Fusion window problem in time-dependent Hartree-Fock theory revisited

M. Tohyama
Kyorin University School of Medicine, Mitaka, Tokyo 181-8611, Japan

A. S. Umar
Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee 37235
(Received 25 September 2001; published 15 February 2002)

Head-on collisions of $^{16}O+^{16}O$ are calculated using an extended version of the time-dependent Hartree-Fock theory known as the time-dependent density-matrix theory (TDDM). TDDM deals with both one-body and two-body dissipation in a quantum-mechanical way. A newly developed TDDM code, which includes spin-orbit force in a mean field, is used. It is found that the threshold energy above which the colliding nuclei do not fuse increases due to two-body dissipation. Head-on collisions of $^{22}O+^{22}O$ are also calculated to investigate the effects of two-body collisions in exotic nuclei. In contrast to the case of $^{16}O+^{16}O$, no additional dissipation due to two-body collisions is found in the head-on collisions of $^{22}O+^{22}O$.

DOI: 10.1103/PhysRevC.65.037601 PACS number(s): 24.10.Cn, 25.60.Pj

With the ever increasing availability of radioactive ion beams, the studies of heavy-ion fusion for exotic nuclei are becoming possible. In terms of theoretical studies it is generally acknowledged that the time-dependent Hartree-Fock (TDHF) method provides a useful foundation for a fully microscopic many-body theory of low-energy—heavy-ion reactions [1]. It is well known that in TDHF, colliding nuclei do not fuse in a low L (orbital angular momentum) region when the incident energy is higher than a certain threshold value E_{th} [2]. This is known as the fusion window anomaly. Experimental search for nonfusion events in nearly central collisions has been done but no evidence for the low L nonfusion window has been found [3]. It has been noted that three effects could resolve this anomaly. (i) Inclusion of two-nucleon collision terms into the description; (ii) modification of effective interactions; and (iii) relaxation of symmetry assumptions employed in calculations. The effects (i) and (iii) were independently shown to introduce enough dissipation to resolve this problem, whereas the dependence on effective interaction resulted in moderate changes to the fusion cross section. In particular, it was found that the inclusion of the spin-orbit force, which had been neglected in the conventional TDHF calculations, introduced enough one-body dissipation to more than double the E_{th} for $^{16}O+^{16}O$ collisions [4]. In a different calculation, the effects of two-body dissipation were quantum mechanically taken into account using an extended version of TDHF known as the time-dependent density-matrix theory (TDDM) [5]. This calculation also resulted in the doubling of the E_{th} for the same system without incorporating the spin-orbit interaction. In this report we examine the combined effects of (i) and (iii) for $^{16}O+^{16}O$ and $^{22}O+^{22}O$.

We have recently developed a new TDDM program [6] based on the TDHF code with spin-orbit force [7], which now enables us to deal with one-body and two-body dissipation in a quantal way. TDDM determines the time evolution of one-body and two-body density matrices ρ and ρ_2 in a self-consistent manner; the equations of motion for ρ and ρ_2 can be derived by truncating the well-known Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy for reduced density matrices [8]. TDDM, therefore, includes both effects of one-body dissipation due to the mean-field potential and the two-body dissipation due to nucleon-nucleon collisions. To solve the coupled equations for ρ and ρ_2, we expand ρ and C_2, a correlated part of ρ_2, with a finite number of single-particle states ψ_α, which satisfy a TDHF-like equation

$$\rho(11',t)=\sum_{\alpha\alpha'} n_{\alpha\alpha'}(t)\psi_\alpha(1,t)\psi^*_{\alpha'}(1',t), \tag{1}$$

$$\begin{aligned} C_2(121'2',t)&=\rho_2-A(\rho\rho)\\ &=\sum_{\alpha\beta\alpha'\beta'} C_{\alpha\beta\alpha'\beta'}(t)\\ &\quad\times\psi_\alpha(1,t)\psi_\beta(2,t)\psi^*_{\alpha'}(1',t)\psi^*_{\beta'}(2',t), \end{aligned} \tag{2}$$

where $A(\rho\rho)$ is an antisymmetrized product of the one-body density matrices and the numbers denote the space, spin and isospin coordinates. The equations of motion of TDDM thus consist of the following three coupled equations [9]:

