

Extended $M1$ sum rule for excited symmetric and mixed-symmetry states in nuclei

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A generalized $M1$ sum rule for orbital magnetic dipole strength from excited symmetric states to mixed-symmetry states is considered within the proton-neutron interacting boson model of even-even nuclei. Analytic expressions for the dominant terms in the $B(M1)$ transition rates from the $2_{1,2}^+$ states are derived in the $U(5)$ and $SO(6)$ dynamic symmetry limits of the model, and the applicability of a sum rule approach is examined at and between these limits. Lastly, the sum rule is applied to the new data on mixed-symmetry states of ^{94}Mo and a quadrupole d -boson ratio $n_d(0_1^+)/n_d(2_2^+) \approx 0.6$ is obtained in a largely parameter-independent way.

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I. INTRODUCTION

Heavy atomic nuclei exhibit both single-particle and collective excitations. However, the coupling between these degrees of freedom can lead to strong fragmentation of the collective modes. Under such circumstances sum rules are useful, since they do not depend on the exact details of the fragmentation and remain applicable in cases where the collective modes are not exact eigenstates of the Hamiltonian. Sum rules generally express direct observables in terms of basic control parameters (e.g., deformation) which dominate the formation of the collective mode. In cases where the relevant control parameter has natural boundaries, one can obtain quantitative limits for observables in a largely model-independent way. Accordingly, sum rules are used both to judge what fraction of a collective mode is present in a given ensemble of quantum states and as a tool to exploit the link between direct observables and properties of a given excitation mode.

One particular collective mode which has been studied extensively in recent years is the orbital magnetic dipole scissors mode [1], which has by now been established experimentally as a general phenomenon in nuclei [2]. The systematics of $M1$ strength from the ground state to the scissors state and its deformation dependence have been extensively measured and corroborated with a variety of sum rules both in even-mass [3–7] and odd-mass nuclei [8]. Within the proton-neutron version of the interacting boson model (IBM-2) [9–11], a sum rule [3] has related this strength to the number of quadrupole d bosons in the ground state wave function. For deformed nuclei the latter can be expressed in terms of the quadrupole deformation determined from $B(E2)$ values, and the measured $M1$ strength was shown to

be in good agreement with this sum rule [12] as well as with its counterpart [8] in odd-mass nuclei [13].

Initially, most of the experimental and theoretical effort has focused on properties of the $J^\pi=1^+$ scissors state in even-even nuclei. However, the latter is only one particular example of mixed-symmetry states predicted by the IBM-2 [14–17], which possess a lower symmetry with respect to the interchange of proton and neutron bosons (F spin) [9]. Observations of other mixed-symmetry states with total angular momentum $J \neq 1$ were subsequently claimed in the literature, e.g., Refs. [18–21], but in some of these cases lifetime information was lacking, leaving the mixed-symmetry assignments without the knowledge of absolute transition strengths less persuasive. Later on, a few mixed-symmetry $J^\pi=2^+$ states in near vibrational nuclei could be identified on the basis of measured absolute $E2$ and $M1$ transition matrix elements [22–28]. Very recently the investigation of mixed-symmetry states was pushed a step forward by extensive data [29–32] on mixed-symmetry states in the nucleus ^{94}Mo , which were obtained by the application of a variety of classical but state-of-the-art γ -spectroscopic measurements. The measurement of the large transition rate between the mixed-symmetry (ms) $1_{1,ms}^+$ and $2_{1,ms}^+$ states [29] represents the first direct evidence for the similar character of their wave functions. The discovery of the $3_{1,ms}^+$ mixed-symmetry state [30] and of the $2_{2,ms}^+$ state [31] on the basis of electromagnetic transition matrix elements added confidence to the general existence of mixed-symmetry states (even off-yrast) and allowed for the first time to judge the energy splitting of the mixed-symmetry two-phonon quintuplet [33]. This new data provides knowledge about $M1$ transition strengths from a set of mixed-symmetry states

$$\mathcal{M} = \{1_{1,ms}^+, 2_{1,ms}^+, 2_{2,ms}^+, 3_{1,ms}^+\} \quad (1)$$

to the symmetric $J=0_1^+$ ground state *and* to the symmetric $J=2_1^+, 2_2^+$ excited states in ^{94}Mo . This data can now be ex-

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ploited in a new way, namely, the total $M1$ strengths between mixed-symmetry states and different low-lying symmetric states of the same nucleus can be compared.

The empirical identification of the states in Eq. (1) relied on specific signatures as predicted by the IBM-2. These IBM-2 predictions and assignments for mixed-symmetry states were found to be in an impressive agreement with the data [29–32] and are further supported by microscopic calculations [34,35]. This motivates us to use the IBM-2 to extract structure information out of this extensive set of measured $M1$ decay strengths in ^{94}Mo . In the present paper we investigate how far a sum rule approach in the IBM-2 may be used for that purpose, relying on the mixed-symmetry interpretation for the states in Eq. (1). In Sec. II we present a sum rule for the $M1$ excitation strength from an arbitrary symmetric state which generalizes an earlier expression for the total $M1$ ground state excitation strength within the IBM-2. Section III discusses the applicability of the sum rule in the U(5) and SO(6) dynamic symmetry (DS) limits of the model as well as in transitional cases preserving the SO(5) symmetry. In Sec. IV we apply the sum rule to the new data on ^{94}Mo and extract the relative quadrupole d -boson content of the $J=0_1^+$ and $J=2_2^+$ states. The sum rule analysis is critically examined in Sec. V and the paper is summarized in Sec. VI.

II. GENERALIZED $M1$ SUM RULE

The standard one-body magnetic dipole ($M1$) operator in the IBM-2 has the form [11]

$$\begin{aligned} \hat{T}(M1) &= \sqrt{\frac{3}{4\pi}} (g_\pi \hat{J}_\pi + g_\nu \hat{J}_\nu) \\ &= \sqrt{\frac{3}{4\pi}} \left[\frac{(g_\pi + g_\nu)}{2} \hat{J} + \frac{(g_\pi - g_\nu)}{2} (\hat{J}_\pi - \hat{J}_\nu) \right], \end{aligned} \quad (2)$$

where \hat{J}_ρ are the individual boson angular momentum operators for protons ($\rho=\pi$) or neutrons ($\rho=\nu$), g_ρ the respective boson g factors, and $\hat{J}=\hat{J}_\pi+\hat{J}_\nu$ the total angular momentum operator. We are interested in a sum rule for the $M1$ strength from an excited state in an even-even nucleus with angular momentum J and maximal F spin, $F_{\max}=N/2=(N_\pi+N_\nu)/2$. The integers N_ρ denote the total number of proton or neutron bosons of monopole (s) or quadrupole (d) type. These IBM-2 bosons represent correlated monopole and quadrupole pairs of identical valence nucleons in the shell model [9,10]. The derivation of the sum rule follows the same steps as for the ground state (which has $J=0^+$) given in Ref. [3], except that the terms proportional to the total angular momentum are not dropped. The derivation has been sketched already in Ref. [8] in which a sum rule for $M1$ ground state excitation strength of odd-mass nuclei is derived. The sum rule we are interested in here corresponds to the part due to the core in Eq. (6) of Ref. [8]. It is given by

$$S_J = \sum_{f \neq i} B(M1; i, J \rightarrow f, J_f) = 6C \left[\langle J | \hat{n}_d | J \rangle - \frac{J(J+1)}{6N} \right], \quad (3)$$

