Disoriented chiral condensate in the presence of dissipation and noise

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(Received 21 June 2001; published 3 December 2001)

We have investigated the phase transition and disoriented chiral condensate domain formation in linear σ model. Solving the Langevin equation for the linear σ model, we have shown that, for zero mass pions, the σ-model fields undergo a phase transition above a certain temperature (T_c). For finite mass pions, there is no phase transition. It was also shown that for zero mass pions, σ-model fields, thermalized at a temperature above T_c, when cooled down rapidly, disoriented chiral condensate domains are formed, quite late in the evolution. For massive pions, no large disoriented chiral condensate domain is formed.

DOI: 10.1103/PhysRevC.65.014905 PACS number(s): 25.75.−q, 12.38.Mh, 11.30.Rd

I. INTRODUCTION

Equilibrium high temperature QCD exhibit chiral symmetry if the quarks are assumed to be massless. At a critical temperature T_c, chiral symmetry is spontaneously broken by the formation of a scalar ($\langle \bar{q}q \rangle$) condensate. In recent years there has been considerable excitement about the possibility of the formation of disoriented chiral condensate (DCC) domains [1–7]. The basic idea is simple. In hadron-hadron or in heavy ion collisions, a macroscopic region of space time may be created, within which the chiral order parameter is not oriented in the same direction in the internal $O(4) = \mathrm{SU}(2)\times\mathrm{SU}(2)$ space, as the ordinary vacuum. In heavy ion collisions some region can thermalize at a temperature greater than T_c, the critical temperature for the chiral symmetry restoration. In that region the quark condensate, $\langle \bar{q}q \rangle$ vanishes. Once the system cools below T_c, chiral symmetry is spontaneously broken, and the system evolve back to the true ground state, where the condensate $\langle \bar{q}q \rangle \neq 0$. Rajagopal and Wilczek [1–3] argued that in the nonlinear environment (as will be the case in heavy ion collisions), as the temperature drops below T_c, the chiral symmetry may begin to break by developing domains, in which the chiral field is misaligned from its true vacuum value. In hadronic collisions, the space-time picture of DCC formation is different [4–7]. In hadron-hadron collisions, most of the models of particle production, stringy or partonic, put the bulk of the space-time activity near the light cone. The flow of produced quanta are concentrated in a rather thin shell expanding from the collision point at the speed of light. As the interior is separated from the true vacuum, by rapidly expanding hot shell of partons, well inside it, quark condensate may be chirally rotated from its usual (vacuum) direction. This is the so-called Baked Alaska model of DCC production. The misaligned condensate has the same quark content and quantum numbers as do pions and essentially constitute a classical pion field. The system will finally relax back to the true vacuum and in the process can emit coherent pions. Possibility of producing classical pion fields in heavy ion collisions had been discussed earlier by Anslem [8]. DCC formation in hadronic or in heavy ion collisions can lead to the spectacular events that some portion of the detector will be dominated by charged pions or by neutral pions only. In contrast, in a general event, all the three pions (π^+, π^-, and π^0) will be equally well produced. This may then be the natural explanation of the so-called Centauro events [9].

If a collision results in a single, large DCC domain, the phenomena may be easily detected. Then comparing the measured spectra of neutral and charged pions on an event-by-event basis, one could have identified the events dominated by DCC formation. The probability for a domain to yield a particular fraction (f) of neutral pions is $P(f) = 1/2\sqrt{f}$, provided all the isospin orientations are equally likely [4,8,10,11]. In contrast, a typical hadronic reaction, without DCC, will produce a binomial distribution of f, peaked at the isospin symmetric value of 1/3. Sizes of domains are then of great importance. A collision resulting in a single large domain can be easily identified from the ratio f. On the other hand, if a collision results in a number of small randomly oriented domains, the result will again be a binomial distribution, typical of a hadronic collision.

After the conjecture about DCC formation by Rajagopal and Wilczek [1–3] and also by Bjorken [4] and Kowalski and Tayolor [5], several authors have studied the phenomena [12–18]. Microscopic physics governing DCC phenomena is not well known. It is in the regime of nonperturbative QCD, and also a nonlinear phenomenon, theoretical understanding of both of which is limited. One thus uses some effective field theory, such as linear σ models with various approximations, to simulate DCC. Linear σ model, usually with Hartree-type of approximation, has been used extensively to simulate DCC phenomena [12–14]. In the linear σ model chiral degrees of freedom are described by the real $O(4)$ field $\Phi = (\sigma, \vec{\pi})$. Because of the isomorphism between the groups $O(4)$ and $\mathrm{SU}(2)\times\mathrm{SU}(2)$, the later being the appropriate group for QCD with two flavors, linear σ model can effectively model the low energy dynamic of QCD [19]. Rajagopal and Wilczek [1–3] used quench initial condition to simulate the nonequilibrium evolution of the chirally symmetric fields. The quench scenario assumes that the effective potential, governing the evolution of the long wave length modes, immediately after the phase transition at T_c, turns into a, zero temperature, classical potential. It can happen

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only in case of very rapid cooling and expansion of the fireball. In heavy ion collisions quenchlike initial conditions are unlikely. Gavin and Mueller [13] considered the annealing scenario, in which the condensate evolve from $\langle\phi\rangle\sim 0$ to $\langle\phi\rangle\sim f_\pi$ slowly (as in the standard annealing) but in which the nonequilibrium conditions, which allow amplification of long wavelength modes, are maintained nevertheless. Explicit simulation with linear σ model, indicate that DCC depends, critically on the initial field configuration. With quenchlike initial condition DCC domains of 4–5 fm in size can form [14]. With initial conditions other than quench, much smaller domains are formed.

