Nonlocal calculation for nonstrange dibaryons and tribaryons

R. D. Mota,[1,2] A. Valcarce,[3,4] F. Fernández,[3] D. R. Entem,[3,5] and H. Garcilazo[2]
[1]*Unidad Profesional Interdisciplinaria en Ingeniería y Tecnologías Avanzadas, A. Instituto Politécnico Nacional No. 2580, Col. La Laguna Ticomán, 07740 México D.F., Mexico*
[2]*Escuela Superior de Física y Matemáticas, Instituto Politécnico Nacional, Edificio 9, 07738 México D.F., Mexico*
[3]*Grupo de Física Nuclear, Universidad de Salamanca, E-37008 Salamanca, Spain*
[4]*Departamento de Física Teórica, Universidad de Valencia, E-46100 Valencia, Spain*
[5]*Department of Physics, University of Idaho, Moscow, Idaho 83844*
(Received 13 September 2001; published 15 February 2002)

We study the possible existence of nonstrange dibaryons and tribaryons by solving the bound-state problem of the two- and three-body systems composed of nucleons and deltas. The two-body systems are NN, $N\Delta$, and $\Delta\Delta$, while the three-body systems are NNN, $NN\Delta$, $N\Delta\Delta$, and $\Delta\Delta\Delta$. We use as input the nonlocal NN, $N\Delta$, and $\Delta\Delta$ potentials derived from the chiral quark cluster model by means of the resonating group method. We compare with previous results obtained from the local version based on the Born-Oppenheimer approximation.

DOI: 10.1103/PhysRevC.65.034006 PACS number(s): 14.20.Gk, 13.75.Cs, 14.20.Pt, 12.40.Yx

I. INTRODUCTION

Systems without strangeness are those which involve only nucleons and nonstrange mesons like the pion or the eta. In a series of recent papers the suggestion has been made that it may be possible to observe unstable nonstrange two- and three-baryon states corresponding to the bound-state solutions of the various systems composed of nucleons and deltas [1–4]. These are the systems NN, $N\Delta$, $\Delta\Delta$, NNN, $NN\Delta$, $N\Delta\Delta$, and $\Delta\Delta\Delta$. The bound states involving one or more unstable particles will show up in nature as dibaryon or tribaryon resonances. In the case of two-body systems (dibaryons) they will decay mainly into two nucleons and either one or two pions, while for the three-body case (tribaryons) they will decay mainly into three nucleons and either one, two, or three pions.

In the previous calculations of our group [1–4], the Born-Oppenheimer approximation was used in order to obtain a local potential for the baryon-baryon interactions ($NN,N\Delta,\Delta\Delta$). In this work, we will overcome the Born-Oppenheimer approximation by working directly with a nonlocal potential derived within the resonating group method (RGM) formalism. This method allows us, once the Hilbert space for the six-body problem has been fixed, to treat the intercluster dynamics in an exact way.

In order to perform the NNN, $NN\Delta$, $N\Delta\Delta$, and $\Delta\Delta\Delta$ calculations we will take advantage of the experience gained in the three-nucleon bound-state problem [5,6]. In that case one knows that the dominant configuration of the system is that in which all particles are in S-wave states. However, in order to get reasonable results for the binding energy, the S-wave two-body amplitudes used as input in the Faddeev equations must already contain the effect of the tensor force. Thus, for example, in the case of the Reid soft-core potential if one considers only the S-wave configurations but neglects the tensor force in the two-body subsystems the triton is unbound. However, if one includes the effect of the tensor force in the nucleon-nucleon $^3S_1$-$^3D_1$ channel, but uses only the $^1S_0$ and $^3S_1$ components of the two-body amplitudes in the three-body equations (two-channel calculation), one gets a triton binding energy of 6.58 MeV. Notice that including the remaining configurations (34-channel calculation), leads to a triton binding energy of 7.35 MeV [7]. This means that the S-wave truncated T-matrix approximation leads to a binding energy which differs from the exact value by less than 1 MeV. Therefore by means of our approach we will not study exact binding energies but which are the best candidates for bound states and the ordering of the different NNN, $NN\Delta$, $N\Delta\Delta$, and $\Delta\Delta\Delta$ states.

NN, $N\Delta$, and $\Delta\Delta$ interactions have been derived in the past in the framework of meson-exchange models or phenomenological potentials [8,9]. These models have been used over the years to fit the NN data very accurately. However, in the $N\Delta$ and $\Delta\Delta$ sectors experimental data are so scarce that it is not possible to obtain reliable values of the parameters involved in the interaction. The situation is different in the case of quark cluster models [10–12]. In these models the basic interaction is at the level of quarks involving only a quark-quark-field (pion or gluon) vertex. Therefore its parameters (coupling constants, cutoff masses, etc.) are independent of the baryon to which the quarks are coupled, the difference among them being generated by SU(2) scaling, as explained in Ref. [13]. Moreover, quark models provide a definite framework to treat the short-range part of the interaction. The Pauli principle between quarks determines the short-range behavior of the different channels without additional phenomenological assumptions. In this way, even in the absence of experimental data, one has a complete scheme which starting from the NN sector allows us to make predictions in the $N\Delta$ and $\Delta\Delta$ sectors. This fact is even more important if one takes into account that the short-range dynamics of the $N\Delta$ and $\Delta\Delta$ systems is to a large extent driven by quark Pauli blocking effects, that do not appear in the NN sector. Pauli blocking acts in a selective way in those channels where the spin-isospin-color degrees of freedom are not enough to accommodate all the quarks of the system [14,15]. Therefore meson-exchange models cannot fully include the effect of quark Pauli blocking through its purely phenomenological short-range channel-independent part.

The lifetime of the bound states involving one or more deltas should be similar to that of the Δ in the case of very weakly bound systems and larger if the system is very strongly bound. Therefore these dibaryon and tribaryon resonances will have widths similar or smaller than the width of the Δ so that, in principle, they are experimentally observable. Also, we want to emphasize that the possible detection of dibaryon and tribaryon resonances does not constitute an exotic subject since, in principle, any nucleus with at least three nucleons can serve as test system that may be excited by forming a tribaryon [16].

The paper is organized as follows. In Sec. II we present the basic quark-quark interaction and we describe the method to obtain the resonating group method baryon-baryon potentials. Section III is dedicated to discussing the formalism to solve the bound state problem for the cases of systems of identical and nonidentical particles, respectively. In Sec. IV we give our results, and we present the conclusions in Sec. V.

