

## Light $\Lambda\Lambda$ hypernuclei and the onset of stability for $\Lambda\Xi$ hypernuclei

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New Faddeev-Yakubovsky calculations for light  $\Lambda\Lambda$  hypernuclei are presented in order to assess the self-consistency of the  $\Lambda\Lambda$ -hypernuclear binding-energy world data and the implied strength of the  $\Lambda\Lambda$  interaction, in the wake of recent experimental reports on  ${}_{\Lambda\Lambda}^4\text{H}$  and  ${}_{\Lambda\Lambda}^6\text{He}$ . Using Gaussian soft-core simulations of Nijmegen one-boson-exchange model interactions, the Nijmegen soft-core model NSC97 simulations are found close to reproducing the recently reported binding energy of  ${}_{\Lambda\Lambda}^6\text{He}$ , but not those of other species. For stranger systems, Faddeev calculations of light  $\Lambda\Xi$  hypernuclei, using a simulation of the strongly attractive  $\Lambda\Xi$  interactions due to the same model, suggest that  ${}_{\Lambda\Xi}^6\text{He}$  marks the onset of nuclear stability for  $\Xi$  hyperons.

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Very little is known experimentally on doubly strange hypernuclear systems, and virtually nothing about systems with higher strangeness content. Multistrange hadronic matter in finite systems and in bulk is predicted on general grounds to be stable, up to strangeness violating weak decays (Ref. [1], and references therein). Hyperons ( $Y$ ) must contribute macroscopically to the composition of neutron-star matter (Ref. [2], and references therein). Over the years the Nijmegen group has constructed a number of one-boson-exchange (OBE) models for the baryon-baryon interaction, fitting the abundant scattering and bound-state  $NN$  data plus the scarce and poorly determined low-energy  $YN$  data using SU(3)-flavor symmetry to relate baryon-baryon-meson coupling constants and phenomenological short-distance hard or soft cores (Ref. [3], and references therein). Data on multistrange systems could help distinguish between these models. The recently reported events from AGS experiment E906 suggest production of light  $\Lambda\Lambda$  hypernuclei [4], perhaps as light even as  ${}_{\Lambda\Lambda}^4\text{H}$ , in the  $(K^-, K^+)$  reaction on  ${}^9\text{Be}$ . If  ${}_{\Lambda\Lambda}^4\text{H}$  is confirmed in a future extension of this experiment, this four-body system  $pn\Lambda\Lambda$  would play as fundamental a role for studying theoretically the  $YY$  forces as  ${}^3_\Lambda\text{H}$  ( $pn\Lambda$ ) has played for studying theoretically the  $YN$  forces [5].

Until recently only three  $\Lambda\Lambda$  hypernuclear candidates fitted events seen in emulsion experiments [6–8]. The  $\Lambda\Lambda$  binding energies deduced from these “old” events suggest a strongly attractive  $\Lambda\Lambda$  interaction in the  ${}^1S_0$  channel [9]. This outlook might be changing substantially following the very recent report from the hybrid-emulsion KEK experiment E373 on a new event [10] uniquely interpreted as  ${}_{\Lambda\Lambda}^6\text{He}$ , with binding energy considerably smaller than that reported for the older event [7].

In this Rapid Communication we report on new Faddeev-Yakubovsky calculations for light  $\Lambda\Lambda$  hypernuclei, using generic  $s$ -wave  $\Lambda\Lambda$  interaction potentials which simulate the low-energy  $s$ -wave scattering parameters produced by the Nijmegen OBE models. The purpose of these calculations is twofold: to check the self-consistency of the data, particularly for  ${}_{\Lambda\Lambda}^6\text{He}$  and  ${}_{\Lambda\Lambda}^{10}\text{Be}$  which are treated here as clusters of  $\alpha$ 's and  $\Lambda$ 's, and to find out which of the Nijmegen OBE models is the most appropriate one for describing these  $\Lambda\Lambda$  hypernuclei.

