Percolation of strings and the relativistic energy data on multiplicity and transverse momentum distributions

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(Received 25 May 2001; published 25 January 2002)

The dependence of the multiplicity on the number of collisions and the transverse momentum distribution for central and peripheral Au-Au collisions are studied in the model of percolation of strings relative to the experimental conditions at RHIC. The comparison with the first RHIC data shows a good agreement.

DOI: 10.1103/PhysRevC.65.024907 PACS number(s): 25.75.−q, 12.38.Mh, 13.87.Ce, 24.85.+p

The first experimental data from the relativistic heavy ion collider show several interesting features [1–7]. The measured multiplicity lies in the lower range of the values predicted by various models and shows a smooth dependence on the number of participants [5,6]. The experimental p_T distributions in Au-Au central collisions reveal a large departure from the naive perturbative quantum chromodynamics (PQCD) predictions [2,7,8]. Also, some differences are seen for peripheral collisions. In this paper, we compare these experimental results with the predictions deduced from the percolation of strings.

Multiparticle production is currently described in terms of color strings stretched between the projectile and target, which decay into new strings and subsequently hadronize to produce observed hadrons. Color strings may be viewed as small areas in the transverse space, πr_0^2, $r_0=0.2$ fm, filled with color field created by the colliding partons [9,10]. Particles are produced via emision of $q\bar{q}$ pairs in this field. With growing energy and/or atomic number of colliding particles, the number of strings grows, and they start to overlap, forming clusters, very much similar to disks in the two-dimensional percolation theory [9,11]. At a certain critical density a macroscopic cluster appears that marks the percolation phase transition. The influence of the clustering of strings and their percolation on the transverse momentum spectra was studied in [12] in the thermodynamical limit of a very large interaction area S and very large number N of formed strings, corresponding to very high energies and atomic numbers of participants. In this limit the study allows for an analytical treatment. Here we apply this approach to realistic energies and colliding nuclei at RHIC using Monte Carlo simulations.

The percolation theory governs the geometrical pattern of the string clustering. Its observable implications, however, require introduction of some dynamics to describe string interaction, i.e., the behavior of a cluster formed by several overlapping strings.

There are several possibilities [12–14]. Here we assume that a cluster behaves as a single string with a higher color field $\vec{Q}_n$ corresponding to the vectorial sum of the color charge of each individual $\vec{Q}_1$ string. The resulting color field covers the area S_n of the cluster. As $\vec{Q}_n=\Sigma_1^n\vec{Q}_1$, and the individual string colors may be oriented in an arbitrary manner respective to one another, the average $\vec{Q}_{1i}\vec{Q}_{1j}$ is zero, and $\vec{Q}_n^2=n\vec{Q}_1^2$.

Knowing this charge color $\vec{Q}_n$, one can compute the particle spectra produced by a single color string of area S_n using the Schwinger formula [15]. For the multiplicity μ_n and the average p_T^2 of particles $\langle p_T^2\rangle_n$ produced by a cluster of n strings one finds

$$\mu_n=\sqrt{\frac{nS_n}{S_1}}\mu_1;\quad \langle p_T^2\rangle_n=\sqrt{\frac{nS_1}{S_n}}\langle p_T^2\rangle_1, \tag{1}$$

where μ_1 and $\langle p_T^2\rangle_1$ are the mean multiplicity and p_T^2 of particles produced by a simple string with a transverse area $S_1=\pi r_0^2$. For strings just touching each other $S_n=nS_1$, and hence $\mu_n=n\mu_1$, $\langle p_T^2\rangle_n=\langle p_T^2\rangle_1$, as expected. In the opposite case of maximum overlapping $S_n=S_1$, and therefore, $\mu_n=\sqrt{n}\mu_1$, $\langle p_T^2\rangle_n=\sqrt{n}\langle p_T^2\rangle_1$, so that the multiplicity results maximally supressed and the mean p_T^2 maximally enhanced. Notice that a certain conservation rule holds for the total $\langle p_T^2\rangle$, $\mu_n\langle p_T^2\rangle_n=n\mu_1\langle p_T^2\rangle_1$.

Equation (1) is the main tool of our calculation. In order to compute the multiplicities we generate strings according to the quark-gluon string model and using the Monte Carlo code described in [10]. Each string is produced at an identified impact parameter. From this, knowing the transverse area of each string, we identify all the clusters formed in each collision and subsequently compute for each of them its multiplicity in units μ_1. The value of μ_1 was fixed by normalizing our results to the SPS WA98 results for central Pb-Pb collisions [16]. Note that in color string models both μ_1 and $\langle p_T^2\rangle$ are assumed to rise with energy due to the increase of the rapidity interval available and hard scattering contribution. However, this rise is very weak in the the energy range $\sqrt{s}=17$–200 GeV. Therefore we can neglect the energy dependence of both μ_1 and $\langle p_T^2\rangle_1$ in the first approximation.