II. TWO-BODY INTERACTIONS

The basic two-body interactions, $V_{AB\to AB}$, between baryons A and B that are going to be used in this work are the nucleon-nucleon interaction $V_{NN\to NN}$, the nucleon-delta interaction $V_{N\Delta\to N\Delta}$, and the delta-delta interaction $V_{\Delta\Delta\to\Delta\Delta}$. These baryon-baryon interactions were obtained from the chiral quark cluster model developed elsewhere [11]. In this model baryons are described as clusters of three interacting massive (constituent) quarks, the mass coming from the breaking of chiral symmetry. The ingredients of the quark-quark interaction are confinement, one-gluon (OGE), one-pion (OPE), and one-sigma (OSE) exchange terms, and whose parameters are fixed from the NN data. Explicitly, the quark-quark (qq) interaction is

$$V_{qq}(\vec{r}_{ij})=V_{\rm con}(\vec{r}_{ij})+V_{\rm OGE}(\vec{r}_{ij})+V_{\rm OPE}(\vec{r}_{ij})+V_{\rm OSE}(\vec{r}_{ij}), \tag{1}$$

where $\vec{r}_{ij}$ is the ij interquark distance and

$$V_{\rm con}(\vec{r}_{ij})=-a_c\vec{\lambda}_i\cdot\vec{\lambda}_j r_{ij}, \tag{2}$$

$$V_{\rm OGE}(\vec{r}_{ij})=\frac{1}{4}\alpha_s\vec{\lambda}_i\cdot\vec{\lambda}_j\left\{\frac{1}{r_{ij}}-\frac{\pi}{m_q^2}\left[1+\frac{2}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j\right]\delta(\vec{r}_{ij}) -\frac{3}{4m_q^2 r_{ij}^3}S_{ij}\right\}, \tag{3}$$

$$V_{\rm OPE}(\vec{r}_{ij})=\frac{1}{3}\alpha_{ch}\frac{\Lambda^2}{\Lambda^2-m_\pi^2} \times m_\pi\left\{\left[Y(m_\pi r_{ij})-\frac{\Lambda^3}{m_\pi^3}Y(\Lambda r_{ij})\right]\vec{\sigma}_i\cdot\vec{\sigma}_j +\left[H(m_\pi r_{ij})-\frac{\Lambda^3}{m_\pi^3}H(\Lambda r_{ij})\right]S_{ij}\right\}\vec{\tau}_i\cdot\vec{\tau}_j, \tag{4}$$

$$V_{\rm OSE}(\vec{r}_{ij})=-\alpha_{ch}\frac{4m_q^2}{m_\pi^2}\frac{\Lambda^2}{\Lambda^2-m_\sigma^2} \times m_\sigma\left[Y(m_\sigma r_{ij})-\frac{\Lambda}{m_\sigma}Y(\Lambda r_{ij})\right], \tag{5}$$

where

$$Y(x)=\frac{e^{-x}}{x};\quad H(x)=\left(1+\frac{3}{x}+\frac{3}{x^2}\right)Y(x). \tag{6}$$

Although taken to be linear for consistency with the baryon and meson spectra, the detailed radial structure and strength of the confining potential is meaningless for the two-baryon interaction [17]. a_c is the confinement strength, the $\vec{\lambda}$'s are the SU(3) color matrices, the $\vec{\sigma}$'s ($\vec{\tau}$'s) are the spin (isospin) Pauli matrices, S_{ij} is the usual tensor operator, m_q (m_π, m_σ) is the quark (pion, sigma) mass, α_s is the qq-gluon coupling constant, α_{ch} is the qq-meson coupling constant, and Λ is a cutoff parameter.

For the present study we make use of the nonlocal potentials derived through a Lippmann-Schwinger formulation of the RGM equations in momentum space [12]. The formulation of the RGM for a system of two baryons, B_1 and B_2, needs the wave function of the two-baryon system constructed from the one-baryon wave functions. The two-baryon wave function can be written as

$$\Psi_{B_1B_2}=\mathcal{A}[\chi(\vec{P})\Psi^{ST}_{B_1B_2}] =\mathcal{A}\{\chi(\vec{P})\phi_{B_1}(\vec{p}_{\xi_{B_1}})\phi_{B_2}(\vec{p}_{\xi_{B_2}})\chi^{ST}_{B_1B_2}\xi_c[2^3]\}, \tag{7}$$

where $\mathcal{A}$ is the antisymmetrizer of the six-quark system, $\chi(\vec{P})$ is the relative wave function of the two clusters, $\phi_{B_1}(\vec{p}_{\xi_{B_1}})$ is the internal spatial wave function of the baryon B_1, ξ_{B_1} are the internal coordinates of the three quarks of baryon B_1. $\chi^{ST}_{B_1B_2}$ denotes the spin-isospin wave function of the two-baryon system coupled to total spin (S) and isospin (T), and finally, $\xi_c[2^3]$ is the product of two color singlets.

The dynamics of the system is governed by the Schrödinger equation

$$(\mathcal{H}-E_T)|\Psi\rangle=0\Rightarrow\langle\delta\Psi|(\mathcal{H}-E_T)|\Psi\rangle=0, \tag{8}$$

where

$$\mathcal{H}=\sum_{i=1}^{N}\frac{\vec{p}_i^{\,2}}{2m_q}+\sum_{i<j}V_{ij}-T_{\rm c.m.}, \tag{9}$$

$T_{\rm c.m.}$ being the center of mass kinetic energy, V_{ij} the quark-quark interaction described above, and m_q the constituent quark mass.

Assuming the functional form

$$\phi_B(\vec{p})=\left(\frac{b^2}{\pi}\right)^{3/4}e^{-b^2p^2/2}, \tag{10}$$

where b is related to the size of the nucleon quark core, Eq. (8) can be written in the following way:

$$\left(\frac{\vec{P}'^2}{2\mu}-E\right)\chi(\vec{P}')+\int K(\vec{P}',\vec{P}_i)\chi(\vec{P}_i)d\vec{P}_i=0, \tag{11}$$

where

$$K(\vec{P}',\vec{P}_i)={}^{RGM}V_D(\vec{P}',\vec{P}_i)+{}^{RGM}V_{EX}(\vec{P}',\vec{P}_i) \tag{12}$$

TABLE I. Quark model parameters.

m_q(MeV)	313
b(fm)	0.518
α_s	0.498
a_c(MeV fm^{-1})	67.0
α_{ch}	0.027
m_σ(fm^{-1})	3.513
m_π(fm^{-1})	0.70
Λ(fm^{-1})	4.3

contains the direct and exchange RGM potentials, the later one coming from quark antisymmetry. $K(\vec{P}',\vec{P}_i)$ is the nonlocal potential. From Eq. (11) a set of coupled Lippmann-Schwinger equations can be obtained and solved using standard techniques. The parameters of the model are summarized in Table I. They have been fixed in order to obtain the best fit of the two-nucleon sector (deuteron binding energy and S-wave NN scattering phase shifts) and the $\Delta-N$ mass difference. In particular, the mass of the quark (m_q) is taken to be 1/3 of the nucleon mass. The pion mass (m_π) is its experimental value. The chiral coupling constant (α_{ch}) has been determined to reproduce the long-range OPE interaction and is given by $\alpha_{ch}=(\frac{3}{5})^2(g^2_{\pi NN}/4\pi)(m^2_\pi/4m^2_N)$, where the πNN coupling constant is taken to be $g^2_{\pi NN}/4\pi=13.87$. The sigma mass is fixed by the chiral symmetry relation $m^2_\sigma\approx m^2_\pi+(2m_q)^2$. The parameter b, which determines the size of the nucleon quark content, was determined by comparing the adiabatic NN potential calculated from the wave function solution of the bound state problem for the potential given by Eq. (1) to the NN potential calculated with a single Gaussian of parameter b. Λ, which controls the pion-gluon proportion in the model and, as a consequence, the strength of the tensor force, has been taken to reproduce the deuteron binding energy in the presence of $\Delta\Delta$ channels. As the OPE provides part of the $\Delta-N$ mass difference, the value of the strong coupling constant (α_s) is determined to obtain the remaining $\Delta-N$ mass difference. Finally, the value of a_c, quoted for completeness because its contribution to the baryon-baryon potential is negligible [17], is obtained from the stability condition for the nucleon $\partial M_N(b)/\partial b=0$ [11].

