Charge symmetry breaking in NN scattering with an interaction from effective field theory

J. A. Niskanen*
Department of Physics, P. O. Box 64, FIN-00014 University of Helsinki, Finland
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The effect of isospin breaking pion s-wave rescattering is included in elastic NN scattering at low energies using an interaction obtained from effective field theory. Although this mechanism gave a large contribution to charge symmetry breaking in $np \to d\pi^0$, the effect is rather small in pp vs nn scattering parameters and in the ^{3}H-^{3}He binding energy difference. This smallness is caused by large cancellation of the up-down quark mass difference contribution and electromagnetic effects to the np mass difference.

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Charge symmetry is the least broken special case of general flavor symmetry. It is, however, trivially broken by the electromagnetic interaction, notably the Coulomb force in comparisons of the pp and nn systems and by the magnetic interaction in the np system. Other well known sources are the np mass difference and $\eta\pi$- as well as $\rho\omega$-meson mixing. These in turn may be related to the up- and down-quark mass difference—the microscopic flavor symmetry breaking in QCD. One might consider remarkable the fact that, although the relative quark mass difference is large ($\geqslant 10\%$), the symmetry breaking at the observable hadron level is two orders of magnitude smaller.

Charge symmetry breaking (CSB) has been studied for the mirror system pp vs nn for many decades [1], while its appearance in the np system was first seen only a decade ago [2] as the difference $\Delta A = A_n - A_p$ elastic analyzing powers and is presently being searched for also in pionic inelasticity in the reaction $np \to d\pi^0$ [3]. The CSB observables have been seen in calculations to be sensitive to different combinations of sources. For example, in np scattering above 300 MeV the np mass difference in OPE dominates, while at ≈ 200 MeV $\rho\omega$ meson mixing and the magnetic interaction become about equally important [4]. Of traditional CSB mechanisms in pion production $\eta\pi$ mixing is important and was seen to dominate at threshold [5], while at higher energies the np mass difference becomes more important [6]. The CSB effects in the np system change the isospin of the two baryons (class IV in the terminology of Ref. [7]), while in pp and nn the isospin is conserved (class III). In class III the main contribution is expected to be the $\rho\omega$ meson mixing [1,8].

Two-meson exchange in CSB has been studied earlier extensively by Coon and collaborators [9,10] and in charge dependence in, e.g., Refs. [9,11–13].

Recently a new mechanism related to the ud quark mass difference in QCD based effective field theory was suggested for the CSB forward-backward asymmetry of the cross section in $np \to d\pi^0$ [14]. It consists of CSB s-wave rescattering of the pion from the second nucleon. This πN scattering [depicted in Fig. 1(a)] can be described in effective field theory by the second term of the isospin symmetry violating Lagrangian [15,16]

$$\mathcal{L} = \frac{\delta m_N}{2}\left(N^\dagger \tau_0 N - \frac{2}{DF_\pi^2} N^\dagger \phi_0 \boldsymbol{\phi}\cdot\boldsymbol{\tau} N \right), \tag{1}$$

where the nucleon isospin is represented by the Pauli matrices $\boldsymbol{\tau}$, $F_\pi = 186$ MeV is the pion decay constant, and δm_N is the up and down quark mass difference effect in the nuclear masses. The denominator is in principle $D = 1 + \boldsymbol{\phi}^2/F_\pi^2$, but $D = 1$ is used here. The isospin violation here originates from the rather significant quark mass difference $m_d - m_u \approx$ a few MeV. In addition to the bare quark mass difference one should include an electromagnetic contribution $\overline{\delta m}_N$ to the nucleon mass difference changing the effective CSB strength parameter [14]. We shall come to this correction later.

The above interaction embedded in $np \to d\pi^0$ as rescattering was seen to be a major contributor to the asymmetry in CSB pion production. However, it is clear that returning the emitted pion back to the first nucleon it can also contribute to elastic scattering as shown in Figs. 1(b) and 1(c). The question is only whether its contribution really is isospin violating and of what type. The aim of this paper is to investigate this interaction and its effect to the difference of the $^1S_0$ scattering lengths (experimentally estimated to be $\Delta a = a_{pp} - a_{nn} = 1.5 \pm 0.5$ fm). Furthermore, a simple estimate of this effect to the ^{3}H-^{3}He binding energy difference is made.

