Structure of the doubly midshell nucleus $^{170}_{66}Dy_{104}$

P. H. Regan,[1] F. R. Xu,[1,2] P. M. Walker,[1] M. Oi,[1] A. K. Rath,[1,3] and P. D. Stevenson[1]
[1]*Department of Physics, University of Surrey, Guildford GU2 7XH, United Kingdom*
[2]*Department of Technical Physics, Peking University, Beijing 100871, China*
[3]*PG Department of Physics, Sambalpur University, Burla 768 109, India*
(Received 2 November 2001; published 15 February 2002)

Potentional energy surface calculations for the doubly midshell nucleus $^{170}_{66}Dy_{104}$ support a variety of extreme properties. The ground-state deformation is among the largest in the region, consistent with it having the maximal value of valence particles for any nucleus below the ^{208}Pb doubly closed shell. The energy minimum is found to be remarkably constant in the (β_2,γ) plane as a function of angular momentum. The nucleus is predicted to undergo a dual alignment with midshell high-j protons and neutrons aligning simultaneously at spin $\approx 14\hbar$. Configuration-constrained calculations for the two-quasiparticle configurations predict the presence of a low-lying $K^\pi = 6^+$ state with a similar axially symmetric shape to the highly deformed ground state.

DOI: 10.1103/PhysRevC.65.037302 PACS number(s): 21.10.Tg, 21.10.Re, 21.60.−n

The existence of shell closures is a cornerstone of our understanding of the atomic nucleus. While much recent research has focussed on the search for exotic ''doubly magic'' nuclei [1,2] formed by assuming the standard shell model gaps at nucleon numbers 2, 8, 20, 28, 50, 82, and 126, their even-even ''doubly midshell'' counterparts are arguably even more rare. Above the *sd* shell, the corresponding doubly midshell, even-even systems which are particle bound are limited to $^{28}_{14}Si_{14}$, $^{38}_{14}Si_{24}$ [3], $^{48}_{24}Cr_{24}$ [4], and $^{170}_{66}Dy_{104}$. (One might also include near-midshell nuclei $^{62,64}Cr_{38,40}$ [5], $^{76}_{38}Sr$, $^{80}_{40}Zr$ [6], $^{104}Sr_{66}$, and $^{106}Zr_{66}$ [7,8] in such a discussion).

It has long been recognized that the nature of nuclear collectivity at low spins is linked to the number of valence protons *and* neutrons. Casten and collaborators have demonstrated that many of the low-lying collective nuclear properties, such as the excitation energy of the first 2^+ level (2_1^+), the energy ratio $4_1^+/2_1^+$ [9–12] and the $B(E2)$ of the first excited state [13] have a smooth dependence on N_pN_n, the product of the number of valence proton and neutron particles or holes outside of the magic numbers. However, even in this simple scheme, in some cases, subshell closures must be envoked, such as those at $Z=64$ [10,14] and at $Z=76$ [11] for the simple N_pN_n relationship to remain applicable. Assuming the standard spherical shell gaps at $Z=50$ and 82 and $N=82$ and 126, ^{170}Dy has the largest number of valence particles for any nucleus lighter than ^{208}Pb, and as such, might naively be expected to be amongst the most collective of all nuclei. However experimentally, one observes the well known saturation of $B(E2)$ values when approaching the double midshell in this region [15,16] which has been explained in terms of the spatial overlap of specific proton and neutron Nilsson orbitals [15].

The single-particle structure of nuclei in the deformed rare-earth region is also of longstanding interest. This is in part due to the preponderance of states with large single-particle components of angular momentum along the axis of nuclear symmetry [17–20]. These so called ''high-K'' states are hindered in their decay to the predominantly $K=0$ ground-state band by the K-selection rule, which states that the multipole of decay, λ, must be at least as large as the change in the K value between the two states [17,20]. Thus decays between states of widely differing K values require high order multipoles, giving rise to long-lived high-K isomers. In practice, the constancy of the value of the K-quantum number is eroded both by fluctuations away from the pure axially symmetric shape and by Coriolis mixing of other K values [21] which increases as a function of rotational frequency. Walker [22] demonstrated a dependence of the K-isomer hindrance factors on N_pN_n, which implied a greater degree of K hindrance as the valence product increased, corresponding to more deformed, axially symmetric nuclei. The high degree of axial symmetry and large deformation expected for a doubly midshell nucleus means that one might expect ^{170}Dy to be an excellent candidate for having a very long lived ''pure'' K-isomeric state.