where

$$C = \frac{3}{4\pi} (g_\pi - g_\nu)^2 \frac{N_\pi N_\nu}{N(N-1)}. \quad (4)$$

Here $J(J_f)$ is the angular momentum of the initial (final) state and the labels $i(f)$ indicate all quantum numbers that may be needed to specify uniquely the states. In general $J \neq 0$ and, therefore, there can be a magnetic dipole transition to the initial state proportional to the magnetic moment. This elastic transition is not measured in the reported experiments on ^{94}Mo [29–32], hence, it is subtracted out in Eq. (3). Since the IBM-2 states are assumed to have pure F spin and the initial state J has $F=F_{\max}$, then only the $(\hat{J}_\pi - \hat{J}_\nu)$ term in Eq. (2) (which is an F -spin vector) can contribute to $M1$ transitions, and the sum in Eq. (3) involves all final states subject to F spin and angular momentum selection rules: $F_{\max} \rightarrow F_{\max}-1$ and $J_f = J-1, J, J+1$ ($M1$ transitions between states with $F=F_{\max}$ are forbidden due to the symmetry of their wave functions [36]). The total $M1$ strength S_J in Eq. (3) depends on the boson numbers, N_ρ , the boson effective g factors, g_ρ , and involves the expectation value of the d -boson number operator, $\hat{n}_d = \hat{n}_{d_\pi} + \hat{n}_{d_\nu}$ in the initial state J . The dependence on N_ρ reflects the local shell structure and their values are fixed to be half the number of valence particles or holes with respect to the nearest closed shell. The boson g factors defining the $M1$ operator of Eq. (2) are model parameters which are needed in order to extract from the sum rule information on $n_d(J) = \langle J | \hat{n}_d | J \rangle$, the average number of d bosons in the IBM-2 wave function. For the $J=0_1^+$ ground state, the sum rule in Eq. (3) reduces to that of Ref. [3]. This special case was used earlier [12] to extract the d -boson content of the ground state from the measured $M1$ excitation strengths. In that analysis the parameters g_ρ were assumed to have the values of bare orbital g factors, namely, $g_\pi = 1\mu_N$ and $g_\nu = 0$. The recent extensive measurements of $M1$ strengths in ^{94}Mo [29–32] provide a way to avoid the assumption of effective-boson g factors by considering ratios of $M1$ excitation strengths from different symmetric states: $R_{J_0}(J) = S_J/S_{J_0}$. Such ratios are independent of g_ρ and are pure functions of the average numbers of d bosons in the states J_0 and J . For example, if $J_0=2^+$ and $J=0_1^+$ it follows from Eq. (3) that

$$\begin{aligned} R_{2^+}(0_1^+) &= \frac{S_{0_1^+}}{S_{2^+}} = \frac{n_d(0_1^+)}{n_d(2^+) - 1/N}, \\ &\approx \frac{n_d(0_1^+)}{n_d(2^+)} \quad [n_d(2^+) \gg 1/N]. \end{aligned} \quad (5)$$

Thus, for N sufficiently large, one can directly extract from the measured $M1$ strengths the relative d -boson contents of the corresponding states in a largely parameter-independent way. The relative d -boson content contains important information on the structure of wave functions and is sensitive to the Hamiltonian parameters, i.e., to the residual interactions.

Before a sum rule approach can be applied to the observed $M1$ strength from low-lying symmetric states, it is crucial to assess to what extent the mixed-symmetry states identified in ^{94}Mo can be expected to exhaust the sum rule. For that purpose we need to examine the following partial strengths Σ_J ,

$$\Sigma_J = \sum_{f \in \mathcal{M}} B(M1; i, J, F_{\max} \rightarrow f, J_f, F_{\max} - 1), \quad (6)$$

to the set of mixed-symmetry states \mathcal{M} of Eq. (1), and compare to the full strengths S_J in Eq. (3). The analysis will be done first for the dynamic symmetries of the IBM-2 and for transitional cases which are of relevance to ^{94}Mo . In the next section we consider specific types of Hamiltonians in order to study the relative contributions to the sum rule of different $M1$ branches from some low-lying states.

III. $M1$ SUM RULES FOR F -SPIN INVARIANT HAMILTONIANS

In order to clarify the discussion we analyze in this section the contribution to the $M1$ sum rule from the two

lowest-lying 2^+ states. Since we aim at the application of the sum rule to the γ -soft nucleus ^{94}Mo , we pay particular attention to F -scalar Hamiltonians with $\text{SO}(5)$ symmetry. We consider first the $\text{U}(5)$ and $\text{SO}(6)$ DS limits of the IBM-2 which contain the $\text{SO}(5)$ subgroup, and derive analytic expressions for the relevant $M1$ excitation strengths (total strengths S_J and partial strengths Σ_J) on top of the excited 2_1^+ and 2_2^+ states. Next we address the evolution of $B(M1; 2_{1,2}^+ \rightarrow J_f)$ values in a $\text{SO}(5)$ -preserving transition path between the $\text{U}(5)$ and $\text{SO}(6)$ DS limits. The results from this section will serve as a guideline for judging whether the currently available experimental data in ^{94}Mo contains sufficient information to qualify for a sum rule analysis.

A. $\text{U}(5)$ and $\text{SO}(6)$ DS limits

In the $\text{U}(5)$ and $\text{SO}(6)$ DS limits, the eigenstates of the Hamiltonian have quantum numbers which are the labels of irreducible representations (irreps) of the groups in the chains [11],

$$\begin{array}{ccccccc} \text{U}_{\pi}(6) & \otimes & \text{U}_{\nu}(6) & \supset & \text{U}_{\pi\nu}(6) & \supset & \text{U}_{\pi\nu}(5) & \supset & \text{SO}_{\pi\nu}(5) & \supset & \text{SO}_{\pi\nu}(3) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ [N_{\pi}] & & [N_{\nu}] & & [N_1, N_2] & & (n_1, n_2) & & (\tau_1, \tau_2) & & \{\alpha_i\} & J \end{array} \quad (7)$$

and

$$\begin{array}{ccccccc} \text{U}_{\pi}(6) & \otimes & \text{U}_{\nu}(6) & \supset & \text{U}_{\pi\nu}(6) & \supset & \text{SO}_{\pi\nu}(6) & \supset & \text{SO}_{\pi\nu}(5) & \supset & \text{SO}_{\pi\nu}(3) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ [N_{\pi}] & & [N_{\nu}] & & [N_1, N_2] & & \langle \sigma_1, \sigma_2 \rangle & & (\tau_1, \tau_2) & & \{\alpha_i\} & J \end{array} \quad (8)$$

respectively. Here $N_1 + N_2 = N$, and $F = (N_1 - N_2)/2 = N/2 - k$ ($k = 0, 1, \dots$), while α_i ($i = 1, 2$) are missing labels, necessary to completely classify the $\text{SO}(5) \supset \text{SO}(3)$ reduction. Within these DS limits the average number of d bosons in any F -spin symmetric state is given by

$$\text{U}(5): \quad n_d[F = F_{\max}, (n_d, 0), (\tau, 0), J] = n_d,$$

$$\begin{aligned} \text{SO}(6): \quad n_d[F = F_{\max}, \langle \sigma = N, 0 \rangle, (\tau, 0), J] \\ = \frac{N(N-1)}{2(N+1)} + \frac{\tau(\tau+3)}{2(N+1)}. \end{aligned} \quad (9)$$