A novel piece of work concluding this report concerns multistrange hypernuclei consisting, in addition to  $\Lambda$ 's, also of a (doubly strange  $S=-2$ )  $\Xi$  hyperon. Schaffner *et al.* [11] observed that  $\Xi$  hyperons would become particle stable against the strong decay  $\Xi N \rightarrow \Lambda\Lambda$  if a sufficient number of bound  $\Lambda$ 's Pauli blocked this decay mode, highlighting  ${}_{\Lambda\Lambda\Xi}^7\text{He}$  ( $S=-4$ ) as the lightest system of its kind. Here we study the possibility of stabilizing a  $\Xi$  hyperon in the isodoublet  ${}_{\Lambda\Xi}^6\text{H} - {}_{\Lambda\Xi}^6\text{He}$  ( $S=-3$ ) hypernuclei due to the particularly strong  $\Lambda\Xi$  attraction in the Nijmegen soft-core NSC97 model [12]. This three-body  $\alpha\Lambda\Xi$  system may provide the onset of  $\Xi$  nuclear stability.

In our calculations, the bound states of three- and four-body systems are obtained by solving the differential  $s$ -wave Faddeev-Yakubovsky equations [13], using the cluster reduction method [14] in which the various channel wave functions are decomposed in terms of eigenfunctions of the Hamiltonians of the two- or three-particle subsystems. A fairly small number of terms, generally less than 10, is sufficient to generate a stable and precise numerical solution. This method has been recently applied to  ${}^9_\Lambda\text{Be}$  and  ${}_{\Lambda\Lambda}^6\text{He}$  in terms of three-cluster  $\alpha\alpha\Lambda$  and  $\alpha\Lambda\Lambda$  systems, respectively [15].

The hyperon-hyperon interaction potentials in the  ${}^1S_0$  channel which are used as input to the above equations are of a three-range Gaussian form,

$$V_{YY'} = \sum_i^3 v_{YY'}^{(i)}(r) \exp(-r^2/\beta_i^2), \quad (1)$$

following the work of Hiyama *et al.* [16] where a  $\Lambda\Lambda$  potential of this form was fitted to the Nijmegen model D (ND) hard-core interaction [17] assuming the same hard core for the  $NN$  and  $\Lambda\Lambda$  potentials in the  ${}^1S_0$  channel. For other models we have renormalized the medium-range attractive component ( $i=2$ ) of this potential such that it yields values as close to the values produced by Nijmegen model interaction potentials for these low-energy parameters. Several  $YY$  potentials fitted to the low-energy parameters of the soft-core NSC97 model [12] are shown in Fig. 1. We note that the  $\Lambda\Xi$

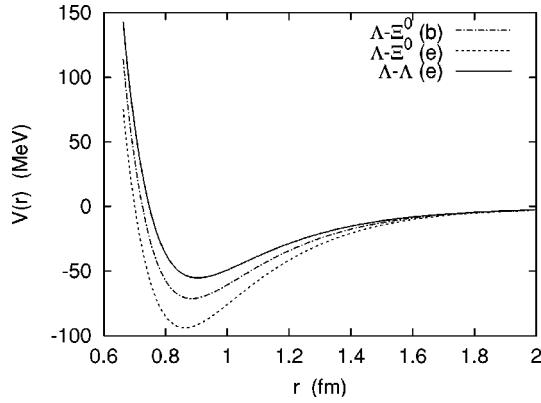


FIG. 1. Selected hyperon-hyperon potentials, simulating versions *b* and *e* of the NSC97 model interactions (Ref. [12]).

interaction is rather strong, considerably stronger within the same version of the model (here, *e*) than the  $\Lambda\Lambda$  interaction. The  $\Lambda\Lambda$  interaction is fairly weak for all of the six versions (*a*–*f*) of model NSC97.

The  $\alpha\alpha$  short-range interaction, and the  $\Lambda\alpha$  and  $\Xi\alpha$  interactions, are given in terms of a two-range Gaussian (Isle) potential

$$V_{Y\alpha} = V_{\text{rep}}^{(Y)} \exp(-r^2/\beta_{\text{rep}}^2) - V_{\text{att}}^{(Y)} \exp(-r^2/\beta_{\text{att}}^2). \quad (2)$$

Here the superscript *Y* extends also for  $\alpha$ . For the  $\alpha\alpha$  short-range potential we used the *s*-wave component of the Ali-Bodmer potential [18]. A finite-size Coulomb potential was added. The  $\Lambda\alpha$  potential, fitted to the binding energy  $B_{\Lambda}({}_{\Lambda}^5\text{He}) = 3.12 \pm 0.02$  MeV [19], was taken from Ref. [20]. For the  $\Xi\alpha$  potential we assumed  $V_{\text{rep}}^{(\Xi)} = V_{\text{rep}}^{(\Lambda)}$  while reducing the depth  $V_{\text{att}}^{(\Lambda)}$  to get the  $\Xi^0\alpha$  binding energy 2.09 MeV. This  $B_{\Xi}$  value was obtained using a Woods-Saxon (WS) potential for  ${}^4\text{He}$  with a depth parameter scaled by the ratio of central densities with respect to a depth of  $\sim 15$  MeV in  ${}^{11}\text{B}$ , as suggested by studying the excitation spectrum in the ( $K^-$ ,  $K^+$ ) reaction on  ${}^{12}\text{C}$  [21].

