

Toroidal dipole resonances in the relativistic random phase approximation

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The isoscalar toroidal dipole strength distributions in spherical nuclei are calculated in the framework of a fully consistent relativistic random phase approximation. It is suggested that the recently observed “low-lying component of the isoscalar dipole mode” might correspond to the toroidal giant dipole resonance. Although predicted by several theoretical models, the existence of toroidal resonances has not yet been confirmed in experiment. The strong mixing between the toroidal resonance and the dipole compression mode might help to explain the large discrepancy between theory and experiment on the position of isoscalar giant dipole resonances.

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Experimental information on the nuclear matter incompressibility is obtained from isoscalar giant monopole resonances (IS GMR) in spherical nuclei. Complementary data could be obtained from studies of the isoscalar dipole mode. The isoscalar giant dipole resonance (IS GDR) is a second order effect, built on $3\hbar\omega$, or higher configurations. It corresponds to a compression wave traveling back and forth through the nucleus along a definite direction. Recent data on IS GDR obtained by using inelastic scattering of α particles on ²⁰⁸Pb [1], and on ⁹⁰Zr, ¹¹⁶Sn, ¹⁴⁴Sm, and ²⁰⁸Pb [2], have been analyzed in the nonrelativistic Hartree-Fock plus RPA framework [3], and with relativistic mean-field plus RPA (RRPA) calculations [4]. The results of both analyses are in disagreement with the reported experimental data on the position of the IS GDR centroid energies. Effective interactions, both nonrelativistic and relativistic, which reproduce the experimental excitation energies of the IS GMR, predict centroid energies of the IS GDR that are 4–5 MeV higher than those extracted from small angle α -scattering spectra. This disagreement between theory and experiment is an order of magnitude larger than for other giant resonances. Another puzzling result is the theoretical prediction of the splitting of the IS GDR strength distribution into two broad structures, one in the high-energy region above 20 MeV, and one in the low-energy window between 8 MeV and 14 MeV. In Ref. [4] we have shown that the RRPA peaks in this region do not correspond to a compression mode, but rather to a kind of toroidal motion with dynamics determined by surface effects. In a very recent article [5], Clark *et al.* reported new experimental data on the isoscalar dipole strength functions in ⁹⁰Zr, ¹¹⁶Sn, and ²⁰⁸Pb, measured with inelastic scattering of α particles at small angles. They found that the isoscalar $E1$ strength distribution in each nucleus consists of a broad component at $E_x \approx 114/A^{1/3}$ MeV containing approximately 100% of the $E1$ EWSR, and a narrower one at $E_x \approx 72/A^{1/3}$ MeV containing 15–28% of the total isoscalar $E1$ strength. The higher component is identified as the $E1$ compression mode, whereas the lower component may be the new mode predicted by the RRPA analysis of Ref. [4]. In a new experiment [6] the IS GDR has been investigated in ²⁰⁸Pb using inelastic scattering of 400 MeV α particles at extremely forward angles. The observed strength distribution

has two components, with centroid energies at 13.0 ± 0.5 MeV and 23.0 ± 0.5 MeV. In the present work we suggest that the observed low-lying $E1$ component may correspond to the toroidal giant dipole resonance (TGDR).

The role of toroidal multipole form factors and moments in the physics of electromagnetic and weak interactions has been extensively discussed in Refs. [7,8]. They appear in multipole expansions for systems containing convection and induction currents. In particular, the multipole expansion of a four-current distribution gives rise to three families of multipole moments: charge moments, magnetic moments, and electric transverse moments. The later are related to the toroidal multipole moments and result from the expansion of the transverse electric part of the current. The toroidal dipole moment, in particular, describes a system of poloidal currents on a torus. Since the charge density is zero for this configuration, and all the turns of the torus have magnetic moments lying in the symmetry plane, both the charge and magnetic dipole moments of this configuration are equal to zero. The simplest model is an ordinary solenoid bent into a torus.