III. INTEGRAL EQUATIONS

We will describe in this section the formalism required in the cases of the two-body systems NN, $N\Delta$, and $\Delta\Delta$ and the three-body systems NNN, $NN\Delta$, $N\Delta\Delta$, and $\Delta\Delta\Delta$.

TABLE II. NN channels (l_{NN},s_{NN}), $N\Delta$ channels $(l_{N\Delta},s_{N\Delta})$, and $\Delta\Delta$ channels $(l_{\Delta\Delta},s_{\Delta\Delta})$ that are coupled together in the $^3S_1$-$^3D_1$ and $^1S_0$ NN states.

NN state	j	i	(l_{NN},s_{NN})	$(l_{N\Delta},s_{N\Delta})$	$(l_{\Delta\Delta},s_{\Delta\Delta})$
$^3S_1$-$^3D_1$	1	0	(0,1),(2,1)		(0,1),(2,1),(2,3)
$^1S_0$	0	1	(0,0)	(2,2)	

TABLE III. Coupled channels (l,s) that contribute to a given $N\Delta$ state with total angular momentum j and isospin i.

j	i	(l,s)
1	1	(0,1),(2,1),(2,2)
1	2	(0,1),(2,1),(2,2)
2	1	(0,2),(2,1),(2,2)
2	2	(0,2),(2,1),(2,2)

A. Two-body systems

If we consider two baryons A and B in a relative S-state interacting through a potential that contains a tensor force, then there is a coupling to the AB D-wave so that the Lippmann-Schwinger equation of the system is of the form

$$t^{l_is_il''_is''_i}_{i;j_ii_i}(p_i,p''_i;E) = V^{l_is_il''_is''_i}_{i;j_ii_i}(p_i,p''_i)+\sum_{l'_is'_i}\int_0^\infty p'^2_i dp'_i V^{l_is_il'_is'_i}_{i;j_ii_i}(p_i,p'_i) \times\frac{1}{E-p'^2_i/2\eta_i+i\epsilon}t^{l'_is'_il''_is''_i}_{i;j_ii_i}(p'_i,p''_i;E), \tag{13}$$

where j_i and i_i are the angular momentum and isospin of the system, while l_is_i, $l'_is'_i$, and $l''_is''_i$ are the initial, intermediate, and final orbital angular momentum and spin of the system, respectively. p_i and η_i are, respectively, the relative momentum and reduced mass of the two-body system. We give in Tables II–IV the corresponding NN, $N\Delta$, and $\Delta\Delta$ two-body channels in a relative S wave that are coupled together for the two possible values of j and i (since the NN state is the one with the lowest mass, in the case of this system we have considered also the possibility of transitions to higher mass states like $N\Delta$ and $\Delta\Delta$). In the cases of the NN and $\Delta\Delta$ systems the Pauli principle requires that $(-)^{l_i+s_i+i_i}=-1$.

As mentioned before, for the solution of the three-body system we will use only the component of the T matrix obtained from the solution of Eq. (13) with $l_i=l''_i=0$, so that for that purpose we define the S-wave truncated amplitude

TABLE IV. Coupled channels (l,s) that contribute to a given $\Delta\Delta$ state with total angular momentum j and isospin i.

j	i	(l,s)
0	1	(0,0),(2,2)
0	3	(0,0),(2,2)
1	0	(0,1),(2,1),(2,3)
1	2	(0,1),(2,1),(2,3)
2	1	(0,2),(2,0),(2,2)
2	3	(0,2),(2,0),(2,2)
3	0	(0,3),(2,1),(2,3)
3	2	(0,3),(2,1),(2,3)

$$t_{i;s_ii_i}(p_i,p_i'';E)\equiv t_{i;s_ii_i}^{0s_i0s_i}(p_i,p_i'';E). \tag{14}$$

B. Three-body systems

If we restrict ourselves to the configurations where all three particles are in S-wave states, the Faddeev equations for the bound-state problem in the case of three particles with total spin S and total isospin I are

$$T_{i;SI}^{s_ii_i}(p_iq_i)=\sum_{j\neq i}\sum_{s_ji_j} h_{ij;SI}^{s_ii_is_ji_j}\frac{1}{2}\int_0^\infty q_j^2dq_j \times\int_{-1}^{1}d\cos\theta\, t_{i;s_ii_i}(p_i,p_i';E-q_i^2/2\nu_i) \times\frac{1}{E-p_j^2/2\eta_j-q_j^2/2\nu_j}T_{j;SI}^{s_ji_j}(p_jq_j), \tag{15}$$

where p_i and q_i are the usual Jacobi coordinates and η_i and ν_i the corresponding reduced masses

$$\eta_i=\frac{m_jm_k}{m_j+m_k}, \tag{16}$$

$$\nu_i=\frac{m_i(m_j+m_k)}{m_i+m_j+m_k}, \tag{17}$$

with ijk an even permutation of 123. The momenta p_i' and p_j in Eq. (15) are given by

$$p_i'^2=q_j^2+\frac{\eta_i^2}{m_k^2}q_i^2+2\frac{\eta_i}{m_k}q_iq_j\cos\theta, \tag{18}$$

$$p_j^2=q_i^2+\frac{\eta_j^2}{m_k^2}q_j^2+2\frac{\eta_j}{m_k}q_iq_j\cos\theta. \tag{19}$$

$h_{ij;SI}^{s_ii_is_ji_j}$ are the spin-isospin coefficients,

$$h_{ij;SI}^{s_ii_is_ji_j}=(-)^{s_j+\sigma_j-S}\sqrt{(2s_i+1)(2s_j+1)}W(\sigma_j\sigma_kS\sigma_i;s_is_j) \times(-)^{i_j+\tau_j-I}\sqrt{(2i_i+1)(2i_j+1)}W(\tau_j\tau_kI\tau_i;i_ii_j), \tag{20}$$

where W is the Racah coefficient and σ_i, s_i, and S (τ_i, i_i, and I) are the spin (isospin) of particle i, of the pair jk, and of the three-body system, respectively.