$$i\hbar\frac{\partial}{\partial t}\psi_\alpha(1,t)=h(1,t)\psi_\alpha(1,t), \tag{3}$$

$$i\hbar\dot{n}_{\alpha\alpha'}=\sum_{\beta\gamma\delta}[\langle\alpha\beta|v|\gamma\delta\rangle C_{\gamma\delta\alpha'\beta}-C_{\alpha\beta\gamma\delta}\langle\gamma\delta|v|\alpha'\beta\rangle], \tag{4}$$

$$i\hbar\dot{C}_{\alpha\beta\alpha'\beta'}=B_{\alpha\beta\alpha'\beta'}+P_{\alpha\beta\alpha'\beta'}+H_{\alpha\beta\alpha'\beta'}, \tag{5}$$

where h is the mean-field Hamiltonian and v the residual interaction. The term $B_{\alpha\beta\alpha'\beta'}$ on the right-hand side of Eq. (5) represents the Born terms (the first-order terms of v). The terms $P_{\alpha\beta\alpha'\beta'}$ and $H_{\alpha\beta\alpha'\beta'}$ in Eq. (5) contain $C_{\alpha\beta\alpha'\beta'}$ and represent higher-order particle-particle (and hole-hole) and particle-hole-type correlations, respectively. Thus full two-body correlations including those induced by the Pauli exclusion principle are taken into account in the equation of

TABLE I. Threshold energy E_{th} in the center-of-mass frame for the head-on collisions of $^{16}\mathrm{O}+^{16}\mathrm{O}$. Fusion occurs below E_{th}.

Method	E_{th} (MeV)
TDHF without $\vec{l}\cdot\vec{s}$	30
TDDM without $\vec{l}\cdot\vec{s}$	66
TDHF with $\vec{l}\cdot\vec{s}$	69
TDDM with $\vec{l}\cdot\vec{s}$	80

motion for $C_{\alpha\beta\alpha'\beta'}$. The explicit expressions for $B_{\alpha\beta\alpha'\beta'}, P_{\alpha\beta\alpha'\beta'}$, and $H_{\alpha\beta\alpha'\beta'}$ are given in Ref. [9].

In the following, we explain some details of numerical calculations. We use the Skyrme II force (SKII) [10] as an effective interaction to calculate the mean-field potential; SKII has frequently been used in TDHF calculations [11]. The most time consuming part of the TDDM calculations is the evaluation of the right-hand side of Eq. (5), which becomes formidable with increasing number of single-particle states. In the present calculations, the single-particle states are restricted to the $1s_{1/2}, 1p_{1/2}, 1p_{3/2}, 2s_{1/2}, 1d_{3/2}$, and $1d_{5/2}$ states for both protons and neutrons in each nucleus. This corresponds to using 40 single-particle indices in the evaluation of $C_{\alpha\beta\alpha'\beta'}$. The TDDM code [6] has been written for use with a realistic residual interaction, such as the Skyrme-type force. Since the Skyrme-type force contains momentum-dependent terms, the computation time of a matrix element of the Skyrme-type force is about one order of magnitude larger than that of the simple force of the δ-function form $v=v_0\delta^3(\vec{r}-\vec{r}\,')$. Since the number of the single-particle states used in the present calculation is large, the use of the Skyrme-type force as the residual interaction is not practical. Therefore, we use the simple force as the residual interaction. We choose v_0 to be $-350\ \mathrm{MeV\,fm^3}$. The values of v_0 ranging from $-230\ \mathrm{MeV\,fm^3}$ to $-420\ \mathrm{MeV\,fm^3}$ have been found to give reasonable damping of giant resonances [6,12]. We assume that the colliding nuclei are initially in the Hartree-Fock ground state and place them at the separation distance of 10 fm. Then we start solving the coupled equations, Eqs. (3)—(5). The single-particle wave functions are confined to a cylinder of length 33 fm and radius 10 fm. The mesh size and time step size used are 0.5 fm and 0.5 fm/c, respectively. Changing the center-of-mass (c.m.) energy by 1 MeV, we search the threshold energy E_{th} for four different calculation schemes for $^{16}\mathrm{O}+^{16}\mathrm{O}$: TDHF calculations with and without spin-orbit force, and TDDM calculations with and without spin-orbit force. The colliding nuclei are considered to fuse when they survive the first separation phase as in the conventional TDHF calculations [2].

