How many σ's is the solar neutrino effect?

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The minimal standard electroweak model can be tested by allowing all the solar neutrino fluxes, with undistorted energy spectra, to be free parameters in fitting the measured solar neutrino event rates, subject only to the condition that the total observed luminosity of the Sun be produced by nuclear fusion. The rates of the five experiments prior to SNO (chlorine, Kamiokande, SAGE, GALLEX, Super-Kamiokande) cannot be fit by an arbitrary choice of undistorted neutrino fluxes at the level of 2.5σ (formally 99% C.L.). Considering just SNO and Super-Kamiokande, the discrepancy is at the 3.3σ level (10^{-3} C.L.). If all six experiments are fit simultaneously, the formal discrepancy increases to 4σ (7×10^{-5} C.L.). If the relative scaling in temperature of the nuclear reactions that produce ^{7}Be and ^{8}B neutrinos is taken into account, the formal discrepancy is at the 7.4σ level.

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I. INTRODUCTION

The Sudbury Neutrino Observatory (SNO) has reported an epochal measurement [1] of the rate of charged current interactions in deuterium due to ^{8}B solar neutrinos. The precise Super-Kamiokande measurement [2] of neutrino-electron scattering (charged plus some neutral current sensitivity) by ^{8}B neutrinos reveals a total neutrino flux that is about 3.3σ [1] larger than the ν_e flux measured by SNO. The combined SNO and Super-Kamiokande result seems to have convinced most physicists that neutrino oscillations are occurring in the solar neutrino domain.

Why is a single 3σ result so convincing? We all know of examples in particle and nuclear physics where 3σ results have not been verified. The purpose of this paper is to quantify the role of the additional information on solar neutrino event rates that, when taken together with the SNO/Super-Kamiokande result, makes the inference of neutrino oscillations so compelling.

I will not discuss the precise helioseismological verification, better than 0.1% rms throughout the Sun, of the sound speeds predicted by the standard solar model [3] (hereafter, BP2000). I will also not discuss nonquantifiable effects such as the manifestly great care with which both the SNO and Super-Kamiokande experiments were performed. The excellent agreement between the standard solar model predictions and the helioseismological measurements and the rigorous calibrations of the SNO and Super-Kamiokande experiments are undoubtedly important factors in convincing many in the physics community that solar neutrino oscillations occur, but I will focus here only on the measurements of solar neutrino event rates in different detectors.

Suppose we allow the solar neutrino fluxes to have arbitrary (positive) amplitudes, subject only to the conditions that the fusion energy associated with these fluxes equal the precisely measured solar luminosity and that the energy spectra be undistorted by neutrino oscillations. If the minimal standard electroweak model is valid (no neutrino oscillations occur), solar neutrino energy spectra differ from their laboratory shapes by the order of 1 part in 10^5 for β decays (like ^{8}B or ^{13}N decay) and less than 1 part in 10^2 for the p-p reaction (1 part in 10^3 for the hep reaction) [4]. Suppose we ignore all other information about the Sun, including helioseismology. How well can we then fit the observed set of solar neutrino event rates in different experiments?

I report here on a simultaneous fit with arbitrary neutrino fluxes to all the available neutrino event rates, chlorine [5], Kamiokande [6], SAGE [7], GALLEX +GNO [8], Super-Kamiokande [2], and SNO [1]. We shall see that including all of the available experiments (not just SNO and Super-Kamiokande) increases by an order of magnitude the stringency of the formal C.L. by which one can conclude that neutrino oscillations are required (a 4σ effect for all six experiments). If the temperature scaling of the nuclear reactions giving rise to ^{7}Be and ^{8}B neutrinos is imposed as an additional condition on the fitting procedure, then the no-oscillation hypothesis is rejected at the 7.4σ level (compared to 6.9σ in the pre-SNO era).

In the present paper, I also provide a physical explanation of why the $\chi^2_{\rm min}$ method leads to the unphysical requirement that some solar neutrino fluxes (^{7}Be, ^{13}N, and ^{15}O) be completely absent while the p-p neutrino flux is enhanced over the standard solar model prediction.

The calculations described in this paper utilize an improved formulation of the solar luminosity constraint on neutrino fluxes [9].

There have been a number of pre-SNO investigations of the failure of free-flux, no-oscillation fits to solar neutrino data. The first such study stressed as early as 1990 [10] the apparent incompatability of the chlorine and Kamiokande experiments, if new physics did not affect the shape of the solar neutrino energy spectra. The seminal studies in the mid-1990s by Hata, Bludman, and Langacker [11], Parke [12], and Heeger and Robinson [13], and related discussions [14], helped convince many physicists of the necessity of solar neutrino oscillations. The inadequacy of free-flux fits was reinforced by the most recent pre-SNO studies [15,16].

