

Dissociation of a heavy quarkonium at high temperatures

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We examine three different ways a heavy quarkonium can dissociate at high temperatures. The heavy quarkonium can dissociate spontaneously when it becomes unbound at a temperature above its dissociation temperature. Following the recent work of Digal, Petreczky, and Satz, we calculate the dissociation temperatures of heavy quarkonia taking into account the angular momentum selection rules and using a temperature-dependent potential inferred from lattice gauge calculations. We find that the selection rules change the dissociation temperatures substantially for charmonia but only slightly for bottomia. A quarkonium system in thermal equilibrium with the medium can dissociate by thermalization. The fraction of quarkonium lying above the dissociation threshold increases as temperature increases. A quarkonium can also dissociate by colliding with light hadrons. We evaluate the cross sections for the dissociation of J/ψ and Υ in collision with π as a function of the temperature of the hadron medium, using the quark-interchange model of Barnes and Swanson. We find that as the temperature increases, the threshold energy decreases and the dissociation cross section increases.

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I. INTRODUCTION

The suppression of heavy quarkonium production in a quark-gluon plasma has been a subject of intense interest since the pioneering work of Matsui and Satz [1]. Initial insight into the dissociation temperatures of heavy quarkonium was further provided by Karsch, Mehr, and Satz [2]. Recently, Digal, Petreczky, and Satz [3,4] reported theoretical results on the dissociation temperatures of heavy quarkonia in hadron and quark-gluon plasma phases. These are interesting results as they are related to the use of the suppression of heavy quarkonia production as a signal for the quark-gluon plasma [1].

The basic input of Digal *et al.* is the temperature dependence of the Q - \bar{Q} interaction as inferred from lattice gauge calculations [5]. We can understand conceptually such a temperature dependence of the Q - \bar{Q} potential by placing Q and \bar{Q} as an external source in a QCD medium. If the temperature of the medium is zero, the linear confining potential between Q and \bar{Q} is the result of the alignment of the color electric fields at neighboring sites inside the flux tube, in analogy with the alignment of spins in a ferromagnet [Fig. 1(a)].

When the medium is at a high temperature, the gluon and light quark fields fluctuate and the alignment of the color electric fields due to the QCD interaction is reduced by the thermal motion for a random orientation of the color electric fields [Fig. 1(b)]. The greater the temperature, the stronger the tendency for thermal disorientation, and the weaker the resultant linear interaction between Q and \bar{Q} . This tendency continues until deconfinement sets in at the phase transition temperature T_c .

The variation of the Q - \bar{Q} potential gives rise to three different modes of heavy quarkonium dissociation at high temperatures: (1) spontaneous dissociation, (2) dissociation by thermal thermalization, and (3) dissociation by collision

with particles in the medium. We shall discuss them in turn.

A heavy quarkonium dissociates spontaneously when the binding energy of the quarkonium relative to a pair of final open charm or open bottom mesons vanishes. Utilizing the temperature-dependent potential for charmonia and bottomia, Digal *et al.* found that the dissociation temperatures for ψ' , χ_c , and Y' in units of T_c are, respectively, 0.1 to 0.2, 0.74, and ≥ 0.83 , while the dissociation temperatures for J/ψ , Y , and χ_b are above T_c [3,4].

Even at $T=0$ a proper description of the heavy quarkonium state should be based on a screening potential [2,6–8], as the heavy quarkonium becomes a pair of open charm or open bottom mesons when r becomes very large, due to the action of dynamical quark pairs. Previously, a screening potential was obtained to provide a good description of char-

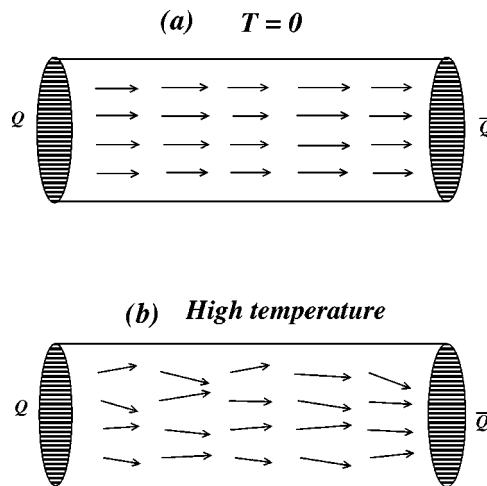


FIG. 1. (a) A schematic configuration of the color electric field between Q and \bar{Q} at $T=0$. (b) A possible configuration of the color electric field in a hot medium. Many different configurations of the color electric fields are possible, with a distribution centered about the mean color electric field configuration which is aligning along the flux tube.

monium bound states, resonances, and e^+e^- decay widths at $T=0$ [8]. This screening potential can be used as the starting point for studying heavy quarkonia at high temperatures. Using a potential different from that of Digal *et al.* [3] will help us find out the sensitivity of the values of dissociation temperatures on the potentials. We would also like to include spin-dependent interactions. Their inclusion is useful because they lead to a better description of the energy splittings of the quarkonium states which affect the dissociation temperatures. Furthermore, these spin and angular momentum quantum numbers give rise to selection rules for the final meson states and alter the dissociation threshold energies and the dissociation temperatures.

The $Q\bar{Q}$ potential at temperature T refers to the situation of placing an external quarkonium (at rest) in a medium of temperature T . The changes of the gluon and light quark fields between the Q and \bar{Q} due to the temperature of the medium give rise to a change of the $Q\bar{Q}$ interaction and the subsequent change of the binding energy. The external quarkonium needs not be in thermal equilibrium with the medium.

The quarkonium placed in a hot medium however will collide with particles in the medium and become thermalized. When the quarkonium reaches thermal equilibrium with the medium, the quarkonium will be in a mixed state whose occupation probabilities for the different energy levels will be distributed statistically according to the Bose-Einstein distribution. There is a finite probability for this system to populate states whose energies lie above their respective dissociation thresholds. This fraction of the quarkonium system above the thresholds will dissociate into open charm or open bottom mesons. Dissociation of this type can be called dissociation by thermalization. Because the binding energies of the quarkonium states decrease as temperature increases, the probability of dissociation by thermalization also increases as the temperature increases.

Finally, a heavy quarkonium can also be dissociated by collision with particles in the medium. The threshold energies and dissociation cross sections will be affected by the temperature dependence of the heavy quarkonium states. We would like to investigate how these dissociation cross sections depend on the temperature of the medium, using the quark-interchange model of Barnes and Swanson [9] which has been applied successfully to many hadron-hadron [9–14] and hadron-(heavy quarkonia) reactions at $T=0$ [15–18].

In Sec. II, we describe the Schrödinger equation and the potential used to calculate the energies and the wave functions of quarkonium states. We introduce a single-particle potential with simple spatial and temperature dependence to represent the results of the lattice gauge calculations. The single-particle states are calculated and exhibited in Sec. III. We discuss the selection rules for the spontaneous dissociation of heavy quarkonia in Sec. IV. In Sec. V, the dissociation temperatures are listed and compared with previous results of Digal *et al.* [3]. In Sec. VI, we introduce the concept of dissociation by thermalization and estimate the fraction of the quarkonium system which can dissociate spontaneously as a function of temperature. In Sec. VII, the cross sections

for the dissociation of ψ and Y in collision with pions are calculated as a function of temperature. We estimate the survival probability of a heavy quarkonium in collision with pions in Sec. VIII. We present our conclusions and discussions in Sec. IX.

