Synthesis of superheavy nuclei: How accurately can we describe it and calculate the cross sections?

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A thorough analysis of all stages of heavy ion fusion reaction leading to the formation of a heavy evaporation residue has been performed. The main goal of the analysis was to gain better understanding of the whole process and to find out what factors and quantities, in particular, bring major uncertainty into the calculated cross sections, how reliable the calculation of the cross sections of superheavy element formation may be and what additional theoretical and experimental studies should be made in this field.

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I. INTRODUCTION

The interest in the synthesis of superheavy nuclei has lately grown due to the new experimental results [1–4] demonstrating a real possibility of producing and investigating the nuclei in the region of the so-called ''island of stability.'' The new reality demands a more substantial theoretical support of these expensive experiments which will allow a more reasonable choice of fusing nuclei and collision energies as well as a better estimation of the cross sections and unambiguous identification of evaporation residues (ER). Unfortunately, at present it is quite difficult (and hardly possible) to make an accurate qualitative analysis of the complex dynamics of the heavy ion fusion reaction leading to the formation in the exit channel of ER of easily fissile superheavy nucleus. However, lately a number of speculative papers have appeared in which predictions are made in terms of rather simplified models concerning the values of formation cross sections of new superheavy elements in reactions with different colliding nuclei. A thorough analysis of all reaction stages is made in the present study. It is aimed at better understanding of how well we can describe them in the framework of existing theoretical models, what quantities the cross sections are sensitive to and how accurate the values of these quantities have been determined, and finally, how accurate are the predictions of the cross sections of the heavy ER produced in reactions for which no experimental data are available.

The production cross section of a cold residual nucleus B, which is the product of light particle evaporation and γ emission from an excited compound nucleus C, formed in the fusion process of two heavy nuclei $A_1+A_2\to C\to B+n,p,\alpha,\gamma$ at a center-of mass energy close to the Coulomb barrier in the entrance channel, is usually decomposed over partial waves and given by

$$\sigma_{\rm ER}^{A_1+A_2\to B}(E)\approx\frac{\pi\hbar^2}{2\mu E}\sum_{l=0}^{\infty}(2l+1)T(E,l)\times P_{CN}(A_1+A_2\to C;E,l)P_{\rm ER}(C\to B;E^*,l). \quad (1)$$

Here $T(E,l)$ is the probability of the colliding nuclei to overcome the potential barrier in the entrance channel and reach the point of contact $R_{\rm cont}=R_1+R_2$, which is, as a rule, smaller than the radius of the Coulomb barrier R_C^B by 2 or 3 fm, R_1 and R_2 are the radii of the nuclei. P_{CN} is the probability that the nuclear system will evolve from a configuration of two touching nuclei into a spherical or nearly spherical form of the compound mononucleus. In the course of this evolution the heavy system may, in principle, fall again into two fragments without forming the compound nucleus (quasifission) and, thus, $P_{CN}\leqslant 1$. The last term in Eq. (1), $P_{\rm ER}(C\to B)$, defines the probability of producing the cold evaporation residue B in the process of the compound nucleus C decay. It has the initial excitation energy $E^*=E-Q_{gg}^{\rm fus}$, where E is the beam energy in the center-of-mass system, $Q_{gg}^{\rm fus}=M(C)c^2-M(A_1)c^2-M(A_2)c^2$, and $M(C)$, $M(A_1)$, $M(A_2)$ are the nuclear masses. In order to avoid hereinafter a confusion in terminology, we shall also define the ''capture cross section'' and the ''fusion cross section'' as follows: $\sigma_{\rm capt}(E)=(\pi\hbar^2/2\mu E)\Sigma_{l=0}^{\infty}(2l+1)T(E,l)$, $\sigma_{\rm fus}(E)=(\pi\hbar^2/2\mu E)\Sigma_{l=0}^{\infty}(2l+1)T(E,l)\cdot P_{CN}(E,l)$.

Equation (1) is an approximation since the whole process of the compound nucleus formation and decay is divided here into three individual reaction stages even if connected with each other but treated and calculated separately: (1) approaching the point of contact $R_1+R_2\leqslant r<\infty$, (2) formation of the compound mononucleus $A_1+A_2\to C$, (3) decay (''cooling'') of the compound nucleus C.