We first applied, for a test, these  $\alpha\alpha$  and  $\Lambda\alpha$  potentials (2) in a three-body *s*-wave Faddeev calculation for the  $\alpha\alpha\Lambda$  system. We will comment below on the restriction to *s* waves. The calculated ground-state binding energy,  $B_{\Lambda}({}_{\Lambda}^9\text{Be}) = 6.67$  MeV, is in excellent agreement with the measured value  $6.71 \pm 0.04$  MeV [19] without need for renormalization [16] or for introducing three-body interactions [22]. We then applied these potentials in Faddeev-Yakubovsky calculations for several  $\Lambda\Lambda$  hypernuclei, using  $\Lambda\Lambda$  interactions generically of the form (1) which simulate some of the Nijmegen OBE interaction potentials. The results are stable against reasonable variations in the  $\Lambda\Lambda$  potential shape, provided the underlying low-energy parameters are kept fixed. The ground-state  $\Lambda\Lambda$  binding energies  $B_{\Lambda\Lambda}$  obtained by solving the *s*-wave three-body ( $\alpha\Lambda\Lambda$ ) Faddeev equations for  ${}_{\Lambda\Lambda}^6\text{He}$  and the *s*-wave four-body ( $\alpha\alpha\Lambda\Lambda$ ) Yakubovsky equations for  ${}_{\Lambda\Lambda}^{10}\text{Be}$  are given in Table I. Using the ND-simulated  $\Lambda\Lambda$  interaction our results may be compared with those of Ref. [16] which were not limited to the dominant *s*-wave channels. For  ${}_{\Lambda\Lambda}^6\text{He}$ , and with similar  $\Lambda\alpha$

TABLE I. Calculated ground-state binding energies ( $B_{\Lambda\Lambda}$  in MeV with respect to the nuclear core).

Model	${}_{\Lambda\Lambda}^6\text{He}$	${}_{\Lambda\Lambda}^{10}\text{Be}$
ESC00	10.7	19.5
ND	9.10	17.8
NSC97 <i>e</i>	6.82	15.5
NSC97 <i>b</i>	6.60	15.3
$V_{\Lambda\Lambda} = 0$	6.27	14.9
ND <sup>a</sup>	9.34	17.24
Experiment	$10.9 \pm 0.6^b$	$17.7 \pm 0.4^c$
	$7.25 \pm 0.19_{-0.11}^{+0.18^d}$	$(14.6 \pm 0.4)^e$

<sup>a</sup>Reference [16].

<sup>b</sup>Reference [7].

<sup>c</sup>Reference [6].

<sup>d</sup>Reference [10].

<sup>e</sup>Assuming  ${}_{\Lambda\Lambda}^{10}\text{Be} \rightarrow \pi^- + p + {}_{\Lambda}^9\text{Be}^*$ .

potentials, the inclusion of higher (*d*) partial waves amounts to additional 0.2 MeV binding. For  ${}_{\Lambda\Lambda}^{10}\text{Be}$  the effect of the higher partial waves is largely compensated by keeping  $B_{\Lambda}({}_{\Lambda}^9\text{Be})$  at its experimental value, whether or not including *d* waves. This was also the practice in Ref. [16]; the comparison in Table I suggests an effect of order 0.5 MeV, which is similar to the effect of model dependence due to using different underlying  $\Lambda N$  interaction potentials in that work. Focusing on our own calculations, Table I shows that the strongest  $\Lambda\Lambda$  binding is provided by the simulation of the very recent extended soft-core (ESC00) model [3] which was in fact motivated by the relatively large  $B_{\Lambda\Lambda}$  value for the  ${}_{\Lambda\Lambda}^6\text{He}$  “old” event [7]. A significantly smaller  $B_{\Lambda\Lambda}$  value is obtained for our simulation of model ND which, however, reproduces well the  $B_{\Lambda\Lambda}$  value reported for  ${}_{\Lambda\Lambda}^{10}\text{Be}$  [6]. Down the list, the simulation of the NSC97 model gives yet smaller  $B_{\Lambda\Lambda}$  values, which for  ${}_{\Lambda\Lambda}^6\text{He}$  are close to the very recent experimental report [10] almost independently of which version of the model is used.