The comparison of our results for the dependence of the multiplicity on the number of participants the SPS WA98 data, with the RHIC PHENIX [5] and PHOBOS [6] data at $\sqrt{s}=130$ GeV, and with PHOBOS data at $\sqrt{s}=200$ GeV is

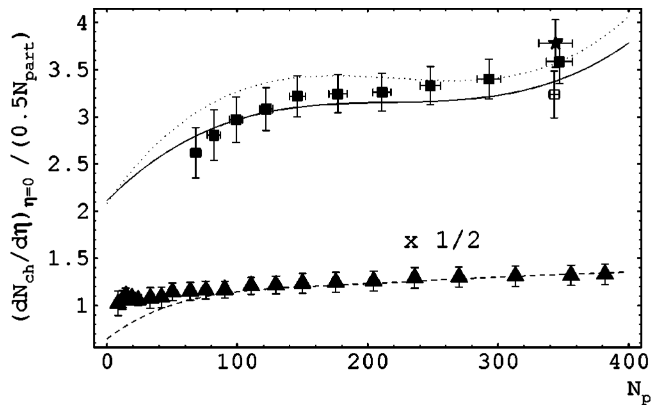

FIG. 1. Comparison of the dependence of the multiplicity on the number of participants the SPS WA98 [16] data (filled triangles), the RHIC PHENIX [5] (filled boxes) PHOBOS [6] (nonfilled boxes) data at $\sqrt{s}=130$ GeV, and with PHIC PHENIX data at $\sqrt{200}$ GeV (filled stars). The dashed, solid, and dotted lines are our predictions for the relevant energies.

presented in Fig. 1. The agreement with the data at $\sqrt{s}=130$ GeV is quite good. Our predictions for $\sqrt{s}=200$ GeV lie a little below the data. This could be a consequence of the mentioned weak energy dependence of μ_1. On the whole the agreement with the data indicates that the percolation of strings mechanism correctly describes the behavior of multiplicities in nucleus-nucleus collisions. This conclusion has also been reached in [17].

In order to compute the transverse momentum distributions, we make use of the parametrization of the pp UA1 data at 130 GeV by Drees [8]

$$\frac{dN}{dp_t^2}=\frac{a}{(p_0+p_T)^{\alpha}} \tag{2}$$

where a, p_0, and α are parameters fitted to data [2]. The standardly expected distribution for central Au-Au collision is

$$\left.\frac{dN}{dp_T^2}\right|_{\mathrm{AuAu}}=\langle N_{\mathrm{bin\ coll}}\rangle \left.\frac{dN}{dp_T^2}\right|_{pp}. \tag{3}$$

However, if we take into account percolation of strings then from Eq. (1) we conclude

$$\langle p_T^2\rangle_{\text{Au-Au}}=\left\langle\frac{nS_1}{S_n}\right\rangle_{\text{Au-Au}}^{1/2}\langle p_T^2\rangle_1 \tag{4}$$

where, by definition,

$$\left\langle\frac{nS_1}{S_n}\right\rangle_{\text{Au-Au}}^{1/2}=\frac{\sum\left(\frac{nS_1}{S_n}\right)^{1/2}\mu_n}{\sum\ \mu_n}. \tag{5}$$

Here the sums go over all clusters formed in Au-Au collisions and μ_n are given by Eq. (1). To exclude the unknown $\langle p_T^2\rangle_1$ we compare Eq. (4) with the one for pp collisions

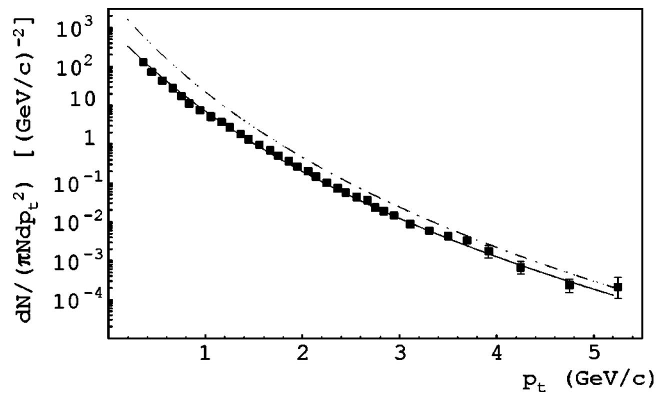

FIG. 2. Expected p_T distribution using the percolation of string model (solid line) for central (5%) Au-Au collisions compared with PHENIX experimental data [2,18] (filled boxes). Also it is shown the expected distribution [8] given by formula (3) and (2) (dotted-dashed line).