Recent studies of the *fp*, doubly midshell nuclei ^{38}Si and ^{48}Cr [3,4] have shown that these systems exhibit significant quadrupole collectivity. Due to the limited number of valence particles in these light systems, theoretical descriptions of their structure have been presented which are based on the constrained-basis shell model (see, e.g., Ref. [23]). The yrast decay schemes of the even-even neighbors of the $N=Z=39$ doubly midshell nucleus ^{78}Y, namely, $^{76}_{38}Sr$ and $^{80}_{40}Zr$ have recently been extended into the medium-spin regime [6]. Although the energies of the 2_1^+ states suggest large deformations for these nuclei ($\beta_2 \sim 0.4$), the $E(4_1^+)/E(2_1^+)$ ratios of 2.84 and 2.86, respectively, suggest that these structures are not pure prolate rotors, but have some degree of axial asymmetry [24]. This may be explained by the existence of other deformation minima which are present in the potential-energy surfaces for these nuclei [25] and arise due to the presence of deformed shell gaps in the single-particle potentials at both prolate, spherical and oblate shapes for proton and neutron numbers in this region. It may also reflect the rather small number of valence particles (compared to ^{170}Dy).

Due to the much larger valence space associated with ^{170}Dy, constrained-basis shell-model methods are not viable. Consequently, prior theoretical work on the excited states of this nucleus has been limited mostly to Monte Carlo shell

model [26], the generator co-ordinate method [27,28], the Strutinsky shell correction [16,29], and relativistic-mean-field [30] calculations. Nevertheless, the behavior as a function of angular momentum has not yet been adequately explored. The current paper addresses this issue.

The neutron-rich nature of ^{170}Dy has meant that until now, it has been experimentally inaccessible for spectroscopic study. The heaviest β-stable dysprosium isotope is $^{164}\mathrm{Dy}_{98}$, while the lightest stable $N=104$ isotone is $^{174}_{70}$Yb. Recently, however, some progress has been made on the production of neutron-rich nuclei in this region. These include ^{170}Dy being synthesized and tentatively identified following the projectile fragmentation of a ^{208}Pb beam at 1 GeV per nucleon [31]. This result, coupled with advances in the instrumentation associated with experiments using multinucleon transfer reactions [32] with stable ions when coupled with high-efficiency γ-ray spectrometers [33], opens up the intriguing possibility of studying the structure of this unique nucleus up to medium spins in the near future. To date, the $N=102$ isotone $^{168}_{66}$Dy is the most neutron-rich dysprosium isotope where the 2_1^+ state has been identified [34]. This was populated following the β decay of $^{168}_{65}$Tb, which was in turn produced as a product of proton induced fission of ^{238}U. A similar isotope separation on-line (ISOL) technique could be used to identify states in ^{170}Dy. Experimental access to the energy of even the first excited state in the doubly midshell system will provide a useful guide to the quadrupole deformation [21,35], and if the yrast 4^+ excited state can also be identified, information on the degree of triaxial deformation, or lack thereof, can also be inferred [36,37].

Motivated by the recent experimental advances in production and identification of heavy neutron-rich rare-earth nuclei, we present calculations predicting the evolution of the yrast and low-lying single particle-structure of ^{170}Dy. In the current work, the nonaxial deformed Woods-Saxon (WS) potential [25] was employed. Collective rotation was investigated in the framework of the cranked shell model (CSM) by means of total-Routhian-surface (TRS) calculations [38] in the three-dimensional deformation β_2,β_4,γ space. Both monopole and quadrupole pairings are included. The monopole pairing strength G is determined by the average gap method [39] and quadrupole strengths are obtained by restoring the Galilean invariance broken by the seniority pairing force [40]. To avoid the spurious phase transition encountered in the BCS approach, we used approximate particle-number projection, described by Lipkin-Nogami pairing [41]. Pairing correlations are dependent on the rotational frequency (ω) and deformation. In order to include such dependence in the TRS, we have performed pairing-deformation-frequency self-consistent TRS calculations, i.e., for any given deformation and frequency, pairings are self-consistently calculated by the HFB-like method [41]. At a given frequency, the deformation of a state is determined by minimizing the calculated TRS.