Very recently, effect of external media on possible DCC is being investigated [20–28]. Indeed, in a heavy ion collision, even if some region is created where chiral symmetry is restored, that region will be continually interacting with surrounding medium (mostly pions). Biro and Greiner [20], using a Langevin equation for the linear σ model, investigated the interplay of friction and white noise on the evolution and stability of collective pion fields. In general friction and noise reduces the amplification of zero modes (they considered the zero modes only). But in some trajectories, large amplification may occur. We have also investigated the interplay of friction and noise on possible DCC phenomena [23–27]. Simulation studies in (2+1) dimension indicate that with quench-like initial condition, large disoriented chiral condensate domains can still be formed. We have also performed a simulation in (3+1) dimension. It was seen that with zero mass pions, even with thermalized fields as initial condition, DCC domains may be formed, late in the evolution. However, as will be shown in the present paper, with finite mass pions, different results are obtained. We find no DCC domain structure of appreciable size, even late at evolution.

In the present paper, we have investigated, in detail, the properties of phase transition in linear σ model and the subsequent DCC domain formation. The paper is organized as follows. In Sec. II, we briefly describe the Langevin equation for linear σ model and show that for zero mass pions, the model undergoes a second order phase transition at a finite temperature. For massive pions there is no exact phase transition in the model, though the symmetry is restored to a large extent at high temperature. In Sec. III, growth of disoriented chiral condensate domains will be studied. It will be shown that, for zero mass pions, if the fields are cooled rapidly, large DCC domains can form. For massive pions, we find no indication of DCC domain formation. In Sec. IV, we will vindicate the results obtained in Sec. III, from pion-pion correlation study. The probability distribution function for the neutral to total pion ratio will be presented in Sec. IV. Lastly, in Sec. V, summary and conclusions will be drawn.

II. LINEAR σ MODEL AND PHASE TRANSITION

The linear σ-model Lagrangian can be written as

$$\mathcal{L}=\frac{1}{2}(\partial_\mu\Phi)^2-\frac{\lambda}{4}(\Phi^2-v^2)^2+H\sigma, \tag{1}$$

where chiral degrees of freedom are the $O(4)$ fields $\phi_a=(\sigma,\vec{\pi})$. In Eq. (1) H is the explicit symmetry breaking term. This term is responsible for finite pion masses. The parameters of the model, λ, v, and H can be fixed using the pion decay constant f_π,σ, and pion masses. With standard parameters, $f_\pi=92$ MeV, $m_\sigma=600$ MeV, and $m_\pi=140$ MeV, one obtains

$$\lambda=\frac{m_\sigma^2-m_\pi^2}{2f_\pi^2}\sim 20,\quad v^2=f_\pi^2-\frac{m_\pi^2}{\lambda}=(87\ \text{MeV})^2,$$
$$H=f_\pi m_\pi^2=(122\ \text{MeV})^3.$$

The potential in Eq. (1) corresponds to zero temperature. At finite temperature (T), to leading order in λ, thermal fluctuations $\langle\delta\phi^2\rangle$ of the pion and σ mesons generate an effective Hartree-type dynamical mass giving rise to an effective, temperature dependent potential. In the high temperature expansion, this results in [19]

$$m_{th}^2\rightarrow\frac{\lambda}{2}T^2. \tag{2}$$

As told in the beginning, if in a heavy ion collision, a certain region undergoes chiral symmetry restoration, that region must be in contact with some environment or background. Exact nature of the environment is difficult to determine but presumably it consists of mesons and hadrons (pions, nucleons etc.). Recognizing the uncertainty in the exact nature of the environment, we choose to represent it by a white noise source, i.e., a heat bath. To analyze the effect of environment or heat bath, on the possible disoriented chiral condensate, we propose to study Langevin equation for linear σ model. Langevin equation for $O(4)$ fields, has been used previously, to investigate, the interplay of noise and dissipation, on the evolution and stability DCC. Recently, it has been shown that in the ϕ^4 theory, hard modes can be integrated out on a two-loop basis, resulting in Langevin-type equations for the soft modes [21,22]. The resulting equation is nonlocal in time and quite complicated. If a Markovian limit exists to this equation, then one may hope to obtain a simpler equation (as the present equation), which can be used for practical purposes.

We write the Langevin equation for linear σ model as

$$\left[\frac{\partial^2}{\partial\tau^2}+\left(\frac{1}{\tau}+\eta\right)\frac{\partial}{\partial\tau}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-\frac{1}{\tau^2}\frac{\partial^2}{\partial Y^2}\right.$$
$$\left.+\lambda(\Phi^2-f_\pi^2-T^2/2)\right]\Phi=\zeta(\tau,x,y,Y), \tag{3}$$

where τ is the proper time and Y is the rapidity, the appropriate coordinates for heavy ion scattering. To be consistent with fluctuation-dissipation theorem, we have included a dissipative term η in the equation. As told earlier, the environment or the heat bath (ζ) was represented as a white noise source with zero average and correlation as demanded by fluctuation-dissipation theorem,

$$\langle \zeta(\tau,x,y,Y)\rangle = 0, \tag{4a}$$

$$\langle \zeta_a(\tau,x,y,Y)\zeta_b(\tau',x',y',Y')\rangle = 2T\eta\frac{1}{\tau}\delta(\tau-\tau')\delta(x-x') \times \delta(y-y')\delta(Y-Y')\delta_{ab}, \tag{4b}$$

where a,b corresponds to π or σ fields. The noise term will continuously heat the system, while the dissipative term will counteract it. Equilibrium is achieved when the system is thermalized at the temperature dictated by the heat bath. We note that Eq. (3) cannot be derived formally, but as will be shown below, it does describe the equilibrium physics correctly. Thus at least on phenomenological level, its use can be justified, and we can hope that it will describe correctly the nonequilibrium physics, as required for the DCC phenomena.