For the lowest symmetric states with $J = 0_1^+$, 2_1^+ , and 2_2^+ and $\tau = 0, 1$, and 2 , respectively, which are of particular interest to the present discussion, we have

$$\begin{array}{ccc} & \text{U}(5) & \text{SO}(6) \\ n_d(0_1^+) & 0 & \frac{N(N-1)}{2(N+1)} \\ n_d(2_1^+) & 1 & \frac{N(N-1)}{2(N+1)} + \frac{2}{N+1} \\ n_d(2_2^+) & 2 & \frac{N(N-1)}{2(N+1)} + \frac{5}{N+1}. \end{array} \quad (10)$$

As can be seen, $n_d(2^+)$ is of order unity in the $\text{U}(5)$ limit and is of order N in the $\text{SO}(6)$ limit. Therefore, the condition $n_d(2^+) \gg 1/N$, mentioned in Eq. (5), is satisfied already for $N \geq 4$ within 16% for $J = 2_1^+$ and 11% for $J = 2_2^+$ near the $\text{SO}(6)$ limit. Substituting the values of $n_d(J)$ into Eq. (3) we obtain the following expressions for the total $M1$ strength, S_J , from these states

$$\begin{array}{lcl}
 & \text{U(5)} & \text{SO(6),} \\
 S_{0_1^+} & 0 & C \frac{3N(N-1)}{N+1} \\
 S_{2_1^+} & C \frac{6(N-1)}{N} & C \frac{3(N^2+2)(N-1)}{N(N+1)} \\
 S_{2_2^+} & C \frac{6(2N-1)}{N} & C \frac{3(N^3-N^2+8N-2)}{N(N+1)}
 \end{array} \tag{11}$$

where C is given in Eq. (4). The states which contribute to these $M1$ strengths have the following classification in the U(5) or SO(6) limits:

	U(6) F	U(5) (n_1, n_2)	SO(6) $\langle \sigma_1, \sigma_2 \rangle$	SO(5) (τ_1, τ_2)
0_1^+	F_{\max}	(0,0)	$\langle N, 0 \rangle$	(0,0)
2_1^+	F_{\max}	(1,0)	$\langle N, 0 \rangle$	(1,0)
2_2^+	F_{\max}	(2,0)	$\langle N, 0 \rangle$	(2,0)
$2_{1,ms}^+$	$F_{\max} - 1$	(1,0)	$\langle N-1, 1 \rangle$	(1,0)
$1_{1,ms}^+, 3_{1,ms}^+$	$F_{\max} - 1$	(1,1)	$\langle N-1, 1 \rangle$	(1,1)
$2_{2,ms}^+$	$F_{\max} - 1$	(2,0)	$\langle N-1, 1 \rangle$	(2,0)
$1_{2,ms}^+, 2_{3,ms}^+, 3_{2,ms}^+$	$F_{\max} - 1$	(2,1)	$\langle N-1, 1 \rangle$	(2,1)
$1_{3,ms}^+, 2_{4,ms}^+, 3_{3,ms}^+, 3_{4,ms}^+$	$F_{\max} - 1$	(3,1)	$\langle N-1, 1 \rangle$	(3,1).

The $M1$ operator of Eq. (2) transforms as a $T^{[2,1^4]\{2,1^3\}(1,1)1}$ tensor under the U(5) chain, Eq. (7), and as a $T^{[2,1^4](1,1)(1,1)1}$ tensor under the SO(6) chain, Eq. (8). Using standard techniques for coupling irreps [37] it is possible to show that Eq. (12) lists *all* mixed-symmetry states which are relevant for $M1$ transitions from the chosen initial states, $J=0_1^+, 2_1^+$, and 2_2^+ . The conservation of d parity [26,38] further restricts the allowed $M1$ transitions. For SO(5) symmetry each state can be characterized by a definite value of d parity, $\pi_d = (-1)^{\tau_1 + \tau_2}$, and the $M1$ operator has $\pi_d = +1$. Altogether the only allowed $M1$ transitions $J(\tau, 0) \rightarrow J_f(\tau_1, \tau_2)$ in the U(5) or SO(6) DS limits are

$$\begin{array}{l}
 \text{U(5),} \\
 2_1^+(1,0) \rightarrow 2_{1,ms}^+(1,0), \\
 2_2^+(2,0) \rightarrow 2_{2,ms}^+(2,0); \quad 1_{1,ms}^+, 3_{1,ms}^+(1,1), \\
 \text{SO(6),} \\
 0_1^+(0,0) \rightarrow 1_{1,ms}^+(1,1), \\
 2_1^+(1,0) \rightarrow 2_{1,ms}^+(1,0); \quad 1_{2,ms}^+, 2_{3,ms}^+, 3_{2,ms}^+(2,1), \\
 2_2^+(2,0) \rightarrow 2_{2,ms}^+(2,0); \quad 1_{1,ms}^+, 3_{1,ms}^+(1,1); \\
 1_{3,ms}^+, 2_{4,ms}^+, 3_{3,ms}^+, 3_{4,ms}^+(3,1).
 \end{array} \tag{13b}$$

In the U(5) DS limit there are fewer allowed $M1$ transitions due to an additional selection rule, namely, that $M1$ transitions can only connect U(5) states (n_1, n_2) such that $n_d = n_1 + n_2$ is preserved. Analytic expressions of $B(M1)$ values for $M1$ transitions in the U(5) and SO(6) DS limits, which are relevant for the present discussion, are collected in Table I.

In general, we see from Eq. (13) that the total strengths, S_J , of Eq. (11) are sums of contributions involving different SO(5) multiplets. Specifically, if we denote by $S_J^{(\tau_1, \tau_2)}$ the summed $M1$ strength from the initial state J to all final states with $J_f = J, J \pm 1$ in a given SO(5) irrep (τ_1, τ_2) , we then find

$$S_{0_1^+} = S_{0_1^+}^{(1,1)}, \tag{14a}$$

$$S_{2_1^+} = S_{2_1^+}^{(1,0)} + S_{2_1^+}^{(2,1)}, \tag{14b}$$

$$S_{2_2^+} = S_{2_2^+}^{(1,1)} + S_{2_2^+}^{(2,0)} + S_{2_2^+}^{(3,1)}. \tag{14c}$$

From Table I we deduce the following expressions for the separate SO(5) contributions to these strengths:

TABLE I. Some relevant analytic expressions for $B(M1)$ values in the U(5) and SO(6) DS limits for $M1$ transitions from symmetric states ($F=F_{\max}$) to mixed-symmetry states ($F=F_{\max}-1$) and SO(5) quantum numbers (τ_1, τ_2) as indicated. The factor C is given in Eq. (4).

Transition	U(5) ^a	SO(6)
$0_1^+(0,0) \rightarrow 1_{1,ms}^+(1,1)$	0	$C \frac{3N(N-1)}{N+1}$ ^b
$2_1^+(1,0) \rightarrow 2_{1,ms}^+(1,0)$	$C \frac{6(N-1)}{N}$	$C \frac{3(N+4)(N+2)(N-1)}{4N(N+1)}$ ^b
$2_1^+(1,0) \rightarrow 1_{2,ms}^+(2,1)$	0	$C \frac{3(N-1)(N-2)}{10(N+1)}$
$2_1^+(1,0) \rightarrow 2_{3,ms}^+(2,1)$	0	$C \frac{3(N-1)(N-2)}{4(N+1)}$
$2_1^+(1,0) \rightarrow 3_{2,ms}^+(2,1)$	0	$C \frac{6(N-1)(N-2)}{5(N+1)}$
$2_2^+(2,0) \rightarrow 1_{1,ms}^+(1,1)$	$C \frac{21}{5}$	$C \frac{3(N+5)(N+4)}{10(N+1)}$ ^b
$2_2^+(2,0) \rightarrow 3_{1,ms}^+(1,1)$	$C \frac{24}{5}$	$C \frac{12(N+5)(N+4)}{35(N+1)}$
$2_2^+(2,0) \rightarrow 2_{2,ms}^+(2,0)$	$C \frac{3(N-2)}{N}$	$C \frac{3(N+5)(N+2)(N-2)}{10N(N+1)}$

^aFrom Ref. [39].