Since the variable p_i, in Eqs. (13) and (15), runs from 0 to ∞, it is convenient to make the transformation

$$x_i=\frac{p_i-d}{p_i+d}, \tag{21}$$

where the new variable x_i runs from -1 to 1, and d is a scale parameter. With this transformation Eq. (15) takes the form

$$T_{i;SI}^{s_ii_i}(x_iq_i)=\sum_{j\neq i}\sum_{s_ji_j} h_{ij;SI}^{s_ii_is_ji_j}\frac{1}{2}\int_0^\infty q_j^2dq_j \times\int_{-1}^{1}d\cos\theta\, t_{i;s_ii_i}(x_i,x_i';E-q_i^2/2\nu_i) \times\frac{1}{E-p_j^2/2\eta_j-q_j^2/2\nu_j}T_{j;SI}^{s_ji_j}(x_jq_j). \tag{22}$$

Since in the amplitude $t_{i;s_ii_i}(x_i,x_i';e)$ the variables x_i and x_i' run from -1 to 1, one can expand this amplitude in terms of Legendre polynomials as

$$t_{i;s_ii_i}(x_i,x_i';e)=\sum_{nm}P_n(x_i)\tau_{i;s_ii_i}^{nm}(e)P_m(x_i'), \tag{23}$$

where the expansion coefficients are given by

$$\tau_{i;s_ii_i}^{nm}(e)=\frac{2n+1}{2}\frac{2m+1}{2}\int_{-1}^{1}dx_i \times\int_{-1}^{1}dx_i'P_n(x_i)t_{i;s_ii_i}(x_i,x_i';e)P_m(x_i'). \tag{24}$$

Applying expansion (23) in Eq. (22) one gets

$$T_{i;SI}^{s_ii_i}(x_iq_i)=\sum_n T_{i;SI}^{ns_ii_i}(q_i)P_n(x_i), \tag{25}$$

where $T_{i;SI}^{ns_ii_i}(q_i)$ satisfies the one-dimensional integral equation

$$T_{i;SI}^{ns_ii_i}(q_i)=\sum_{j\neq i}\sum_{ms_ji_j}\int_0^\infty dq_jA_{ij;SI}^{ns_ii_ims_ji_j}(q_i,q_j;E)T_{j;SI}^{ms_ji_j}(q_j), \tag{26}$$

with

$$A_{ij;SI}^{ns_ii_ims_ji_j}(q_i,q_j;E)=h_{ij;SI}^{s_ii_is_ji_j}\sum_l\tau_{is_ii_i}^{nl}(E-q_i^2/2\nu_i)\frac{q_j^2}{2} \times\int_{-1}^{1}d\cos\theta\frac{P_l(x_i)P_m(x_j)}{E-p_j^2/2\eta_j-q_j^2/2\nu_j}. \tag{27}$$

The three amplitudes $T_{1;SI}^{ls_1i_1}(q_1)$, $T_{2;SI}^{ms_2i_2}(q_2)$, and $T_{3;SI}^{ns_3i_3}(q_3)$ in Eq. (26) are coupled together. The number of coupled equations can be reduced, however, since some of the particles are identical. In the case of three identical particles (NNN and $\Delta\Delta\Delta$ systems) we have that all three amplitudes are equal and therefore Eq. (26) becomes, in this case,

$$T_{SI}^{ns_ii_i}(q_i)=2\sum_{ms_ji_j}\int_0^\infty dq_jA_{ij;SI}^{ns_ii_ims_ji_j}(q_i,q_j;E)T_{SI}^{ms_ji_j}(q_j). \tag{28}$$

TABLE V. Two-body NN channels (j,i) that contribute to a given NNN state with total spin S and isospin I.

S	I	(j,i)
1/2	1/2	(1,0),(0,1)
1/2	3/2	(0,1)
3/2	1/2	(1,0)

We give in Table V the three NNN states characterized by total spin and isospin (S,I) that are possible as well as the two-body NN channels that contribute to each state. In Table VI we give the 25 $\Delta\Delta\Delta$ states characterized by total spin and isospin (S,I) that are possible as well as the two-body $\Delta\Delta$ channels that contribute to each state.

In the case where two particles are identical and one different ($NN\Delta$ and $N\Delta\Delta$ systems) two of the amplitudes are equal. The reduction procedure for the case where one has two identical fermions has been described before [18,19] and will not be repeated here. With the assumption that particle 1 is the different one and particles 2 and 3 are the two identical, only the amplitudes $T_{1;SI}^{ns_1i_1}(q_1)$ and $T_{2;SI}^{ms_2i_2}(q_2)$ are independent from each other and they satisfy the coupled integral equations

$$T_{1;SI}^{ls_1i_1}(q_1)=2\sum_{ns_2i_2}\int_0^\infty dq_3 A_{13;SI}^{ls_1i_1ns_2i_2}(q_1,q_3;E)T_{2;SI}^{ns_2i_2}(q_3), \tag{29}$$

TABLE VI. Two-body $\Delta\Delta$ channels (j,i) that contribute to a given $\Delta\Delta\Delta$ state with total spin S and isospin I.

S	I	(j,i)
1/2	1/2	(1,2),(2,1)
1/2	3/2	(1,0),(1,2),(2,1),(2,3)
1/2	5/2	(1,2),(2,1),(2,3)
1/2	7/2	(1,2),(2,3)
1/2	9/2	(2,3)
3/2	1/2	(0,1),(1,2),(2,1),(3,2)
3/2	3/2	(0,1),(0,3),(1,0),(1,2),(2,1),(2,3),(3,0),(3,2)
3/2	5/2	(0,1),(0,3),(1,2),(2,1),(2,3),(3,2)
3/2	7/2	(0,3),(1,2),(2,3),(3,2)
3/2	9/2	(0,3),(2,3)
5/2	1/2	(1,2),(2,1),(3,2)
5/2	3/2	(1,0),(1,2),(2,1),(2,3),(3,0),(3,2)
5/2	5/2	(1,2),(2,1),(2,3),(3,2)
5/2	7/2	(1,2),(2,3),(3,2)
5/2	9/2	(2,3)
7/2	1/2	(2,1),(3,2)
7/2	3/2	(2,1),(2,3),(3,0),(3,2)
7/2	5/2	(2,1),(2,3),(3,2)
7/2	7/2	(2,3),(3,2)
7/2	9/2	(2,3)
9/2	1/2	(3,2)
9/2	3/2	(3,0),(3,2)
9/2	5/2	(3,2)
9/2	7/2	(3,2)
9/2	9/2	

TABLE VII. Two-body $N\Delta$ channels $(j_{N\Delta},i_{N\Delta})$ and two-body NN channels (j_{NN},i_{NN}) that contribute to a given $NN\Delta$ state with total spin S and isospin I.