Before we proceed further, a few words are necessary about the applicability of Langevin equation, in describing DCC, when the noise term contains temperature. DCC is basically a nonequilibrium phenomena. By using temperature, we are approximating it as a equilibrium one. This approximation is valid when the system is not far from equilibrium. Fluctuationdissipation relation is also valid for such a system only. In such a system, it is possible to define a temperature at each point of time, if the time scale of the constituents of the system is large compared to the time scale of the collective variable (in this case expansion of the system).

Set of partial differential Eqs. (3) were solved on a 32^3 lattice, using a lattice spacing of 1 fm and with periodic boundary conditions. Solving Eqs. (3) require initial conditions (ϕ and $\dot{\phi}$). We distribute the initial fields according to a random Gaussian, with

$$\langle\sigma\rangle = [1-f(r)]f_\pi, \tag{5a}$$

$$\langle \pi_i\rangle = 0, \tag{5b}$$

$$\langle\sigma^2\rangle - \langle\sigma\rangle^2 = \langle\pi_i^2\rangle - \langle\pi_i\rangle^2 = f_\pi^2/4f(r), \tag{5c}$$

$$\langle\dot{\sigma}\rangle = \langle\dot{\pi}_i\rangle = 0, \tag{5d}$$

$$\langle\dot{\sigma}^2\rangle = \langle\dot{\pi}\rangle^2 = f_\pi^2 f(r). \tag{5e}$$

The interpolation function

$$f(r) = [1+\exp(r-r_0)/\Gamma)]^{-1} \tag{6}$$

separates the central region from the rest of the system. We have taken $r_0=11$ fm and $\Gamma=0.5$ fm. The initial field configuration corresponds to a quenchlike condition [1,2], but it is important to note that the field configuration at equilibrium will be independent of the initial configuration. The other parameter of the model is the friction. The friction coefficient was assumed to be $\eta=\eta_\pi+\eta_\sigma$, where $\eta_{\pi,\sigma}$ are the friction coefficient of the pion and the σ fields. They have been calculated by Rischke [28],

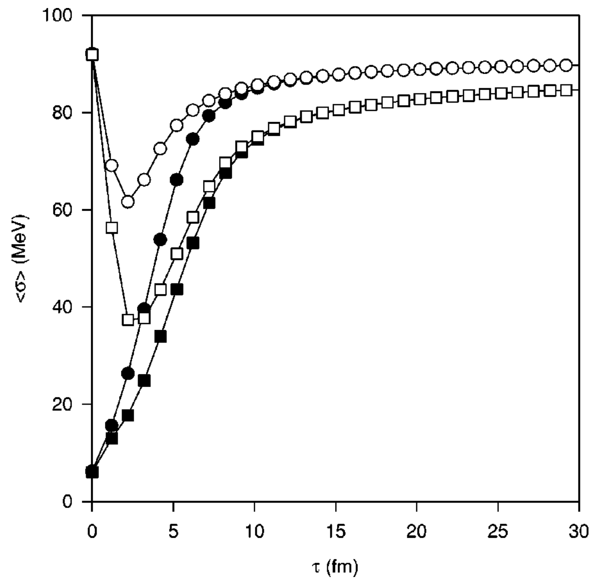

FIG. 1. Evolution of the σ condensate with (proper) time for two different fixed temperature 20 MeV and 40 MeV of the heat bath. Two different initial condition evolve to the same condensate value (see text).

$$\eta_\pi = \left(\frac{4\lambda f_\pi}{N}\right)^2 \frac{m_\sigma^2}{4\pi m_\pi^3}\sqrt{1-\frac{4m_\pi^2}{m_\sigma^2}}\frac{1-\exp(-m_\pi/T)}{1-\exp(-m_\sigma^2/2m_\pi T)} \times \frac{1}{\exp[(m_\sigma^2-2m_\pi^2)/2m_\pi T]-1}., \tag{7a}$$

$$\eta_\sigma = \left(\frac{4\lambda f_\pi}{N}\right)^2 \frac{N-1}{8\pi m_\sigma}\sqrt{1-\frac{4m_\pi^2}{m_\sigma^2}}\coth\frac{m_\sigma}{4T} \tag{7b}$$

Equilibrium physics will be independent of the exact value of the friction also. Friction just determines the rate of approach towards the equilibrium.

To show that the equilibrium physics is independent of the initial field configuration, in Fig. 1, we have shown the (proper) time evolution of the condensate value of σ field,

$$\langle\sigma\rangle = \frac{\int \sigma dx dy dY}{\int dx dy dY}, \tag{8}$$

for two different initial conditions. In one of them, the initial configuration was chosen such that $\langle\sigma\rangle\sim f_\pi$, (depicted in Fig. 1 as white symbol) and in the other, the initial configuration was chosen such that $\langle\sigma\rangle\sim 0$ (depicted in the figure by the filled symbol). We have shown the evolution of the condensate for two fixed temperatures of the heat bath, 20 MeV and 40 MeV. Equilibrium configuration is reached after ~ 10 fm of evolution. For both the temperatures, the equilibrium condensate value is independent of the initial value. It is also noted that at higher temperature, the $\langle\sigma\rangle$ condensate (as expected) settles to a lower value.