^bFrom Ref. [36].

$$S_{0_1^+}^{(1,1)} \quad \begin{array}{cc} \text{U(5)} & \text{SO(6),} \\ 0 & C \frac{3N(N-1)}{N+1} \end{array} \quad \Sigma_{0_1^+} = S_{0_1^+}^{(1,1)}, \quad (16a)$$

$$S_{2_1^+}^{(1,0)} \quad \begin{array}{cc} C \frac{6(N-1)}{N} & C \frac{3(N-1)(N+2)(N+4)}{4N(N+1)} \end{array} \quad \Sigma_{2_1^+} = S_{2_1^+}^{(1,0)}, \quad (16b)$$

$$S_{2_2^+}^{(2,1)} \quad \begin{array}{cc} 0 & C \frac{9(N-2)(N-1)}{4(N+1)} \end{array} \quad \Sigma_{2_2^+} = S_{2_2^+}^{(1,1)} + S_{2_2^+}^{(2,0)}. \quad (16c)$$

$$S_{2_1^+}^{(2,1)} \quad \begin{array}{cc} 0 & C \frac{9(N-2)(N-1)}{4(N+1)} \end{array}$$

$$S_{2_2^+}^{(1,1)} \quad \begin{array}{cc} 9C & C \frac{9(N+4)(N+5)}{14(N+1)} \end{array}$$

$$S_{2_2^+}^{(2,0)} \quad \begin{array}{cc} C \frac{3(N-2)}{N} & C \frac{3(N+5)(N+2)(N-2)}{10N(N+1)} \end{array}$$

$$S_{2_2^+}^{(3,1)} \quad \begin{array}{cc} 0 & C \frac{72(N-2)(N-3)}{35(N+1)} \end{array} \quad (15)$$

where C is given in Eq. (4). As mentioned in Sec. I, so far only the mixed-symmetry states shown in Eq. (1) have been identified in ^{94}Mo . Using their SO(5) classification given in Eq. (12), we see that empirical information is available only for $M1$ strengths to SO(5) multiplets with $(\tau_1, \tau_2) = (1,1)$, $(1,0)$, and $(2,0)$. No comparable firm information exists at present on the multiplets with SO(5) quantum numbers $(2,1)$ and $(3,1)$. Considering the $M1$ strengths that have been measured, the partial (yet observed) strengths, Σ_J , of Eq. (6) can be transcribed as

Comparing with the corresponding expressions for total strengths in Eq. (14), we see that the $M1$ strength from the $J=0_1^+$ ground state arises entirely from the transition $0_1^+(0,0) \rightarrow 1_{1,ms}^+(1,1)$, i.e., $\Sigma_{0_1^+} = S_{0_1^+}$ in Eq. (16a). However, the $M1$ transitions from the symmetric excited states, 2_1^+ and 2_2^+ , to the mixed-symmetry states of Eq. (1) exhaust only part of the total strengths. Using Eq. (15) these partial strengths in the U(5) and SO(6) DS limits are found to be

$$\begin{array}{cc} \text{U(5)} & \text{SO(6),} \\ \Sigma_{2_1^+} & C \frac{6(N-1)}{N} \quad C \frac{3(N-1)(N+2)(N+4)}{4N(N+1)} \\ \Sigma_{2_2^+} & C \frac{6(2N-1)}{N} \quad C \frac{3(N+5)(11N^2+30N-14)}{35N(N+1)} \end{array} \quad (17)$$

where C is given in Eq. (4). In order to be able to apply the sum rule of Eq. (3) to the existing data, we need to assess the goodness of the approximation in replacing the total $M1$

strengths S_J by the partial strengths Σ_J . From Eqs. (11) and (17) we get the following expressions for the ratios $Y(J) = \Sigma_J/S_J$:

$$\begin{aligned}
 Y(2_1^+) = \Sigma_{2_1^+}/S_{2_1^+} & \quad \text{U(5)} & \quad \text{SO(6),} \\
 & 1 & \quad \frac{(N+2)(N+4)}{4(N^2+2)} \\
 Y(2_2^+) = \Sigma_{2_2^+}/S_{2_2^+} & \quad 1 & \quad \frac{(N+5)(11N^2+30N-14)}{35(N^3-N^2+8N-2)}.
 \end{aligned} \tag{18}$$

We see that in the U(5) limit $Y(J)=1$ for $J=2_1^+, 2_2^+$ and hence the set \mathcal{M} of mixed-symmetry states in Eq. (1) exhausts the total $M1$ strength. This is not the case in the SO(6) limit in which $Y(J)<1$. In this case, the fraction $Y(J)$ of exhausted strength depends on N and is seen to be a monotonic decreasing function of the boson number. For example, from Eq. (18) we have $Y(2_1^+)=0.58, 0.41,$ and 0.35 and $Y(2_2^+)=0.85, 0.61,$ and 0.51 for $N=5, 10,$ and $15,$ respectively. For large N the asymptotic values are $Y(2_1^+)\rightarrow 0.25$ and $Y(2_2^+)\rightarrow 0.31$. We conclude that for nuclei near the SO(6) DS limit, the approximation involved in the substitution $\Sigma_J \leftrightarrow S_J$ is better for small N and becomes less justified for large values of N . To exhaust at least 75% of the total strength requires $N \leq 3$ for $J=2_1^+$ and $N \leq 6$ for $J=2_2^+$. Furthermore, for a given $N, \Sigma_{2_2^+}$ is seen to provide a better approximation to the total strength $S_{2_2^+}$, than $\Sigma_{2_1^+}$ to $S_{2_1^+}$. This suggests that near the SO(6) limit, a sum rule approach, based on the currently available data, is likely to be reliable for moderate values of N when applied to the $J=0_1^+$ and $J=2_2^+$ states but not for the $J=2_1^+$ state.

B. U(5) to SO(6) transition

The majority of transitional γ -soft nuclei lie between the U(5) and SO(6) limits and retain good SO(5) symmetry. The main features of the evolution in structure accompanying the transition between these two DS limits can be studied by considering the following schematic F -scalar Hamiltonian:

$$\hat{H} = a \left[(1-\zeta)\hat{n}_d - \frac{\zeta}{4N}(\hat{Q}_\pi + \hat{Q}_\nu) \cdot (\hat{Q}_\pi + \hat{Q}_\nu) + \lambda \hat{M} \right]. \tag{19}$$

Here $\hat{Q}_\rho = [d_\rho^\dagger \times s_\rho + s_\rho^\dagger \times \tilde{d}_\rho]^{(2)}$ ($\rho = \pi, \nu$) is the quadrupole operator relevant for this transition region and \hat{M} is the Majorana operator in Casimir form [11]. The Majorana term is diagonal and determines the energy shift (proportional to λ) between eigenstates in accord with their F spin quantum numbers. Neither the parameter λ nor the parameter a in Eq. (19), which sets the overall energy scale, affects the structure of wave functions. The latter are completely determined by the parameter ζ of \hat{H} . For $\zeta=0$ the Hamiltonian possesses the U(5) DS, while for $\zeta=1$ it attains the SO(6) DS. By varying ζ from 0 to 1 we can study in a simple way the transition between the two limits. The calculations presented