The obtained results for E_{th} in the c.m. frame are summarized in Table I. E_{th} dramatically increases when either spin-orbit force or the effects of two-body collisions are considered: from 30 MeV to 69 MeV due to spin-orbit force and from 30 MeV to 66 MeV due to two-body collisions. However, an increase in E_{th} due to the effects of two-body collisions is rather small (from 69 MeV to 80 MeV) when spin-orbit force is already included in the mean-field potential.

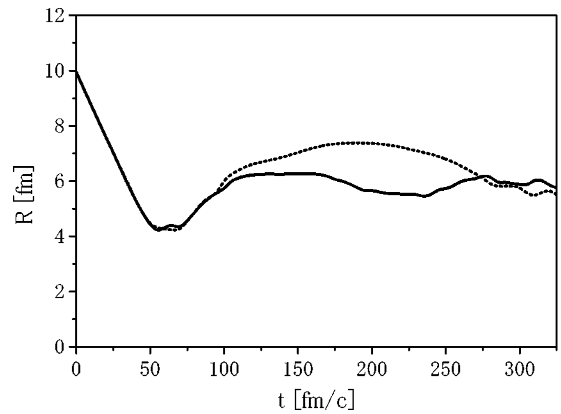

FIG. 1. Time evolution of the relative distance in a head-on collision of $^{16}\mathrm{O}+^{16}\mathrm{O}$ at $E_{\mathrm{c.m.}}=65$ MeV calculated in TDHF (dotted line) and TDDM (solid line).

The threshold energy E_{th} may depend on the residual interaction used and the truncation of the single-particle space. To investigate the effect of the residual interaction, we performed a TDDM calculation at $E_{\mathrm{c.m.}}=81$ MeV using a stronger residual interaction with $v_0=-500\ \mathrm{MeV\,fm^3}$ and found no fusion. This indicates that E_{th} is affected by the truncation of the single-particle space. Since it is impractical to extend the number of the single-particle states, we made a TDDM calculation using reduced single-particle space in which the $2s_{1/2}$ and $1d_{3/2}$ states were excluded for both protons and neutrons. These states are in the continuum while the $1d_{5/2}$ states included are bound. The obtained E_{th} is 79 MeV, which is only 1 MeV less than the result with all the single-particle states. The small contribution of the $2s_{1/2}$ and $1d_{3/2}$ states to two-body dissipation is related to the fact that the wave functions of the $2s_{1/2}$ and $1d_{3/2}$ orbits are spatially extended. This makes the two-body matrix elements involving these states small. The above investigation suggests that although E_{th} may increase with increasing single-particle space, the increase is quite moderate.

Although the colliding system fuses both in TDHF and TDDM below $E_{\mathrm{c.m.}}=69$ MeV when spin-orbit force is included, there is a difference in the approach toward equilibrium. We show in Fig. 1 the time evolution of the relative distance R between the two nuclei. R is defined as a distance between the c.m. of the left-half mass distribution and that of the right-half one. $E_{\mathrm{c.m.}}$ is chosen to be 65 MeV at which the system fuses both in TDHF and TDDM. The maxima around 200 fm/c in TDHF (dotted line) and around 130 fm/c in TDDM (solid line) correspond to the first separation phase. It is clearly seen from Fig. 1 that the translational motion damps faster in TDDM than in TDHF. This is due to two-body dissipation mechanism considered in TDDM. Since the low-L nonfusion region disappears, the fusion cross sections calculated in TDHF and TDDM increase with increasing incident energy and overestimate experimental values [4]. This is largely due to the fact that the Skyrme interaction used in these codes is made a finite-range approximation to the surface terms of the single-particle Hamiltonian without refitting parameters [4].