$$\langle p_T^2\rangle_{pp}=\left\langle\frac{nS_1}{S_n}\right\rangle_{pp}^{1/2}\langle p_T^2\rangle_1 \tag{6}$$

to finally get

$$\langle p_T^2\rangle_{\text{Au-Au}}=\langle p_T^2\rangle_{pp}\frac{\langle nS_1/S_n\rangle_{\text{Au-Au}}^{1/2}}{\langle nS_1/S_n\rangle_{pp}^{1/2}}. \tag{7}$$

This equation implies that the same parametrization (2) for the transverse momentum distribution can be used for nucleus-nucleus collisions, with the only change

$$p_0\rightarrow p_0\left(\frac{\langle nS_1/S_n\rangle_{\text{Au-Au}}}{\langle nS_1/S_n\rangle_{pp}}\right)^{1/4}. \tag{8}$$

Note that at the energies considered, only two strings are exchanged in pp collisions on the average, with very small fusion probablilty, so that $\langle nS_1/S_n\rangle_{pp}\simeq 1$.

In Fig. 2, we show the distribution (2) with the change (8) for central (5%) Au-Au collisions (solid line) compared to the PHENIX experimental data [2,18] (black squares). Also the distribution expected from the independent string picture [Eq. (3)] is shown (dash-dotted line). For peripheral (80–92 %) Au-Au collisions the same results are presented in Fig. 3 (with the same notations). A very good agreement is observed in both cases. We stress that this result is obtained using a very simple formula deduced in a simple way from the dynamics, with practically a single parameter, the string transverse radius.

Our predictions for central (5%) Ag-Ag and central Au-Au collisions at 200 GeV/nucleon are presented in Fig. 4.

In the thermodynamic limit one approximately obtains [14]

$$\left\langle\frac{nS_1}{S_n}\right\rangle=\frac{\eta}{1-\exp(-\eta)}\equiv 1/F^2(\eta) \tag{9}$$

where $\eta=N\pi r_0^2/S$ is the percolation parameter assumed to be finite with both N and S large. Then one obtains an ana-

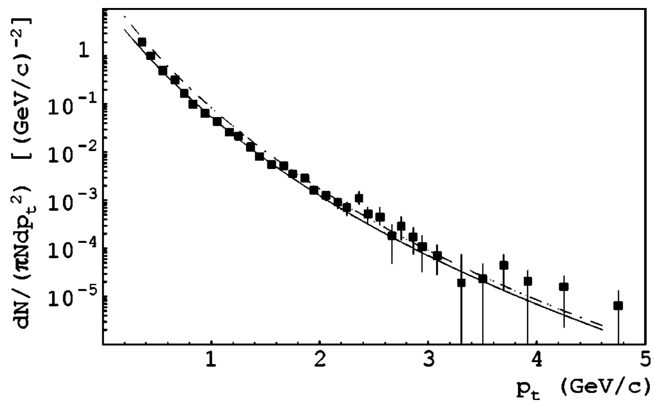

FIG. 3. Expected p_T distribution using the percolation of string model (solid line) for peripheral (80–90 %) Au-Au collisions compared with PHENIX experimental data [2,18] (filled boxes). Also it is shown the expected distribution [8] given by formula (3) and (2) (dotted-dashed line).

lytic expression for the transverse momentum distribution for all colliding particles in terms of η

$$\frac{dN}{dp_t^2}=\frac{a}{[p_0\sqrt{F(\eta_{pp})/F(\eta)}+p_T]^{\alpha}}. \tag{10}$$

In Eq. (10) a and p_0 are parameters of the experimentally observed spectrum in pp collisions at a given energy and η and η_{pp} are percolation parameters for nucleus-nucleus and pp collisions respectively, to be obtained from the geometrical percolation picture. Naturally all parameters depend on energy, although, as mentioned, at energies below 200 GeV η_{pp} is quite small so that $F(\eta_{pp})\simeq 1$. Equation (10) implies a certain universality of the distributions at fixed energy, for all participant nuclei and different centralities, the distribution results a universal function of η.