The principal results of this paper are presented in Table II and summarized in Sec. VI. The reader is urged to look

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TABLE I. Solar neutrino rates: standard theory versus experiment. The unit is SNU (10^{-36} interactions per target atom per sec) for the radiochemical experiments: chlorine [5], SAGE [7], and GALLEX + GNO [8]. The unit is 10^6 cm^{-2} s^{-1} for the water Cherenkov experiments, SNO [1], SuperKamiokande [2], and Kamiokande [6], which measure the ^{8}B neutrino flux. Results are also shown in the last two rows for the weighted average of the SAGE and GALLEX/GNO experiments and for the weighted average of the Kamiokande and Super-Kamiokande experiments. The BP2000 predictions for the combined standard solar and electroweak model are taken from Ref. [3]. The errors quoted for Measured/BP2000 are the quadratically combined statistical and systematic uncertainties. The larger experimental error was used here when asymmetric errors were quoted in the original publications.

Experiment	BP2000	Measured	Measured/BP2000
Chlorine	$7.6[1.00\ ^{+0.17}_{-0.14}]$	$2.56[1.00\pm0.088]$	0.337 ± 0.030
Kamiokande	$5.05[1.00\ ^{+0.20}_{-0.16}]$	$2.80[1.00\pm0.136]$	0.554 ± 0.075
SAGE	$128[1.00\ ^{+0.07}_{-0.05}]$	$77.0[1.00\pm0.087]$	0.602 ± 0.052
GALLEX + GNO	$128[1.00\ ^{+0.07}_{-0.05}]$	$74.1[1.00\pm0.092]$	0.579 ± 0.053
Super-Kamiokande	$5.05[1.00\ ^{+0.20}_{-0.16}]$	$2.32[1.00\pm0.037]$	0.459 ± 0.017
SNO	$5.05[1.00\ ^{+0.20}_{-0.16}]$	$1.75[1.00\pm0.084]$	0.3465 ± 0.029
K + SK	$5.05[1.00\ ^{+0.20}_{-0.16}]$	$2.34[1.00\pm0.035]$	0.464 ± 0.016
GALLIUM	$128[1.00\ ^{+0.07}_{-0.05}]$	$75.6[1.00\pm0.063]$	0.590 ± 0.037

first at the table and discussion section and then to decide whether to read the more detailed discussion in the main body of the text.

Section II summarizes in a convenient form the data on the measured solar neutrino event rates and Sec. III describes the way the calculations were done. Using the data and methods described in the previous sections, Sec. IV presents the principal results for the pre-SNO era, for just the water Cherenkov solar neutrino detectors, for all six detectors, and for the effect of taking account of the scaling of the fusion reactions that produce ^{7}Be and ^{8}B solar neutrinos. Section V explains physically why the $\chi^2_{\rm min}$ solutions eliminate all ^{7}Be and CNO neutrinos. I summarize the main results briefly in Sec. VI.

All of the results presented in this paper depend upon the validity of the published solar neutrino measurements. The formal statistical estimates given here are not to be taken literally when the probabilities become extremely small because of the likely effects of unknown systematic errors.

II. MEASURED RATES

Table I summarizes the data from measurements of solar neutrino event rates. For each of the six experiments listed in the first column, the table gives in the second column the combined prediction [3] of the standard solar model and the simplest version of standard electroweak theory (BP2000 solar model, no ν oscillations). The third column shows the measured values for each experiment [1,2,5–8]. In the last column, the table gives the ratio of each measured value to the predicted standard value (no theoretical errors included in the last column). The dimensionless ratios are convenient to use in calculations. The last two rows present the weighted average rate for the two ν-e scattering experiments (K and SK) and the weighted average rate for the two gallium experiments (GALLIUM). In this paper, we shall use the theoretical predictions only for one very special case, when we compare the standard solar model with all the measurements.

III. CALCULATIONS

The predicted event rates are linear functions of the seven important neutrino fluxes: p-p, pep, hep, ^{7}Be, ^{8}B, ^{13}N, and ^{15}O. From purely physics considerations, any or all of these fluxes could be important. In fact, at one time or another in the history of solar neutrino research, each of these fluxes has been hypothesized to be important for solar neutrino measurements [17].

A. Equations and uncertainties

The equations for the neutrino event rates can be written conveniently in terms of the ratios of the actual fluxes to the predicted BP2000 fluxes. In this case, the linear coefficients of the predicted solar neutrino interaction rates can be read directly from Table 7 of Ref. [3] and the observed rates can be taken from the last column of Table I of the present paper.

