Nos. 13640281 and 12640281).

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