S	I	$(j_{N\Delta},i_{N\Delta})$	(j_{NN},i_{NN})
1/2	1/2	(1,1)	
1/2	3/2	(1,1),(1,2)	(1,0)
1/2	5/2	(1,2)	
3/2	1/2	(1,1),(2,1)	(0,1)
3/2	3/2	(1,1),(1,2),(2,1),(2,2)	(1,0),(0,1)
3/2	5/2	(1,2),(2,2)	(0,1)
5/2	1/2	(2,1)	
5/2	3/2	(2,1),(2,2)	(1,0)
5/2	5/2	(2,2)	

$$T_{2;SI}^{ms_2i_2}(q_2)=\sum_{ns_3i_3}(-)^{Iden}\int_0^\infty dq_3 \times A_{23;SI}^{ms_2i_2ns_3i_3}(q_2,q_3;E)T_{2;SI}^{ns_3i_3}(q_3) + \sum_{ls_1i_1}\int_0^\infty dq_1 A_{31;SI}^{ms_2i_2ls_1i_1}(q_2,q_1;E)T_{1;SI}^{ls_1i_1}(q_1), \tag{30}$$

with the identical-particles phase

$$Iden=1+\sigma_1+\sigma_3-s_2+\tau_1+\tau_3-i_2. \tag{31}$$

Substitution of Eq. (29) into Eq. (30) yields an equation with only the amplitude T_2

$$T_{2;SI}^{ms_2i_2}(q_2)=\sum_{ns_3i_3}\int_0^\infty dq_3 K_{23;SI}^{ms_2i_2ns_3i_3}(q_2,q_3;E)T_{2;SI}^{ns_3i_3}(q_3), \tag{32}$$

where

$$K_{23;SI}^{ms_2i_2ns_3i_3}(q_2,q_3;E) =(-)^{Iden}A_{23;SI}^{ms_2i_2ns_3i_3}(q_2,q_3;E)+2\sum_{ls_1i_1}\int_0^\infty dq_1 \times A_{31;SI}^{ms_2i_2ls_1i_1}(q_2,q_1;E)A_{13;SI}^{ls_1i_1ns_3i_3}(q_1,q_3;E). \tag{33}$$

We give in Table VII the nine $NN\Delta$ states characterized by total spin and isospin (S,I) that are possible as well as the two-body $N\Delta$ and NN channels that contribute to each state. In Table VIII we give the 16 $N\Delta\Delta$ states characterized by total spin and isospin (S,I) that are possible as well as the two-body $N\Delta$ and $\Delta\Delta$ channels that contribute to each state.

C. Numerical solutions

In order to find the bound-state solutions of Eqs. (13), (28), and (32) we drop the inhomogeneous term in Eq. (13) [of course, in the solution of the three-body problem we use as input the solutions of the inhomogeneous Eq. (13)] and replace the integral by a sum applying a numerical integration quadrature [20]. In this way, Eqs. (13), (28), and (32)

TABLE VIII. Two-body $N\Delta$ channels $(j_{N\Delta},i_{N\Delta})$ and two-body $\Delta\Delta$ channels $(j_{\Delta\Delta},i_{\Delta\Delta})$ that contribute to a given $N\Delta\Delta$ state with total spin S and isospin I.

S	I	$(j_{N\Delta},i_{N\Delta})$	$(j_{\Delta\Delta},i_{\Delta\Delta})$
1/2	1/2	(1,1),(1,2),(2,1),(2,2)	(1,0),(0,1)
1/2	3/2	(1,1),(1,2),(2,1),(2,2)	(0,1),(1,2)
1/2	5/2	(1,1),(1,2),(2,1),(2,2)	(0,3),(1,2)
1/2	7/2	(1,2),(2,2)	(0,3)
3/2	1/2	(1,1),(1,2),(2,1),(2,2)	(1,0),(2,1)
3/2	3/2	(1,1),(1,2),(2,1),(2,2)	(1,2),(2,1)
3/2	5/2	(1,1),(1,2),(2,1),(2,2)	(1,2),(2,3)
3/2	7/2	(1,2),(2,2)	(2,3)
5/2	1/2	(1,1),(1,2),(2,1),(2,2)	(2,1),(3,0)
5/2	3/2	(1,1),(1,2),(2,1),(2,2)	(2,1),(3,2)
5/2	5/2	(1,1),(1,2),(2,1),(2,2)	(2,3),(3,2)
5/2	7/2	(1,2),(2,2)	(2,3)
7/2	1/2	(2,1),(2,2)	(3,0)
7/2	3/2	(2,1),(2,2)	(3,2)
7/2	5/2	(2,1),(2,2)	(3,2)
7/2	7/2	(2,2)	

become a set of homogeneous linear equations. This set of linear equations has solutions only if the determinant of the matrix of the coefficients (the Fredholm determinant) vanishes for certain energies. Thus the procedure to find the bound states of the system consists simply in searching for the zeroes of the Fredholm determinant as a function of energy. We checked our program by comparing with known results for the three-nucleon bound-state problem with the Reid soft-core potential [5]. We found very stable results taking for the scale parameter $d=3\ \ \mathrm{fm}^{-1}$, a number of Legendre polynomials $L=10$, and a number of Gauss-Legendre points $N=12$.

IV. RESULTS

We will now present the results of our nonlocal calculations for the seven systems corresponding to the two- and three-body bound-state problem of nucleons and deltas, and compare them to previous calculations which have been done by our group based on the local potentials obtained from the Born-Oppenheimer approximation.

The two body interaction in the $N\Delta$ states $(j,i)=(1,1)$ and $(2,2)$, and those of the $\Delta\Delta$ states $(j,i)=(2,3)$ and $(3,2)$ present quark Pauli blocking. As a consequence, a strong repulsive core appears in the baryon-baryon potential. The reason for that is based on the fast decrease of the norm of the six-quark wave function when $R\to 0$ [14]. A similar analysis performed in terms of the SU(4) symmetry shows the presence of a forbidden state. From the physical point of view, it is connected with the lack of enough degrees of freedom to accommodate all the quarks. It is important to note that the origin of this repulsion is not the same as in the NN channels, because they do not show a forbidden state but a mixing of [6] with the [4,2] six-quark orbital symmetry. Technically, the reason for such a strong repulsive core is the presence of nodes in the inner region of the relative wave function of Eq. (7). This behavior originates essentially from the condition that the relative wave function should be orthogonal to the forbidden state due to the Pauli principle [21]. The forbidden state should then be eliminated from the relative wave function for each partial wave. This procedure is tedious both from the conceptual and numerical point of view [21,22]. It has been demonstrated [23] that for the Pauli blocked channels the local $N\Delta$ and $\Delta\Delta$ potentials reproduce the qualitative behavior of the RGM kernels after the subtraction of the forbidden states. This is why we used in our calculations the local version of the quark Pauli blocked channels mentioned above.