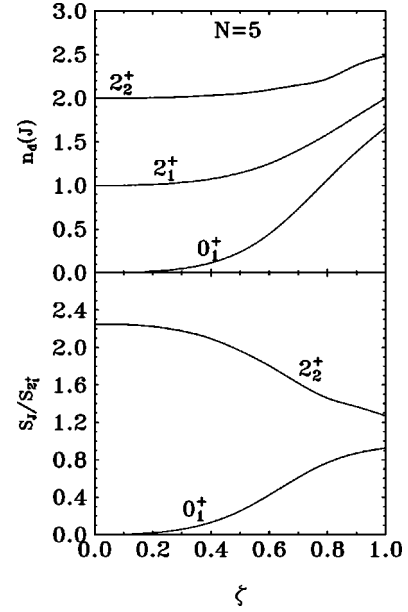


FIG. 1. d -boson content, $n_d(J)$, of the $J=0_1^+, 2_1^+$, and 2_2^+ states (top) and the corresponding ratios of total $M1$ excitation strengths $S_{0_1^+}/S_{2_1^+}$ and $S_{2_2^+}/S_{2_1^+}$ (bottom) as a function of ζ . Calculations are done with the Hamiltonian of Eq. (19) with boson numbers $N_\pi=4$ and $N_\nu=1$ ($N=5$).

below are done with $N_\pi=4$ and $N_\nu=1$ ($N=5$) which are the appropriate boson numbers for ^{94}Mo .

The top part of Fig. 1 shows the d -boson content, $n_d(J)$, of the symmetric $J=0_1^+, 2_1^+$, and 2_2^+ states with $F=F_{\text{max}}$ as a function of ζ . The curves shown interpolate between the U(5) and SO(6) values of Eq. (10) for $N=5$. The lower part of Fig. 1 shows the corresponding ratios of strengths, $R_{2_1^+}(J) = S_J/S_{2_1^+}$, evaluated as in Eq. (3). For given boson numbers these ratios depend only on the structural parameters of the Hamiltonian (in this case only on ζ) and not on parameters of the $M1$ operator in Eq. (2). The sensitivity of such ratios to the transition path between the U(5) and the SO(6) DS limits can be used to determine the location of a given γ -soft nucleus along the transition leg between these two DS limits.

The Hamiltonian of Eq. (19) is F scalar and although it does not have dynamic symmetry for an arbitrary value of ζ , it still always has SO(5) symmetry. Away from the U(5) and SO(6) DS limits, the eigenstates are no longer pure with respect to U(5) and SO(6). However, they do retain good SO(5) quantum numbers and, consequently, the pattern of allowed $M1$ transitions shown in Eq. (13b) persists also in the transition region. In particular, the SO(5) and d -parity selection rules for $M1$ transitions are still in effect and the total strengths, S_J , maintain the same SO(5) decomposition as in Eq. (14). Figure 2 displays the ratios of partial to total strengths, $Y(J) = \Sigma_J/S_J$, as a function of ζ for $J=0_1^+, 2_1^+$, and 2_2^+ . As shown, the partial strengths Σ_J , to the set \mathcal{M} of mixed-symmetry states of Eq. (1) exhaust the sum rules $S_{0_1^+}$ completely and $S_{2_2^+}$ to a large extent (more than 85%) throughout the transition region. Less than 15% of the $M1$ strength from the $J=2_2^+$ state goes into the SO(5) irrep (3,1) which is not included in the partial strength $\Sigma_{2_2^+}$ of Eq.

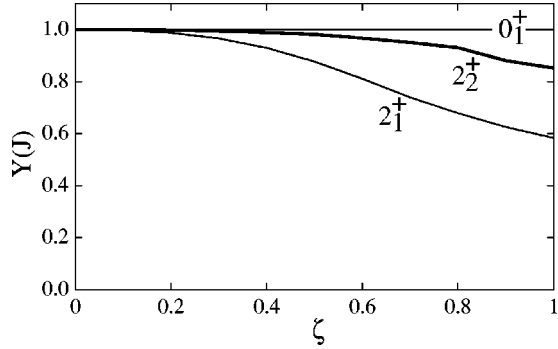


FIG. 2. Calculated ratios $Y = \Sigma_J / S_J$ of partial to total $M1$ strengths as a function of ζ for $J = 0_1^+$, 2_1^+ , and 2_2^+ and $N = 5$. The partial strengths Σ_J to the mixed-symmetry states of Eq. (1) exhaust the sum rules of Eq. (3), $S_{0_1^+}$ completely, and $S_{2_2^+}$ to more than 85% in the whole U(5)-to-SO(6) transition path.

(16c). On the other hand, in most of the transition region, a considerable fraction of $M1$ strength from the $J = 2_1^+$ state is not concentrated in the above set of mixed-symmetry states. About 40% of the total strength $S_{2_1^+}$ goes into the SO(5) irrep (2,1) which is left out of the partial strength $\Sigma_{2_1^+}$ in Eq. (16b). We conclude that for $N = 5$ throughout the transition region between the U(5) and SO(6) DS limits, the partial strengths Σ_J of Eq. (6) provide an adequate approximation to the total strengths S_J of Eq. (3) for the $J = 0_1^+$ and $J = 2_2^+$ states but not for the $J = 2_1^+$ state. This identifies the initial states J in ^{94}Mo which qualify for a sum rule analysis based on the measured $M1$ strengths to the mixed symmetry states of Eq. (1).

IV. APPLICATION TO ^{94}MO

The primary goal of the present investigation is to extract structure information, via a sum rule approach, out of the recent extensive data on mixed-symmetry states in ^{94}Mo . Table II displays a compilation of the available data on $M1$ transitions from the $J = 0_1^+$, 2_1^+ , and 2_2^+ states in ^{94}Mo [29–32]. This data has been used to identify the set \mathcal{M} of mixed-symmetry states listed in Eq. (1). The experimental summed $M1$ strengths, $S(J)_{\text{expt}}$, given in Table II correspond to the calculated partial strengths Σ_J of Eq. (6) to these mixed-symmetry states. In accord with the discussion of the previous section (see in particular Fig. 2), for a γ -soft nucleus such as ^{94}Mo with $N = 5$, these partial strengths exhaust to a large extent the $M1$ sum rule for $J = 0_1^+$ and $J = 2_2^+$. For these states, it is therefore justified to compare the measured ratio

$$R_{2_2^+}(0_1^+)_{\text{expt}} = \frac{S(0_1^+)_{\text{expt}}}{S(2_2^+)_{\text{expt}}} = 0.58_{-14}^{+11} \quad (20)$$

with the calculated ratio $S_{0_1^+} / S_{2_2^+}$ of total strengths S_J obtained from the sum rule in Eq. (3). Since between the U(5) and SO(6) DS limits the value of $n_d(2_2^+)$ varies in the range 2–2.5 for $N = 5$ [see Fig. 1 and Eq. (10)], we can neglect

TABLE II. Measured $M1$ transition strengths in ^{94}Mo in units of μ_N^2 (Refs. [29–32]). The notation “n.o.” denotes cases where the corresponding transitions were too weak to be observed, although other decay branches of the issuing level were detected. The states 1_1^+ , 3_2^+ , 2_3^+ , and 2_6^+ are the main fragments of the $1_{1,\text{ms}}^+$, $3_{1,\text{ms}}^+$, $2_{1,\text{ms}}^+$, and $2_{2,\text{ms}}^+$ mixed-symmetry states, respectively. $S(J)_{\text{expt}}$ is the experimental summed $M1$ strength from the initial state J .