II. SCHRÖDINGER EQUATION FOR HEAVY QUARKONIUM STATES

The energy ϵ of the heavy quarkonium single-particle state $(Q\bar{Q})_{\text{JLS}}$ can be obtained by solving the eigenvalue of the Schrödinger equation

$$\left\{ -\nabla \cdot \frac{\hbar^2}{2\mu_{12}} \nabla + V_{12}(r, T) + \Delta(r, T) \right\} \psi_{\text{JLS}}(\mathbf{r}, T) = \epsilon(T) \psi_{\text{JLS}}(\mathbf{r}, T), \quad (2.1)$$

where

$$\Delta(r, T) = m_1(r, T) + m_2(r, T) - m_1(\infty, T) - m_2(\infty, T). \quad (2.2)$$

We have followed Ref. [8] and describe the system as a two-body system whose properties and masses vary with r . At a distance $r \lesssim R$ where $R \sim 0.8 - 1.0$ fm, the system consists of a quark and an antiquark, and their masses $\{m_1(r, T), m_2(r, T)\} = \{m_Q(T), m_{\bar{Q}}(T)\}$.

The nature of the two-body system at $r \gtrsim R$ depends on the temperature. If the temperature is below the phase transition temperature T_c for light quark deconfinement, a heavy quark Q and antiquark \bar{Q} separated at large distances will lead to the production of a light quark pair $\bar{q}q$ in between and the subsequent formation of open charm or open bottom mesons $(Q\bar{q})$ and $(q\bar{Q})$. At temperatures below T_c , the system at $r \gtrsim R$ therefore consists of a pair of heavy open flavor mesons $(Q\bar{q})$ and $(q\bar{Q})$ and their masses $\{m_1(r, T), m_2(r, T)\} = \{M_{Q\bar{q}}(T), M_{q\bar{Q}}(T)\}$. By including dynamical quarks, the lattice gauge calculations of Karsch *et al.* [5] incorporate this change of the configuration at large distances. In order to match with the lattice gauge calculations, the pair of mesons at large distances $(Q\bar{q})$ and $(q\bar{Q})$ should be taken to be the lowest-mass pair, because the lattice gauge calculation results refer to those of the lowest-energy states of the system. The quantity μ_{12} is the reduced mass. The interaction V_{12} is the interaction between Q and \bar{Q} for $r \lesssim R$ and between $(Q\bar{q})$ and $(q\bar{Q})$ for $r \gtrsim R$. As the interaction between mesons has a short range, the interaction $V_{12}(r, T)$ vanishes as r approaches infinity.

Above T_c , light quarks are deconfined and the system is a separated Q and \bar{Q} . Thus, $\{m_1(r, T), m_2(r, T)\} = \{m_Q(T), m_{\bar{Q}}(T)\}$ and $\Delta(r, T) = 0$. The interaction $V_{12}(r, T)$ also vanishes as r approaches infinity because the interaction between Q and \bar{Q} is screened at large distances in the deconfined phase.

In Eq. (2.1) the energy $\epsilon(T)$ is measured relative to the two-body pair at $r \rightarrow \infty$. The quarkonium is bound if $\epsilon(T)$ is negative. The quarkonium is unbound and dissociates spon-

taneously into two particles if $\epsilon(T)$ is positive, subject to selection rules which we shall discuss in Sec. IV.

In the present manuscript, we shall limit our attention to $T \leq T_c$ for which the free energy contains contributions only from the color-singlet component and the extraction of the color-singlet potential from the lattice gauge calculations is without ambiguity. For T slightly greater than T_c , it is necessary to make assumptions on the relative fractions of the color-singlet and color-octet contributions in order to extract the $Q\bar{Q}$ potential [4] and the extracted potential there may depend on the assumed color-singlet and color-octet fractions.

Limiting our attention to $T < T_c$ we note that the mass term $\Delta(r, T)$ can be approximately represented as

$$\Delta(r, T) \approx [2m_Q(T) - 2M_{Q\bar{q}}(T)] \theta(R-r). \quad (2.3)$$

The meson mass $M_{Q\bar{q}}(T)$ vary with temperature [20–22]. Neglecting the kinetic energy of the heavy quark, the meson mass at temperature T is

$$M_{Q\bar{q}}(T) = m_Q(T) + \sqrt{\mathbf{p}_q^2 + [m_{\bar{q}}(T)]^2} + \langle V_{Q\bar{q}}(T) \rangle \quad (2.4)$$

which leads to

$$\begin{aligned} & [2m_Q(T) - 2M_{Q\bar{q}}(T)] - [2m_Q(T=0) - 2M_{Q\bar{q}}(T=0)] \\ &= -2 \left[\sqrt{\mathbf{p}_q^2 + [m_{\bar{q}}(T)]^2} + \langle V_{Q\bar{q}}(T) \rangle \right]_{T=0}^T. \end{aligned} \quad (2.5)$$

In the evaluation of the Polyakov loop one obtains the free energy of the system from the lattice calculations. This free energy contains the interaction V_{12} between Q and \bar{Q} or between $(Q\bar{q})$ and $(q\bar{Q})$, depending on the separation r . In addition, the free energy includes also the term on the right hand side of Eq. (2.5) due to the variation of light quark q with temperature, since the light quark fields are dynamical variables and they change the energy of the light quark, and the interaction between the light quark and the heavy quark $\langle V_{Q\bar{q}}(T) \rangle$. The free energy of the lattice gauge calculations actually yields an effective interaction $V(r, T)$ which includes $V_{12}(r, T)$ and the temperature dependence of $\Delta(r, T) - \Delta(r, 0)$:

$$V(r, T) = V_{12}(r, T) + [\Delta(r, T) - \Delta(r, T=0)]. \quad (2.6)$$

This effective $Q\bar{Q}$ interaction $V(r, T)$ vanishes at large distances.

In terms of this potential $V(r, T)$ extracted from the lattice gauge calculation, the Schrödinger Eq. (2.1) is then

$$\begin{aligned} & \left\{ -\nabla \cdot \frac{\hbar^2}{2\mu_{12}} \nabla + V(r, T) + \Delta(r, T=0) \right\} \psi_{\text{JLS}}(\mathbf{r}, T) \\ &= \epsilon(T) \psi_{\text{JLS}}(\mathbf{r}, T). \end{aligned} \quad (2.7)$$

Our first task is to obtain a convenient representation of $V(r, T)$ from the results of lattice gauge calculations of Karsch *et al.* [5]. Following Karsch *et al.* [2], we represent $V(r, T)$ by a Yukawa plus an exponential potential [2,6–8]

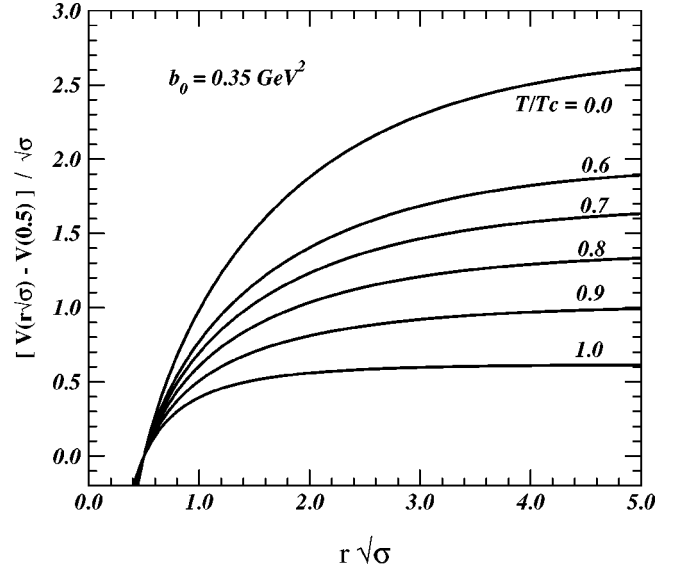


FIG. 2. Quark-antiquark potential as a function of r obtained by using Eqs. (2.8)–(2.10) and $b_0 = 0.35 \text{ GeV}^2$.

$$V(r, T) = -\frac{4}{3} \frac{\alpha_s e^{-\mu(T)r}}{r} - \frac{b(T)}{\mu(T)} e^{-\mu(T)r}, \quad (2.8)$$

where $b(T)$ is the effective string-tension coefficient, $\mu(T)$ is the effective screening parameter, and the potential is calibrated to vanish as r approaches infinity. The results of the lattice calculations of Ref. [5] for $T \leq T_c$, which has been normalized to the Cornell potential [19] at short distances, can be described by

$$b(T) = b_0 [1 - (T/T_c)^2] \theta(T_c - T) \quad (2.9)$$

and

$$\mu(T) = \mu_0 \theta(T_c - T), \quad (2.10)$$

where $b_0 = 0.35 \text{ GeV}^2$, $\mu_0 = 0.28 \text{ GeV}$, and θ is the step function. The value of μ_0 has been fixed to be the same as in $T=0$, and the value of b_0 is close to the value of $b = 0.335 \text{ GeV}^2$ obtained earlier at $T=0$ [8].