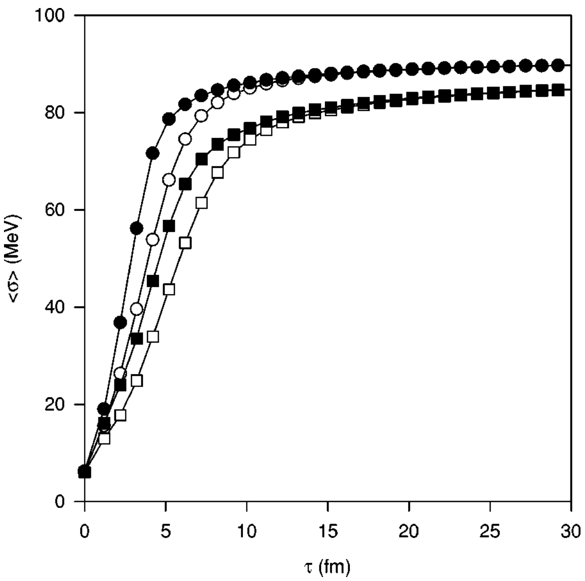

FIG. 2. Evolution of the σ condensate with (proper) time for two different fixed temperature 20 MeV and 40 MeV of the heat bath, showing that equilibrium value is independent of friction constant. Two different friction constant results into same equilibrium value (see text).

Equilibrium configuration is also independent of the exact value of the friction. In Fig. 2, we have plotted the evolution of σ condensate for two different values of the friction; (i) η as given in Eq. (5) and (ii) half of its value. Here again we perform the calculation for two different temperatures of the heat bath, 20 and 40 MeV. It can be seen the equilibrium value of the condensate does not depend on the exact value of the friction. Friction determines the rate at which equilibrium is achieved.

Having thus shown that the equilibrium value of the fields are independent of the initial condition or the exact value of the friction, we now proceed to study the phase transition in the model. To this end, we keep the heat bath at fixed temperature (T) and evolve the fields for sufficiently long time such that equilibrium is reached. The condensate value of the σ field at the equilibrium can be considered as the order parameter for the chiral phase transition. In the symmetry broken phase it will have nonzero value while in the symmetric phase it will vanish. We note that for symmetry restoration phase transition the order parameter should *exactly* vanish for temperatures greater and equal to T_c [29]. Small but nonzero value of the condensate will indicate approximate symmetry restoration.

In Fig. 3, we have shown the equilibrium condensate value of the σ field at different temperatures. With the explicit symmetry breaking term in Eq. (1) the pions are massive. Thus one should not get exact symmetry restoring phase transition in the model. Indeed, in our simulation also, exact phase transition is not obtained. At low temperature, condensate value as expected, is around f_π, and it decreases with temperature. However, though it became very small, it do not vanish even at very large temperature. Thus there is no exact phase transition in the model. However, very small value of $\langle\sigma\rangle$ at high temperature, in comparison to its zero temperature value, indicate that the symmetry is restored to a large extent.

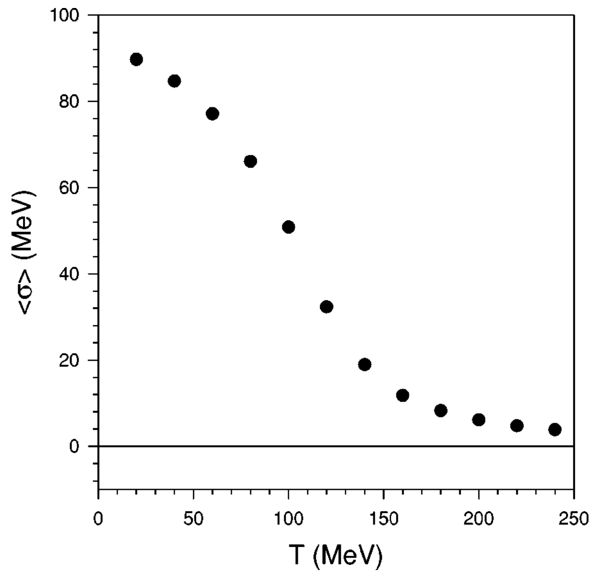

FIG. 3. Equilibrium value of the σ condensate as a function of temperature. The pions are massive.

If the explicit symmetry breaking term is neglected, then there is exact chiral symmetry in the model. In Fig. 4, we have presented the equilibrium condensate values of the σ field as a function of temperature for zero mass pions. Now we find that σ condensate exactly vanishes for temperature higher than 120 MeV. The model shows that with zero mass pions, there is a symmetry restoring phase transition at high temperature.

III. DISORIENTED CHIRAL CONDENSATE DOMAIN FORMATION

In the last section, we have shown that the Langevin equation correctly reproduces the equilibrium behavior of the

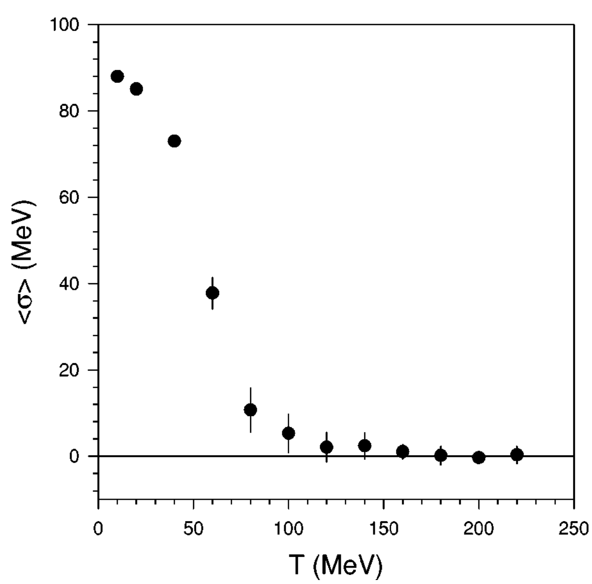

FIG. 4. Equilibrium value of the σ condensate as a function of temperature. The pions are massless.

linear σ model fields. For zero mass pions, simulation of the Langevin equation shows symmetry restoring phase transition at appropriate temperature. For massive pions it was shown that, though there is no exact symmetry restoring phase transition, nevertheless symmetry is restored to a great extent.