The luminosity constraint can be written also as a convenient linear equation in the neutrino fluxes. One has

$$1=\sum_i \left(\frac{\alpha_i}{10\ \ \mathrm{MeV}}\right)\left(\frac{\phi_i}{8.532\times10^{10}\ \ \mathrm{cm^{-2}\ s^{-1}}}\right), \qquad (1)$$

where accurate values of the energy coefficients α_i are given in Ref. [9] and the ϕ_i are the individual neutrino fluxes ($i=pp$, pep, hep, ^{7}Be, ^{8}B, ^{13}N, and ^{15}O).

The best-fit neutrino fluxes were obtained by minimizing χ^2 for each case considered (see Table II in the following section for a description of the different cases). The χ^2 can be written symbolically as

$$\chi^2=\sum_i \frac{\left(\mathrm{rate}_i-\sum_j c_{ij}\phi_j\right)^2}{\sigma^2_{\rm expt}+\sigma^2_{\rm c.s.}}, \qquad (2)$$

where the c_{ij} are numerical coefficients for each experiment (cf. Table 7 of Ref. [3]) and ϕ_j are the neutrino fluxes. The experimental errors σ_{expt} were taken from the last column of Table I. It is important to include also the theoretical uncertainties for the calculated interaction cross sections $\sigma_{\text{c.s.}}$. For each neutrino flux, the cross section errors are take from Ref. [18] for the gallium experiments and from Ref. [19] for the chlorine experiment. The cross section uncertainties are included in the reported rates for the other experiments listed in Table I. Since the neutrino fluxes are treated as free parameters, the uncertainties in the predicted fluxes are not included in the calculations (except for the special case of testing the standard solar model fit; cf. the last two rows of Table II).

For a given number of degrees of freedom, n[$n=$ (number of experiments + 1) − (number of free fluxes)], the value of χ^2_{min} corresponds to a probability P that a worse fit would have been obtained by chance if the model being tested is correct. In our case, the model is that the measured experiments plus the luminosity constraint are described by a theory in which the undistorted neutrino energy spectra can have arbitrary amplitudes. When the number of experiments plus the luminosity condition is one more than the number of free-parameter neutrino fluxes, then there is a particularly simple relation between χ^2_{min} and the effective number of σ's. For this special case ($n=1$), $\sigma=\chi_{\text{min}}$. Here σ is the number of sigmas for a normal distribution such that the two-sided probability of getting a value greater than σ is equal to P. More generally, for $\chi^2_{\text{min}} \gg n$, one can show[1] that

$$\sigma^2=\chi^2_{\text{min}}-\ln\sigma^2+(2-n)\ln\chi^2_{\text{min}}+\ln g(n), \qquad (3)$$

where $g(n)=\Gamma^2(n/2)/(2^{1-n}\pi)$ and Γ is the gamma (generalized factorial) function. The result given in Eq. (3) is exact (not just asymptotically correct) for n equal to 1. For practical cases with $n\neq 1$, Eq. (3) can be solved simply by iteration with a hand calculator.

B. Supplementary conditions

We fit the results for at most six experiments and the luminosity constraint. Therefore, we cannot use all seven of the neutrino fluxes as free parameters. In previous free-flux analyses of solar neutrino rates, nearly all authors have followed Hata *et al.* [11] in taking the ratio of the *pep* to *p-p* fluxes to be the same as in the standard solar model. The justification for this assumption is that the *pep* to *p-p* ratio is practically independent of details of the solar model, depending upon just the weighted average of the density over the square root of the temperature [21]. For the 12 variant and deviant solar models listed in Table 10 of Ref. [3], the ratio of *pep* to *p-p* fluxes is $(2.25\pm 0.1)\times 10^{-3}$. The model that gives the most extreme ratio (and also the most conservative result) is ruled out by helioseismological data, giving a rms fit to the helioseismological data that is more than 100 times worse than the standard solar model. In the calculations described in the following section, I chose the value of the *pep* fluxes to be within the range $(2.25\pm 0.1)\times 10^{-3}$ *p-p*, varying the exact value to give the most conservative result.

Many authors [11,13,15,16] have also assumed that the CNO nuclear reactions are in equilibrium and have therefore taken the ^{13}N and ^{15}O neutrino fluxes to be exactly equal (or in some cases both to be equal to zero [12]). Any nonzero value for the CNO neutrino fluxes increases the discrepancy with the standard electroweak model.

Finally, all previous authors (except for Hata *et al.* [11]) have neglected the *hep* flux, although *hep* neutrinos could in principle contribute significantly to the chlorine and gallium experiments (see Ref. [3]). At the 3σ upper limit corresponding to the Super-Kamiokande result [2], the *hep* flux contributes $1.4\sigma_{\text{expt}}$(Cl) (0.31 SNU) to the chlorine rate but only $0.2\sigma_{\text{expt}}$(Ga) (0.8 SNU) to the gallium rate. Here σ_{expt}(Cl) is the total experimental error for the chlorine experiment [5] and σ_{expt}(Ga) is the total weighted average of the SAGE and the GALLEX/GNO experiments.