Early cluster calculations [22,23] noted that the calculated  $B_{\Lambda\Lambda}$  values for  ${}_{\Lambda\Lambda}^6\text{He}$  and for  ${}_{\Lambda\Lambda}^{10}\text{Be}$  are correlated nearly linearly with each other, such that the two events reported in the 1960s could not be reproduced simultaneously. Our calculations also produce such a correlation, as demonstrated in Fig. 2 by the solid circles along the dotted line. This line precludes any joint theoretical framework in terms of two-body interactions alone for the  ${}_{\Lambda\Lambda}^6\text{He}$  and  ${}_{\Lambda\Lambda}^{10}\text{Be}$  experimental candidates listed in Table I. For  $V_{\Lambda\Lambda} = 0$ , the lower-left point on the dotted line corresponds to approximately zero incremental binding energy  $\Delta B_{\Lambda\Lambda}$  for  ${}_{\Lambda\Lambda}^6\text{He}$ , where

$$\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^AZ) = B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^AZ) - 2B_{\Lambda}({}_{\Lambda}^{A-1}Z). \quad (3)$$

This is easy to understand owing to the rigidity of the  $\alpha$  core. However, the corresponding  $\Delta B_{\Lambda\Lambda}$  value for  ${}_{\Lambda\Lambda}^{10}\text{Be}$  is fairly substantial, about 1.5 MeV, reflecting a basic difference between the four-body  $\alpha\alpha\Lambda\Lambda$  calculation and any three-body approximation in terms of a nuclear core and two  $\Lambda$ 's as in  ${}_{\Lambda\Lambda}^6\text{He}$ . To demonstrate this point we show by the open

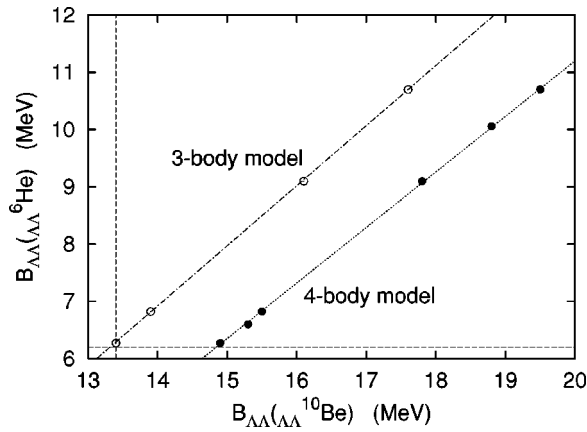


FIG. 2. Calculated binding energies ( $B_{\Lambda\Lambda}$  in MeV) for  ${}^6_{\Lambda\Lambda}\text{He}$  in a three-body  $\alpha\Lambda\Lambda$  model, and for  ${}^{10}_{\Lambda\Lambda}\text{Be}$  in a four-body  $\alpha\alpha\Lambda\Lambda$  model and in a three-body  ${}^8\text{Be}\Lambda\Lambda$  model. The origin of the dashed axes corresponds to  $\Delta B_{\Lambda\Lambda} = 0$ .