Figure 1 shows the results of TRS calculations for the yrast sequence of ^{170}Dy. Note the presence of a very deep, axially symmetric, prolate minimum corresponding to $\beta_2=0.288$. The ground state ($\omega=0$) deformation compares with calculated values of $\beta_2=0.292$, 0.281, 0.288, and 0.287 for the neighboring isotopes and isotones $^{168}\mathrm{Dy}_{102}$, $^{172}\mathrm{Dy}_{106}$, $^{168}_{64}$Gd, and $^{172}_{68}$Er, respectively. This is consistent with the results of relativistic mean-field calculations for this region [30] which predicted a saturation of quadrupole deformation around neutron number 102. As Fig. 1 shows, the deformation parameters predicted for this minimum in ^{170}Dy are found to be remarkably constant as a function of rotational frequency (which is related to the projection of the average spin on the axis of rotation I_x).

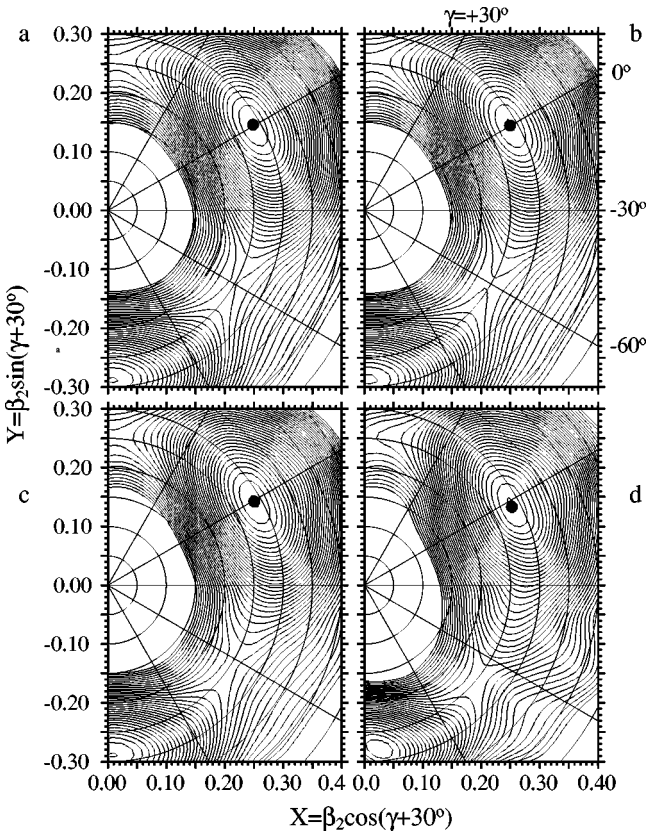

FIG. 1. TRS calculations for the yrast sequence in ^{170}Dy as calculated in the present work for rotational frequencies of $\omega=$ (a) 0.0, (b) 0.1, (c) 0.2, and (d) 0.3 MeV/$\hbar$. The minima correspond to deformation values (β_2,γ) of (a) (0.288,0.3°), (b) (0.288,0.1°), (c) (0.288,$-$0.4°), and (d) (0.286,$-$2.2°), respectively. The contours are shown at 200 keV intervals.

Figure 2 shows the calculated kinematic moment of inertia for the yrast sequence of ^{170}Dy as a function of rotational frequency. The figure highlights the almost simultaneous alignments of both protons and neutrons, with the predicted gradual alignment starting at $\hbar\omega\approx0.35$, corresponding to $I\approx14\hbar$. This simultaneous alignment arises from the breaking of a pair of $i_{13/2}$ neutrons *and* a pair of $h_{11/2}$ protons. The nature of the simultaneous alignments in the two fluids can be easily qualitatively understood since the neutrons and protons have the same status, namely in the middle of high-j shells, and hence experience similar Coriolis forces.

In the present work, we also investigate noncollective excitations, i.e., multiquasiparticle (multi-qp) states. For the