Vortex waves in nuclei were analyzed in a hydrodynamic model [9]. By relaxing the assumption of irrotational motion, in this pioneering study solenoidal toroidal vibrations were predicted, which correspond to the toroidal giant dipole resonance at excitation energy $E_x \approx (50-70)/A^{1/3}$ MeV. It was suggested that the vortex excitation modes should appear in electron backscattering. The isoscalar 1^- toroidal dipole states were studied in the framework of the time-dependent Hartree-Fock theory by analyzing the dynamics of the moments of the Wigner transform of the density matrix [10], and excitations with dipole toroidal structure were also found in semiclassical studies based on nuclear fluid dynamics [11].

In this Rapid Communication the toroidal dipole strength distributions are calculated in the relativistic random phase approximation (RRPA). The RRPA represents the small amplitude limit of the time-dependent relativistic mean-field theory [12]. A self-consistent calculation ensures that the same correlations which define the ground-state properties, also determine the behavior of small deviations from the equilibrium. The same effective Lagrangian generates the Dirac-Hartree single-particle spectrum and the residual

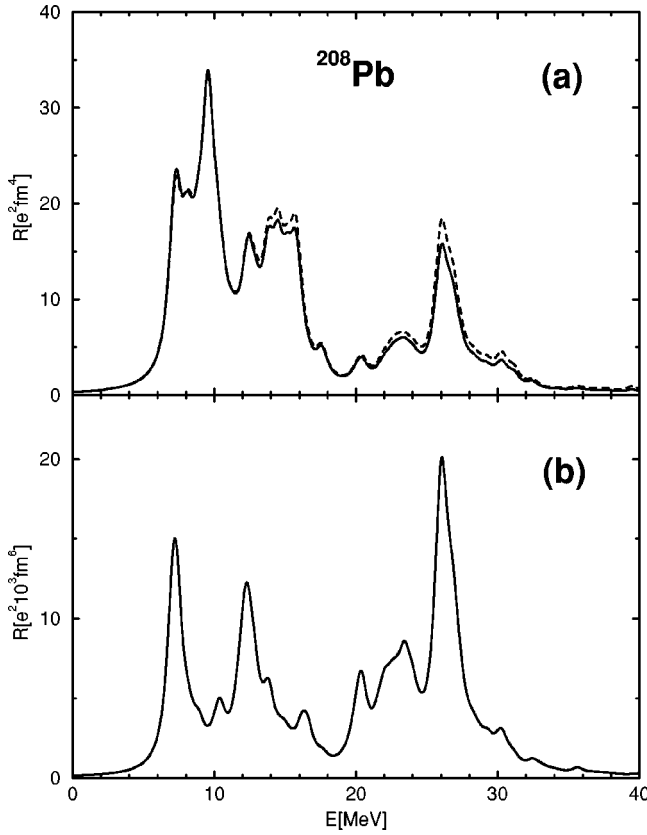


FIG. 1. (a) Toroidal dipole strength distributions in ^{208}Pb , calculated without (dashed) and with (solid) projection of spurious center-of-mass components. (b) IS GDR strength distributions in ^{208}Pb .

particle-hole interaction. In a number of recent applications [4,13–17] it has been shown that, by using effective Lagrangians which in the mean-field approximation provide an accurate description of ground-state properties, excellent agreement with experimental data is also found for the excitation energies of low-lying collective states and of giant resonances. Two points are essential for the successful application of the RRPA in the description of dynamical properties of finite nuclei: (i) the use of effective Lagrangians with nonlinear terms in the meson sector, and (ii) the fully consistent treatment of the Dirac sea of negative energy states. In particular, in Ref. [16] it has been shown that configurations which include negative-energy states have an especially pronounced effect on isoscalar excitation modes.

In Fig. 1(a) we display the RRPA toroidal dipole strength distribution for ^{208}Pb :

$$R(E) = \sum_i B^{T=0}(E1, 1_i^- \rightarrow 0_f) \frac{\Gamma^2}{4(E - E_i)^2 + \Gamma^2}, \quad (1)$$

where Γ is the width of the Lorentzian distribution, and

$$B^{T=0}(E1, 1_i^- \rightarrow 0_f) = \frac{1}{3} |\langle 0_f || \hat{T}_1^{T=0} || 1_i^- \rangle|^2. \quad (2)$$