circles along the dot-dashed line in Fig. 2 the results of a three-body calculation for  ${}^{10}_{\Lambda\Lambda}\text{Be}$  in which the  ${}^8\text{Be}$  core is not assigned an  $\alpha\alpha$  structure. In this calculation, the geometry and depth of the  $\Lambda$ - ${}^8\text{Be}$  WS potential were fitted to reproduce (i) the measured  $B_{\Lambda}({}^9\text{Be})$  value and (ii) the rms distance between the  $\Lambda$  and the c.m. of the two  $\alpha$ 's as obtained in the  $\alpha\alpha\Lambda$  model calculation for  ${}^9\text{Be}$ . This three-body  ${}^8\text{Be}\Lambda\Lambda$  calculation gives about 1.5 MeV less binding for  ${}^{10}_{\Lambda\Lambda}\text{Be}$  than the four-body calculation does. The difference is due to the  $\alpha\alpha$  correlations which are absent in the three-body calculation, and which are built in within the Yakubovsky equations of the four-body calculation. The other calculations mentioned above [22,23] found smaller values, not exceeding 0.5 MeV, for the binding-energy gain due to having a four-body calculation for  ${}^{10}_{\Lambda\Lambda}\text{Be}$ . An obvious merit of our four-body Faddeev-Yakubovsky calculation is that it automatically accounts for *all* possible rearrangement channels in the  $\alpha\alpha\Lambda\Lambda$  system. In particular, by breaking up  ${}^8\text{Be}$  into two  $\alpha$ 's in the four-body calculation, substantial attraction is gained due to several additional bound subsystems such as the  ${}_{\Lambda\Lambda}\text{He}-\alpha$  and  ${}^5_{\Lambda}\text{He}-{}^5_{\Lambda}\text{He}$  clusters which almost saturate the corresponding rearrangement channels  $(\alpha\Lambda\Lambda)-\alpha$  and  $(\alpha\Lambda)-(\alpha\Lambda)$ , respectively.

Our calculations confirm, if not aggravate, the incompatibility of the “old” experimental determination of the binding energy of  ${}^6_{\Lambda\Lambda}\text{He}$  [7] with that of  ${}^{10}_{\Lambda\Lambda}\text{Be}$  [6]. The “new” experimental determination of the binding energy of  ${}^6_{\Lambda\Lambda}\text{He}$  [10] is found to be still incompatible with that of  ${}^{10}_{\Lambda\Lambda}\text{Be}$ , even if an unobserved  $\gamma$  deexcitation involving either  ${}^{10}_{\Lambda\Lambda}\text{Be}^*$  or  ${}^9_{\Lambda}\text{Be}^*$  is allowed; one of these possibilities, involving  ${}^9_{\Lambda}\text{Be}^*$  at 3.1 MeV [24], is recorded in Table I. Since no particle-stable excited states are possible for  ${}^6_{\Lambda\Lambda}\text{He}$  or for its  $\Lambda$  hypernuclear core  ${}^5_{\Lambda}\text{He}$ , and since  ${}^6_{\Lambda\Lambda}\text{He}$  is also ideally suited for three-body cluster calculations such as the  $s$ -wave Faddeev equations here solved for the  $\alpha\Lambda\Lambda$  system, we continue by discussing the implications of accepting the E373 KEK experiment [10] determination of  $\Delta B_{\Lambda\Lambda} \sim 1$  MeV for  ${}^6_{\Lambda\Lambda}\text{He}$ . We have shown that model NSC97 is the only one

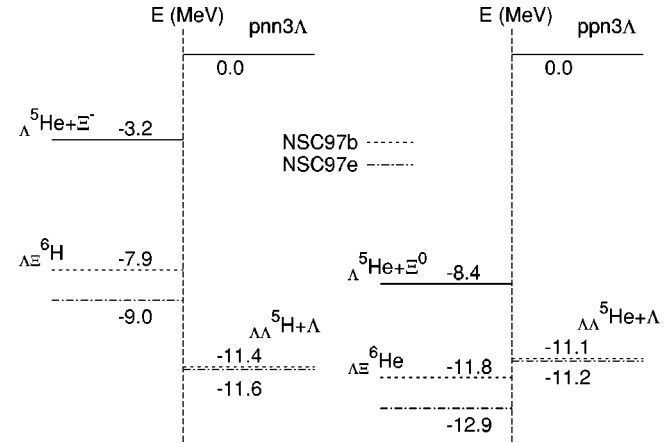


FIG. 3. Calculated level scheme of  ${}^6_{\Lambda\Xi}\text{H}$  and  ${}^6_{\Lambda\Xi}\text{He}$  hypernuclei.

capable of getting close to this “new” binding-energy value, short by about 0.5 MeV. In fact, we estimate the theoretical uncertainty of our Faddeev calculation for  ${}^6_{\Lambda\Lambda}\text{He}$  as bounded by 0.5 MeV, such that a more precisely calculated binding energy would be *larger* by a fraction of this bound, at most, than the  $B_{\Lambda\Lambda}$  values shown in Table I. Taking into account such possible corrections would bring our calculated  $B_{\Lambda\Lambda}$  values to within the error bars of the reported  $B_{\Lambda\Lambda}$  value. There are two possible origins for this theoretical uncertainty. One, which was already mentioned above, is the restriction to  $s$  waves in the partial-wave expansion of the Faddeev equations; the other one is ignoring the off-diagonal  $\Lambda\Lambda-\Xi N$  interaction which admixes  $\Xi$  components into the  ${}^6_{\Lambda\Lambda}\text{He}$  wave function. A recent work [25] using two  $YN$  and  $YY$  models finds an increase of 0.1 to 0.4 MeV in the calculated  $B_{\Lambda\Lambda}({}^6_{\Lambda\Lambda}\text{He})$  value due to a 0.1% to 0.3% (probability)  $\Xi$  component, respectively.