To investigate the effects of two-body collisions in exotic nuclei, we also calculated the head-on collisions of ^{22}O $+^{22}$O both in TDHF and TDDM. In ^{22}O the neutron $2s_{1/2}$ and $1d_{3/2}$ states are in the continuum and the proton $2s_{1/2}$ and $1d_{3/2}$ states are barely bound. The obtained $E_{\rm th}$ in the c.m. frame is 103 MeV in TDHF and 102 MeV in TDDM. The little difference between the TDHF and TDDM results means that two-body collisions play no essential role in damping the tranlational motion of the colliding nuclei. To evaluate the contribution of the $2s_{1/2}$ and $1d_{3/2}$ states, we made a TDDM calculation with reduced number of single-particle states, where the $2s_{1/2}$ and $1d_{3/2}$ states were excluded for protons and neutrons, and found no change in $E_{\rm th}$, indicating their much smaller roles in ^{22}O than in ^{16}O. The negligible effect of the two-body collisions in ^{22}O can be understood by the spatial extension of the neutron single-particle wave functions. This makes two-body matrix elements involving neutron single-particle states quite small in ^{22}O. The slight decrease in $E_{\rm th}$ obtained in TDDM originates in the partial occupation of the proton $1d_{5/2}$ state due to ground-state correlations. Since the $1d_{5/2}$ state has high momentum components, its partial occupation increases the high momentum components in the beam direction and makes the colliding system more transparent than in TDHF. The role of the high momentum components in nonfusion window problem in TDHF has been discussed in Ref. [2] and a similar decrease in $E_{\rm th}$ has been found in an extended TDHF calculation done by Wong and Davies [13].

In summary, we calculated the head-on collisions of ^{16}O $+^{16}$O using the newly developed TDDM code, which includes spin-orbit force in the nuclear mean field. Both the effects of one-body and two-body dissipation were quantally taken into account in this approach. It was found that the threshold energy $E_{\rm th}$, above which the colliding nuclei do not fuse, increases from the TDHF value when two-body collisions are taken into account. However, the increase in $E_{\rm th}$ remained small when spin-orbit force was already included in the mean field. This might be due to the truncation of the single-particle space. It was pointed out that the anticipated increase in $E_{\rm th}$ with increasing number of single-particle states would be rather moderate. It was also found that the translational motion calculated in TDDM damps faster than that in TDHF in the energy region where both TDHF and TDDM make the colliding nuclei fuse. The head-on collisions of ^{22}O$+^{22}$O were also calculated both in TDHF and TDDM. It was found that two-body dissipation in ^{22}O$+^{22}$O is negligible, indicating that the TDHF method may be a good description of low-energy fusion reactions of exotic nuclei.

This work was partially supported under U.S. Department of Energy Grant No. DE-FG02-96ER40963 to Vanderbilt University.

[1] J.W. Negele, Rev. Mod. Phys. **54**, 913 (1982).
[2] K.T.R. Davies *et al.*, in *Treatise on Heavy Ion Science*, edited by D.A. Bromley (Plenum, New York, 1985), Vol. 3.
[3] A. Lazzarini *et al.*, Phys. Rev. C **24**, 309 (1981); S. Kox *et al.*, Phys. Rev. Lett. **44**, 1204 (1980); A. Szanto de Toledo *et al.*, *ibid.* **47**, 1881 (1981); H. Ikezoe *et al.*, Nucl. Phys. **A456**, 298 (1986).
[4] A.S. Umar *et al.*, Phys. Rev. Lett. **56**, 2793 (1986).
[5] M. Tohyama, Phys. Rev. C **36**, 187 (1987).
[6] M. Tohyama and A.S. Umar, Phys. Lett. B **516**, 415 (2001).
[7] A.S. Umar *et al.*, Phys. Rev. C **40**, 706 (1989).
[8] S.J. Wang and W. Cassing, Ann. Phys. **159**, 328 (1985).
[9] M. Gong and M. Tohyama, Z. Phys. A **335**, 153 (1990).
[10] D. Vautherin and D.M. Brink, Phys. Rev. C **5**, 626 (1972).
[11] K.T.R. Davies and S.E. Koonin, Phys. Rev. C **23**, 2042 (1981).
[12] F.V. De Blasio *et al.*, Phys. Rev. Lett. **68**, 1663 (1992).
[13] C.Y. Wong and K.T.R. Davies, Phys. Rev. C **28**, 240 (1983).