"Super-radiant" states and narrow resonances in the Δ-nucleus system

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We discuss the possibility that narrow states observed in the Δ_{33}-C system are due to a mechanism of coupling through the continuum when long-lived states are formed in addition to the broad "super-radiant" state. Expressions and estimates for the narrow widths are presented.

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I. INTRODUCTION

In the early 1950s the possibility of forming a "super-radiant" (SR) state in a gas of atoms confined to a volume of a size smaller than the wave length of radiation was suggested by Dicke [1]. In the absence of direct interaction, the atoms are coupled through their common radiation field. This interaction through the continuum leads to the redistribution of widths among unstable intrinsic states. A rapidly decaying SR state is created at the expense of the rest of the states of the system that are "robbed" of their decay probability and become narrow. Later it was understood that such a mechanism has a general origin and analogous phenomena should appear in many situations when quasistationary states are strongly coupled through their common decay channels [2–5]. The concept of the SR state was applied to several fields including chemical physics [6], atomic physics [7], low-energy nuclear physics [4,8–10], and intermediate-energy nuclear physics [11]. Several two-level examples from nuclear and particle physics have been studied in detail [12,13]. It was pointed out in Ref. [11] that in certain physical situations pertaining to intermediate-energy nuclear physics, broad SR states may be formed along with the observable narrow resonances.

A recent experimental work [14] provided evidence from the $^{12}\mathrm{C}(e,e'p\pi^-)^{11}\mathrm{C}$ reaction for narrow (several MeV wide) resonances in the Δ_{33}-C system in addition to a broad peak of approximately 100 MeV width. Subsequently, one of the coauthors published a paper with an attempt to explain the emergence of such narrow resonances [15] as a manifestation of a coherent pion state.

In the present paper we show that the conditions required for the formation of the broad SR state along with narrow resonances are well satisfied in the case of the Δ-nucleus system. In fact, such narrow states were seen in the direct numerical calculations of Δ-hole excitations for ^{16}O [16] in the random-phase approximation (RPA) framework. Similar narrow states were shown to be possible for a quasibound nucleon-antinucleon pair in nuclear matter [17].

II. COUPLING THROUGH CONTINUUM AND THE SUPER-RADIANT STATE

Below we present a short description of the theory of the SR state. We limit ourselves to a simple case relevant to the Δ-nucleus system. The approach used for the estimates goes back to Refs. [3,5,11,18].

Consider a system that contains a set $\{|q\rangle\}$ of "internal" states, for example, of shell-model type. These states can decay into "external" decay channels $|c\rangle$. We will refer to those subspaces of the system as q space and P space, respectively. The internal states $|q\rangle$ with the same exact quantum numbers, such as spin, isospin, and parity, may couple to each other directly via a Hermitian interaction $\langle q_1|V|q_2\rangle$ but also indirectly in the second order via the channel states $|c\rangle$ serving as intermediate states along the coupling path $\langle q_1|V|c\rangle\langle c|V|q_2\rangle$. The effective Hamiltonian $\mathcal{H}$ for the $|q\rangle$ states can be written as

$$\mathcal{H}_{qq}=H_{qq}+H_{qP}\frac{1}{E^{(+)}-H_{PP}}H_{Pq}, \tag{1}$$

with the notation $H_{ab}\equiv aHb$, a and b being operators projecting the full original Hamiltonian H onto subspaces q and P. Total energy E belongs to the continuum so that one needs to introduce $E^{(+)}=E+i\eta,\ \eta\to+0$. The first term, H_{qq}, describes the direct coupling between the internal states, while the second term describes their coupling through the states outside the internal subspace.

The effective Hamiltonian, Eq. (1), is non-Hermitian since the second term contains a real and an imaginary part. The real part (the principal value) describes a Hermitian coupling through the continuum, whereas the imaginary part $(-1/2)W$,

$$W=2\pi\sum_{c,\mathrm{open}}V|c\rangle\langle c|V, \tag{2}$$

is obtained from the δ function of energy in the Green's function $G^{(+)}=[E^{(+)}-H_{PP}]^{-1}$ and corresponds to the decay into channels $|c\rangle$ that are open at a given energy.