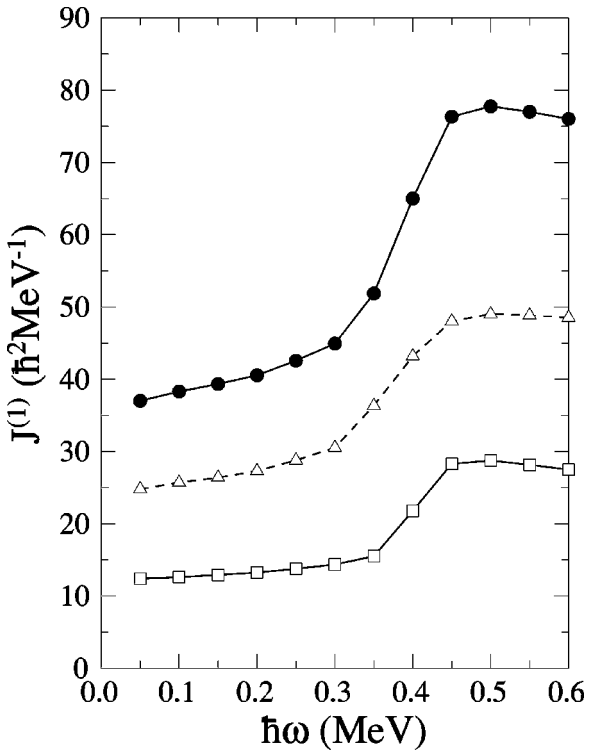

FIG. 2. Predicted kinematic moment of inertia for ^{170}Dy as a function of rotational frequency. Note the dramatic rise at $\hbar\omega \approx 0.35$ MeV, which corresponds to the *simulataneous* alignment of both midshell protons and neutrons. The lowest curve is for protons, the middle one for neutrons and the upper one for the two combined.

potential-energy-surface (PES) calculations of multi-qp states, the configuration-constrained method [42] has been used. In this method, the orbitals involved in a given configuration are identified and diabatically blocked by calculating the average Nilsson numbers of the orbitals. The shape and excitation energy of a multi-qp state can be obtained from the configuration-constrained PES. It is well known that a large axial deformation and high K value provide good conditions for the formation of isomeric states [17]. Our calculations suggest the presence of a $K^\pi = 6^+$ two quasineutron state at a low excitation energy of approximately 1.2 MeV. This state is based on the $\nu(\frac{5}{2}^-[512] \otimes \frac{7}{2}^-[514])$ configuration. Note, that similar $K^\pi = 6^+$ instrinsic states have been experimentally observed in the heavier $N = 104$ isotones ^{174}Yb, ^{176}Hf, and ^{178}W [43].

Figure 3 shows the configuration-constrained potential energy surface calculation [42] for the $K^\pi = 6^+$ state in ^{170}Dy. The deformation predicted for this state is essentially identical with that predicted for the low-lying members of the ground-state band. The large deformation and high degree of axial symmetry predicted for both the two-quasineutron state and ground state band, suggest that this may represent an extremely good example of a K-isomeric state, with little mixing of either the initial or final K value due to either Coriolis mixing or softness in the triaxial plane. Indeed, if the transition-rate hindrance systematics [22] as a function of N_pN_n can be extrapolated in a simple way to N_pN_n

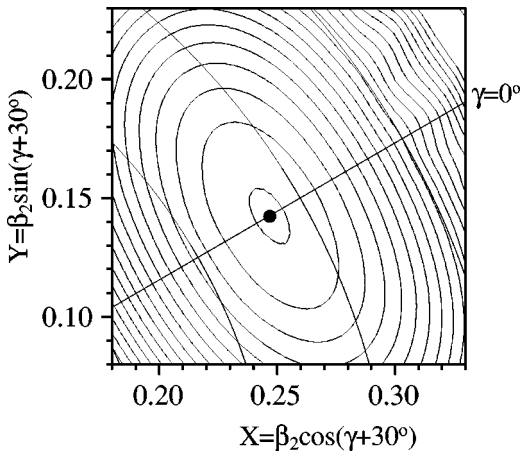

FIG. 3. Configuration constrained potential energy surface calcualtion for the $K^\pi = 6^+$ state in ^{170}Dy (see text for details). The predicted minimum lies at deformation values of $\beta_2 = 0.286$ and $\gamma = 0.0°$.

$= 352$ for ^{170}Dy (see Fig. 4), then the corresponding γ-decay half-life of the $K^\pi = 6^+$ isomer could be as long as several hours. [The reduced hindrance, $f_\nu = (T/T_W)^{1/\nu}$, where T and T_W are the experimental and Weisskopf single particle estimate for the decay half life, respectively, $\nu = \lambda - \Delta K$, where λ is the multipole of decay and ΔK is the difference in K value between the intial and final state.]