Total or partially chiral symmetric phase (as is the case for zero or finite mass pions) at high temperature, will roll back to symmetry broken phase as the system cools and temperature drops below the transition temperature. As told earlier, it is has been conjectured [1,2] that during the roll down period, pseudoscalar condensate, $\langle \overline{q}\tau\gamma_5 q\rangle$ can assume nonvanishing values (instead of remaining zero as in the ground state). In the process, domainlike structures, with definite isospin orientations, may emerge during the roll down period. Numerical simulations of linear σ model, with quenchlike initial field configuration, shows domainlike structures [14]. It was also seen that $\pi\pi$ correlation is also increased [14]. However, in heavy ion collision, quench is not a natural initial condition. $\langle\phi\rangle$ and $\langle\dot{\phi}\rangle$ are in a configuration appropriate for high temperature but that of $\langle\phi^2\rangle$ and $\langle\dot{\phi}^2\rangle$ are characteristic of a lower temperature. On the contrary, thermalized fields are better suited to mimic initial conditions that may arise in heavy ion collisions. Here, $\langle\phi\rangle,\langle\dot{\phi}\rangle$ as well as $\langle\phi^2\rangle$ and $\langle\dot{\phi}^2\rangle$ are in configurations appropriate for high temperature.

To see whether domainlike structure emerges or not, with more appropriate initial condition such as the thermalized fields, we proceed as follows. Using the Langevin equation [Eq. (3)], we thermalize the fields at temperature $T=200$ MeV. Next we use the thermalized fields as the initial condition, and now we allow the heat bath to cool and follow the evolution of the thermalized fields. At this stage, evolution of the thermalized fields will depend on the exact nature of the friction, however, we choose to use the same friction as before. We assume the following cooling law for the heat bath,

$$T(t)=T_0\frac{1}{t^n}, \tag{9}$$

with $n=1$, appropriate for three-dimensional (3D) scaling expansion, and we call it fast cooling law. We have also used $n=1/3$ (slow cooling), appropriate for one-dimensional scaling expansion of massless ideal gases for some demonstrative calculations.

Assuming that the number density is proportional to the square of the fields amplitude ($N_\pi \propto \phi_\pi^2$), at each lattice point, we calculate the neutral to total pion ratio according to

$$f(x,y,Y)=\frac{N_{\pi_0}}{N_{\pi_1}+N_{\pi_2}+N_{\pi_0}}. \tag{10}$$

Very large or small value of the ratio, over an extended spatial zone will be definite indication of disoriented chiral condensate domain formation.

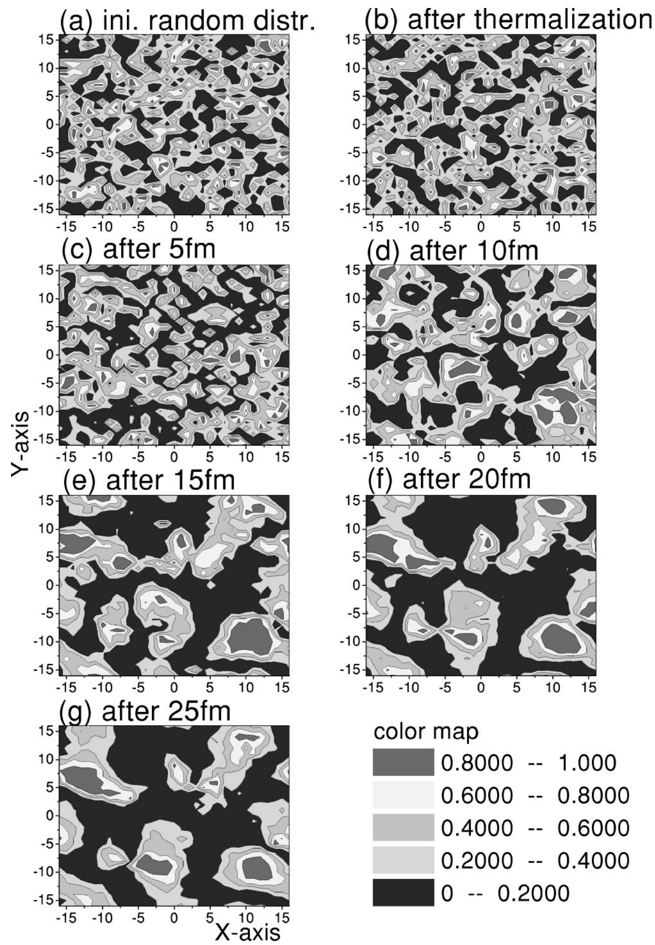

FIG. 5. Contour plot of the neutral to total pion ratio, at rapidity $Y=0$. The explicit symmetry breaking term is omitted, i.e., pions are massless. The cooling law corresponds to fast cooling law. Different panels shows the evolution of the ratio at different times. Domainlike structure is evident at late times.

A. DCC domains without explicit symmetry breaking term

We first consider the case of zero mass pions, i.e., when the symmetry breaking term in Eq. (1) is neglected. For zero mass pions there is a exact symmetry restoring phase transition, the condensate value of the $\langle\sigma\rangle$ field vanishes around $T=120$ MeV. We have performed simulations for 100 events. In the following we have shown results for a single, representative event, which belong to the average class. No (pre)selection has been done, e.g., for events containing one large domain in the analysis. While good DCC events are expected to be rare, and they might produce extraordinary signals [20,23].