As a comparison, we plot in Fig. 2 the quantity $[V(r\sqrt{\sigma}) - V(0.5)]/\sqrt{\sigma}$ calculated with the above potential, Eqs. (2.8)–(2.10), as a function of $r\sqrt{\sigma}$ for charmonium at different temperatures. The scale constant $\sqrt{\sigma} = 0.425 \text{ GeV}$ is used to convert distance and energy to dimensionless numbers as in the lattice gauge calculations of Ref. [5]. The general features of the spatial and temperature dependence of the potential in Eqs. (2.8)–(2.10) agree well with the lattice gauge results (Fig. 6 of Ref. [5]). In particular, there is good agreement between the variation of the potential difference between $r\sqrt{\sigma} = 5$ and $r\sqrt{\sigma} = 0.5$ as a function of the temperature calculated for charmonium, as shown in Fig. 3.

In our calculation, we use a running coupling constant obtained in another meson spectrum study [18]

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \ln(A + Q^2/B^2)}, \quad (2.11)$$

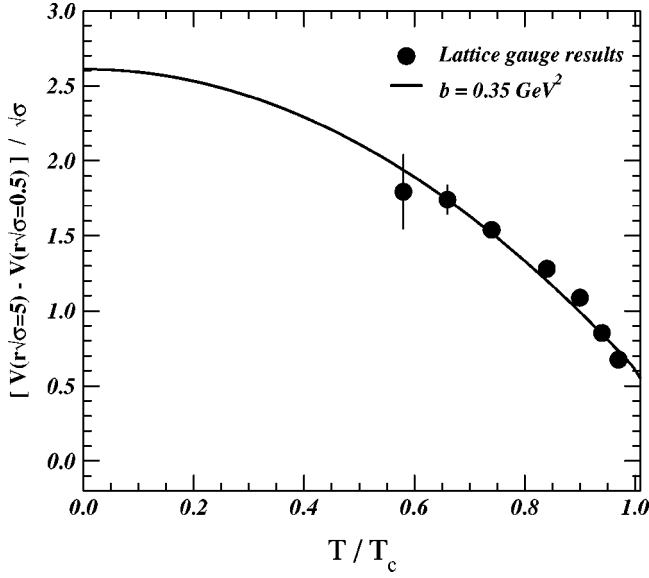


FIG. 3. The potential difference between $r\sqrt{\sigma}=5$ and $r\sqrt{\sigma}=0.5$ as a function of the temperature. The curve gives the results of Eqs. (2.8) and (2.10) with the parameter $b_0=0.35 \text{ GeV}^2$ and the solid points are lattice gauge results of Ref. [5].

where $A=10$ and $B=0.31 \text{ GeV}$, with Q identified as the mass of the meson. The above formula gives $\alpha_s \sim 0.32$ for charmonium and $\alpha_s \sim 0.24$ for bottomium. We include, in addition, the spin-spin, spin-orbit, and tensor interactions

$$V_{\text{spin-spin}} + V_{\text{spin-orbit}} + V_{\text{tensor}} = \frac{4}{3} \times \frac{8\pi\alpha_s}{3m_1m_2} s_1 \cdot s_2 \left(\frac{d^3}{\pi^{3/2}} \right) e^{-d^2r^2} + C_{LS} \mathbf{L} \cdot \mathbf{S} + C_T S_{12}, \quad (2.12)$$

where s_i are the spins of the interacting constituents, \mathbf{L} is the orbital angular momentum, $\mathbf{S} = \mathbf{s}_1 + \mathbf{s}_2$, and $S_{12} = \{3(\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}}) \times (\boldsymbol{\sigma}_2 \cdot \hat{\mathbf{r}}) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\}$. For our calculations, we take $\{C_{LS}, C_T\} = \{34.6, 9.78\} \text{ MeV}$ for charmonium, $\{C_{LS}, C_T\} = \{14.25, 2.98\} \text{ MeV}$ for bottomium. The width parameter d is the range of the spin-spin interaction. Following Godfrey and Isgur [23], we use a running width parameter to include the relativistic effects on the width of the spin-spin interaction

$$d^2 = \sigma_0^2 \left\{ \frac{1}{2} + \frac{1}{2} \left(\frac{4m_1m_2}{(m_1+m_2)^2} \right)^4 \right\} + \sigma_1^2 \left(\frac{2m_1m_2}{m_1+m_2} \right)^2, \quad (2.13)$$

where we obtain the values of $\sigma_0 = 0.591 \text{ GeV}$ and $\sigma_1 = 1.967$ by examining the hyperfine splittings of π - ρ and η_c - J/ψ .

For $T=0$, $M_{Q\bar{q}}$ are known from experimental B and D meson masses. Using the $V(r, T=0)$ extrapolated from lattice gauge calculations, we vary m_c and m_b and find the zero-temperature constituent quark masses $m_c = 1.89 \text{ GeV}$ and $m_b = 5.22 \text{ GeV}$ which give a good fit to the heavy quarkonium spectrum (Figs. 4 and 5). These heavy quark

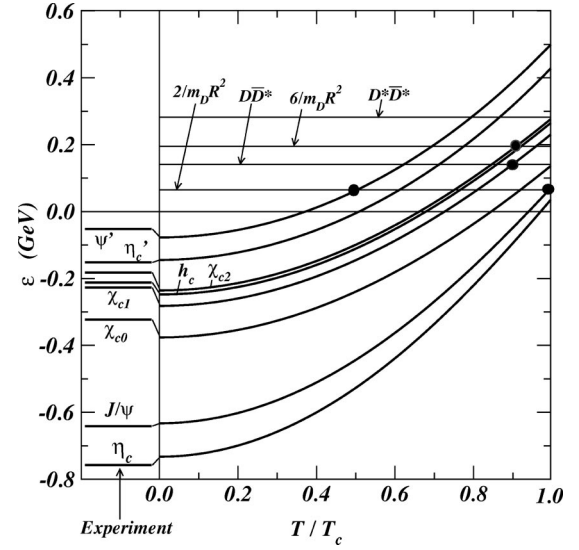


FIG. 4. Charmonium single-particle states as a function of temperature. The threshold energies are indicated as horizontal lines. The solid circles indicate the locations of the dissociation temperatures for states with significant decay branching fractions into J/ψ .

constituent mass values are close to those of $m_c = 1.84 \text{ GeV}$ and $m_b = 5.18 \text{ GeV}$ in the Cornell potential [19], whose short-distance values have been used to normalize the results of the lattice gauge calculations [5].