I want to add a word of reassurance for the mathematically fastidious who may be concerned about the fact that before imposing the supplementary conditions there are more free fluxes than experiments plus constraints. One can find the minimum χ^2 using all the fluxes. As we shall see in Sec. IV and V, this minimum always lies in the region within which the supplementary conditions apply; i.e., there are no ^{7}Be or CNO neutrinos. By choosing to consider the subset of fluxes that give the smallest χ^2, we are making it as difficult as possible to reject the no-oscillation hypothesis.

In the following section, I explore the robustness of the free-flux analyses to the supplementary conditions on *hep* and CNO neutrinos described above. For simplicity, I shall denote in Table II and in Sec. IV these conditions symbolically as (i) $n13=n15$ and (ii) $hep=0.0$.

IV. RESULTS

Table II presents the principal results of this paper. The table gives the formal probability P of obtaining a fit as bad ($\chi^2\geqslant\chi^2_{\text{min}}$) as the best-obtainable fit with arbitrary amplitudes, but undistorted energy spectra, for the solar neutrino fluxes. The table also gives the effective number of standard deviations σ [defined by Eq. (3)] by which the no-neutrino-oscillation hypothesis fails to fit the observed data on solar neutrino event rates. In a number of cases, the probabilities quoted are so small that the distributions from which the probabilities are calculated are not valid in the relevant extreme limits. Therefore, I have included the effective number of σ's because most physicists have, based upon bitter experience with unknown systematic errors, developed their own healthy internal recalibration for the meaning of sigmas.

I also give in Table II the best fit values, in units of the BP2000 standard solar model fluxes [3], for the three most important neutrino fluxes, *p-p*, ^{7}Be, and ^{8}B. Contrary to what one expects on astrophysical grounds, the formal mini-

[1]This asymptotic formula can be derived by integrating the normal distribution by parts to obtain the leading term for the probability to have a value greater than σ and then equating this expression to the leading term in the repeated fraction expansion of the incomplete gamma function that describes [20] the χ^2 probability distribution.

TABLE II. How many σ's? The table shows the effective number of standard deviations, σ, by which the no-oscillation hypothesis fails to account for the total event rates measured in solar neutrino experiments. The abbreviated notation for the six experiments is the same as in Table I. For each case (combination of experiments), the table also gives the probability P for errors distributed normally of obtaining a fit as bad as the best fit found, the neutrino fluxes (in units of the BP2000 fluxes) for the p-p, ^{7}Be, and ^{8}B neutrinos, and the predicted event rate (in SNU) for the chlorine and gallium experiments. The supplementary conditions $pep=pp$, $hep=0.0$, $n13=o15$, and $n13=o15=0.0$ are defined in Sec. III B.

Case	P	σ's	pp	^{7}Be	^{8}B	Cl	Ga
Pre-SNO							
Cl,K,gallium, SK [a]	1×10^{-2}	2.5	1.0917	0.000	0.4550	2.9	84.6
Only water Cherenkov experiments							
SK and SNO	8×10^{-4}	3.35	—	—	0.4311	—	—
K, SK, and SNO	1×10^{-3}	3.3	—	—	0.4356	—	—
Six experiments in different combinations							
Cl,gallium,K+SK,SNO [a]	7×10^{-5}	4.0	1.0917	0.000	0.4333	2.7	84.4
Cl,K,gallium,SK,SNO [b]	3.5×10^{-5}	4.1	1.0917	0.000	0.4315	2.7	84.3
Cl,K,gallium,SK,SNO [c]	3.5×10^{-5}	4.1	1.0917	0.000	0.4315	2.7	84.3
Six experiments	3×10^{-5}	4.2	1.0917	0.000	0.4314	2.7	84.3
^{7}Be$\geqslant^8$B in units of BP2000 fluxes							
Cl,gallium,K+SK,SNO [a]	1×10^{-13}	7.4	1.039	0.6787	0.4144	3.4	103.6
Pre-SNO: Cl,gallium,K+SK [d]	4×10^{-12}	6.9	1.039	0.6907	0.4312	3.5	104.1
Standard solar model							
Cl,gallium,K+SK,SNO	3×10^{-11}	6.7	1.000	1.000	1.000	7.6	127.8

[a] $hep=0.0$, $n13=o15$.
[b] $hep=0.0$.
[c] $n13=o15$.
[d] $hep=0.0$, $n13=0.0$, $o15=0.0$.

mization process requires in all cases, where they are allowed to vary freely, that the neutrino fluxes from the CNO chain, ^{13}N and ^{15}O, as well as the hep neutrino flux, be identically zero (for an explanation, see Sec. V). Therefore, these fluxes are not given explicitly in Table II. The pep neutrino flux is fixed to have that ratio relative to the basic p-p neutrino flux which produces the most conservative result, given the general form of the ratio that results from weak interaction theory (see discussion in Sec. III B).