In Fig. 5, evolution of the neutral to total pion ratio at rapidity $Y=0$, with fast cooling, is shown. The initial distribution (panel a) do not show any domainlike structure with very high or low value of the ratio (the distribution was random). No domainlike structure is seen even after thermalization of the fields at $T=200$ MeV (panel b). Large domainlike structures with high/low value of the ratio f starts to emerge after 10–15 fm of evolution and cooling. With time domainlike structures grow. It seems that, indeed disoriented chiral condensate domains are formed in presence of noise

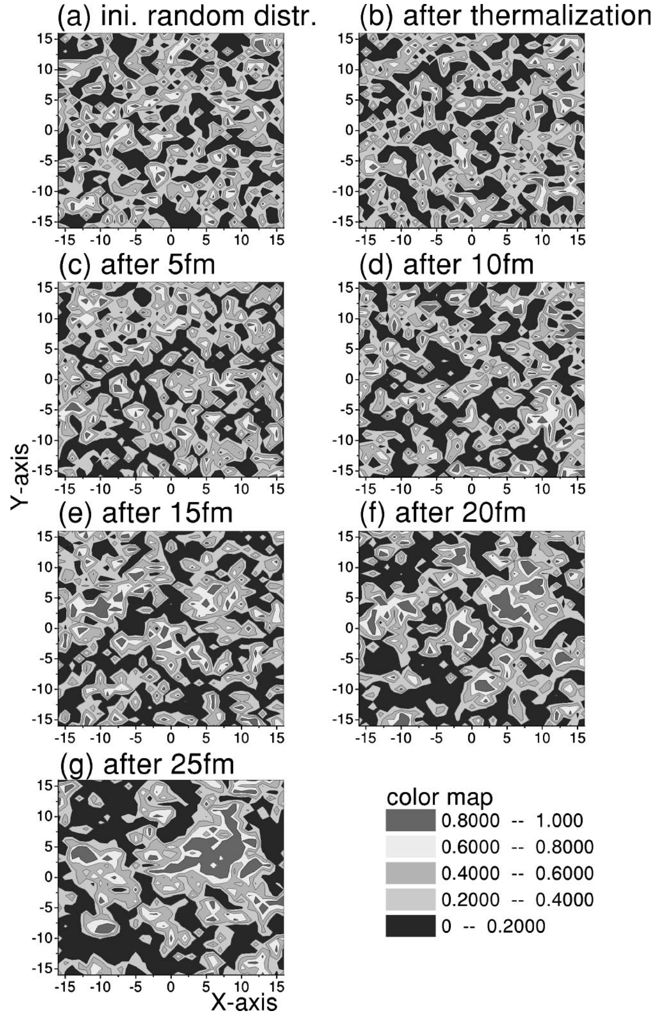

FIG. 6. Contour plot of the neutral to total pion ratio, at rapidity $Y=0$. The explicit symmetry breaking term is omitted, i.e., pions are massless. The cooling law corresponds to slow cooling law. Different panels shows the evolution of the ratio at different times. Domainlike structure is evident at very late times.

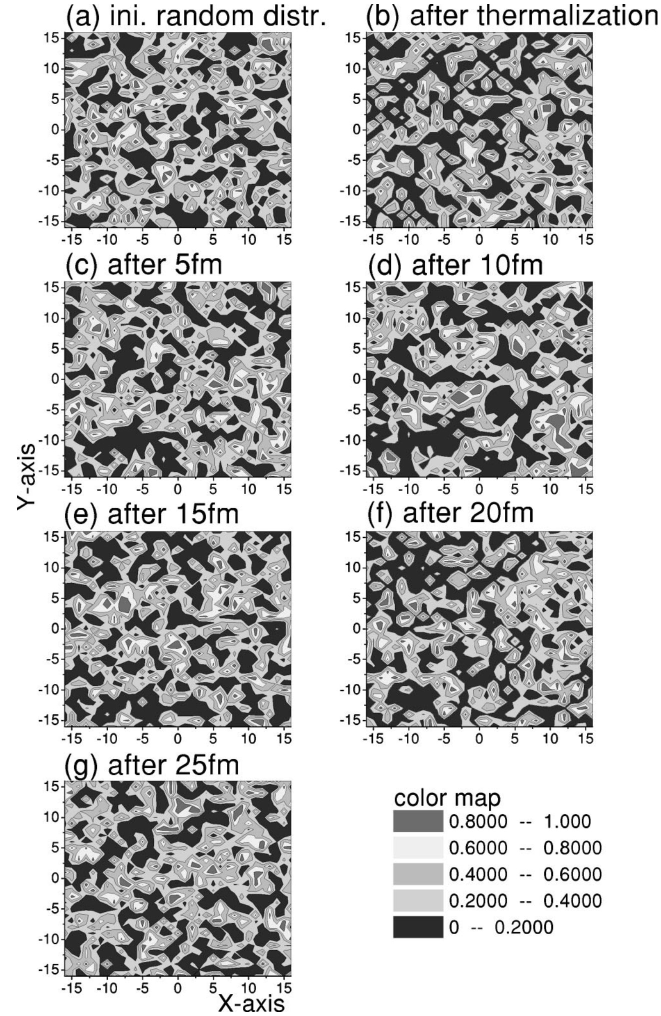

FIG. 7. Contour plot of the neutral to total pion ratio, at rapidity $Y=0$. The explicit symmetry breaking term is included, i.e., pions are massive. The cooling law corresponds to fast cooling law. Different panels shows the evolution of the ratio at different times. No domainlike structure is evident even at late times.

and friction, even with thermalized fields. At other rapidities also, similar behavior is seen. Figure 5, clearly demonstrate that with fast cooling law, multiple disoriented chiral condensate domains, with definite isospin orientations can form, as the chirally symmetric phase roll back to symmetry broken phase. However, domain formation occurs quite late in the evolution. By the time domainlike structures emerge, the system is cooled to $\sim$20 MeV. Applicability of classical field theory down to that low temperature is questionable. Apart from that it is doubtful whether in heavy ion collision, system can be cooled to this extent. Current wisdom is that the hadrons freeze out around 100–160 MeV. With this reservation in mind, it may be said that, even with thermal fields, domains of disoriented chiral condensate can form. But the domain growth occurs at late times.