In summary, we have presented calculations outlining the yrast sequence for the doubly midshell nucleus ^{170}Dy. The TRS calculations predict a highly deformed nucleus, consistent with the maximising of valence particle/hole number. The yrast band deformation is predicted to be remarkably contant over a large spin range, with pure, axially symmetric deformed shapes favored. A simultaneous alignment of midshell $h_{11/2}$ protons and $i_{13/2}$ neutrons is predicted to occur at approximately $\hbar\omega \approx 0.35$ MeV. Configuration constrained PES calculations also predict a $K^\pi = 6^+$ two-quasiparticle state at low excitation energy, which, due to its large defor-

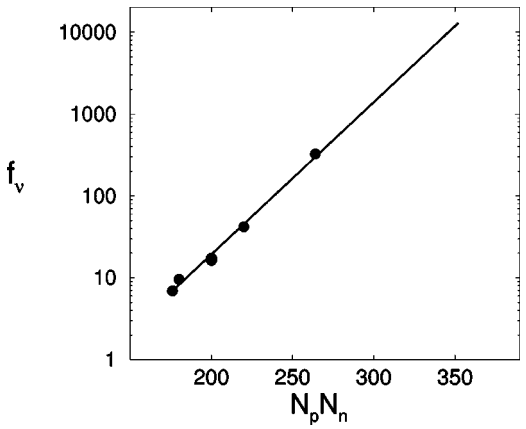

FIG. 4. Systematics of the ($E2$) reduced hindrances for $K^\pi = 6^+$ isomers in ^{172,174,176,178}Hf, ^{174}Yb, and ^{178}W [22,43] as a function of the valence nucleon product. The straight line extrapolation is to $N_pN_n = 352$, appropriate for ^{170}Dy.

mation and axial symmetry is expected to have a considerable decay half-life, possibly in the hours range.

This work was supported by EPSRC (U.K.), F.R.X. acknowledges support from the U.K. Royal Society, the Major State Basic Research Development Program of China (Grant No. G2000077400), and the Chinese Ministry of Education. Support is also ackowledged from the Japan Society of the Promotion of Sciences (M.O.) and The Association of Commonwealth Universities (A.K.R.).