The string tension term in the screening potential Eq. (2.8) needs to have the property that it vanishes at $r \rightarrow \infty$, and the exponential potential has been conveniently chosen to describe such a property. It is of interest to compare the exponential potential $-b_0 \exp\{-\mu_0 r\}/\mu_0$ with the standard linear potential σr at $T=0$. The effective local string tension, as obtained by taking dV/dr , is σ for the linear potential, independent of r . For the exponential potential, it is

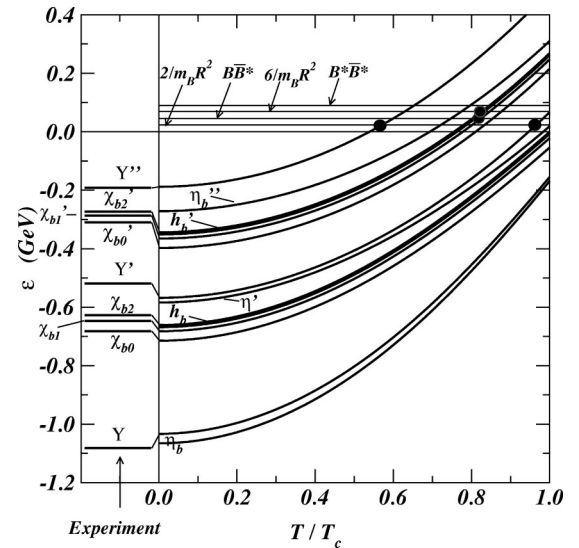


FIG. 5. Bottomium single-particle energies as a function of temperature. The solid circles indicate the locations of the dissociation temperatures for states with significant decay branching fractions into Y .

$b_0 \exp\{-\mu_0 r\}$ which is equal to b_0 at $r=0$ and zero at $r \rightarrow \infty$. At the radius of J/ψ , the local string tension is $0.57b_0 = 0.20 \text{ GeV}^2$, which is close to the standard value of $\sigma = 0.18 \text{ GeV}^2$, as it should be. It is therefore reasonable that the parameter b_0 is approximately two times the standard value of σ [18]. The large value of b_0 in comparison with σ was noted earlier by Ding *et al.* [7].

III. HEAVY QUARKONIUM SINGLE-PARTICLE STATES

The eigenvalues of the Hamiltonian can be obtained by matrix diagonalization using a set of nonorthogonal Gaussian basis states with different widths, as described in Ref. [18]. For the construction of the Hamiltonian matrix, the matrix elements of the Yukawa potential and exponential potential between Gaussian basis states are

$$\begin{aligned} \langle i | e^{-\mu r} | j \rangle &= \left(2 \sqrt{\frac{ij}{i+j}} \right)^{l+3/2} \frac{2^{l+3/2}(l+1)!}{\sqrt{\pi}} \\ &\times \exp\left\{ \frac{\mu^2}{4(i+j)\beta^2} \right\} U\left(2l+5, \frac{\mu}{\sqrt{(i+j)\beta}} \right) \end{aligned} \quad (3.1)$$

and

$$\begin{aligned} \left\langle i \left| \frac{e^{-\mu r}}{r} \right| j \right\rangle &= \left(2 \sqrt{\frac{ij}{i+j}} \right)^{l+3/2} \frac{4\pi\beta\sqrt{i+j}(2l+1)!}{(2\pi)^{3/2}(2l+1)!!} \\ &\times \exp\left\{ \frac{\mu^2}{4(i+j)\beta^2} \right\} U\left(2l+\frac{3}{2}, \frac{\mu}{\sqrt{(i+j)\beta}} \right), \end{aligned} \quad (3.2)$$

where U is the cylinder parabolic function, β is the width parameter of the wave function basis, and we have used the notation and normalization of Ref. [18]. The matrix elements of the kinetic energy operator are

$$\langle i | -\nabla \cdot \frac{\hbar^2}{2\mu_{12}(\mathbf{r})} \nabla | j \rangle = T_{ij>} + T_{ij<}, \quad (3.3)$$

where

$$\begin{aligned} T_{ij>} &= \frac{\hbar^2}{2\mu_{>}} \left(2 \sqrt{\frac{ij}{i+j}} \right)^{l+3/2} (i+j)\beta^2 \left\{ \frac{l}{(2l+1)!!} \right. \\ &\times [-(2l+1)2^l I_l(z) + 2^{l+1} I_{l+1}(z)] \\ &\left. + \frac{ij}{(i+j)^2} \frac{(2l+3)!! - 2^{l+2} I_{l+2}(z)}{(2l+1)!!} \right\}, \end{aligned} \quad (3.4)$$

$$\begin{aligned} T_{ij<} &= \frac{\hbar^2}{2\mu_{<}} \left(2 \sqrt{\frac{ij}{i+j}} \right)^{l+3/2} (i+j) \\ &\times \beta^2 \left\{ \frac{l}{(2l+1)!!} [(2l+1)2^l I_l(z) - 2^{l+1} I_{l+1}(z)] \right. \\ &\left. + \frac{ij}{(i+j)^2} \frac{2^{l+2} I_{l+2}(z)}{(2l+1)!!} \right\}, \end{aligned} \quad (3.5)$$

$$z = \sqrt{\frac{i+j}{2}} \beta R, \quad (3.6)$$

$$\begin{aligned} I_n(z) &= \frac{2}{\sqrt{\pi}} \int_0^z t^{2n} e^{-t^2} dt = (-1)^n \\ &\times \left\{ \left(\frac{\partial}{\partial \lambda} \right)^n [\lambda^{-1/2} \text{erf}(\lambda^{1/2} z)] \right\}_{\lambda=1}, \end{aligned} \quad (3.7)$$

and $\mu_{>}$ and $\mu_{<}$ are the reduced masses in the region $r > R$ and $r < R$, respectively.

After the Hamiltonian matrix is constructed using these matrix elements, the single-particle state energies and wave functions can be obtained by matrix diagonalization. In Figs. 4 and 5, we show charmonium and bottomium single-particle states as a function of temperature. On the left panel of each figure, the experimental single-particle energies at $T=0$ are also shown for comparison. The single-particle energies $\{\epsilon_i\}$ rise with increasing temperatures. At a certain temperature, the energy of a single-particle state will rise above its threshold for dissociation. But what is its dissociation threshold?

IV. SELECTION RULES FOR SPONTANEOUS DISSOCIATION OF HEAVY QUARKONIUM

In considering the dissociation below T_c , it is necessary to find the selection rules for the spontaneous dissociation of a heavy quarkonium state with initial quantum numbers J , L_i , and S_i into two mesons with a total spin S and a relative orbital angular momentum L . We shall only discuss the selection rules for charmonium states, as similar rules apply in the case of bottomium states. We limit our attention to final states consisting of a D or D^* with an antiparticle \bar{D} or \bar{D}^* . The parity of a quarkonium state is $(-1)^{L_i+1}$. On the other hand, the parity of a pair of dissociated open charm mesons is $(-1)^L$. Parity conservation requires $\Delta L = |L - L_i| = 1$ in this dissociation. Hence, J/ψ and ψ' will dissociate into a pair of open charm mesons with $L=1$, while χ states will dissociate into a pair of open charm mesons with $L=0$ or 2 .

The threshold for spontaneous dissociation of J/ψ and ψ' is therefore $M(D) + M(\bar{D}) + 2/2\mu_{D\bar{D}}R^2$ where $\mu_{D\bar{D}}$ is the reduced mass of the final two-meson system and the term $2/2\mu_{D\bar{D}}R^2$ represents the $L=1$ centrifugal barrier height at the distance R at which D and \bar{D} become on the mass shell. The reduced mass $\mu_{D\bar{D}}$ is equal to $m_D/2$. For numerical purposes, we use $R=0.8 \text{ fm}$.

Consider now the dissociation of χ_{c2} . If the final orbital angular momentum of the two-meson state is $L=0$, then

TABLE I. Selection rules for the dissociation of S -wave charmonia.