To see whether domainlike formation occurs if the system is cooled slowly, we have repeated the above calculation with the slow cooling. In Fig. 6, contour plot of the neutral to total pion ratio, at rapidity $Y=0$ is shown. In this case also, at the early stage of the evolution, there is no indication of domainlike structures. Only at very late time, 20 fm or so, there is a indication of domain formation. Similar results are obtained at other rapidities also. If the cooling is slow, the fields get enough time, to adjust to new environment, consequently energy can be equipartitioned among all the modes. In heavy ion collision, 1D scaling expansion is more appropriate than 3D scaling expansion.

Present simulations indicate that for massless pions, DCC domains can form even with thermalized fields as initial condition. However, domain formation occur only at late stage of the evolution.

B. DCC domains with explicit symmetry breaking term

For finite mass pions, there is no exact phase transition in the model. However, symmetry is partially restored. To see whether DCC domains are formed even when there is no exact phase transition, we repeat the above calculation including the symmetry breaking term. In Fig. 7, xy contour plot of the neutral to total pion ratio at rapidity $Y=0$ is

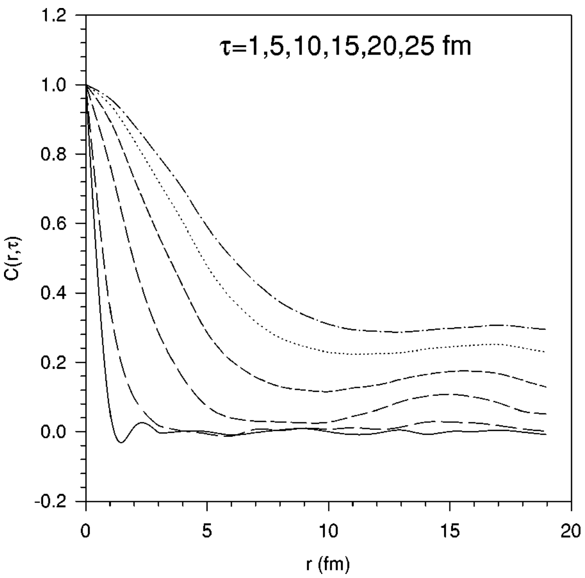

FIG. 8. Evolution of the correlation function with time for zero mass pions at rapidity $Y=0$. Cooling law corresponds to fast cooling law. Long range correlation develops at late times.

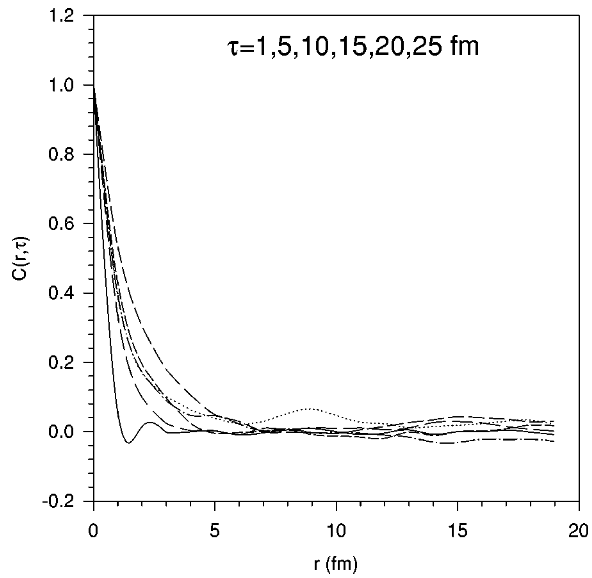

FIG. 9. Evolution of the correlation function with time for massive pions at rapidity $Y=0$. The cooling law corresponds to fast cooling law. No long range correlation develops at late times.

shown. The cooling was fast. We do not find any domainlike configuration. At other rapidities also, similar results are obtained. With slow cooling law also, though not shown, we do not find any indication of DCC domain formation. Indeed, if domains are not formed with fast cooling law, then it is unlikely that they will be formed with a slow cooling law. The simulation results indicate that, for massive pions, when thermalized fields are used as initial condition, disoriented chiral condensate domains are not formed.

IV. PION-PION CORRELATION

We define a correlation function at rapidity Y as [14],

$$C(r,\tau)=\frac{\sum_{i,j} \pi(i)\dot{\pi}(j)}{\sum_{i,j} |\pi(i)||\pi(j)|}, \tag{11}$$

where the sum is taken over those grid points i and j, such that the distance between them is r. In Fig. 8, temporal evolution of the correlation function, at rapidity $Y=0$, for massless pions are shown. The cooling was fast. Initially, the thermalized pions have no correlation length beyond the lattice spacing of 1 fm. With time correlation length increases. But really large, long range correlation builds up quite late in the evolution, after 10–15 fm of evolution. Pions separated by large distances are then correlated. Similar results are obtained at other rapidities also. The results are in accordance with the contour plot of the neutral to total pion ratio, showing DCC domain formation for zero mass pions at late times.

With finite mass pions, results are quite different (Fig. 9). In this case, as shown earlier, there is no exact phase transition. Also, evolution of thermalized fields do not result into large domainlike formation. Similarly, we find that there is not much long range correlation even at the late stage of the evolution.