It is straightforward algebra to see that, with the conventions of Fig. 1 and neglecting the baryon kinetic energies, the diagram Fig. 1(b) yields in the momentum space a CSB interaction of the form

$$V_N(q) = \frac{\delta m_N}{F_\pi^2}\frac{f^2}{\mu^2}\int \frac{d^3k}{(2\pi)^3} \times \frac{(k^2 - q^2/4)(\tau_{10} + \tau_{20})}{[\mu^2 + (\mathbf{k} + \mathbf{q}/2)^2][\mu^2 + (\mathbf{k} - \mathbf{q}/2)^2]}, \tag{2}$$

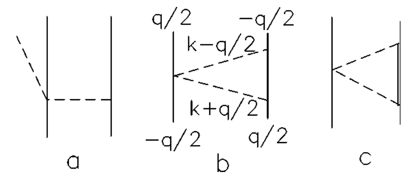

FIG. 1. CSB mechanisms arising from the up-down quark mass difference in pion rescattering: (a) in $np \to d\pi^0$, (b) in NN elastic scattering with a nucleonic, and (c) Δ intermediate state.

*Email address: jouni.niskanen@helsinki.fi

where $f^2/4\pi = 0.076$ is the pion-nucleon coupling constant, μ the pion mass, and τ_{i0} refers to the z component of the isospin operator of the ith nucleon. With the intermediate Δ [Fig. 1(c)] the corresponding result would be

$$V_\Delta(q) = \frac{4}{9}\frac{\delta m_N}{F_\pi^2}\frac{f^{*2}}{\mu^2}\int\frac{d^3k}{(2\pi)^3} \times \frac{(k^2-q^2/4)(\tau_{10}+\tau_{20})(\omega_+ + \omega_- + \Delta)}{\omega_+\omega_-(\omega_+ + \Delta)(\omega_- + \Delta)(\omega_+ + \omega_-)}, \quad (3)$$

where now the $\pi N\Delta$ coupling constant is $f^{*2}/4\pi = 0.35$ from the width of the $\Delta(1232)$ and Δ is the mass difference between the $\Delta(1232)$ isobar and the nucleon (the real part of the Δ pole is used). Also a shorthand notation has been introduced for the pion energy with $\omega_\pm^2 = \mu^2 + (\mathbf{k} \pm \mathbf{q}/2)^2$. In addition, monopole form factors $(\Lambda^2 - \mu^2)/(\Lambda^2 + q_\pi^2)$ are inserted for the pion emission and absorption vertices. Clearly the above potentials belong to class III in the classification of Ref. [7], which violates charge symmetry between pp and nn but not in the np system.[1] For positive δm_N they tend to make the nn interaction more attractive.

An interesting point in these CSB contributions is that the coefficient multiplying the integrals could be numerically large as compared with the coefficients in Refs. [10,17] for CSB arising from the np mass difference. However, the dimension is different (depending also on the integral). One may ask whether the contribution could be even unrealistically large to exclude this mechanism from CSB. An explicit calculation is necessary to answer this question.

One may note that there appears large uncertainty in the exact value of δm_N with estimates ranging mostly between 2 and 3 MeV depending on electromagnetic corrections to the np mass difference. For the moment the value $\delta m_N = 2.4$ MeV has been used in these results, which represented the total CSB strength for the reaction $np \to d\pi^0$ in Ref. [14] (including also the electromagnetic contribution to the np mass difference). Incidentally, this is close to the value obtained from an overall fit to the whole baryon octet [18]. The contribution to Δa scales linearly with δm_N. We shall return to the effect of the electromagnetic corrections later.

The above integrals are numerically easy to perform and, in the same way as in Ref. [17], the resulting potential is then transformed into the coordinate space where the final calculations are done. Simple fits of the integrals with a form

$$V(q) = A\frac{B^2}{B^2+q^2} \quad (4)$$

[1]One might note that there is also a contribution with a structure $i(\boldsymbol{\tau}_1 \pm \boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1 \pm \boldsymbol{\sigma}_2)\cdot\mathbf{k}\times\mathbf{q}$. With the above static approximation for the baryons this vanishes in the integration over $\mathbf{k}$. However, if the baryon kinetic energies are taken into account, there is also an odd term in the angular dependence of the denominators allowing a nonzero class IV part as found in Ref. [17]. At low energies this correction, however, should be significantly smaller than the potentials (2) and (3).