The table also presents the predictions of each of the best fits for the capture rates in the radiochemical chlorine and gallium experiments (in SNU, 10^{-36} interactions per target particle per sec). By hypothesis, we are considering only undistorted energy spectra; all the neutrinos are ν_e. Therefore, the predicted rates for the Kamiokande, Super-Kamiokande, and SNO CC measurements are, when expressed in units of the predicted rates of the combined standard model, all numerically equal to the tabulated value for the ^{8}B neutrino flux (which is given in units of the BP2000 flux).

One can make contradictory plausibility arguments about whether or not one should use the weighted average of the Kamiokande [6] and Super-Kamiokande [2] experiments or the weighted average of the SAGE [7] and GALLEX/GNO [8] experiments. I therefore report calculations performed with the experiments combined in different ways. Fortunately, it turns out not to matter much whether one uses the weighted averages or the individual experiments (see the section labeled ''Six experiments in different combinations'' of Table II). Because it yields the most conservative answer, I adopt as ''standard'' the case in which one combines both Kamiokande and Super-Kamiokande and SAGE and GALLEX/GNO.[2]

[2]My personal preference, however, is to regard the Super-Kamiokande and Kamiokande measurements as two experiments because they have different energy thresholds and because their energy calibrations were performed in different ways. On the other hand, I prefer to combine the SAGE and GALLEX/GNO experiments because they have exactly the same energy sensitivity. One could argue, however, that for purposes of testing the null hypothesis of no new physics the SAGE and GALLEX/GNO experiments should be treated as different because they are located at different places on Earth and they made measurements over different times. The null hypothesis could, in principle, be wrong because of a strong regeneration effect in the Earth or because of a highly time-dependent neutrino flux.

A. Pre-SNO

The situation prior to the announcement of the SNO results is the first case listed in Table II. Considering the five pre-SNO experiments, but using the weighted average of the SAGE and GALLEX/GNO experiments, the no-oscillation hypothesis is rejected at the effective 2.5σ level (99% C.L.). Even this most-favorable solution requires that ^{7}Be neutrinos be entirely missing, a result which many authors have argued is not physically or astrophysically reasonable [10–14,22,16].

Following essentially all previous work on this subject, the case listed in Table II includes the supplementary conditions $pep = p\text{-}p$, $hep = 0.0$, and $n13 = o15$ (see explanation of this notation in Sec. III B). Nearly identical results are obtained if SAGE and GALLEX/GNO are treated as separate experiments and either pair of supplementary conditions, $pep = p\text{-}p$, $hep = 0.0$, or $pep = p\text{-}p$ and $n13 = o15$, is used.[3]

B. Water Cherenkov experiments

The published results of the SNO CC and Super-Kamiokande experiments are inconsistent at the level of 3.35σ, as shown in Ref. [1] and in Table II. One might hope that this result would be strengthened by including the Kamiokande measurement. However, this is not the case. The discrepancy that arises from assuming no neutrino oscillations is essentially unchanged if all three experiments are included; in this case, the fit is acceptable at $P = 1 \times 10^{-3}$, which corresponds to 3.3σ [for $\nu = 2$; cf. Eq. (3)].

Several authors have shown [23–25] how one can choose the energy thresholds for the Super-Kamiokande and SNO experiments such that the response functions for the two experiments are made approximately equal. The advantage of this method is that some of the systematic errors are reduced, but there is some slight loss of statistical power. Also, one must understand the details of the Super-Kamiokande experiment well enough to reevaluate accurately the rate at a different threshold than the published value. Apparently, this has been done successfully. In obtaining the results given in Table II, I have simply used the rates and energy thresholds published by the Super-Kamiokande [2] and SNO [1] Collaborations. The straightforward result given in the present paper is in good agreement with the more sophisticated analysis described in Refs. [25,26].

C. Six experiments in different combinations

Table II shows the results for a variety of different ways of combining all six of the experimental results and of imposing the supplementary conditions. The formal statistical probabilities of obtaining fits as bad as the best fits that were found range from $P = 3 \times 10^{-5}$ to $P = 7 \times 10^{-5}$, about an order of magnitude worse than obtained with the water Cherenkov experiments alone. The corresponding number of σ's at which the best fit is formally rejected is 4.0σ–4.2σ. The SNO experiment contributes 56% of the total χ^2, with the other experiments contributing much less: K + SK (23%), gallium (18%), and chlorine (3%). Even if one omits without justification the chlorine experiment, the result is barely affected; $P = 3 \times 10^{-5}$ (3.9σ).

The best-fit solutions all correspond, as in the pre-SNO case, to the unphysical result with an identically zero ^{7}Be neutrino flux.