Observable	Experiment
$B(M1; 0_1^+ \rightarrow 1_1^+)$	0.47(3) ^a
$B(M1; 0_1^+ \rightarrow 1_2^+)$	0.14(5) ^a
$S(0_1^+)_{\text{expt}}$	0.61(7) ^a
$B(M1; 2_1^+ \rightarrow 1_1^+)$	0.004 ^{+4a} ₋₁
$B(M1; 2_1^+ \rightarrow 1_2^+)$	0.007(4) ^{a,d}
$B(M1; 2_1^+ \rightarrow 2_2^+)$	0.06(2) ^a
$B(M1; 2_1^+ \rightarrow 2_3^+)$	0.48(6) ^a
$B(M1; 2_1^+ \rightarrow 2_4^+)$	0.07(2) ^a
$B(M1; 2_1^+ \rightarrow 2_5^+)$	0.03(1) ^a
$B(M1; 2_1^+ \rightarrow 2_6^+)$	<0.0077 ^b
$B(M1; 2_1^+ \rightarrow 3_2^+)$	0.014 ^{+17c} ₋₈
$S(2_1^+)_{\text{expt}}$	0.67(7)
$B(M1; 2_2^+ \rightarrow 1_1^+)$	0.26(3) ^a
$B(M1; 2_2^+ \rightarrow 1_2^+)$	n.o. ^d
$B(M1; 2_2^+ \rightarrow 2_3^+)$	n.o. ^d
$B(M1; 2_2^+ \rightarrow 2_4^+)$	<0.02 ^b
$B(M1; 2_2^+ \rightarrow 2_5^+)$	0.095(6) ^b
$B(M1; 2_2^+ \rightarrow 2_6^+)$	0.35(11) ^b
$B(M1; 2_2^+ \rightarrow 2_7^+)$	0.009 ^{+7b} ₋₃
$B(M1; 2_2^+ \rightarrow 2_8^+)$	n.o. ^d
$B(M1; 2_2^+ \rightarrow 3_2^+)$	0.34 ^{+20c} ₋₁₀
$S(2_2^+)_{\text{expt}}$	1.05 ⁺²³ ₋₁₅

^aFrom Ref. [29].

^bFrom Ref. [31].

^cFrom Ref. [30].

^dFrom Ref. [32].

$1/N = 0.2$ with respect to $n_d(2_2^+)$ and, as in Eq. (5), extract from the data a relative d -boson content ratio

$$\left[\frac{n_d(0_1^+)}{n_d(2_2^+)} \right]_{^{94}\text{Mo}} \approx 0.58_{-14}^{+11}. \quad (21)$$

We find that the $J = 0^+$ ground state of ^{94}Mo contains more than half as many d bosons as the $J = 2_2^+$ state. This number is considerably higher than that for a spherical vibrator [$n_d(0_1^+) / n_d(2_2^+) = 0$ in the U(5) DS limit] and is in fact closer to the value expected for a γ -unstable rotor [$n_d(0_1^+) / n_d(2_2^+) = 2/3$ in the SO(6) DS limit with $N = 5$]. These findings are consistent with previous observations that the $M1$ and $E2$ strengths involving mixed-symmetry states in ^{94}Mo compare favorably with the SO(6) predictions [29–

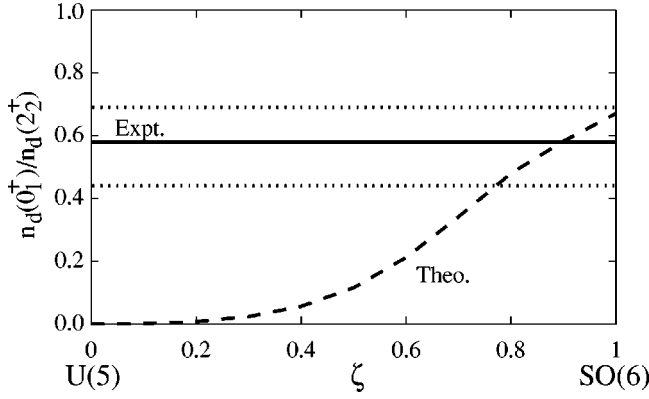


FIG. 3. The calculated d -boson ratio of the $J=0_1^+$ and $J=2_2^+$ states (dashed curve) as a function of ζ compared with the empirical value (solid line with experimental uncertainties indicated by the dotted lines) of Eq. (21) extracted from the measured $M1$ strengths in ^{94}Mo .

31. However, that comparison relied on an assumption for the parameters of the $M1$ operator (boson effective g factors) and $E2$ operator (boson effective quadrupole charges). In the present approach such an assumption is avoided by using ratios of $M1$ strengths. The d -boson ratio of Eq. (21) is extracted directly from the data and its value is independent of any model parameters.

The d -boson content is sensitive to the transition path between the $U(5)$ and $SO(6)$ limits, which are not easy to distinguish otherwise [40]. We can therefore use its empirical value to pin down the location of ^{94}Mo along the transition leg between these limits. For that purpose we show in Fig. 3 the calculated ratio $n_d(0_1^+)/n_d(2_2^+)$ as a function of ζ (dashed line) and the value $[n_d(0_1^+)/n_d(2_2^+)]_{^{94}\text{Mo}} \approx 0.6$ of Eq. (21) extracted from the data (solid line). The comparison between the calculated and empirical values strongly suggests a structural parameter $\zeta > 0.7$ for the IBM-2 description of ^{94}Mo and unambiguously identifies this nucleus to be closer to the $SO(6)$ γ -unstable rotor rather than the $U(5)$ spherical vibrator.

Besides the $M1$ properties, one may attempt to consider the known $E2$ rates in order to determine the appropriate parameter space of the IBM-2 Hamiltonian for ^{94}Mo . An observable which can distinguish between the $U(5)$ and the $SO(6)$ DS limits is the shape invariant K_4 [41,42] which can be well approximated [41] by the experimentally accessible $B(E2)$ ratio $K_4^{\text{appr}} = (7/10)B(E2; 4_1^+ \rightarrow 2_1^+)/B(E2; 2_1^+ \rightarrow 0_1^+)$. For large N , $K_4 = 1.4$ in the $U(5)$ limit and $K_4 = 1$ in the $SO(6)$ limit, with small deviations for finite N [42]. Unfortunately, for ^{94}Mo the measured value [43,44] is $K_4^{\text{appr}} = 1.16(17)$ and hence the large error bars prohibit any definite conclusion about the symmetry character of ^{94}Mo from $E2$ data. More precise lifetime experiments on low-lying symmetric states would be of interest for this issue.

V. CRITICAL EXAMINATION OF THE ANALYSIS

Some critical remarks on the implementation of the $M1$ sum rule are in order. While Eq. (3) is an exact relation in the

IBM-2, the justification for applying it to the new data on ^{94}Mo is less straightforward and relies on the following assumptions. (i) All strong $M1$ transitions between low-lying states of ^{94}Mo can be modeled by the IBM-2. (ii) F spin is a good symmetry for the states considered in ^{94}Mo . (iii) The structure of ^{94}Mo is consistent with a $U(5)$ -to- $SO(6)$ transition path.