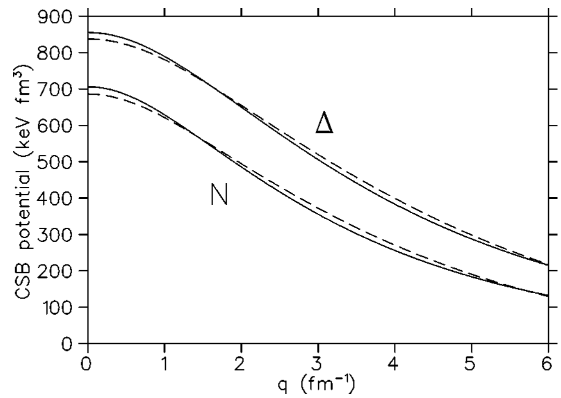

FIG. 2. The momentum space CSB potentials $V_N(q)$ and $V_\Delta(q)$ as functions of the momentum transfer q. Solid curves: exact integrals used; dashed curves: fits with forms (4) and (5).

for the first (k^2 dependent) parts and

$$V(q) = A\frac{B^2}{B^2+q^2}\frac{C^2}{C^2+q^2} \quad (5)$$

lead to a tolerable agreement (although not as perfect as in Ref. [17]) with the exact results for V_N and V_Δ (Fig. 2). In the coordinate representation these turn to Yukawa functions or their derivatives, shown in Fig. 3. These are very large potentials, indeed, for charge asymmetry, an order of magnitude larger than in Ref. [17] for class IV, but this may be in line with chiral power counting arguments, which stipulate that class III should be stronger than class IV [13,15]. In these figures the coefficients of the $(\tau_{10}+\tau_{20})$ operators are shown, so the *total difference* of the pp vs nn interaction will get still another factor of 4.

The charge symmetric interaction between the nucleons is taken to be the phenomenological Reid soft core potential [19]. This is then also supplemented with explicit excitation of $N\Delta$ intermediate states by the coupled channels method [17]. No other CSB effects are included in the present calculation except Eqs. (2) and (3) [Figs. 1(b) and 1(c)].

The results for the effective range parameters in the low energy expansion $p\cot\delta_0 \approx -1/a + \frac{1}{2}r_0p^2$ are presented in Table I. It can be seen that with this model and the monopole

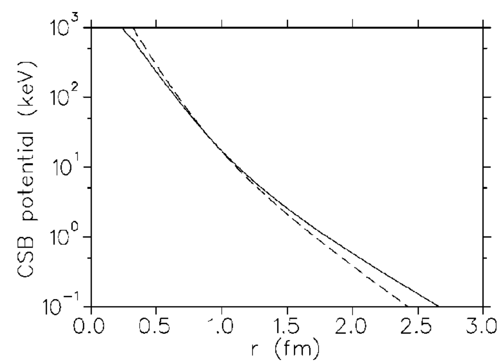

FIG. 3. The coordinate space CSB potentials $V_N(r)$ (solid) and $V_\Delta(r)$ (dashed) obtained from fits of Fig. 2.

TABLE I. CSB effective range parameters and ^{3}H-^{3}He binding energy differences for various models described in the text.

Model	$\Delta a = a_{pp} - a_{nn}$ (fm)	$\Delta r_0 = r_{0,pp} - r_{0,nn}$ (fm)	ΔE (keV)
Reid SC, NN, only Fig. 1(b)	0.28	0.006	20
Reid SC, $NN + N\Delta$, Figs. 1(b) and 1(c)	0.55	0.012	41
Reid SC, Figs. 1(b) and 1(c) dipole ff	0.40	0.009	30
Coupled channels	0.50	0.010	36
Coupled channels, dipole ff	0.37	0.007	27
Experiment [1]	1.5 ± 0.5	0.10 ± 0.12	76 ± 24 [10]

form factor mass $\Lambda = 1$ GeV quite a considerable contribution is obtained to Δa, about one-third of its experimental value and of the same sign (i.e., the nn interaction is the more attractive of the two). The fraction is even larger, if one considers that perhaps 0.4 fm in Δa may be attributed simply to different kinetic energies arising from the np mass difference [20]. One half of the calculated effect here comes from V_N and the other half from the Δ contribution V_Δ.