TABLE IX. Binding energies B_2 of the NN states with total angular momentum j and isospin i. B_2^L are the results of the local model and B_2^{NL} are the results of the nonlocal model.

j	i	B_2^L(MeV)	B_2^{NL}(MeV)
1	0	3.13	2.14
0	1	Unbound	Unbound

In the case of the three-body systems we calculated the binding-energy spectrum (that is, the energy of the states measured with respect to the three-body threshold) as well as the separation-energy spectrum (that is, the energy of the states measured with respect to the threshold of one free particle and a bound state of the other two). The deepest bound three-body state is not the one with the largest binding energy but the one with the largest separation energy, since that state is the one that requires more energy in order to become unbound (that is, to move it from the bound state to the nearest threshold).

A. *NN* system

We found that of the two states of Table II only the one with $(j,i)=(1,0)$, that is the deuteron, is bound. The nonlocal model gives a deuteron binding energy of 2.14 MeV, while the local version gave an energy of 3.13 MeV. These results are shown in Table IX.

The exact chiral quark cluster model NN potential [12] gives a deuteron binding energy of 2.225 MeV. This value was obtained by taking into account the $\Delta\Delta$ partial wave $(l_{\Delta\Delta},s_{\Delta\Delta})=(4,3)$ coupled together in addition to those given in Table II. Since in our calculation we consider only S and D waves, we omit the $\Delta\Delta$ $(l_{\Delta\Delta},s_{\Delta\Delta})=(4,3)$ partial-wave contribution, and we obtain instead a deuteron binding energy of 2.14 MeV, which differs less than 0.1 MeV from the exact calculation.

B. *N*Δ system

We give in Table X the results for the binding energies of the $N\Delta$ system. Out of the four possible $N\Delta$ states of Table III only one, the $(j,i)=(2,1)$, has a bound state which lies exactly at the $N\Delta$ threshold for the local model. However, if we use the nonlocal model we find instead a bound state of 0.141 MeV. The states $(j,i)=(1,1)$ and $(2,2)$ are unbound because they present quark Pauli blocking [14] and therefore

TABLE X. Binding energies B_2 of the $N\Delta$ states with total angular momentum j and isospin i. B_2^L are the results of the local model and B_2^{NL} are the results of the nonlocal model.

j	i	B_2^L(MeV)	B_2^{NL}(MeV)
1	1	Unbound	Unbound
1	2	Unbound	Unbound
2	1	0.0	0.141
2	2	Unbound	Unbound

they have a strong repulsive barrier at short distances in the S-wave central interaction. These two states play an important role in the three-body spectrum. The state $(j,i)=(2,1)$ can also exist in the NN system and there it corresponds to the $^1D_2$ partial wave which has a resonance at an invariant mass of 2.17 GeV [24–26]. This means that the $N\Delta$ bound state may decay into two nucleons and appear in the NN system as a resonance. The $N\Delta$ bound state has for both local and nonlocal models energies very close to the $N\Delta$ threshold, so that the invariant mass of the system is also very close to 2.17 GeV. Thus one or another of our models predict the NN $^1D_2$ resonance as being a $N\Delta$ bound state.

C. $\Delta\Delta$ system

We give in Table XI our results for the $\Delta\Delta$ system. Out of the eight possible $\Delta\Delta$ states given in Table IV with nonlocal interactions five have a bound state, whereas the local interactions bind six of them (in both local and nonlocal models there are no excited states in any of the channels). It is interesting to note that the predicted bound states: $(j,i)=(1,0)$, (0,1), (2,1), and (3,0), also appear in the case of the NN system. In the nonlocal model, we find that the deepest bound state is $(j,i)=(1,0)$, the second $(j,i)=(0,1)$, the third $(j,i)=(3,0)$ and the fourth $(j,i)=(2,1)$. This clearly shows that there is a qualitative similarity between the $\Delta\Delta$ and NN systems (both are systems of identical particles). Three of these states appear also in the case of the NN system. The $(j,i)=(1,0)$ state is of course the deuteron, the $(j,i)=(0,1)$ is the $^1S_0$ virtual state and the $(j,i)=(2,1)$ state is the $^1D_2$ resonance that lies at ≈ 2.17 GeV [24] (note that the $^3F_3$ NN resonance has no counterpart in Table XI because we calculated only even-parity states and $^3F_3$ has odd parity). Thus the $(j,i)=(3,0)$ state, which is also allowed in the case of the NN system, would correspond to a new nucleon-nucleon resonance that is predicted by our model. The $(j,i)=(3,0)$ channel corresponds in the case of the NN system to the $^3D_3$ partial wave. Some indication of the (3,0) resonance can already be seen in the most recent analysis of the NN data by Arndt *et al.* [26] and other theoretical calculations [27].

TABLE XI. Binding energies B_2 of the $\Delta\Delta$ states with total angular momentum j and isospin i. B_2^L are the results of the local model and B_2^{NL} are the results of the nonlocal model.

j	i	B_2^L(MeV)	B_2^{NL}(MeV)
0	1	108.4	159.5
0	3	0.4	0.2
1	0	138.5	190.3
1	2	5.7	Unbound
2	1	30.5	7.4
2	3	Unbound	Unbound
3	0	29.9	7.8
3	2	Unbound	Unbound

TABLE XII. Binding energies B_3 and separation energies B_3-B_2 of the NNN states with total spin S and isospin I. B_2^L and B_3^L are the results of the local model while B_2^{NL} and B_3^{NL} are the results of the nonlocal model.

S	I	B_3^L(MeV)	$B_3^L-B_2^L$(MeV)	B_3^{NL}(MeV)	$B_3^{NL}-B_2^{NL}$(MeV)
1/2	1/2	5.76	2.63	6.52	4.38
1/2	3/2	Unbound		Unbound	
3/2	1/2	Unbound		Unbound	

The channels $(j,i)=(2,3)$ and (3,2) are unbound because they have a strong repulsive barrier at short distances in the S-wave central interaction. This strong repulsion originates from the quark Pauli blocking produced by the saturation of states that occurs when the total spin and isospin are near their maximum values [15]. As we will see later in the discussion of the $\Delta\Delta\Delta$ results, these repulsive cores in the (3,2) and (2,3) channels largely determine the three-body spectrum.

From Table XI we note that the two-body $\Delta\Delta$ bound states which have low quantum numbers are deeper for the nonlocal model than with the local one. This peculiar feature results to be conversely for the case of high quantum numbers.

D. NNN system

As another test of the reliability of our model in the case of the three-baryon system we solved the NNN bound-state problem. We found that of the states of Table V only the state with $(S,I)=(\frac{1}{2},\frac{1}{2})$, that is the triton, has a bound state. By using the local potentials we obtain a binding energy of 5.76 MeV for the triton. On other hand, if we use the nonlocal potentials as input we find a triton binding energy of 6.52 MeV. For comparison, we notice that the triton binding energy for the Reid-soft-core potential in the truncated T-matrix approximation is 6.58 MeV. Since the experimental value is $B_{EXP}=8.49$ MeV the difference with our theoretical result, of about 3 MeV, is a measure of the uncertainty of our calculation in the case of the three-baryon system. We show in Table XII the results of our calculations for the NNN system. There, B_3 is the binding energy of the system and B_3-B_2 is the separation energy, being B_2 the binding energy of the deepest bound two-body channel that contributes to the three-body state (see Table IX).