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In order to estimate typical widths of quasistationary states emerging as a result of the dynamics described by this Hamiltonian, we assume that the real part is diagonalized (the eigenstates will be denoted as $|1\rangle$, and their energies as ϵ_1) and that there is only one important decay channel $|c\rangle$ for the group of those states with the same exact quantum numbers but different energies. The effective non-Hermitian Hamiltonian for decaying levels coupled to a single decay channel is

$$\mathcal{H}_{12}=\epsilon_1\delta_{12}-\frac{i}{2}A_1A_2^*, \tag{3}$$

where the decay amplitudes are

$$A_1=\sqrt{2\pi}\langle 1|V|c\rangle. \tag{4}$$

The diagonalization of this Hamiltonian gives quasistationary states

$$|j)=\sum_1 C_1^j|1\rangle, \tag{5}$$

where the (unnormalized) coefficients satisfy

$$C_1^j=-\frac{i}{2}\frac{A_1}{\mathcal{E}_j-\epsilon_1}\sum_2 C_2^jA_2^*, \tag{6}$$

and the secular equation for the complex energies

$$\mathcal{E}_j=E_j-\frac{i}{2}\Gamma_j \tag{7}$$

acquires the form

$$1=-\frac{i}{2}\sum_1\frac{\gamma_1}{\mathcal{E}_j-\epsilon_1}, \tag{8}$$

where the unperturbed widths are

$$\gamma_1=|A_1|^2. \tag{9}$$

Separating the real and imaginary parts, we come to the set of coupled equations

$$\sum_1\frac{\gamma_1(E_j-\epsilon_1)}{(E_j-\epsilon_1)^2+\Gamma_j^2/4}=0 \tag{10}$$

and

$$\frac{\Gamma_j}{4}\sum_1\frac{\gamma_1}{(E_j-\epsilon_1)^2+\Gamma_j^2/4}=1. \tag{11}$$

We are interested in the situation of overlapping resonances where all original widths γ_1 are of the same order of magnitude and large compared to the real energy spread ΔE of the resonances. It is easy to see that there exist a solution $\mathcal{E}_0=E_0-(i/2)\Gamma_0$, corresponding to the broad SR state; we label this root as $j=0$. Its width Γ_0 is very large and close to the total summed width, the trace of the imaginary part of the Hamiltonian $\mathcal{H}$,

$$\Gamma=\sum_1\gamma_1. \tag{12}$$

The position of the broad pole on the real axis, E_0, is close to the centroid $\overline{\epsilon}$ of the internal levels; this centroid will be determined later. We are looking for the solution $j=0$ in the form

$$E_0=\overline{\epsilon}+x,\quad \Gamma_0=\Gamma-y, \tag{13}$$

where the small deviations x and y are to be found from Eqs. (10) and (11). Note that in the limit of full degeneracy, $\epsilon_1=\text{const}$, the SR state accumulates the total width, $\Gamma_0=\Gamma$.

If the coordinates ξ_1 of the levels with respect to the centroid are defined as

$$\epsilon_1=\overline{\epsilon}+\xi_1, \tag{14}$$

Eq. (11) gives

$$\frac{\Gamma-y}{4}\sum_1\frac{\gamma_1}{(x-\xi_1)^2+(\Gamma-y)^2/4}=1. \tag{15}$$

The expansion up to the second order leads to

$$\Gamma=\sum_1\gamma_1\left[1+\frac{y}{\Gamma}+\frac{y^2-4(x-\xi_1)^2}{\Gamma^2}\right]. \tag{16}$$

Now, comparison with Eq. (12) shows that y is of the second order, the term y^2 can be neglected, and

$$y=-\frac{4}{\Gamma^2}\sum_1\gamma_1(x-\xi_1)^2. \tag{17}$$

Thus, the result is the following: the width of the broad state is smaller than the total summed width Γ by a certain amount that is nothing but the total width Γ_n of the narrow states,

$$\Gamma_0=\Gamma-\Gamma_n,\quad \Gamma_n=\frac{4}{\Gamma^2}\sum_1\gamma_1(x-\xi_1)^2. \tag{18}$$

All differences $x-\xi_1$ are within the interval ΔE on the real axis. If we have N states, with typical individual width γ, so that the total width $\Gamma\sim N\gamma$, then Γ_n can be estimated as

$$\Gamma_n\sim 4N\frac{\gamma(\Delta E)^2}{(N\gamma)^2}\sim\frac{4}{N}\gamma\left(\frac{\Delta E}{\gamma}\right)^2\sim 4\frac{(\Delta E)^2}{\Gamma}, \tag{19}$$

and each narrow width is $\gamma_n\sim(\Gamma_n/N)$.