Initial heavy charmonium	Final mesons	L	Threshold
$J/\psi, \psi'$	$D + \bar{D}$	1	$M(D) + M(\bar{D}) + 2/2\mu_{D\bar{D}}R^2$
	$D + \bar{D}^*$	1	$M(D) + M(\bar{D}^*) + 2/2\mu_{D\bar{D}^*}R^2$
	$D^* + \bar{D}^*$	1	$M(D^*) + M(\bar{D}^*) + 2/2\mu_{D^*\bar{D}^*}R^2$
η_c	$D + \bar{D}$	1	not allowed
	$D + \bar{D}^*$	1	$M(D) + M(\bar{D}^*) + 2/2\mu_{D\bar{D}^*}R^2$
	$D^* + \bar{D}^*$	1	$M(D^*) + M(\bar{D}^*) + 2/2\mu_{D^*\bar{D}^*}R^2$

because $J=L+S=2$, the final two-meson system must have $S=2$ and the two mesons must each have $S=1$. The threshold energy for the dissociation of χ_{c2} into two mesons in the $L=0$ state is $M(D^*) + M(\bar{D}^*)$. On the other hand, if the final state has $L=2$, then any spin combination of the final meson is allowed, and the lowest threshold for the dissociation of χ_{c2} into two mesons in the $L=2$ state is $M(D) + M(\bar{D}) + 6/2\mu_{D\bar{D}}R^2$.

Consider next the dissociation of χ_{c1} . If the final state has $L=0$, then because $J=L+S=1$, the final meson states must have $S=1$. The threshold energy for the dissociation of χ_{c1} into two mesons in the $L=0$ state is $M(D) + M(\bar{D}^*)$. The dissociation of χ_{c1} into $D + \bar{D}$ is not allowed.

One can obtain the selection rules for η_c , χ_{c0} , and h_c in a similar way. We list the selection rules for the S -wave charmonium states (J/ψ , ψ' , and η_c) in Table I, and the P -wave charmonium states (χ_{cJ} and h_c) in Table II.

Similar selection rules can be written down for bottomium dissociation by replacing J/ψ by Y , ψ' by Y' , χ_c by χ_b , h_c by h_b , and D by B in Tables I and II.

V. DISSOCIATION TEMPERATURES FOR CHARMONIA AND BOTTOMIA

Figure 4 shows the charmonium single-particle states as a function of temperature. The dissociation temperatures of different quarkonium states can be determined by plotting the threshold quantities and the state energies of the quarkonia. The points of intercept, as indicated by solid circles in Fig. 4, give the positions of the dissociation temperatures. Because the observed J/ψ receives little feeding from the χ_{c0} and h_c states, and η_c is not observed in dilepton decays, we shall not include the dissociation temperatures of χ_{c0} , h_c , and η_c (and similarly χ_{b0} , h_b , and η_b) in our consideration. Figure 5 gives similar temperature dependence of the bottomium single-particle states as a function of temperature.

We list the dissociation temperatures of charmonia and bottomia in Table III. If the dissociation threshold is just $M(D) + M(\bar{D})$, the dissociation temperatures in units of T_c for ψ' , χ_{c2} , χ_{c1} , and J/ψ in units of T_c would be 0.37, 0.66, 0.72, and 0.94, as listed in the second row of Table III. This is in rough agreement with the results of Digal *et al.* which give the values of 0.1–0.2, 0.74, and 1.1 for ψ' , χ_c , and J/ψ , respectively. One of the main reasons for the observed

TABLE II. Selection rules for the dissociation of P -wave charmonia.

Initial heavy charmonium	Final mesons	L	Threshold
χ_{c2}	$D + \bar{D}$	0	not allowed
		2	$M(D) + M(\bar{D}) + 6/2\mu_{D\bar{D}}R^2$
	$D + \bar{D}^*$	0	not allowed
		2	$M(D) + M(\bar{D}^*) + 6/2\mu_{D\bar{D}^*}R^2$
	$D^* + \bar{D}^*$	0	$M(D^*) + M(\bar{D}^*)$
		2	$M(D^*) + M(\bar{D}^*) + 6/2\mu_{D^*\bar{D}^*}R^2$
χ_{c1}	$D + \bar{D}$	0	not allowed
		2	not allowed
	$D + \bar{D}^*$	0	$M(D) + M(\bar{D}^*)$
		2	$M(D) + M(\bar{D}^*) + 6/2\mu_{D\bar{D}^*}R^2$
	$D^* + \bar{D}^*$	0	$M(D^*) + M(\bar{D}^*)$
		2	$M(D^*) + M(\bar{D}^*) + 6/2\mu_{D^*\bar{D}^*}R^2$
χ_{c0}	$D + \bar{D}$	0	$M(D) + M(\bar{D})$
		2	not allowed
	$D + \bar{D}^*$	0	not allowed
		2	not allowed
	$D^* + \bar{D}^*$	0	$M(D^*) + M(\bar{D}^*)$
		2	$M(D^*) + M(\bar{D}^*) + 6/2\mu_{D^*\bar{D}^*}R^2$
h_c	$D + \bar{D}$	0	not allowed
		2	not allowed
	$D + \bar{D}^*$	0	$M(D) + M(\bar{D}^*)$
		2	$M(D) + M(\bar{D}^*) + 6/2\mu_{D\bar{D}^*}R^2$
	$D^* + \bar{D}^*$	0	$M(D^*) + M(\bar{D}^*)$
		2	$M(D^*) + M(\bar{D}^*) + 6/2\mu_{D^*\bar{D}^*}R^2$

differences is that the locations of the dissociation temperatures depend on the energies of the theoretical single-particle states. Hyperfine interaction, spin-orbit interaction and the shape of the potential affect the positions of the single-particle states. Although the fine structures are small in terms of the separation between major shells, they are nonetheless important in determining the locations of the dissociation temperatures.

With the additional threshold energies due to the angular momentum selection rules, the dissociation temperatures T_d in units of T_c for ψ' , χ_{c2} , χ_{c1} , and J/ψ are shifted to 0.50, 0.91, 0.90, and 0.99, as listed in the first row of Table III. The increase in the dissociation temperature is greatest for χ_{c2} and least for J/ψ .

For the bottomium system, because the mass of the bottom quark is very large, the mass difference between B^* and B and the centrifugal barrier energies are small; the selection rules do not lead to substantial changes of the thresholds. The shifts of the dissociation temperatures are not as large as in the case of charmonium states. The dissociation temperatures for various bottomium states are listed in Table III. We confirm the general features of the results of Digal *et al.* but there are also differences in the details as the dissociation

TABLE III. The dissociation temperatures T_d in units of T_c for various heavy quarkonia.

Heavy quarkonium	ψ'	χ_{c2}	χ_{c1}	J/ψ	Y''	χ'_{b2}	χ'_{b1}	Y'	χ_{b2}	χ_{b1}	Y
T_d/T_c	0.50	0.91	0.90	0.99	0.57	0.82	0.82	0.96	> 1.00	> 1.00	> 1.00
T_d/T_c	0.37	0.66	0.72	0.94	0.54	0.75	0.78	0.95	1.00	> 1.00	> 1.00
(Selection rules not invoked)											
T_d/T_c (Digal <i>et al.</i>)	0.1-0.2	0.74	0.74	1.10	0.75	0.83	0.83	1.10	1.13	1.13	2.31

temperatures depend on the potential and interactions, as well as on the selection rules.

With the temperature-dependent potential of Eqs. (2.8)–(2.12), all the dissociation temperatures of heavy quarkonia are below T_c , except for χ_{b1} , χ_{b2} , and Y . The present analysis uses a potential valid in the range $T \leq T_c$ and cannot be used to determine dissociation temperatures beyond T_c .