D. Temperature scaling of nuclear reactions

The temperature scalings of the neutrino producing reactions can be derived without considering details of a solar model, requiring only energy conservation and a quasistatic equilibrium [27]. The dominant factor is the exponential temperature dependence of the Gamow penetration factor (see, e.g., Chap. 3 of Ref. [17]). In a one-zone model for the present-day sun, with a fixed temperature and density, one finds [27], for the ^{7}Be neutrino flux,

$$\phi(^7\mathrm{Be}) \propto T^{11} \tag{4}$$

and, for the ^{8}B neutrino flux,

$$\phi(^8\mathrm{B}) \propto T^{25}. \tag{5}$$

These results are in excellent agreement with the scaling found in detailed Monte Carlo studies of complete solar models [28]. The scalings are robust because the Gamow factor depends sensitively on temperature in the region of the exponential tail where nuclear fusion reactions occur [17,29–31].

If the deficit of neutrinos observed in the water Cherenkov experiments were due to astrophysical processes, then one would expect that

$$\left[\frac{\phi(^7\mathrm{Be})}{\phi(^7\mathrm{Be})_{\mathrm{BP2000}}}\right] \geqslant \left[\frac{\phi(^8\mathrm{B})}{\phi(^8\mathrm{B})_{\mathrm{BP2000}}}\right]^{11/25}. \tag{6}$$

Table II shows that, using all six experiments, the best fit for $\phi(^7\mathrm{Be})/\phi(^7\mathrm{Be})_{\mathrm{BP2000}} \geqslant \phi(^8\mathrm{B})\phi(^8\mathrm{B})_{\mathrm{BP2000}}$ is awful, corresponding to a rejection level, 7.4σ, that is so small that it has no practical meaning. The gallium experiments (SAGE and GALLEX/GNO) contribute 50% of the total χ^2, with the chlorine and K + SK both contributing $\sim$20%.

For practical purposes, the fit omitting the SNO result is just about as bad, 6.9σ, as the fit including SNO.

Figure 1 shows the dependence of χ^2_{min} on the minimum allowed value of $\phi(^7\mathrm{Be})$. It is obvious from Fig. 1 that a little ^{7}Be goes a long way.

[3]The case in which SAGE and GALLEX/GNO are treated as independent and the supplementary condition $hep = 0.0$ and $n13 = o15$ is imposed leads to a doubly unphysical result for the parameters that correspond to the minimum χ^2. Not only is the ^{7}Be flux required to be zero, but also the best-fit pep neutrino flux is identically zero (cf. the extreme allowed range for pep given in Sec. III B). For this very unphysical case, the $P = 0.07$, $^8\mathrm{B} = 0.4608$, and the predicted chlorine and gallium rates are, respectively, 2.7 SNU and 81.8 SNU.

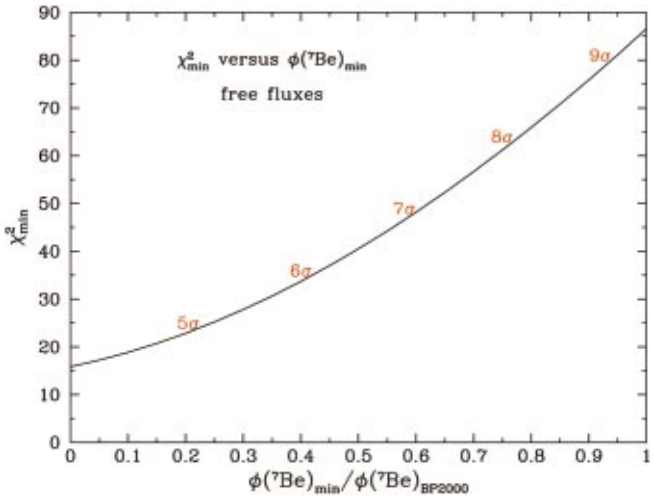

FIG. 1. (Color) A little bit of ^{7}Be goes a long way. The figure shows the best fit $\chi^2_{\rm min}$ versus the minimum allowed value for $\phi(^7\mathrm{Be})$. The neutrino fluxes were treated as free parameters in fitting to the rates of the chlorine, gallium, ν-e scattering, and SNO CC experiments, except that $\phi(^7\mathrm{Be})$ was required to be at least as large as $\phi(^7\mathrm{Be})_{\rm min}$. The neutrino flux is measured in terms of the BP2000 [3] flux. Nuclear fusion scaling arguments give $\phi(^7\mathrm{Be})_{\rm min} \geqslant 0.69$. The figure also shows the corresponding number of standard deviations at which the no-oscillation hypothesis is rejected.