A possibility of such a division is determined first of all by different time scales of all the three reaction stages. The time of overcoming the Coulomb barrier and drawing the nuclei together until they touch does not exceed several units of 10^{-21} s, whereas the characteristic time of neutron emission from a weakly excited compound nucleus is at least by two orders of magnitude longer. The intermediate stage of the compound nucleus formation is not an entirely independent process: it is closely connected with the initial as well as with the final reaction stages. In particular, precompound light particles could be emitted at this stage (though highly improbable) further complicating the whole process. Nevertheless, this reaction stage, namely, its beginning and end, are also well defined in the configuration space of parameters with the help of which the entire process is described, and hence the use of the separate factor P_{CN} for the modelling of that stage is justified in the calculation of the total cross section.

The process of the compound nucleus formation is the least studied reaction stage. It is due to the fact that in the fusion of light and medium nuclei, in which the fissility of the compound nucleus is not very high, the colliding nuclei having overcome the Coulomb barrier form a compound nucleus with a probability close to unity, i.e., $P_{CN}\approx 1$ and $\sigma_{\mathrm{fus}}\approx\sigma_{\mathrm{capt}}$. Thus, this reaction stage does not influence the yield of ER at all. However, in the fusion of heavy nuclei it is the fission channels (normal and quasifission) that substantially determine the dynamics of the whole process; the P_{CN} value can be much smaller than unity, while its accurate calculation is difficult. Moreover, today there is no consensus for the mechanism of the compound nucleus formation itself, and quite different, sometimes opposite in their physics sense, models are used for its description.

Another and quite unexpected result of our analysis is the fact that despite the great available experience (theoretical and experimental) in the study of the initial stage of the heavy ion fusion reactions and the processes of statistical decay of weakly excited compound nuclei, an accurate description of these reaction stages in the synthesis of superheavy elements is also quite difficult, which brings an additional uncertainty into the calculations of the cross sections of ER formation. Here the uncertainty is connected not only with the complexity of the mechanisms of the first and last reaction stages, but also with the fact that a number of quantities and nuclear characteristics are not properly determined in this region.

In Sec. II we analyze the first stage of the heavy ion fusion reaction and the feasibility of the semiphenomenological description of the capture cross section in the cases when the microscopic calculation within the existing algorithms of the coupling channel method turns out to be difficult. In Sec. III we briefly discuss different theoretical approaches to the description of the second reaction stage and a possibility of experimental measurement of the probability of the compound nucleus formation. The major expressions and quantities determining the statistical decay of weakly excited heavy nuclei and the compound nucleus survival probability are presented in Sec. IV. Section V is devoted to the analysis of the available experimental data on the synthesis of heavy easily fissile nuclei, the sensitivity of the calculations to different quantities and the scale of uncertainty in our predictions of cross sections of superheavy nucleus formation.

II. THE STAGE OF APPROACHING AND THE CAPTURE CROSS SECTION

It is well established that in the fusion of heavy ions the barrier penetrability $T(E,l)$ is defined not only by the height and width of the Coulomb barrier but also by the strong channel coupling of relative motion with internal degrees of freedom, which enhances significantly (by several orders of magnitude) the fusion cross section at sub-barrier energies (see, e.g., the review article [5]). In the case when the capture cross section is measured experimentally within a not so narrow near-barrier energy region, the height of the potential barrier and the so-called "barrier distribution function" can be obtained from experimental data, and the transmission coefficients $T(E,l)$ can be easily calculated or approximated. In the synthesis of superheavy elements and in the case of fusion of more or less symmetric nuclei it is difficult to measure the capture cross section $\sigma_{\mathrm{capt}}(E)$ (it is usually done by detecting the total yield of fission fragments) and the barrier penetrability $T(E,l)$ has to be estimated within some theoretical model describing the initial stage of the reaction.

The Bass approximation of the potential energy of the interaction between two heavy spherical nuclei [6] is widely used and reproduces rather well the height of the potential barrier. Coupling with the excitation of nuclear collective states (surface vibrations and/or rotation of deformed nuclei) and with nucleon transfer channels is the second main factor which determines the capture cross section at near-barrier energies. In the case of fusion of relatively light nuclei a few low-excited states can be taken into account within some CC code (e.g., Refs. [7,8]) and a quite good description of the capture cross sections and the barrier distribution function itself can be obtained [5].