VI. DISSOCIATION OF QUARKONIUM BY THERMALIZATION

In addition to spontaneous dissociation, dissociation by interaction with particles in the medium can also lead to dissociation. In order to bring out the salient features, it is useful to present simplifying descriptions of these dissociation processes in terms of dissociation by thermalization and dissociation by collisions. A full description will involve the interplay between the dynamics of thermalization and the dynamics of different types of dissociation.

In the dissociation by thermalization, we consider a two-step process. In the first step, a thermal equilibrium can arise from inelastic reactions of the type

$$h + (Q\bar{Q})_{JLS} \rightarrow h' + (Q\bar{Q})_{J'L'S'} \quad (6.1)$$

in which the collision of a hadron h with a quarkonium in state JLS results in the excitation or deexcitation of the quarkonium state. If the heavy quarkonium reaches thermal equilibrium with the medium, the occupation probabilities of the heavy quarkonium state ϵ_i will be distributed according to the Bose-Einstein distribution

$$n_i = \frac{1}{\exp\{(\epsilon_i - \mu)/T\} - 1}. \quad (6.2)$$

If there is no dissociation of the quarkonium, then the sum of the occupation numbers is

$$n = \sum_i (2J_i + 1)n_i = 1, \quad (6.3)$$

where $2J_i + 1$ is the spin degeneracy. This equation can be used to determine the chemical potential μ . At temperature T , the fraction of heavy quarkonium whose single-particle state energies lie above their dissociation thresholds is

$$f = \sum_i n_i |_{\epsilon_i \geq \epsilon_{\text{th}}}. \quad (6.4)$$

In the second step, after the quarkonium reaches thermal equilibrium with the medium, there is a finite probability for the quarkonium system to be found in excited states. If these excited states lie above their thresholds for spontaneous dissociation, the heavy quarkonium system will have a finite probability to dissociate into open charm or open bottom mesons.

Previously, dissociation by thermalization was studied by Kharzeev, McLerran, and Satz [24] with a temperature-independent dissociation threshold and free-gas continuum states in a Boltzmann distribution. They found that the dissociation by thermalization does not provide a significant amount of J/ψ dissociation at temperature $T = 0.2$ GeV. Results in Secs. III and IV indicate, however, that the dissociation thresholds and the positions of the single-particle states change with temperature. Furthermore, the set of states ϵ_i included in Eqs. (6.2) and (6.3) in the quarkonium thermal equilibration process (6.1) should consist only of bound and resonance states [25], because the thermal equilibration of the heavy quarkonium $Q\bar{Q}$ takes place when Q and \bar{Q} remain in the vicinity of each other. Free-gas continuum states off the resonance represent $Q\bar{q}$ and $q\bar{Q}$ states whose spatial amplitudes lie predominantly outside the quarkonium system and they do not participate significantly in the quarkonium thermal equilibration process (6.1).

To study dissociation of heavy quarkonium by thermalization, we evaluate the fraction f of heavy quarkonium whose single-particle energies lie above their dissociation thresholds, as a function of temperature, using the single-particle states obtained in Secs. III and IV. The results are shown in Fig. 6. The fraction f increases with temperature, due predominantly to the change of the positions of the single-particle energies. In Fig. 6, a state label shown along the curves denotes the onset of spontaneous dissociation of that state. As the temperature approaches T_c , the fraction lying above dissociation thresholds is quite large for charmonium, and is considerably smaller for bottomium.

By the dissociation of the excited heavy quarkonium, the occupation number is no longer constant and the rate of dissociation of the quarkonium is

$$\frac{dn}{dt} = - \sum_i \lambda_i (2J_i + 1) n_i |_{\epsilon_i \geq \epsilon_{\text{th}}}, \quad (6.5)$$

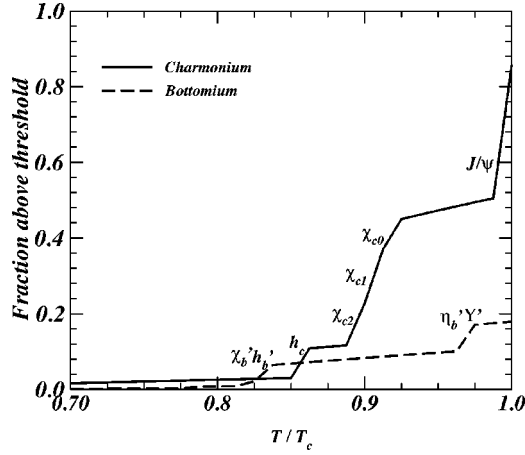


FIG. 6. The fractions of charmonium and bottomonium lying above the dissociation threshold as a function of T/T_c .

where λ_i is the decay rate of the dissociating excited quarkonium state i . These dissociation rates for various excited heavy quarkonium states have not been evaluated. It will be interesting to calculate these rates using explicit quark model wave functions and the formalism discussed by Ackleh, Barnes, and Swanson [26].

In order to appreciate the effect of dissociation by thermalization on quarkonium survival probability, one can consider as an example the case of placing a J/ψ in a medium below the dissociation temperature of J/ψ . If the J/ψ is not in thermal equilibrium with the medium, then this J/ψ system will be stable against spontaneous dissociation. However, with the approach of thermal equilibrium (by collision with particles in the medium) at a temperature T , the initial J/ψ system will evolve into a mixed state with a probability distribution to populate different states of the quarkonium system, some of which lie above their dissociation thresholds at that temperature. As a result, a fraction of the charmonium system will dissociate into open charm mesons, even though the temperature is below the J/ψ dissociation temperature. Thus, a nonequilibrated J/ψ that is stable in the medium may become partially unstable against dissociation when it reaches thermal equilibrium. The dissociation probability of a quarkonium system depends on its state of thermal equilibrium or nonequilibrium.

VII. DISSOCIATION OF HEAVY QUARKONIUM BY COLLISION

In the hadron thermal bath in which a heavy quarkonium is placed, there will be hadrons which will collide with the heavy quarkonium. Dissociation can occur as a result of these collisions.

Previously, we studied the dissociation of heavy quarkonium by collision with light hadrons at $T=0$ [15–18]. We study in this section the dissociation of J/ψ and Y by collision with pions in a medium at temperature T . The heavy quarkonia under consideration can be part of a system in thermal equilibrium with the medium. They can also be nonequilibrated heavy quarkonia introduced into the medium. The kinetic energy of collision would be of the order of the

temperature of the hadron gas. As the temperature of the medium increases, the quarkonium single-particle energy increases and the energy needed to dissociate the heavy quarkonium decreases. If the energy threshold for the dissociation of the heavy quarkonium is comparable to the temperature of the hadron gas, the dissociation cross section will be large.

We can calculate the dissociation cross sections of J/ψ and Y in collision with π as a function of the temperature using the Barnes and Swanson model [9]. The calculation requires the energies and wave functions of the initial and final meson states, as well as the interquark interaction which leads to the dissociation. For the interquark interaction, we generalize the temperature-dependent Yukawa and exponential interaction in Eqs. (2.8) and (2.12) to the form

$$V_{ij} = \frac{\lambda(i)}{2} \cdot \frac{\lambda(j)}{2} \left\{ \frac{\alpha_s e^{-\mu(T)r}}{r} - \frac{3b(T)}{4\mu(T)} e^{-\mu(T)r} - \frac{8\pi\alpha_s}{3m_i m_j} \mathbf{s}_i \cdot \mathbf{s}_j \left(\frac{d^3}{\pi^{3/2}} \right) e^{-d^2 r^2} \right\}. \quad (7.1)$$

For an antiquark, the generator $\lambda/2$ is replaced by $-\lambda^T/2$. We have already calculated the single-particle energies and wave functions of J/ψ and Y at various temperatures in Secs. III and IV. We can calculate the wave functions of π , open charm, and open bottom mesons using the above interaction. The dissociation reactions with the lowest final meson masses are $J/\psi + \pi \rightarrow D\bar{D}^*, D^*\bar{D}, D^*\bar{D}^*$ and $Y + \pi \rightarrow B\bar{B}^*, B^*\bar{B}, B^*\bar{B}^*$. As the final states involves the D^* and the B^* while the above potential of Eqs. (2.8)–(2.12) refers to $D\bar{D}$ or $B\bar{B}$ at $r \rightarrow \infty$, we also need the mass differences of $M(D^*) - M(D)$ and $M(B^*) - M(B)$, which we shall take from the experimental masses at $T=0$. According to the spectral analysis by Hatsuda, the mass of pion does not change substantially as a function of temperature (see Fig. 3 of Ref. [21]). We shall keep the mass of pion to be the same as in $T=0$ in this calculation.