If model NSC97 indeed provides for a valid extrapolation from fits to  $NN$  and  $YN$  data, and recalling the strongly attractive  ${}^1S_0$   $\Lambda\Xi$  potentials in Fig. 1 simulating the NSC97 model, it is tempting to check for stability of  $A=6$ ,  $S=-3$  systems obtained from  ${}^6_{\Lambda\Lambda}\text{He}$  by replacing a  $\Lambda$  by a  $\Xi$  hyperon. The results of a Faddeev calculation for the isodoublet hypernuclei  ${}^6_{\Lambda\Xi}\text{H}$  and  ${}^6_{\Lambda\Xi}\text{He}$ , considered as  $\alpha\Lambda\Xi^-$  and  $\alpha\Lambda\Xi^0$  three-body systems, respectively, are shown in Fig. 3, including the location of the lowest particle-stability thresholds, due to  $\Lambda$  emission into  ${}^5_{\Lambda\Lambda}\text{H}$  and  ${}^5_{\Lambda\Lambda}\text{He}$ , respectively. These  $A=5$  isodoublet  $\Lambda\Lambda$  hypernuclei, considered as three-cluster systems  ${}^3\text{H}\Lambda\Lambda$  and  ${}^3\text{He}\Lambda\Lambda$ , respectively, are found to be particle stable for *all* the  $\Lambda\Lambda$  attractive potentials used in the present calculation. Figure 3 demonstrates that  ${}^6_{\Lambda\Xi}\text{He}$  is particle stable for potentials simulating the NSC97 model. The mirror hypernucleus  ${}^6_{\Lambda\Xi}\text{H}$  is unstable because the  $\Xi^-$  hyperon is heavier by 6.5 MeV than  $\Xi^0$ . Our prediction for the stability of  ${}^6_{\Lambda\Xi}\text{He}$  would hold valid, particularly for potentials simulating model NSC97e (and also *f*), even if the binding energy of  ${}^6_{\Lambda\Lambda}\text{He}$  is increased by a fraction of an MeV to scale it with the recently reported  ${}^6_{\Lambda\Lambda}\text{He}$  binding energy [10]. However, if the  $\Xi\alpha$  WS potential depth is set equal to that for  $\Xi$  in  ${}^{11}\text{B}$  [21],  ${}^6_{\Lambda\Xi}\text{He}$  would become unstable by a fraction of an MeV in version *e*. Lack

of direct experimental evidence on  $\Xi$  interactions in or around  ${}^4\text{He}$  prevents us from reaching a more definitive conclusion on this issue.

In summary, we have shown that  $s$ -wave simulations of the OBE Nijmegen model NSC97, versions  $e$  and  $f$  of which have been shown recently to agree quantitatively with light single  $\Lambda$  hypernuclei [26], are capable of reproducing the recently reported binding energy of  ${}_{\Lambda\Lambda}{}^6\text{He}$ , but are incapable of reproducing previously reported  $\Lambda\Lambda$  binding energies. This inconsistency, for a wide class of  $\Lambda\Lambda$  potentials, was demonstrated on firm grounds by doing the first ever Faddeev-Yakubovsky calculation of  ${}_{\Lambda\Lambda}{}^{10}\text{Be}$  as a  $\alpha\alpha\Lambda\Lambda$  four-cluster system. Accepting the predictive power of model

NSC97, our calculations suggest that  ${}_{\Lambda\Xi}{}^6\text{He}$  may be the lightest particle-stable  $S = -3$  hypernucleus, and the lightest and least strange particle-stable hypernucleus in which a  $\Xi$  hyperon is bound. Unfortunately, the direct production of  $\Lambda\Xi$  hypernuclei is beyond present experimental capabilities, requiring the use of  $\Omega^-$  initiated reactions.

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