The column labeled ΔE is the contribution to the ^{3}H-^{3}He binding energy difference using the simple prescription

$$\Delta E_{\rm GS} = (40\Delta a + 1600\Delta r_0)\,\text{keV/fm} \tag{6}$$

obtained by Gibson and Stephenson for separable potentials [21]. This is likely an overestimate as has been seen with more sophisticated potentials [10], but gives an idea of the order of magnitude of the effect. Here the relevant empirical result is $\Delta E_{\rm expt} \approx 76 \pm 24$ keV after removing the "trivial" Coulomb repulsion and the effect of the np mass difference in the kinetic energy.

For model dependence one can vary the form factors. With softer form factors one normally expects smaller results. On line 3 the form factor has been taken to be of the dipole form with the same cutoff mass (or as well monopole vertices and a dipole form factor in πN scattering). The result is about 25% smaller as might be expected. Also the calculation for only NN [Fig. 1(b)] was repeated with monopole form factors and Λ ranging from 600 to 1400 MeV. This caused a variation of Δa between 0.17 and 0.30 fm, respectively, as compared with 0.28 fm of the table. The cutoff dependence could be rather well described by $\Delta a \approx (0.333\text{–}1.446/\Lambda^2)$ fm (Λ in fm^{-1}).

A more interesting and more fundamental comparison is to a phase-equivalent coupled channels calculation with explicit $N\Delta$ intermediate states included in the charge symmetric scattering. Details of the $NN \leftrightarrow N\Delta$ transition potential including both π and ρ exchanges are given in Ref. [17]. The diagonal $^1S_0$ Reid soft core potential must be adjusted by a repulsion of 381 $e^{-3\mu r}/(\mu r)$ MeV to refit the phase shift from the coupled channels with the original at $E_{\rm lab} = 2$ MeV. By unitarity, the NN wave function should be depleted at short distances and consequently the mechanisms of Figs. 1(b) and 1(c) somewhat suppressed. This is, indeed, the case as seen on lines 4 and 5 of Table I, but the decrease is not very large. Since the $N\Delta$ excitation must be an essential part of isospin one NN scattering, this may be considered as the most realistic estimate.

In principle the presence of the Δ makes new diagrams possible, e.g., those with one or both pion-baryon vertices being $\pi\Delta\Delta$ or pion rescattering off the Δ. The knowledge of these is much inferior to πNN or $\pi N\Delta$. These mechanisms are also of higher order and are not discussed in this work. However, one could note that also the above unitarity depletion effect is of higher order in this sense, so that conservatively one can only say that the effect of coupling to the $N\Delta$ intermediate states is only of the order of 10% in the CSB observables.

The above obtained results appear to indicate that CSB pion rescattering could be potentially an important effect also in elastic NN scattering as it was in $np \to d\pi^0$. However, we have not yet considered another isospin violating term in the effective Lagrangian [15,16]

$$\mathcal{L} = \frac{\overline{\delta m}_N}{2}\left(N^\dagger \tau_0 N + \frac{2}{DF_\pi^2} N^\dagger (\phi_0 \boldsymbol{\phi}\cdot\boldsymbol{\tau} - \boldsymbol{\phi}\cdot\boldsymbol{\phi}\tau_0) N \right), \tag{7}$$

of electromagnetic origin. Here $\overline{\delta m}_N$ is the electromagnetic contribution to the np mass difference, typically estimated to be of the order of -1 or -2 MeV. In Ref. [14] this gave a similar contribution as Eq. (1) and the strength parameter changed there $\delta m_N \to \delta m_N - \overline{\delta m}_N/2$. With $\overline{\delta m}_N$ negative this increased the effect. However, in the present case of NN scattering the effect turns out to be the change $\delta m_N \to \delta m_N + 2\overline{\delta m}_N$. Now the two mass difference terms tend to cancel each other and the above results should be scaled

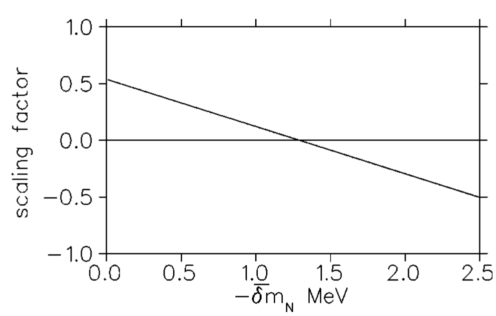

FIG. 4. The scaling factor needed for consistency with the np mass difference and its electromagnetic part.

accordingly by a factor $(\delta m_N + 2\overline{\delta m}_N)/(2.4\ \mathrm{MeV})$ shown as a function of $\overline{\delta m}_N$ in Fig. 4. It can be seen that for the most reasonable range of $|\overline{\delta m}_N|$ between 0.5 and 1.5 MeV [14] the strength of the CSB potentials decreases into a fraction of the original. For example, using $\overline{\delta m}_N = -0.76 \pm 0.30$ MeV from the Cottingham formula [22] yielding the strength 2.4 MeV for Ref. [14], the final results here would be only a quarter of the results in Table I. This means that if the present situation of understanding the pp vs nn difference (in particular Δa) is considered satisfactory, this understanding is not significantly disturbed by the present mechanism even if it is large in pion production in $np \to d\pi^0$.