The reaction matrix element of V_{ij} is a product of the color matrix element, the flavor matrix element, the spin matrix elements, and the spatial matrix elements, as discussed in detail in Refs. [9] and [18]. With our Gaussian basis states, the spatial matrix elements of V_{ij} between the initial and the final states can be reduced to the evaluation of Gaussian integrals of the Fourier transform of the interaction potential, as shown in Eqs. (48)–(50) of Ref. [18]. For the above Yukawa and exponential interactions, the corresponding integrals are

$$\int d\mathbf{q} e^{-(\mathbf{q}-\mathbf{q}_0)^2/2\beta^2} V_{\text{Yukawa}}(\mathbf{q}) = \frac{4\pi\alpha_s}{q_0} (\sqrt{2\pi}\beta)^3 \mathcal{I}_{-1}, \quad (7.2)$$

$$\int d\mathbf{q} e^{-(\mathbf{q}-\mathbf{q}_0)^2/2\beta^2} V_{\text{exponential}}(\mathbf{q}) = \frac{4\pi}{q_0} \frac{3b(T)}{4\mu(T)} (\sqrt{2\pi}\beta)^3 \mathcal{I}_0, \quad (7.3)$$

where

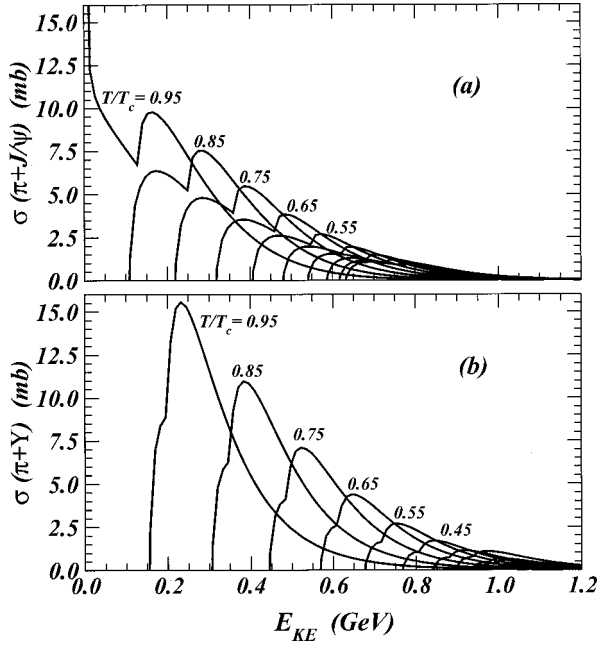


FIG. 7. Total dissociation cross section of J/ψ in collision with π for various temperatures as a function of the kinetic energy E_{KE} .

$$\mathcal{I}_n = \int_0^\infty r^{1+n} e^{-\beta^2 r^2/2 - \mu r} \sin q_0 r \, dr. \quad (7.4)$$

The integrals \mathcal{I}_{-1} and \mathcal{I}_0 are

$$\mathcal{I}_{-1} = -\frac{i}{2\beta} \sqrt{\frac{\pi}{2}} \{ \exp\{z_1^2\} \operatorname{erfc}(z_1) - \exp\{z_2^2\} \operatorname{erfc}(z_2) \} \quad (7.5)$$

and

$$\mathcal{I}_0 = \frac{i}{\sqrt{2}\beta^2} \sqrt{\frac{\pi}{2}} \{ z_1 \exp\{z_1^2\} \operatorname{erfc}(z_1) - z_2 \exp\{z_2^2\} \operatorname{erfc}(z_2) \}, \quad (7.6)$$

where

$$z_1 = \frac{\mu - iq_0}{\sqrt{2}\beta} \quad (7.7)$$

and

$$z_2 = \frac{\mu + iq_0}{\sqrt{2}\beta}. \quad (7.8)$$

Using these results, the reaction matrix elements can be evaluated and the dissociation cross section can be calculated, as in Ref. [18]. The sum of dissociation cross sections for $\pi + J/\psi \rightarrow D\bar{D}^*, D^*\bar{D}, D^*\bar{D}^*$ are shown in Fig. 7 for different temperatures T/T_c , as a function of the kinetic energy, $E_{KE} = \sqrt{s} - M_A - M_B$ where M_A and M_B are the masses of the colliding mesons. In Fig. 8 we show similar total dissociation cross sections for $\pi + Y \rightarrow B\bar{B}^*, B^*\bar{B}, B^*\bar{B}^*$ for

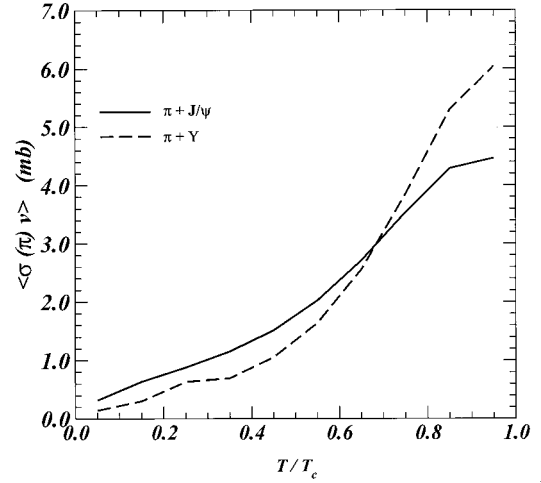


FIG. 8. The average dissociation cross section of J/ψ and Y in collision with π as a function of the temperature.

various temperatures, as a function of E_{KE} . Each cross section curve is the average of the results from the “prior” and “post” formalisms, as explained in Refs. [10,18].

We observe in Fig. 7 that the dissociation cross sections increase as the temperature increases. Such an increase arises from decreasing threshold energies as the temperature increases. At $T/T_c = 0.95$ the reaction $\pi + J/\psi \rightarrow D + \bar{D}^*$ is exothermic and the total dissociation cross section diverges at $E_{KE} = 0$. There is another cross section maximum of about 10 mb at $E_{KE} \sim 0.17$ GeV, which arises from the $\pi + J/\psi \rightarrow D^* + \bar{D}^*$ reaction. As the temperature decreases, the cross section decreases. At the temperature of $T/T_c = 0.7$, the maximum cross section decreases to about 4 mb at $E_{KE} \sim 0.3 - 0.5$ GeV. At $T = 0$, the maximum cross section decreases to about 1 mb at $E_{KE} \sim 0.7$ GeV and the threshold is at about 1.00 GeV [18].

We present similar results for the dissociation of Y in collision with π in Fig. 7(b). The reactions remain endothermic even for T close to T_c . For $T/T_c = 0.95$ the maximum cross section of about 15 mb occurs at $E_{KE} \sim 0.2$ GeV. As the temperature decreases, the maximum of the cross section decreases and moves to higher kinetic energy. For $T/T_c = 0.75$, the maximum cross section is about 4 mb and is located at $E_{KE} = 0.55$ GeV. At $T = 0$, the maximum cross section is about 0.6 mb at $E_{KE} \sim 1.04$ GeV and the threshold is at about 1.00 GeV [18]. The dissociation cross section is a sensitive function of the threshold. The results in Fig. 7 indicate that over a large range of temperatures below the phase transition temperature, dissociation cross sections of J/ψ and Y in collisions with π are large.