The first assumption is necessary to justify the comparison of the experimental summed $M1$ strengths to the sums calculated in the IBM-2. Sizeable, hypothetical contributions to the experimental sums from states and degrees of freedom outside the IBM-2 space could potentially obscure the results. However, the fact that $M1$ transitions in Table II with strengths larger than $\approx 0.1\mu_N^2$ are understandable in the IBM-2 suggests that for ^{94}Mo the excluded degrees of freedom are not likely to have a significant impact on the empirical summed strengths in the considered energy region. Furthermore, eventually existing, additional strength can be accounted for, to a large extent, by renormalizing the parameters of the $M1$ operator in Eq. (2). In the present analysis we avoid any assumption on these effective-boson g factors by considering ratios of strengths.

The primary motivation for the use of the IBM-2 in the present sum rule analysis is the model's impressive success in predicting the experimental data in ^{94}Mo [29–32]. The IBM-2 interpretation of these low-energy structures as mixed-symmetry states implies that the $M1$ excitations involved are predominantly of orbital character, and suggests that in ^{94}Mo the spin contribution to the $M1$ strength at low energy is suppressed. This conjecture is supported by recent microscopic studies of mixed-symmetry states in this nucleus [34,35]. Results of a realistic calculation within the quasiparticle-phonon model (QPM) [35] indicate that quantitatively, the $M1$ strengths resulting from using the standard values for the spin quenching factor ($g_s = 0.6-0.7$) are larger than the experimental ones by at least a factor of 2. The best overall agreement with experiments is reached for $g_s = 0.3$. Even for $g_s = 0.6$ the spin contribution is consistently smaller than the orbital one and is about half the orbital strength in the transition $1_{1,\text{ms}}^+ \rightarrow 2_2^+$ for which the largest discrepancy between theory and experiment occurs. The low-lying spin transitions were found to be very sensitive to small components of the wave functions, yet the appropriate quenching mechanism in the QPM has not been identified so far. A shell model calculation for ^{94}Mo [34] shows the isovector $M1$ ground state excitation strength to be concentrated in the $1_{1,\text{ms}}^+$ state and to be composed of almost equal spin and orbital contributions. This considerable fraction of orbital $M1$ strength is to be regarded as a lower limit on the actual orbital contribution, given the small model space used in the calculation (^{88}Sr core, employing large effective $E2$ charges), neglect of important orbitals, e.g., $\pi(p_{3/2}, g_{7/2})$, and insistence on pure isovector $M1$ transitions. Clearly, large-scale shell model calculations are desirable to pin down the relative orbital and spin contributions to the $M1$ strength in ^{94}Mo . Empirically, properties of the $J=1_{1,\text{ms}}^+$ state observed in ^{94}Mo were found to be consistent with systematics of the scissors mode extrapolated from the deformed rare

earth nuclei [29]. For the latter, the predominantly orbital character was empirically established by a comparison of (γ, γ') , (e, e') , and (p, p') spectra [45]. It will be worthwhile in the future to verify experimentally to what extent such dominance of orbital character for low-lying $M1$ strengths persists also in transitional nuclei such as ^{94}Mo . This can be investigated by a comparison with inelastic hadronic scattering and by exploiting the fact that while the orbital contribution is enhanced by deformation, the spin part has anticorrelation with collectivity [46].

The second assumption of good F spin symmetry is the basis for the derivation of the sum rule in Eq. (3). Various procedures have been proposed to estimate the F -spin purity of low-lying states in nuclei [47]. These involve examining $M1$ transitions (which should vanish between pure $F = F_{\text{max}}$ states [36]), magnetic moments [48,49], the difference in proton-neutron deformations [50], and properties of F spin multiplets [51,52]. In the majority of analyses the F -spin admixtures in low-lying states are found to be a few percent ($< 10\%$), typically 2%–4% [47]. Although the empirical $M1$ strengths shown in Table II are fragmented, the pattern of dominant transitions to the mixed-symmetry states in ^{94}Mo as well as their energy systematics agree favorably with the assignment of F -spin quantum numbers. The smallness of the observed $M1$ rate, $B(M1; 2_2^+ \rightarrow 2_1^+) = 0.06(2)\mu_N$, which is F spin forbidden, is a benchmark for the anticipated F -spin mixing in low-lying states of ^{94}Mo .

The last assumption is adequate for γ -soft nuclei and ensures that the $\text{SO}(5)$ symmetry is preserved. This additional symmetry played a significant role in the current analysis by imposing further constraints on the allowed $M1$ transitions, which in turn enabled the observed four mixed-symmetry states of Eq. (1) to exhaust an appreciable fraction of the sum rules, S_J , for $J=0_1^+$, 2_2^+ .

VI. SUMMARY

The recent extensive data in ^{94}Mo [29–32] on the four mixed-symmetry states listed in Eq. (1) has paved the way for comparing $M1$ strengths between mixed-symmetry states and different low-lying symmetric states in the same nucleus. Sum rules are the proper tool to study the systematics of total strengths in the presence of fragmentation. The success of the IBM-2 in reproducing the data has motivated us to consider a generalized $M1$ sum rule from any symmetric state in the IBM-2 framework. The sum rule is a generalization to excited states of an earlier sum rule for the ground state and it relates the total $M1$ excitation strength to the average number of d bosons, $n_d(J)$, in the IBM-2 wave function of the initial state J . The latter is an important quantity characterizing the state and is linked with its deformation. By applying the sum rule to different initial states and taking ratios of the

total strengths, one can avoid any assumption on the effective-boson g factors and thus eliminate to a large extent a model dependence from the extracted ratios of d boson contents.

Before the sum rule can be applied, one needs, however, to be sure that the experimental summed $M1$ strengths to the mixed-symmetry states of Eq. (1) exhaust a significant fraction of the total $M1$ strengths. This was verified to be the case, analytically, for the $\text{U}(5)$ and $\text{SO}(6)$ DS limits and, numerically, throughout the transition region, between these limits. The analysis employed F -spin scalar and $\text{SO}(5)$ invariant Hamiltonians relevant for γ -soft nuclei. The presence of an additional $\text{SO}(5)$ symmetry restricts the allowed $M1$ transitions and for $N=5$ enables the mixed-symmetry states of Eq. (1) to exhaust more than 85% of the sum rule for the $J=0_1^+$ and 2_2^+ states. We have applied the sum rule to ^{94}Mo and deduced from the data a relative d -boson content ratio $n_d(0_1^+)/n_d(2_2^+) \approx 0.6$. The extracted value is independent of any model parameters and suggests the structure of ^{94}Mo being close to the $\text{SO}(6)$ DS limit of the IBM-2. The results obtained show that existing and future high-quality data on excited mixed-symmetry states in nuclei can qualify for a sum rule analysis from which one can extract valuable model-independent structure information. The present analysis relies on the IBM-2 interpretation of mixed-symmetry states as predominantly orbital $M1$ excitations. This interpretation is consistent with presently available data in ^{94}Mo [29–32] and with microscopic calculations [34,35]. Further theoretical and experimental work on orbital and spin excitations is highly desirable to verify the character of $M1$ excitations in transitional nuclei such as ^{94}Mo .

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[1] D. Bohle, A. Richter, W. Steffen, A.E.L. Dieperink, N. Lo Iudice, F. Palumbo, and O. Scholten, Phys. Lett. **137B**, 27 (1984).

[2] For recent reviews, see A. Richter, Prog. Part. Nucl. Phys. **34**,

261 (1995); U. Kneissl, H.H. Pitz, and A. Zilges, *ibid.* **37**, 349 (1996).

[3] J.N. Ginocchio, Phys. Lett. B **265**, 6 (1991).