In summary, a new πN rescattering contribution has been incorporated in CSB elastic NN scattering using an interaction derived from effective field theory. This is potentially a strong effect as was seen for CSB in pion production. However, contrary to Ref. [14] in this class III interaction the quark mass difference and electromagnetic mass contributions tend to cancel, so that the effect in, e.g., Δa actually becomes rather small. Thus even the large CSB contribution found in $np \to d\pi^0$ can be accommodated without compromising the understanding of pp vs nn differences.

[1] G.A. Miller, B.M.K. Nefkens, and I. Slaus, Phys. Rep. **194**, 1 (1990); G.A. Miller and W.T. van Oers, in *Symmetries and Fundamental Interactions in Nuclei*, edited by W.C. Haxton and E.M. Henley (World Scientific, Singapore, 1995).

[2] R. Abegg *et al.*, Phys. Rev. Lett. **56**, 2571 (1986); Phys. Rev. D **39**, 2464 (1989); L.D. Knutson *et al.*, Phys. Rev. Lett. **66**, 1410 (1991); S.E. Vigdor *et al.*, Phys. Rev. C **46**, 410 (1992); J. Zhao *et al.*, *ibid.* **57**, 2126 (1998).

[3] TRIUMF experiment E704, A. Opper and E. Korkmaz, spokespersons.

[4] A.G. Williams, A.W. Thomas, and G.A. Miller, Phys. Rev. C **36**, 1956 (1987); M.J. Iqbal and J.A. Niskanen, *ibid.* **38**, 2259 (1988).

[5] J.A. Niskanen, Few-Body Syst. **26**, 241 (1999).

[6] J.A. Niskanen, M. Sebestyen, and A.W. Thomas, Phys. Rev. C **38**, 838 (1988).

[7] E.M. Henley and G.A. Miller, in *Mesons and Nuclei*, edited by M. Rho and D.H. Wilkinson (North-Holland, Amsterdam, 1979), Vol. I.

[8] S.A. Coon and R.C. Barrett, Phys. Rev. C **36**, 2189 (1987).

[9] S.A. Coon and M.D. Scadron, Phys. Rev. C **26**, 2402 (1982).

[10] S.A. Coon and J.A. Niskanen, Phys. Rev. C **53**, 1154 (1996).

[11] T.E.O. Ericson and G.A. Miller, Phys. Lett. **132B**, 32 (1983); Phys. Rev. C **36**, 2707 (1987).

[12] C.Y. Cheung and R. Machleidt, Phys. Rev. C **34**, 1181 (1986).

[13] J.L. Friar and U. van Kolck, Phys. Rev. C **60**, 034006 (1999).

[14] U. van Kolck, J.A. Niskanen, and G.A. Miller, Phys. Lett. B **493**, 65 (2000).

[15] U. van Kolck, Ph.D. dissertation, University of Texas, 1993; Few-Body Syst., Suppl. **9**, 444 (1995).

[16] S. Weinberg, in *Chiral Dynamics: Theory and Experiment*, edited by A.M. Bernstein and B.R. Holstein (Springer, Berlin, 1995).

[17] J.A. Niskanen, Phys. Rev. C **45**, 2648 (1992).

[18] S.A. Coon, *Invited talk at the XVIIth European Conference on Few-Body Problems in Physics, Elovora, Portugal, 2000* [Nucl. Phys. **A689**, 119c (2001)].

[19] R.V. Reid, Ann. Phys. (N.Y.) **50**, 411 (1968).

[20] E.M. Henley, in *Isospin in Nuclear Physics*, edited by D.H. Wilkinson (North-Holland, Amsterdam, 1969).

[21] B.F. Gibson and G.J. Stephenson, Jr., Phys. Rev. C **8**, 1222 (1973).

[22] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).