At very high energies both η and η_{pp} become large, so that $F(\eta)\simeq 1/\sqrt{\eta}$. Then we find

$$\frac{F(\eta_{pp})}{F(\eta)}\simeq\sqrt{\frac{\eta}{\eta_{pp}}}. \tag{11}$$

For minimum bias collision of identical nuclei

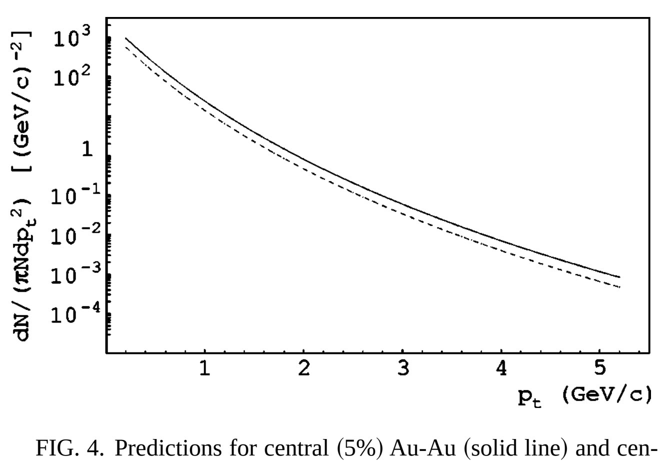

FIG. 4. Predictions for central (5%) Au-Au (solid line) and central (5%) Ag-Ag (dashed line) collisions at 200 GeV/nucleon.

$$\eta=16\frac{\nu N_{pp}\pi r_0^2}{\sigma_{AA}^{tot}}, \tag{12}$$

where ν the average number of collisions and we used the fact that the average overlap area is 1/16 of the total nucleus-nucleus cross section. From this we find an asymptotic expression for the distribution valid at very high energies (large η's),

$$\frac{dN}{dp_t^2}=\frac{a}{(2p_0\sqrt{A\sigma_{pp}/\sigma_{AA}^{tot}}+p_T)^{\alpha}}. \tag{13}$$

Accordingly we find for these energies

$$\langle p_T^2\rangle_{AA}=4\langle p_T^2\rangle_{pp}\frac{A\sigma_{pp}}{\sigma_{AA}^{tot}}, \tag{14}$$

from which it evidently follows that at very high energies the ratio of the averages p_t^2 in nucleus-nucleus and pp collisions should grow as the total pp cross section.

Notice that from Eq. (1) we have for a cluster

$$\langle p_T^2\rangle_n=\frac{S_1}{S_n}\frac{\langle p_T^2\rangle_1}{\mu_1}\mu_n. \tag{15}$$

In the limit of very high density of strings, as observed from the Monte Carlo calculations, practically all strings overlap into a single cluster, which occupies the whole interaction area. This is obviously an alternative description of parton saturation. In this limit from Eq. (15) we find a universal relation between the observed $\langle p_T^2\rangle_{AB}$ and the multiplicity per unit overlap area in the nucleus A–nucleus B collisions, valid for any participant nuclei and any centrality

$$\langle p_T^2\rangle_{AB}=\frac{S_1\langle p_T^2\rangle_1}{\mu_1}\frac{1}{S_{AB}}\mu_{AB}. \tag{16}$$

Here S_{AB} is the overlap area depending on the impact parameter. It is remarkable that a similar relation has been found using the idea of the saturation of gluons in [19]. In their case the proportionality coefficient is given by $\alpha_s P^2(m_h/p_s)$, where m_h is the mass of the produced hadron, p_s is essentially the saturation momentum, and P is some function that tends to a constant as its argument vanishes. At high energies and therefore large p_s this coefficient tends to a constant modulo logarithmic terms coming from the running of the coupling constant. In our case the coefficient also tends essentially to a constant at high energies, since both $S_1\langle p_T^2\rangle$ and μ_1 (per unit rapidity) tend to a constant.

To conclude, some general remarks. The color string model, in principle, was introduced to describe the soft part of the spectra. However, our results remain in good agreement with the experimental data up to p_T of the order 5 GeV/c, where hard scattering is supposed to play the dominant role, standardly desribed by the PQCD approach. This implies that with the parametrization (2) the hard part of the spectrum is also included into our string picture, corresponding to the high-momentum tail of the string spectra.

Clustering and percolation of strings can then be looked upon as an alternative description of nonlinear effects, which in the PQCD language correspond to high partonic densities, such as saturation of partons.

There are several models that explain qualitatively the dependence of the multiplicity on the number of participants [20–24], although not all of them are able to reproduce quantitatively the experimental data. The situation is worse concerning the p_T distributions. With some fitted parameters, the jet quenching effect [8,25] could explain the data, but in this case one expects an enhancement of the multiplicity in the central rapidity region, which is not seen by experimental data. The percolation of strings is a natural and simple way of explaining both the multiplicity and transverse momentum distribution. To further check our approach, experimental information on forward-backward correlations of multiplicity and transverse momentum distributions would be welcome (see [13,14]).

This work was done under Contract No. AEN99-0589-C02-02 from CICYT of SPAIN and Contract No. PGIDTOOPXI20613PN from Xunta de Galicia. It was also supported by the Secretaria de Estado de Educacion y Universidades of Spain and NATO Grant No. PST.CLG.976799.

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