V. PROBABILITY DISTRIBUTION OF NEUTRAL TO PION RATIO

If a single DCC domain is formed in a heavy ion collision, it can be easily detected. Probability to obtain a particular fraction,

$$f=\frac{N_{\pi^0}}{N_{\pi^+}+N_{\pi^-}+N_{\pi^0}}, \tag{12}$$

of neutral pions from a single domain is $P(f)=1/2\sqrt{f}$ [1,2]. However, our simulations indicate that if at all domains are formed, there will be quite a few them. Naturally the resultant distribution will not be $1/\sqrt{f}$ type. In this section we obtain the probability distribution of neutral to total pion ratio.

We calculate the number of pions at rapidity Y by integrating square of the amplitude of the pion fields over the spacetime as

$$N_{\pi}(Y)=\int \pi^2\tau d\tau dx dy. \tag{13}$$

In Fig. 10, probability distribution, f, for neutral to total pion ratio, for 100 events are shown. The pions are massless. In three panels, we have shown the distribution obtained after 10, 20, and 30 fm of evolution. If the fields are evolved up to 10 fm, then the f distribution is sharply peaked around the isospin symmetric average value of 1/3. At the early stage of the evolution, as indicated in Fig. 5, domainlike structures or long range correlations are not developed . As a consequence of that, the neutral to pion ratio is peaked

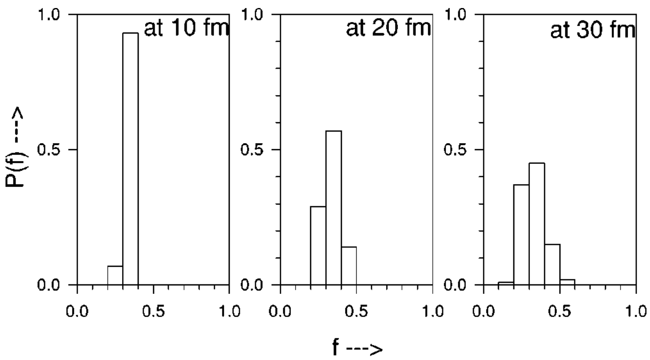

FIG. 10. Probability distribution for the neutral to total pion ratio, at different times. The symmetry breaking term is omitted. The cooling law corresponds to fast cooling law.

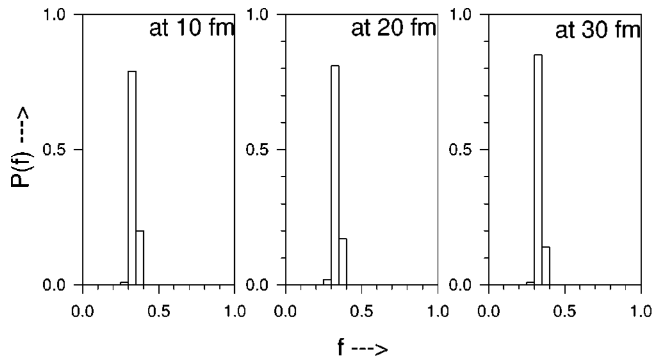

FIG. 11. Probability distribution for the neutral to total pion ratio, at different times. The symmetry breaking term is included. The cooling law corresponds to fast cooling law.

around the isospin symmetric value of 1/3. If the fields are evolved for longer duration (20 or 30 fm), average of the distribution remains 1/3, but it gets broadened. As shown earlier, at late stage of the evolution, there are definite domainlike structures in individual events. Long range correlations also develop. Its effect is seen as the increased width of the probability distribution. However, increase in width is not as dramatic as may be expected from the contour plot of the ratio or the correlation studies. In obtaining the f distribution, we have integrated the pion amplitudes over time. Domainlike structures develop only at late time. Thus in a single event also, non-DCC pions will be mixed up with DCC pions.

However, quite different results are obtained if the symmetry breaking term is included. The pions are massive then. Probability distributions, for massive pions, are shown in Fig. 11. Whether the fields are evolved up to 10, 20, or 30 fm, the distributions are sharply peaked around the isospin symmetric value of 1/3. We have seen that for massive pions, even at late time, domains are not formed. Also long range correlations do not develop. The probability distributions reflect those results.

VI. SUMMARY AND CONCLUSIONS

In summary, we have studied disoriented chiral condensate formation in presence of the background. We have simulated the Langevin equation for linear σ model, in (3+1) dimensions. It was shown that the model reproduces equilibrium physics correctly. For zero mass pions, the model undergoes a second order phase transition at $T_c \sim 120$ MeV. Also the model correctly shows that, for finite mass pions, even though symmetry is restored to a great extent, there is no exact phase transition. Simulation studies indicate that, for zero mass pions, if the thermalized fields are cooled down rapidly, multiple disoriented chiral condensate domains are formed, while for slow cooling no domainlike structure is formed. Long range correlations also develop in case of rapid cooling. However, even with rapid cooling, with thermalized fields, domains or long range correlations develop quite late in the evolution (after 10–15 fm of evolution). Probability distribution of neutral to total pion ratio shows that the distribution is not $1/\sqrt{f}$ type, rather it is a Gaussian with average at the isospin symmetric value of 1/3. The width of the Gaussian increases, when domains are formed.

For massive pions our simulations indicate that thermalized fields do not evolve into domainlike structure, irrespective of slow or fast cooling law. Long range correlations also do not develop, even at late time. The probability distribution of neutral to total pion ratio do not show any broadening of width as was seen for zero mass pions.

To conclude, simulation studies of Langevin equations for linear σ model indicate that for realistic pions, thermalized fields do not evolve into DCC domainlike structure.

ACKNOWLEDGMENTS

The author gratefully acknowledges the kind hospitality of Center for Theoretical Physics, MIT and McGill University, where part of the work was done. The author also thanks K. Rajagopal, C. Gale, J. Randrup, and D. Rischke for discussions and suggestions.

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