E. Standard solar model

The fit with the standard solar model, BP2000 [3], is shown in the last row of Table II. The fit is bad, but the formal level of the discrepancy is not quite as bad as for the free-parameter case with $\phi(^7\mathrm{Be})/\phi(^7\mathrm{Be})_{\rm BP2000} \geqslant \phi(^8\mathrm{B})\phi(^8\mathrm{B})_{\rm BP2000}$. The reason for the somewhat lower apparent discrepancy is that for the standard solar model the relatively large theoretical uncertainties in the flux predictions are included in the quadratically added errors.

Nevertheless, the agreement between the standard model predictions and the experimental measurements is poor, with formal values of $\sigma = 6.7$ and $P = 3 \times 10^{-11}$.

V. WHY ARE THE ^{7}Be, ^{13}N, ^{15}O, AND *hep* NEUTRINOS REQUIRED TO BE MISSING COMPLETELY?

All of the formal best fits of the undistorted solar neutrino energy spectra to the observed event rates require that the ^{7}Be, ^{13}N, ^{15}O, and *hep* neutrino fluxes be identically zero. On the other hand, these same solutions require a ^{8}B flux that is a reasonable compromise fit to the observed fluxes of ^{8}B neutrinos in the different water Cherenkov detectors (no big surprise) and a *p*-*p* neutrino flux that is typically 9% larger than the predicted standard solar model *p*-*p* neutrino flux (cf. Table II).

The requirement that ^{7}Be, ^{13}N, ^{15}O, and *hep* neutrino fluxes be absent contradicts all astrophysical calculations and basic laboratory astrophysics data [10–14,22,16]. For example, the formal solutions require the complete absence of ^{7}Be neutrinos and only a modest reduction in the flux of ^{8}B neutrinos predicted by the standard solar model. But both ^{7}Be and ^{8}B neutrinos are produced by nuclear interactions on the same parent isotope (^{7}Be), with production of ^{7}Be neutrinos being about 1000 times more probable (according to the standard solar model estimates). Moreover, according to simple estimates and to detailed calculations, the fusion reactions that lead to ^{13}N and ^{15}O neutrinos occur more frequently than the reaction which gives rise to ^{8}B neutrinos.

So why do the minimum χ^2 solutions prefer *p*-*p* neutrinos and abhor ^{7}Be, ^{13}N, ^{15}O, and *hep* solar neutrinos? The answer is simple and is contained in the basic equation that describes the nuclear fusion process that is responsible for the solar luminosity: four protons are burned to form an α particle, two positrons, and two electron-type neutrinos. Thus

$$4p \rightarrow \alpha + 2e^+ + 2\nu_e . \tag{7}$$

Equation (7) shows that two neutrinos are emitted every time four protons are burned to an α particle.

The $\chi^2_{\rm min}$ solutions prefer replacing other neutrinos by *p*-*p* neutrinos since *p*-*p* neutrinos have by far the lowest energies ($\leqslant 0.42$ MeV). Because of their low energies, *p*-*p* neutrinos have the smallest interaction cross sections in gallium solar neutrino detectors and are not detected at all in the chlorine and water Cherenkov experiments. Moreover, the amount of thermal energy communicated to the star depends upon which neutrinos are emitted: if high-energy neutrinos are emitted, then less energy is communicated to the star.

If one replaces in a formal fitting process a higher-energy neutrino (like ^{7}Be or ^{13}N) by a *p*-*p* neutrino, then mathematically the replaced solution wins in two ways: more thermal energy is communicated to the star (making it easier to satisfy the luminosity constraint) and the calculated event rates are lower (in agreement with observations).

VI. DISCUSSION

No satisfactory fit can be found that describes well the observed event rates in solar neutrino experiments, if the different sources of solar neutrinos have amplitudes that can be treated as free parameters but energy spectra that are undistorted.

Table II summarizes quantitatively the situation regarding fits with undistorted energy spectra to the measured solar neutrino event rates. For the pre-SNO era, which includes five experiments (chlorine, Kamiokande, SAGE, GALLEX, and Super-Kamiokande), the no-distortion, no-oscillation hypothesis fails at a formal statistical level of 99% (2.5σ). Considering just the water Cherenkov experiments, SNO, Super-Kamiokande, and Kamiokande, the discrepancy with the no-oscillation hypothesis is at the 3.3σ level, where the formal probability of obtaining a fit as bad as observed is $P = 8 \times 10^{-4}$. These results confirm previously published calculations and are included here to provide an appropriate context.

Two new results on the quality of fits to solar neutrino data are presented in this paper. First, if all six of the solar neutrino experiments (SNO, chlorine, Kamiokande, SAGE, GALLEX, and Super-Kamiokande) are included in the fit, the formal statistical level at which the fit is unsatisfactory

rises to 4.0σ ($P=7\times10^{-5}$). Second, if the temperature scaling of the fusion reactions producing the ^{7}Be and ^{8}B solar neutrinos are taken into account, then the failure of the fitting procedure is at the enormously high 7.4σ level (the formal value of P is ridiculously small).