For the strength distributions in Fig. 1 the width is $\Gamma = 1.0$ MeV. The isoscalar toroidal dipole operator is defined [7]

$$\hat{T}_{1\mu}^{T=0} = -\sqrt{\pi} \int \left[r^2 \left(\vec{Y}_{10\mu}^* + \sqrt{\frac{2}{5}} \vec{Y}_{12\mu}^* \right) - \langle r^2 \rangle_0 \vec{Y}_{10\mu}^* \right] \cdot \vec{J}(\vec{r}) d^3 r. \quad (3)$$

In the relativistic framework the expression for the the isoscalar baryon current reads

$$J^\nu = \sum_{i=1}^A \bar{\psi}_i \gamma^\nu \psi_i, \quad (4)$$

where the summation is over all occupied states in the Fermi sea. The resulting toroidal dipole operator is

$$\hat{T}_{1\mu}^{T=0} = -\sqrt{\pi} \sum_{i=1}^A \left[r_i^2 \left(\vec{Y}_{10\mu}^*(\Omega_i) + \sqrt{\frac{2}{5}} \vec{Y}_{12\mu}^*(\Omega_i) \right) \cdot \vec{\alpha}_i - \langle r^2 \rangle_0 \vec{Y}_{10\mu}^*(\Omega_i) \cdot \vec{\alpha}_i \right], \quad (5)$$

where $\vec{Y}_{l\mu}$ denotes a vector spherical harmonic, and $\vec{\alpha}$ are the Dirac α matrices. The calculations have been performed with the self-consistent Dirac-Hartree plus relativistic RPA. The effective mean-field Lagrangian contains nonlinear meson self-interaction terms, and the configuration space includes both particle-hole pairs, and pairs formed from hole states and negative-energy states. The inclusion of the term $-\langle r^2 \rangle_0 \vec{Y}_{10\mu}^*$ in the operator ensures that the TGDR strength distributions do not contain spurious components that correspond to the center-of-mass motion [9]. In Fig. 1(a) we compare the toroidal strength distributions calculated without (dashed) and with (solid) the inclusion of this term in the operator.

The projection of spurious center-of-mass motion components can be also performed by subtracting the spurious transition current, following a procedure similar to that adopted in Refs. [3,18] for the ISGDR. The toroidal dipole strength distribution can be written as

$$R(E) = \left| \int d^3 r \delta \vec{j}(\vec{r}) \cdot \vec{T}(\vec{r}) \right|^2, \quad (6)$$

where $\vec{T}(\vec{r})$ is the vector toroidal operator, and $\delta \vec{j}(\vec{r})$ is the transition current

$$\delta \vec{j}(\vec{r}) = j_-(r) \vec{Y}_{10\mu}^*(\Omega) + j_+(r) \vec{Y}_{12\mu}^*(\Omega). \quad (7)$$

The radial functions $j_-(r)$ and $j_+(r)$ are defined in Ref. [19]. We have verified that identical strength distributions [solid curve in Fig. 1(a)] are obtained when (a) the transition current (7) is used and the toroidal operator is corrected by including the term $-\langle r^2 \rangle_0 \vec{Y}_{10\mu}^*$, or (b) this term is not in-

cluded in the operator, but the spurious component is subtracted from the transition current at each energy

$$\delta\vec{j}(\vec{r}) - a\rho_0\vec{e}_z. \quad (8)$$

The second term in this expression is the spurious transition current [20]. ρ_0 is the ground-state density and \vec{e}_z denotes the unit vector in the direction of the center-of-mass motion. The energy dependent coefficient a is determined by the condition that the integral of the transition current over the nucleus should vanish at each energy

$$\int d^3r(\delta\vec{j}(\vec{r}) - a\rho_0\vec{e}_z) = 0. \quad (9)$$

The toroidal dipole strength distribution [Fig. 1(a)] is compared with the strength function of the usual isoscalar dipole operator [4]

$$\hat{Q}_{1\mu}^{T=0} = \sum_{i=1}^A \gamma_0 \left(r^3 - \frac{5}{3} \langle r^2 \rangle_0 r \right) Y_{1\mu}(\theta_i, \varphi_i) \quad (10)$$