E. $NN\Delta$ system

We show in Table XIII the results of our calculations for the $NN\Delta$ system.

TABLE XIII. Binding energies B_3 and separation energies $B_3 - B_2$ of the $NN\Delta$ states with total spin S and isospin I. B_2^L and B_3^L are the results of the local model while B_2^{NL} and B_3^{NL} are the results of the nonlocal model.

S	I	B_3^L(MeV)	$B_3^L-B_2^L$(MeV)	B_3^{NL}(MeV)	$B_3^{NL}-B_2^{NL}$(MeV)
3/2	1/2	Unbound		0.143	0.002
3/2	3/2	Unbound		2.280	0.144

One may have hoped to find several bound states in this system, due to the fact that the $N\Delta$ two-body subsystem has a bound state in the channel $(j,i)=(2,1)$ and the NN two-body subsystem has a bound state in the channel $(j,i)=(1,0)$ and an almost-bound state in the channel $(j,i)=(0,1)$. This is not the case, however, and as a matter of fact, with the nonlocal potentials as input only two of the nine possible three-body states given in Table VII are bound. Because of the attractive contribution of the $N\Delta$ $(j,i)=(2,1)$ bound state with the nonlocal model, the three-body state $(\frac{3}{2},\frac{1}{2})$ results to be very weakly bound, at an energy of 0.143 MeV, and a separation energy scarcely different from zero. That means that the $(S,I)=(\frac{3}{2},\frac{1}{2})$ state is very near the $NN\Delta$ threshold and therefore it represents the tribaryon resonance with the lowest possible mass since it can decay into three nucleons and one pion. Also, for this case the three-body state $(\frac{3}{2},\frac{3}{2})$ is bound. As it can be seen from Table VII, this state has the contribution of all the two-body $N\Delta$ and NN channels. In spite of the fact that the $N\Delta$ two-body channels $(j,i)=(1,1)$ and $(2,2)$ present Pauli blocking [14], and therefore they have a strong repulsive barrier at short distances in the S-wave central interaction, the attractive contribution of the $N\Delta$ $(j,i)=(2,1)$ and NN $(j,i)=(1,0)$ channels results to be enough to weakly bound this state. We note that neither one of the three-body states $(S,I)=(\frac{3}{2},\frac{1}{2})$ and $(\frac{3}{2},\frac{3}{2})$ is bound with local interactions.

F. $N\Delta\Delta$ system

The results for the $N\Delta\Delta$ system are shown in Table XIV. Similarly to the case just discussed, in our calculations with nonlocal interactions we found that three of the 16 possible $N\Delta\Delta$ states given in Table VIII are bound. They are the $(S,I)=(\frac{1}{2},\frac{5}{2})$, $(\frac{5}{2},\frac{1}{2})$, and $(\frac{5}{2},\frac{5}{2})$ states and their corresponding bound state energies are 0.630, 8.158, and 0.181 MeV, respectively. In the case of the states $(S,I)=(\frac{1}{2},\frac{5}{2})$ and $(S,I)=(\frac{5}{2},\frac{1}{2})$ the repulsive barrier due the quark Pauli blocking in the $N\Delta$ states $(j,i)=(1,1)$ and $(2,2)$ is less strong than the attraction due to the state $(j,i)=(2,1)$, so that they result to be bound states in the nonlocal model. The state $(S,I)=(\frac{5}{2},\frac{5}{2})$ is the weakest bound state of this system, since in addition to the contribution of the $N\Delta$ quark Pauli blocking channels, there exists that of the $\Delta\Delta$ quark Pauli blocking channels $(j,i)=(2,3)$ and $(3,2)$. This confirms what we have mentioned before that it is the structure of the interaction of the two-body system which largely determines the three-body spectrum. Thus the nonlocal interactions predict the bound states $(S,I)=(\frac{1}{2},\frac{5}{2})$, $(\frac{5}{2},\frac{1}{2})$, and $(\frac{5}{2},\frac{5}{2})$, which in principle may be observable as tribaryon resonances which decay into three nucleons and two pions with masses close to the $N\Delta\Delta$ threshold.

TABLE XIV. Binding energies B_3 and separation energies $B_3 - B_2$ of the $N\Delta\Delta$ states with total spin S and isospin I. B_2^L and B_3^L are the results of the local model while B_2^{NL} and B_3^{NL} are the results of the nonlocal model.

S	I	B_3^L(MeV)	$B_3^L-B_2^L$(MeV)	B_3^{NL}(MeV)	$B_3^{NL}-B_2^{NL}$(MeV)
1/2	5/2	Unbound		0.630	0.43
5/2	1/2	Unbound		8.158	0.358
5/2	5/2	Unbound		0.181	0.04

TABLE XV. Binding energies B_3 and separation energies $B_3 - B_2$ of the $\Delta\Delta\Delta$ states with total spin S and isospin I. B_2^L and B_3^L are the results of the local model while B_2^{NL} and B_3^{NL} are the results of the nonlocal model.

S	I	B_3^L(MeV)	$B_3^L-B_2^L$(MeV)	B_3^{NL}(MeV)	$B_3^{NL}-B_2^{NL}$(MeV)
1/2	1/2	84.0	53.5	16.6	9.2
1/2	3/2	139.2	0.7	Unbound	
1/2	7/2	6.3	0.6	Unbound	
3/2	1/2	109.5	1.1	Unbound	
5/2	1/2	39.1	8.6	9.3	1.9
7/2	1/2	31.7	1.2	7.8	0.4
7/2	3/2	35.1	4.6	9.8	2.0

G. $\Delta\Delta\Delta$ system

We show in Table XV the results for the $\Delta\Delta\Delta$ system. The system has four bound states while by using the local interactions the system had seven bound states. From Table XV we observe that the three states which are missing in the nonlocal version are barely bound in the local version, i.e., they have very small separation energies. Since the nonlocal interaction tends to lower the attraction in all the $\Delta\Delta\Delta$ channels it is not surprising that those which were barely bound have now disappeared. The more strongly bound three-body state (that is, the one with the largest separation energy) is the $(S,I)=(\frac{1}{2},\frac{1}{2})$ state which has precisely the quantum numbers of the triton. This shows again, like in the $\Delta\Delta$ and NN systems, the similarity between the $\Delta\Delta\Delta$ and NNN systems.