To achieve even the unsatisfactory fits described in Table II, the $\chi^2_{\rm min}$ solutions require that the ^{7}Be, ^{13}N, and ^{15}O neutrinos be entirely missing. Section V describes the physical reasons leading to this unphysical effect. Since the formal best fits are achieved at the expense of eliminating the ^{7}Be, ^{13}N, and ^{15}O neutrinos, we should really regard the (improbable) solutions found here as even more unlikely because of their physical implausibility.

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[1] SNO Collaboration, Q. R. Ahmad *et al.*, Phys. Rev. Lett. **87**, 071301 (2001).
[2] Super-Kamiokande Collaboration, S. Fukuda *et al.*, Phys. Rev. Lett. **86**, 5651 (2001).
[3] J. N. Bahcall, M. H. Pinsonneault, and S. Basu, Astrophys. J. **555**, 990 (2001).
[4] J. N. Bahcall, Phys. Rev. D **49**, 3923 (1994).
[5] B. T. Cleveland *et al.*, Astrophys. J. **496**, 505 (1998).
[6] Y. Fukuda *et al.*, Phys. Rev. Lett. **77**, 1683 (1996).
[7] SAGE Collaboration, J. N. Abdurashitov *et al.*, Phys. Rev. C **60**, 055801 (1999). The latest results, from 16 April 2001, are given at the SAGE web site:http://EWIServer.npl.washington.edu/SAGE/SAGE.html
[8] GNO Collaboration, M. Altmann *et al.*, Phys. Lett. B **490**, 16 (2000); GALLEX Collaboration, W. Hampel *et al.*, *ibid.* **447**, 127 (1999).
[9] J. N. Bahcall, Phys. Rev. D (to be published).
[10] J. N. Bahcall and H. A. Bethe, Phys. Rev. Lett. **65**, 2233 (1990).
[11] N. Hata, S. Bludman, and P. Langacker, Phys. Rev. D **49**, 3622 (1994).
[12] S. Parke, Phys. Rev. Lett. **74**, 839 (1995).
[13] K. M. Heeger and R. G. H. Robertson, Phys. Rev. Lett. **77**, 3720 (1996).
[14] M. Spiro and D. Vignaud, Phys. Lett. B **242**, 279 (1990); A. Dar and S. Nussinov, Part. World **2**, 117 (1991); V. Castellani *et al.*, Phys. Lett. B **324**, 425 (1994); **329**, 525(E) (1994); V. Berezinsky, Comments Nucl. Part. Phys. **21**, 249 (1994); G. L. Fogli, E. Lisi, and D. Montanino, Phys. Rev. D **49**, 3626 (1994); see also G. L. Fogli and E. Lisi, Astropart. Phys. **13**, 185 (1995).
[15] J. N. Bahcall and P. I. Krastev, Phys. Rev. D **53**, 4211 (1996).
[16] J. N. Bahcall, P. I. Krastev, and A. Yu. Smirnov, Phys. Rev. D **58**, 096016 (1998).
[17] J. N. Bahcall, *Neutrino Astrophysics* (Cambridge University Press, Cambridge, England, 1989).
[18] J. N. Bahcall, Phys. Rev. C **56**, 3391 (1997).
[19] J. N. Bahcall, L. Lisi, D. E. Alburger, L. De Braeckeleer, S. J. Freedman, and J. Napolitano, Phys. Rev. C **54**, 411 (1996).
[20] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vettering, *Numerical Recipes* (Cambridge University Press, Cambridge, England, 1986).
[21] J. N. Bahcall and R. M. May, Astrophys. J. **155**, 501 (1969).
[22] W. Kwong and S. P. Rosen, Phys. Rev. Lett. **73**, 369 (1994); J. N. Bahcall, Phys. Lett. B **338**, 276 (1994).
[23] F. L. Villante, G. Fiorentini, and E. Lisi, Phys. Rev. D **59**, 013006 (1999).
[24] G. L. Fogli, E. Lisi, A. Palazzo, and F. L. Villante, Phys. Rev. D **63**, 113016 (2001).
[25] G. L. Fogli, E. Lisi, D. Montanino, and A. Palazzo, hep-ph/0106247.
[26] C. Giunti, hep-ph/0107310.
[27] J. N. Bahcall and A. Ulmer, Phys. Rev. D **53**, 4202 (1996).
[28] J. N. Bahcall and R. K. Ulrich, Rev. Mod. Phys. **60**, 297 (1988).
[29] H. A. Bethe, Phys. Rev. **55**, 434 (1939).
[30] D. D. Clayton, *Principles of Stellar Evolution and Nucleosynthesis* (University of Chicago Press, Chicago, 1983).
[31] W. A. Fowler, Rev. Mod. Phys. **56**, 149 (1984).