plotted in Fig. 1(b). Both strength distributions have been calculated with the NL3 [21] effective interaction. In Ref. [22] it has been shown that isoscalar giant monopole resonances calculated with this effective force ($K_{\text{nm}} = 271.8$ MeV) are in excellent agreement with experimental data, and in Ref. [4] this interaction was used in the RRP analysis of the IS GDR. By using effective interactions with different values of the nuclear matter compressibility modulus, it was shown that only the high-energy (above 20 MeV) portion of the isoscalar dipole strength distribution corresponds to a compression mode. The same effect is observed for the toroidal strength function: the position of the peaks in the low-energy region (below 20 MeV) depends only weakly on the incompressibility, while the structure in the high-energy region is strongly affected by the choice of the compression modulus of the interaction. Both dipole strength distributions, toroidal in Fig. 1(a) and compressional in Fig. 1(b), display two broad structures: one at low energies between 8 and 15 MeV, and one in the high-energy region 25–30 MeV. Obviously, one could expect a strong coupling between the two isoscalar 1^- modes. This coupling becomes even more evident if one rewrites the expression in square brackets of the toroidal operator (3) as [9]

$$\nabla \times (\vec{r} \times \nabla) \left(r^3 - \frac{5}{3} \langle r^2 \rangle_0 r \right) Y_{1\mu}, \quad (11)$$

and compares it with the isoscalar dipole operator of the compression mode (10). The relative position of the two resonance structures will, therefore, depend on the interaction between the toroidal and compression modes.

In Fig. 2 we display the RRP toroidal dipole strength distributions in ^{90}Zr , ^{116}Sn , and ^{208}Pb , calculated with the NL3 effective interaction. In all three nuclei a broad, strongly fragmented structure is found in the low-energy region, where “the low-lying component of the IS GDR” has been observed [5,6]. The toroidal strength distributions in

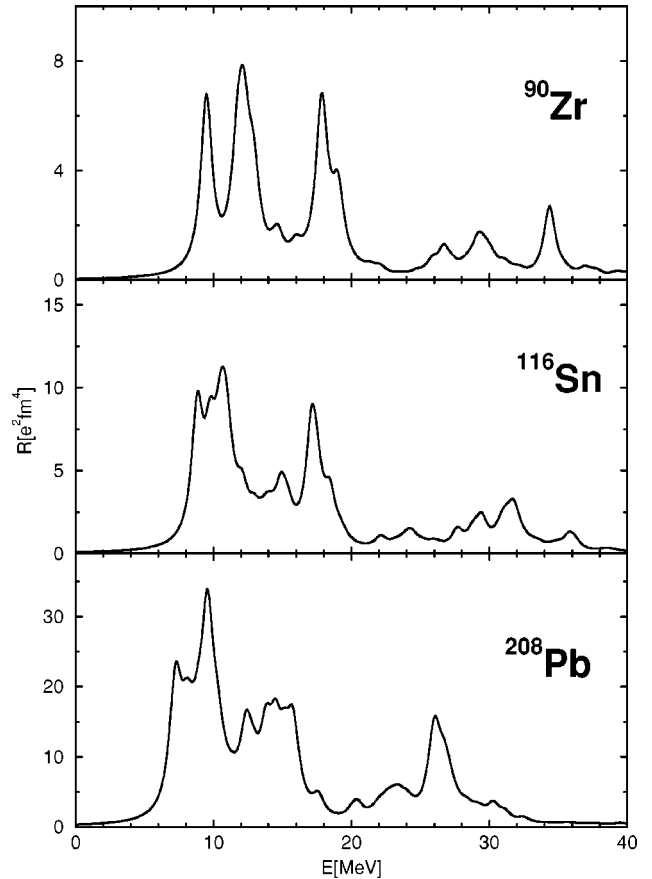


FIG. 2. Toroidal dipole strength distributions in ^{90}Zr , ^{116}Sn , and ^{208}Pb , calculated with the NL3 effective interaction.