In a hadron gas, pions collide with the heavy quarkonium at different energies. We can get an idea of the energy-averaged magnitude of the dissociation cross section by treating the pions as a Bose-Einstein gas at temperature T . In Fig. 8, we show the quantity $\langle \sigma v \rangle$ which is the product of the dissociation cross section and the relative velocity, averaged over the energies of the pions. The quantity $\langle \sigma v \rangle$ is about 3 mb at $T/T_c = 0.70$ and rises to about 4.5 mb at $T/T_c = 0.95$ where the value of T_c has been taken to be 0.175 GeV [5].

VIII. DISSOCIATION OF HEAVY QUARKONIUM IN COLLISION WITH PIONS

We can estimate the survival probability of a heavy quarkonium in a hot pion gas in the presence of this type of collisional dissociation. If we represent the survival probability S by $\exp\{-I\}$, the exponential factor I is given by

$$I = \int_{\tau_0}^{\tau_{\text{freeze}}} \langle \sigma v \rangle (\tau) \rho(\tau) d\tau, \quad (8.1)$$

where σ is the dissociation cross section, v is the relative velocity between π and the heavy quarkonium, $\rho(\tau)$ is the density of π at the proper time τ , and τ_0 and τ_{freeze} are the initial proper time and the freeze-out proper time, respectively. The quantity $\langle \sigma v \rangle$ in Fig. 8 can be represented approximately by

$$\langle \sigma v \rangle \approx \langle \sigma v \rangle_c (T/T_c), \quad (8.2)$$

where $\langle \sigma v \rangle_c \sim 4.5$ mb for J/ψ or Υ . Assuming a Bjorken type of expansion in which the density $\rho(\tau)$ is proportional to τ^{-1} and $T(\tau)$ is proportional to $\tau^{-1/3}$, we obtain

$$I = \langle \sigma v \rangle_c \rho_0 \tau_0 \left(\frac{T_0}{T_c} \right) 3 \left\{ 1 - \left(\frac{\rho_{\text{freeze}}}{\rho_0} \right)^{1/3} \right\}, \quad (8.3)$$

where ρ_0 is the pion density after all the pions are initially formed and the heavy quarkonium is thermalized in the medium, T_0 is the corresponding temperature, and ρ_{freeze} is the freeze-out pion density. Continuing the assumption of Bjorken expansion, the initial pion density is given by

$$\rho_0 = \frac{dN}{\tau_0 dy \mathcal{A}}, \quad (8.4)$$

where dN/dy is the rapidity density of pions and \mathcal{A} is the overlapping area in the heavy-ion collision, which depend on centrality. We therefore obtain

$$I = \langle \sigma v \rangle_c \frac{dN}{dy \mathcal{A}} \left(\frac{T_0}{T_c} \right) 3 \left\{ 1 - \left(\frac{\rho_{\text{freeze}} \tau_0 \mathcal{A}}{dN/dy} \right)^{1/3} \right\}. \quad (8.5)$$

To make some estimates on the survival probability of J/ψ and Υ in a hot hadron gas, we can take the quantity τ_0 to be $(1 \text{ fm}/c + 2 \text{ fm}/c + 2R/\gamma)$, where $1 \text{ fm}/c$ is the formation time for pions and heavy quarkonium, the additional $2 \text{ fm}/c$ is the time for the thermalization of hadrons, and $2R/\gamma$ is the approximate time spread of the first and the last nucleon-nucleon collisions in a nucleus-nucleus collision (R is the radius of the colliding nucleus and γ is the relativistic factor in the c.m. frame). We can take ρ_{freeze} , the freeze-out pion density, to be $0.5/\text{fm}^3$, corresponding to an average pion separation of about 1.3 fm , and $T_0/T_c \sim 1$.

For the most central Pb-Pb collision at $158A \text{ GeV}$, $dN_{\text{ch}}/dy \sim 450$ [27], and $dN/dy \sim 675$. Then for this central collision, $I = \langle \sigma v \rangle_c (2.40/\text{fm}^2)$ and the heavy quarkonium survival probability is $S = e^{-1.08} = 0.34$.

For the most central Au-Au collision RHIC at $\sqrt{s_{NN}} = 200 \text{ GeV}$, $dN_{\text{ch}}/dy \sim 650$ [28], and $dN/dy \sim 975$. Then

for this central collision, $I = \langle \sigma v \rangle_c (7.13/\text{fm}^2)$ and the heavy quarkonium survival probability is $S = e^{-3.21} = 0.04$. There is a substantial absorption of both J/ψ and Υ by the hot pion gas at this temperature.

IX. DISCUSSIONS AND CONCLUSIONS

We study the dissociation of a heavy quarkonium in high temperatures. The temperature of the medium alters the gluon and quark fields and changes the interaction between a heavy quark Q and antiquark \bar{Q} placed in the medium. We first examine the effects of the temperature of the medium on the quarkonium single-particle states by using a potential inferred from lattice gauge calculations and evaluate the dissociation temperature at which the quarkonium dissociate spontaneously. We include spin-dependent interactions and take into account the selection rules. We confirm the general features of the results of Digal *et al.* [3] but there are some differences in the dissociation temperatures, which depend on the potential and interactions, as well as on the selection rules. We find that the selection rules change the dissociation temperatures substantially for charmonia but only slightly for bottomia.

The results of Digal *et al.* and our work indicate that most, if not all, of the dissociation temperatures of the charmonium states are below T_c , but the dissociation temperatures of a number of low-lying bottomium states lie above T_c because of the strong binding of these states.

The quarkonium placed in a medium can collide with particles in the medium to reach thermal equilibrium with the medium. A heavy-quarkonium in thermal equilibrium can dissociate by thermalization as there is a finite probability for the system to be in an excited state lying above its dissociation threshold. We find that the fraction of the quarkonium lying above the threshold increases with increasing temperatures and is quite large for charmonium at T slightly below T_c .

Dissociation of a heavy quarkonium can occur in collision with hadrons at temperatures below the dissociation temperatures and the phase transition temperature. As the temperature increases, the quarkonium single-particle energies increase, and the threshold energies for collisional dissociation decrease. As a consequence, the dissociation cross sections increase as the temperature increases, reaching an average of $\langle \sigma v \rangle \sim 3.0$ mb for the dissociation of J/ψ and Υ in collision with pions just below the phase transition temperature. We have estimated the survival probability of J/ψ and Υ in collision with pions in central Pb-Pb collisions at SPS and RHIC energies and found the survival probability to be small. Dissociation of J/ψ by collisions with pions at high temperatures may be an important source of the anomalous J/ψ suppression in Pb-Pb collisions observed at CERN [29].

It has been suggested that collision of J/ψ with ‘‘comovers’’ was the source of the anomalous J/ψ suppression in Pb-Pb collisions [30–32]. There were some uncertainties as the dissociation cross section of the ‘‘comover’’ with J/ψ was not known and the dissociation cross sections for different hadrons are very different. The large threshold energy of 0.64 GeV for J/ψ dissociation in collision with pions raised

questions whether the π - J/ψ dissociation cross section could be large enough to lead to the anomalous J/ψ suppression. The new information from lattice gauge calculations of Karsch *et al.* [5] provides valuable input to infer the in-medium dissociation energy of J/ψ . The large cross sections for the in-medium dissociation in collision with pions obtained here suggest that further microscopic investigations of the J/ψ dissociation in collision with hadrons at high hadron temperatures will be of great interest.

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