[4] K. Heyde and C. De Coster, Phys. Rev. C **44**, R2262 (1991);

- K. Heyde, C. De Coster, and D. Ooms, *ibid.* **49**, 156 (1994).
- [5] E. Moya de Guerra and L. Zamick, *Phys. Rev. C* **47**, 2604 (1993).
- [6] N. Lo Iudice and A. Richter, *Phys. Lett. B* **304**, 193 (1993); N. Lo Iudice, *Phys. Rev. C* **57**, 1246 (1998).
- [7] J. Enders, H. Kaiser, P. von Neumann-Cosel, C. Rangacharyulu, and A. Richter, *Phys. Rev. C* **59**, R1851 (1999).
- [8] J.N. Ginocchio and A. Leviatan, *Phys. Rev. Lett.* **79**, 813 (1997).
- [9] A. Arima, T. Otsuka, F. Iachello, and I. Talmi, *Phys. Lett.* **66B**, 205 (1977).
- [10] T. Otsuka, A. Arima, F. Iachello, and I. Talmi, *Phys. Lett.* **76B**, 139 (1978).
- [11] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge, 1987).
- [12] P. von Neumann-Cosel, J.N. Ginocchio, H. Bauer, and A. Richter, *Phys. Rev. Lett.* **75**, 4178 (1995).
- [13] N. Huxel *et al.*, *Nucl. Phys.* **A645**, 239 (1999).
- [14] F. Iachello, *Lecture Notes on Theoretical Physics*, Groningen, 1976; T. Otsuka, Ph.D. thesis, University of Tokyo, 1978.
- [15] F. Iachello, *Phys. Rev. Lett.* **53**, 1427 (1984).
- [16] T. Otsuka and J.N. Ginocchio, *Phys. Rev. Lett.* **54**, 777 (1985).
- [17] N.A. Smirnova, N. Pietralla, T. Mizusaki, and P. Van Isacker, *Nucl. Phys.* **A678**, 235 (2000).
- [18] W.D. Hamilton, A. Irbäck, and J.P. Elliott, *Phys. Rev. Lett.* **53**, 2469 (1984).
- [19] G. Molnár, R.A. Gatenby, and S.W. Yates, *Phys. Rev. C* **37**, 898 (1988).
- [20] R. De Leo *et al.*, *Phys. Lett. B* **226**, 5 (1989).
- [21] A. Giannatiempo, A. Nannini, and P. Sona, *Phys. Rev. C* **58**, 3316 (1998); **58**, 3335 (1998).
- [22] W.J. Vermeer, C.S. Lim, and R.H. Spear, *Phys. Rev. C* **38**, 2982 (1988).
- [23] B. Fazekas, T. Belgya, G. Molnár, A. Veres, R.A. Gatenby, S.W. Yates, and T. Otsuka, *Nucl. Phys.* **A548**, 249 (1992).
- [24] J.R. Vanhoy, J.M. Anthony, B.M. Haas, B.H. Benedict, B.T. Meehan, Sally F. Hicks, C.M. Davoren, and C.L. Lundstedt, *Phys. Rev. C* **52**, 2387 (1995).
- [25] P.E. Garrett, H. Lehmann, C.A. McGrath, Minfang Yeh, and S.W. Yates, *Phys. Rev. C* **54**, 2259 (1996).
- [26] I. Wiedenhöver, A. Gelberg, T. Otsuka, N. Pietralla, J. Gableske, A. Dewald, and P. von Brentano, *Phys. Rev. C* **56**, R2354 (1997).
- [27] Sally F. Hicks, C.M. Davoren, W.M. Faulkner, and J.R. Vanhoy, *Phys. Rev. C* **57**, 2264 (1998).
- [28] N. Pietralla *et al.*, *Phys. Rev. C* **58**, 96 (1998).
- [29] N. Pietralla *et al.*, *Phys. Rev. Lett.* **83**, 1303 (1999).
- [30] N. Pietralla, C. Fransen, P. von Brentano, A. Dewald, A. Fit- zler, C. Frießner, and J. Gableske, *Phys. Rev. Lett.* **84**, 3775 (2000).
- [31] C. Fransen, N. Pietralla, P. von Brentano, A. Dewald, J. Gableske, A. Gade, A. Lisetskiy, and V. Werner, *Phys. Lett. B* **508**, 219 (2001).
- [32] C. Fransen, Ph.D. thesis, University of Cologne, 2000.
- [33] N. Pietralla, in *Nuclear Structure Physics*, Proceedings of the International Symposium, Göttingen, edited by R. F. Casten, J. Jolie, U. Kneissl, and P. Lieb (World Scientific, Singapore, 2001), p. 243.
- [34] A.F. Lisetskiy, N. Pietralla, C. Fransen, R.V. Jolos, and P. von Brentano, *Nucl. Phys.* **A677**, 100 (2000).
- [35] N. Lo Iudice and Ch. Stoyanov, *Phys. Rev. C* **62**, 047302 (2000).
- [36] P. Van Isacker, K. Heyde, J. Jolie, and A. Sevrin, *Ann. Phys. (N.Y.)* **171**, 253 (1986).
- [37] P. Van Isacker, H. Harter, A. Gelberg, and P. von Brentano, *Phys. Rev. C* **36**, 441 (1987).
- [38] N. Pietralla, P. von Brentano, A. Gelberg, T. Otsuka, A. Richter, N. Smirnova, and I. Wiedenhöver, *Phys. Rev. C* **58**, 191 (1998).
- [39] A. Giannatiempo, G. Maino, A. Nannini, and P. Sona, *Phys. Rev. C* **48**, 2657 (1993).
- [40] A. Leviatan, A. Novoselsky, and I. Talmi, *Phys. Lett. B* **172**, 144 (1986).
- [41] R.V. Jolos, P. von Brentano, N. Pietralla, and I. Schneider, *Nucl. Phys.* **A618**, 126 (1997).
- [42] V. Werner, N. Pietralla, P. von Brentano, R.F. Casten, and R.V. Jolos, *Phys. Rev. C* **61**, 021301(R) (2000).
- [43] J. Barrette, M. Barrette, A. Boutard, R. Haroutunian, G. Lamoureaux, and S. Monaro, *Phys. Rev. C* **6**, 1339 (1972).
- [44] S. Raman, C.H. Malarkey, W.T. Milner, C.W. Nestor, Jr., and P.H. Stelson, *At. Data Nucl. Data Tables* **36**, 1 (1987).
- [45] C. Wesselborg *et al.*, *Z. Phys. A* **323**, 485 (1986).
- [46] N. Auerbach, D.C. Zheng, L. Zamick, and B.A. Brown, *Phys. Lett. B* **304**, 17 (1993).
- [47] P.O. Lipas, P. von Brentano, and A. Gelberg, *Rep. Prog. Phys.* **53**, 1355 (1990), and references therein.
- [48] J.N. Ginocchio, W. Frank, and P. von Brentano, *Nucl. Phys.* **A541**, 211 (1992).
- [49] A. Wolf, O. Scholten, and R.F. Casten, *Phys. Lett. B* **312**, 372 (1993).
- [50] A. Leviatan, J.N. Ginocchio, and M.W. Kirson, *Phys. Rev. Lett.* **65**, 2853 (1990).
- [51] H. Harter, P. von Brentano, A. Gelberg, and R.F. Casten, *Phys. Rev. C* **32**, 631 (1985).
- [52] A. Leviatan, and J.N. Ginocchio, *Phys. Rev. C* **61**, 024305 (2000).