Equation (10) for the real part of energy of the broad state $j=0$ reads

$$\sum_1\frac{\gamma_1(x-\xi_1)}{(x-\xi_1)^2+\Gamma_0^2/4}=0. \tag{20}$$

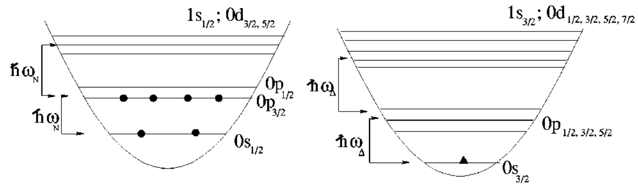

FIG. 1. Schematic presentation of the ground state configuration of the ^{12}C nucleus (left, only one kind of nucleons is shown) and the levels for the Δ in the nuclear potential (right). The $0\hbar\overline{\omega}$ excitations uniquely combine Δ in the $0s_{3/2}$ level with the hole in the nucleon $0p_{3/2}$ orbit; the $1\hbar\overline{\omega}$ excitations are built either with the promotion of the Δ to the $0p$ shell and a hole in the nucleon $0p$ shell, or with the Δ in the ground level but a hole in the nucleon $0s$ level.

With the expansion up to the second order,

$$\sum_{1} \gamma_1(x-\xi_1)\left[1-4\frac{(x-\xi_1)^2}{\Gamma_0^2}\right]=0. \tag{21}$$

Let us fit the centroid $\overline{\epsilon}$ of the levels in such a way that they are weighted by their widths,

$$\overline{\epsilon}=\frac{\sum_{1} \gamma_1\epsilon_1}{\Gamma}. \tag{22}$$

Then

$$\sum_{1} \gamma_1\xi_1=\sum_{1} \gamma_1(\epsilon_1-\overline{\epsilon})=0, \tag{23}$$

and Eq. (21) gives for x

$$x=4\frac{\sum_{1} \gamma_1(x-\xi_1)^3}{\Gamma\Gamma_0^2}, \tag{24}$$

or, since x turns out to be small and $\Gamma\approx\Gamma_0$ up to small corrections,

$$x=4\frac{\sum_{1} \gamma_1\xi_1^3}{\Gamma_0^3}. \tag{25}$$

This means that the broad state is located very close to the centroid determined by the decay width weighting (22).

III. THE Δ-NUCLEUS SYSTEM

The free pion-nucleon resonance Δ_{33} with spin 3/2 and isospin 3/2 has a width $\Gamma_\Delta\simeq 120$ MeV, and its four charge components have an average centroid energy of about 1235 MeV, 300 MeV above the nucleon mass. Excitation of nuclei with various high energy probes (pions, protons, electrons) reveals a wide peak at excitation energies of about 300 MeV. This peak is usually described [15,16] in terms of Δ particle—nucleon hole (ΔN^{-1}) configurations. The Δ is treated as a particle bound in a potential similar to the nuclear mean field. The width of the Δ resonance is determined by the coupling of the ΔN^{-1} configuration to the decay channel with pion emission; the nonpionic processes $\Delta+N\rightarrow N+N$ are also possible.

In a recent ^{12}C$(e,e'p\pi^-)^{11}$C experiment at the Mainz microtron MAMI [14], in addition to a broad peak, at least two narrow peaks with widths of the order of 4 MeV were observed at excitation energies just below 300 MeV. The Δ-nucleus system has the qualities that make it a favorable case to be described by the SR theory [11]. The available ΔN^{-1} configurations are strongly coupled to a *single* channel $|c\rangle=|A-1;\,(N+\pi)_\Delta\rangle$ corresponding to the decay into a nucleon plus pion and the residual $(A-1)$ nucleus. The partial width γ of this decay is a fraction of the total width of the free Δ_{33}, being of the order of few tens of MeV. On the other hand, the energy spacings $\epsilon_1-\epsilon_2$ between the various relevant ΔN^{-1} configurations are of the order of several MeV. Thus, realistically the energy spread of the relevant intrinsic configurations is small compared to their characteristic unperturbed widths, $\Delta E<\gamma$. This overlap creates the conditions for the redistribution of the widths.

We take the simplest shell-model description of the ^{12}C target nucleus assuming that the nucleons occupy $0s_{1/2}$ and $0p_{3/2}$ orbits. We denote the energy distance between the s and p shells by $\hbar\omega_N$. The Δ is assumed to move in a potential well with major shell spacings $\hbar\omega_\Delta$ that are not much different from $\hbar\omega_N$ so that we can classify the excitations by the average quantity $\hbar\overline{\omega}$, limiting ourselves to $0\hbar\overline{\omega}$ and $1\hbar\overline{\omega}$ configurations. The lowest, $0\hbar\omega_\Delta$ and $1\hbar\omega_\Delta$ single-particle orbits are $0s_{3/2}$; $0p_{1/2}$, $0p_{3/2}$, and $0p_{5/2}$, as seen in the level scheme in Fig. 1. The possible total spins and configurations in this space are given in Table I.