[1] B. Blank and P. H. Regan, Nucl. Phys. News **11**, 15 (2001).
[2] R. F. Casten and B. M. Sherrill, Prog. Part. Nucl. Phys. **45**, 171 (2000).
[3] R. W. Ibbotson *et al.*, Phys. Rev. Lett. **80**, 2081 (1998); F. Sarazin *et al.*, *ibid.* **84**, 5062 (2000).
[4] S. Lenzi *et al.*, Phys. Scr. **T88**, 100 (2000); F. Brandolini *et al.*, Nucl. Phys. **A642**, 387 (1998); J. A. Cameron *et al.*, Phys. Lett. B **387**, 266 (1996).
[5] O. Sorlin *et al.*, Nucl. Phys. **A660**, 3 (1999); **A669**, 351(E) (2000).
[6] S. M. Fischer *et al.*, Phys. Rev. Lett. **87**, 132501 (2001).
[7] J. Skalski, S. Mitutori, and W. Nazarewicz, Nucl. Phys. **A617**, 282 (1997).
[8] M. Bernas *et al.*, Phys. Lett. B **331**, 19 (1994).
[9] R. F. Casten, D. D. Warner, D. S. Brenner, and R. L. Gill, Phys. Rev. Lett. **47**, 1433 (1981).
[10] R. F. Casten, Phys. Lett. **152B**, 145 (1985).
[11] H. Mach, Phys. Lett. B **185**, 20 (1987).
[12] Y. M. Zhao, A. Arima, and R. F. Casten, Phys. Rev. C **63**, 067302 (2000).
[13] R. F. Casten and N. V. Zamfir, Phys. Rev. Lett. **70**, 402 (1993).
[14] J. A. Cizewski and E. Gülmez, Phys. Lett. B **175**, 11 (1986).
[15] R. F. Casten, K. Heyde, and A. Wolf, Phys. Lett. B **208**, 33 (1988).
[16] G. L. Long, Chin. Phys. Lett. **14**, 340 (1997).
[17] P. M. Walker and G. D. Dracoulis, Nature (London) **399**, 35 (1999).
[18] K. Jain, O. Burglin, G. D. Dracoulis, B. Fabricius, N. Rowley, and P. M. Walker, Nucl. Phys. **A591**, 61 (1995).
[19] K. E. G. Löbner, Phys. Lett. **26B**, 369 (1968).
[20] G. D. Dracoulis, Phys. Scr. **T88**, 54 (2000).
[21] F. S. Stephens, R. M. Diamond, J. R. Leigh, T. Kammuri, and K. Nakai, Phys. Rev. Lett. **29**, 438 (1972).
[22] P. M. Walker, J. Phys. G **16**, L233 (1990).
[23] V. G. Gueorguiev *et al.*, Phys. Rev. C **63**, 014318 (2001); K. Hara *et al.*, Phys. Rev. Lett. **83**, 1922 (1999); T. Otsuka *et al.*, J. Phys. G **25**, 699 (1999); A. Poves, *ibid.* **25**, 589 (1999); J. Retamosa *et al.*, Phys. Rev. C **55**, 1266 (1997); R. Rodrigues-Guzman *et al.*, Phys. Lett. B **474**, 15 (2000); G. A. Lalazissis *et al.*, Phys. Rev. C **60**, 014310 (1999); E. Caurier *et al.*, Phys. Rev. Lett. **75**, 2466 (1995); A. Juodagalvis and S. Aberg, Phys. Lett. B **428**, 227 (1998); K. Hara *et al.*, Phys. Rev. Lett. **83**, 1922 (1999).
[24] D. Bucurescu *et al.*, Phys. Rev. C **56**, 2497 (1998).
[25] W. Nazarewicz, J. Dudek, R. Bengtsson, T. Bengtsson, and I. Ragnarsson, Nucl. Phys. **A435**, 397 (1985).
[26] D. J. Dean, S. E. Koonin, G. H. Lang, W. E. Ormond, and P. B. Radha, Phys. Lett. B **317**, 275 (1993).
[27] B. Nerlo-Pomorska, K. Pomorski, M. Brack, and E. Werner, Nucl. Phys. **A462**, 252 (1987).
[28] B. Nerlo-Pomorska, Z. Phys. A **328**, 11 (1987).
[29] W. Nazarewicz, M. A. Riley, and J. D. Garrett, Nucl. Phys. **A512**, 61 (1990).
[30] G. A. Lalazissis, M. M. Sharma, and P. Ring, Nucl. Phys. **A597**, 35 (1996).
[31] M. Caamaño, Ph.D. thesis, University of Surrey, 2002.
[32] L. Corradi *et al.*, Phys. Rev. C **63**, 021601(R) (2001), and references therein.
[33] R. D'Alarcao *et al.*, Phys. Rev. C **59**, R1227 (1999); C. Wheldon *et al.*, *ibid.* **63**, 011304(R) (2001); X. Liang *et al.*, Eur. Phys. J. A **10**, 41 (2001); S. J. Asztalos *et al.*, Phys. Rev. C **60**, 044307 (1999).
[34] M. Asai *et al.*, Phys. Rev. C **59**, 3060 (1999).
[35] L. Grodzins, Phys. Lett. **2**, 88 (1962); L. Goettig and W. Nazarewicz, J. Phys. G **16**, L51 (1990).
[36] L. Esser, U. Neuneyer, R. F. Casten, and P. von Brentano, Phys. Rev. C **55**, 206 (1997).
[37] Richard F. Casten, *Nuclear Structure from a Simple Perspective* (Oxford University Press, Oxford, 1990).
[38] W. Nazarewicz, R. Wyss, and A. Johnson, Phys. Lett. B **225**, 208 (1989).
[39] P. Möller and J. R. Nix, Nucl. Phys. **A536**, 20 (1992).
[40] H. Sakamoto and T. Kishimoto, Phys. Lett. B **245**, 321 (1990).
[41] W. Satula, R. Wyss, and P. Magierski, Nucl. Phys. **A578**, 45 (1994).
[42] F. R. Xu, P. M. Walker, J. A. Sheikh, and R. Wyss, Phys. Lett. B **435**, 257 (1998); F. R. Xu, P. M. Walker, and R. Wyss, Phys. Rev. C **59**, 731 (1999); **62**, 014301 (2000).
[43] T. L. Khoo, J. C. Waddington, and M. W. Johns, Can. J. Phys. **51**, 2307 (1973); C. L. Dors, F. M. Bernthal, T. L. Khoo, C. H. King, J. Borggreen, and G. Sletten, Nucl. Phys. **A314**, 61 (1979); M. J. Canty, N. E. Davidson, D. A. Dohan, and P. Yuen, *ibid.* **A265**, 1 (1976); R. B. Firestone and V. S. Shirley, *Table of Isotopes*, 8th ed. (Wiley, New York, 1996), Vol. II.