The reason why the $(S,I)=(\frac{1}{2},\frac{1}{2})$ state is the more strongly bound is very simple. As shown in Table VI, this is the only state where none of the two-body channels with a strong repulsive core $(j,i)=(2,3)$ or $(3,2)$ contribute. In all the other three-body states the strong repulsion of the $(j,i)=(2,3)$ and $(3,2)$ channels either completely destroys the bound state or allows just a barely bound one. The state $(S,I)=(\frac{7}{2},\frac{3}{2})$ comes next with respect to separation energy. This state $(S,I)=(\frac{7}{2},\frac{3}{2})$ has a somewhat anomalous behavior since it has a relatively large separation energy. This behavior is sort of accidental and it can be understood as follows. As seen in Table VI, there are four two-body channels contributing to the $(S,I)=(\frac{7}{2},\frac{3}{2})$ state, the two attractive ones $(j,i)=(2,1)$ and (3,0) and the two repulsive ones $(j,i)=(2,3)$ and (3,2). However, as one can see in Table XI the attractive channels (2,1) and (3,0) have bound states at $E=-7.4$ MeV and $E=-7.8$ MeV, respectively, for the nonlocal version, and $E=-30.5$ MeV and $E=-29.9$ MeV, respectively, for the local version, so that the poles in the scattering amplitudes of these two channels are very close together and therefore there is a reinforcement between them, which gives rise to the anomalously large separation energy in both versions.

V. CONCLUSIONS

By using both the local and nonlocal models we have studied the bound-state solutions of the two- and three-body systems composed of nucleons and deltas. First of all we would like to emphasize the goodness of the Born-Oppenheimer approximation, producing results very similar to the usually more involved RGM results. We conclude that the more realistic nonlocal interactions produce in the two-body systems NN, $N\Delta$, and $\Delta\Delta$ one, one, and five bound states, respectively. The bound states of the unstable systems $N\Delta$ and $\Delta\Delta$ correspond to dibaryon resonances that decay mainly into two nucleons and one pion and two nucleons and two pions, respectively. The $N\Delta$ bound state with $(j,i)=(2,1)$ and $M\approx 2.17$ GeV is the dibaryon resonance with the lowest possible mass and the one which seems to be well confirmed by experiment. The five $\Delta\Delta$ bound states of the nonlocal potentials correspond to dibaryon resonances with masses between 2.4 and 2.5 GeV. The $(j,i)=(3,0)$ $\Delta\Delta$ state would correspond to a new nucleon-nucleon resonance predicted by our model. A possible signal of this resonance appears in a recent analysis of NN data up to 3 GeV by Arndt *et al.* [26]. With respect to the three-body systems we found that the NNN has one bound state, the $\Delta\Delta\Delta$ has four bound states, the $NN\Delta$ has two bound states, and the $N\Delta\Delta$ has three bound states. The predicted $NN\Delta$ states with $(S,I)=(\frac{3}{2},\frac{1}{2})$ and $(S,I)=(\frac{3}{2},\frac{3}{2})$ which correspond to $M\approx 3.4$ GeV are the tribaryon resonances with the lowest mass and therefore the ones that would be more easy to detect experimentally.

ACKNOWLEDGMENTS

A.V. thanks the hospitality of the Instituto Politécnico Nacional of Mexico during several visits when part of this work has been done. He also thanks the Ministerio de Educación, Cultura y Deporte of Spain for financial support through the Salvador de Madariaga program. This work has been partially funded by COFAA-IPN (Mexico), by Dirección General de Investigación Científica y Técnica (DGICYT) under Contract No. PB97-1401, and by Junta de Castilla y León under Contract No. SA-109/01, by the University of Salamanca, and by the Ramón Areces Foundation (Spain).

[1] H. Garcilazo, F. Fernández, A. Valcarce, and R.D. Mota, Phys. Rev. C **56**, 84 (1997).
[2] R.D. Mota, A. Valcarce, F. Fernández, and H. Garcilazo, Phys. Rev. C **59**, 46 (1999).
[3] H. Garcilazo, A. Valcarce, and F. Fernández, Phys. Rev. C **60**, 044002 (1999).
[4] A. Valcarce, H. Garcilazo, R.D. Mota, and F. Fernández, J. Phys. G **27**, L1 (2001).
[5] E.P. Harper, Y.E. Kim, and A. Tubis, Phys. Rev. Lett. **28**, 1533 (1972).
[6] G.H. Berthold, H. Zankel, L. Mathelitsch, and H. Garcilazo, Nuovo Cimento A **93**, 89 (1986).
[7] C.R. Chen, G.L. Payne, J.L. Friar, and B.F. Gibson, Phys. Rev. Lett. **55**, 374 (1985).
[8] M.T. Peña, H. Henning, and P.U. Sauer, Phys. Rev. C **42**, 855 (1990).
[9] R.B. Wiringa, R.A. Smith, and T.L. Ainsworth, Phys. Rev. C **29**, 1207 (1984).
[10] A. Faessler, F. Fernández, G. Lübeck, and K. Shimizu, Phys. Lett. **112B**, 201 (1982); M. Oka and K. Yazaki, Prog. Theor. Phys. **66**, 556, 572 (1980).
[11] F. Fernández, A. Valcarce, U. Straub, and A. Faessler, J. Phys. G **19**, 2013 (1993); A. Valcarce, A. Buchmann, F. Fernández, and A. Faessler, Phys. Rev. C **50**, 2246 (1994).
[12] D.R. Entem, F. Fernández, and A. Valcarce, Phys. Rev. C **62**, 034002 (2000).
[13] G.E. Brown and W. Weise, Phys. Rep. **22**, 279 (1975).
[14] A. Valcarce, F. Fernández, P. González, and V. Vento, Phys. Rev. C **52**, 38 (1995).
[15] A. Valcarce, F. Fernández, and P. González, Phys. Rev. C **56**, 3026 (1997).
[16] G.M. Huber *et al.*, Phys. Rev. C **62**, 044001 (2000).
[17] K. Shimizu, Rep. Prog. Phys. **52**, 1 (1989).
[18] I.R. Afnan and A.W. Thomas, Phys. Rev. C **10**, 109 (1974).
[19] H. Garcilazo and T. Mizutani, *πNN Systems* (World Scientific, Singapore, 1990).
[20] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1972).
[21] S. Saito, Prog. Theor. Phys. **41**, 706 (1968).
[22] S. Otsuki, R. Tamagaki, and M. Yasuno, Suppl. Prog. Theor. Phys. **Extra number 578**, 1 (1965).
[23] H. Toki, Z. Phys. A **294**, 173 (1980).
[24] N. Hoshizaki, Prog. Theor. Phys. **60**, 1796 (1978); **61**, 129 (1979).
[25] A. Yokosawa, Phys. Rep. **64**, 47 (1980).
[26] R.A. Arndt, C.H. Oh, I.I. Strakovski, R.L. Workman, and F. Dohrmann, Phys. Rev. C **56**, 3005 (1997); R.A. Arndt, I.I. Strakovski, and R.L. Workman, *ibid.* **62**, 034005 (2000).
[27] D. Quing, H. Sun, and F. Wang, Phys. Rev. C **62**, 018201 (2000); J. Ping, F. Wang, and T. Goldman, Nucl. Phys. A **688**, 871 (2001).