For the $0\hbar\overline{\omega}$ configurations coupled to the $N\pi$ channel there is no mixing through the continuum. Therefore, for the negative parity states we expect only a broad peak with the width of the free Δ_{33} resonance and no narrow states. The situation for the $1\hbar\overline{\omega}$ configurations is different. The channel $J^\pi=0^+$ with the single configuration allowed at low excitation energy will, via its coupling to the exit channel, have a large width. At the same time the $J^\pi=1^+$ states include four low-lying ΔN^{-1} configurations. Applying the SR theory, we expect the appearance of a SR state that should be identified with the broad peak with the width, $\sim$100 MeV, observed in the experiment. The remaining three $J^\pi=1^+$ states should be narrow. With the use of Eq. (19) we find the average width of the narrow 1^+ states $\overline{\gamma_n}=\Gamma_n/3=\frac{4}{3}(\Delta E)^2/\Gamma$. Taking for an estimate $\Delta E=10$ MeV and $\Gamma\approx 100$ MeV, we obtain $\overline{\gamma_n}\simeq 1.3$ MeV. Similarly, for the $J^\pi=2^+$ states, one expects

TABLE I. Possible configurations of the type ΔN^{-1} that belong to $0\hbar\overline{\omega}$ and $1\hbar\overline{\omega}$ classes.

	J^{π}	ΔN^{-1} configurations
$0\hbar\overline{\omega}$	0^-	$(0s_{3/2},0p_{3/2}^{-1})$
	1^-	$(0s_{3/2},0p_{3/2}^{-1})$
	2^-	$(0s_{3/2},0p_{3/2}^{-1})$
	3^-	$(0s_{3/2},0p_{3/2}^{-1})$
$1\hbar\overline{\omega}$	0^+	$(0p_{3/2},0p_{3/2}^{-1})$
	1^+	$(0s_{3/2},0s_{1/2}^{-1})$; $(0p_{1/2},0p_{3/2}^{-1})$; $(0p_{3/2},0p_{3/2}^{-1})$; $(0p_{5/2},0p_{3/2}^{-1})$
	2^+	$(0s_{3/2},0s_{1/2}^{-1})$; $(0p_{1/2},0p_{3/2}^{-1})$,$(0p_{3/2},0p_{3/2}^{-1})$; $(0p_{5/2},0p_{3/2}^{-1})$
	3^+	$(0p_{3/2},0p_{3/2}^{-1})$; $(0p_{5/2},0p_{3/2}^{-1})$
	4^+	$(0p_{5/2},0p_{3/2}^{-1})$

to find, in addition to the broad SR, three narrow states that should again have an average width of a few MeV. One narrow state is possible for $J^{\pi}=3^+$.

The resulting picture is expected to show several narrow resonances in the vicinity of the $1\hbar\overline{\omega}$ excitation superimposed on a background of the SR states for various channels. This picture should repeat itself at higher excitation energies. Such a prediction is in a qualitative agreement with the results of the experiment of Ref. [14]. One should of course bear in mind that other mechanisms, especially at higher excitation energies, might contribute to the width of quasistationary states. For example, the nucleon hole states acquire a spreading width due to the residual nucleon-nucleon interaction. This width would add to the widths of the narrow states.

In conclusion, we can mention that our interpretation does not necessarily contradict the idea of the coherent RPA-like pionic state discussed in Ref. [15]. The two approaches emphasize two different aspects of the mixing of ΔN^{-1} configurations. Internal mixing by the real part of the effective Hamiltonian creates a collective state similar to a giant resonance. This leads to the redistribution of strengths (electromagnetic for giant resonances or pionic in our case) and energies on the real axis. Mixing through the continuum leads to the redistribution of the widths (imaginary parts of the complex energies) and creates the SR state. These resonances can in fact be combined into one giant state if the two types of mixing are "parallel," as discussed in Refs. [8,10]. This happens, for instance, in the γ-ray decay of the giant dipole resonance when the same dipole matrix elements are responsible for the collectivization of the strength and the super radiance (collectivization of the widths). A similar situation apparently takes place for the ΔN^{-1} mixing where emission and absorption of pions lead both to internal mixing and to real decay in the continuum. RPA calculations [16] show that the state with the maximum strength also has the maximum pionic width. It is not necessarily so for nonpionic decay modes. The effect of coherent coupling through the continuum, which was frequently ignored, is what we tried to demonstrate in the present paper.

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