this region should be compared with the experimental centroid energies of the “low-lying component”: 16.2 ± 0.8 MeV for ^{90}Zr [5], 14.7 ± 0.5 MeV for ^{116}Sn [5], 12.2 ± 0.6 MeV [5], and 13.0 ± 0.5 MeV [6] for ^{208}Pb . The calculated peaks in the high-energy region, on the other hand, correspond to the compression mode. The dynamics of the solenoidal toroidal vibrations is illustrated in Fig. 3, where we plot the velocity fields for the four most pronounced peaks of the toroidal dipole strength distributions in ^{116}Sn (see Fig. 2). The velocity distributions are derived from the corresponding transition currents, following the procedure described in Ref. [19]. A vector of unit length is assigned to the largest velocity. All the other velocity vectors are normalized accordingly. Since the collective flow is axially symmetric, we plot the velocity field in cylindrical coordinates. The z axis corresponds to the symmetry axis of a torus. We note that the two lowest peaks at 8.82 MeV and 10.47 MeV are completely dominated by vortex collective motion. The velocity fields in the (z, r_{\perp}) plane correspond to poloidal currents on a torus with vanishing inner radius. The poloidal currents determine the dynamical toroidal moment. The high-energy peak at 30.97 MeV displays the dynamics of dipole compression mode. The “squeezing” compression mode is identified by the flow lines which concentrate in the two “poles” on the symmetry axis. The velocity field corresponds to a density distribution which is being compressed in the lower half plane, and expands in the upper half plane.

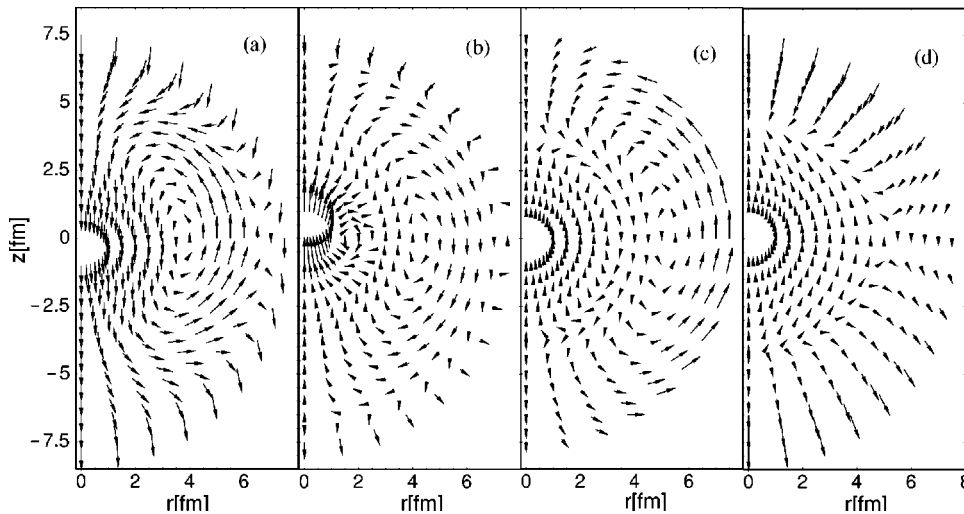


FIG. 3. Velocity distributions for the most pronounced dipole peaks in ^{116}Sn (see Fig. 2). The velocity fields correspond to the peaks at 8.82 MeV (a), 10.47 MeV (b), 17.11 MeV (c), and 30.97 MeV (d).

The centers of compression and expansion are located on the symmetry axis, at approximately half the distance between the center and the surface of the nucleus. Finally, the intermediate peak at 17.11 MeV clearly displays the coupling between the toroidal and compression dipole modes. A very similar behavior of the velocity distributions as a function of excitation energy is also observed for ^{90}Zr and ^{208}Pb .

We suggest, therefore, that the recently observed “low-lying component of the isoscalar dipole mode” [5,6] might correspond to the toroidal giant dipole resonance. By employing the fully consistent relativistic random phase approximation, in Ref. [4] and in the present analysis we have shown that the toroidal dipole strength is concentrated in the low-energy region around 10 MeV, while the isoscalar dipole excitations in the high-energy region above 20 MeV correspond to the “squeezing” compression mode. States in the intermediate region display a strong mixing between the two

dipole resonances. The pronounced coupling between the toroidal resonance and the IS GDR, predicted by the RRP calculations, might also help to explain the strong discrepancy between theory and the experimental position of the IS GDR centroid energies [3–6]. Namely, the interaction causes a repulsion of the two 1^- isoscalar modes and this effect might explain the observation of Ref. [5] that “The centroids of the higher (compression) mode calculated with interactions which reproduce GMR energies are about 4 MeV higher than the experimental centroids while the calculated centroids for the lower mode lie 1–2 MeV below the experimental values.”

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