However, for heavy and rather "soft" nuclei (low energy values of the vibrational excitations) a realistic nucleus-nucleus interaction leads to very large dynamic deformations and thus to a necessity of taking into account a large number of coupled channels [9], which significantly complicates the microscopic calculation of $T(E,l)$ and makes it unreliable. In such cases standard CC calculations cannot reproduce experimental fusion cross sections at sub-barrier energies [10].

In Fig. 1 the experimental capture cross sections are shown for three fusion reactions. They are compared with theoretical calculations made within a model of the one-dimensional barrier penetrability (dashed curves). In all the three cases a substantial increase in the barrier penetrability is observed in the sub-barrier energy region. However, the character of this increase significantly changes: the shift of the barrier and the distribution width, in particular, grow with the increase in the masses of fusing nuclei. In the case of $^{16}\mathrm{O}+{}^{208}\mathrm{Pb}$ it becomes possible within the CC approach to describe rather well an enhancement in the sub-barrier fusion due to the coupling of the relative motion with the vibrations of nuclear surfaces [11]. It appears to be a difficult task in the case of $^{48}\mathrm{Ca}+{}^{208}\mathrm{Pb}$ and especially in the case of $^{48}\mathrm{Ca}+{}^{244}\mathrm{Pu}$.

In order to take into account explicitly the main effect of a decrease in the height of the potential barrier and, therefore, an increase in the penetration probability at sub-barrier energies due to dynamic deformation of nuclear surfaces, we use here the following nucleus-nucleus potential energy for nuclei with quadrupole deformations

$$V_{1,2}(r,\beta_1,\beta_2,\theta_1,\theta_2) = V_C(r,\beta_1,\beta_2,\theta_1,\theta_2)+V_{\mathrm{prox}}(r,\beta_1,\beta_2,\theta_1,\theta_2) + \frac{1}{2}C_1(\beta_1-\beta_1^0)^2+\frac{1}{2}C_2(\beta_2-\beta_2^0)^2. \quad (2)$$

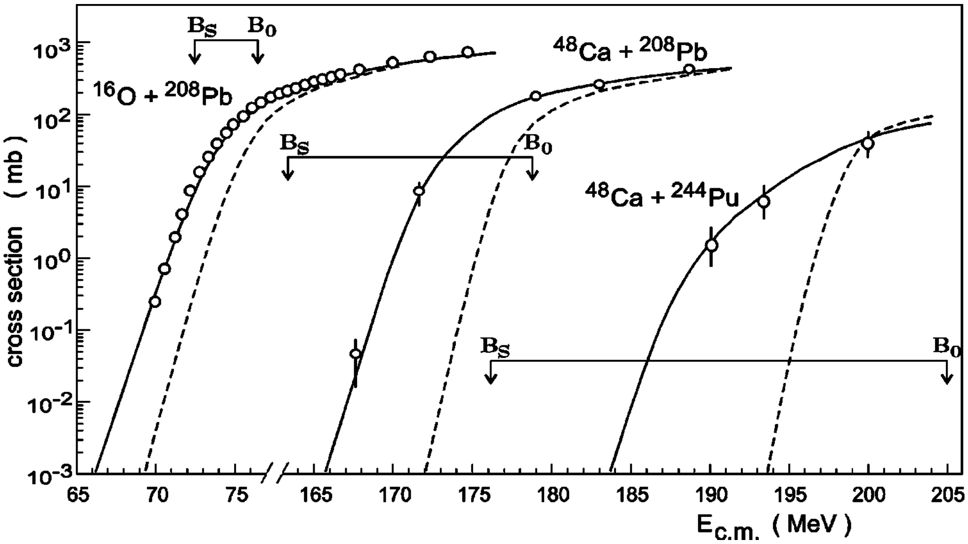

FIG. 1. Capture cross sections in the $^{16}O+^{208}Pb$ [11], $^{48}Ca+^{208}Pb$ [12], and $^{48}Ca+^{244}Pu$ [13] fusion reactions. Dashed lines represent one-dimensional barrier penetration calculations. Solid lines show the effect of dynamic deformation of nuclear surfaces (see the text). The arrows marked by B_0 and B_S show the positions of the corresponding Coulomb barrier at zero deformation and at the saddle point.

Here 1 and 2 denote the projectile and the target, $\beta_{1,2}$ are the parameters of the dynamic quadrupole deformation, $\beta^0_{1,2}$ are the parameters of static deformation, $\theta_{1,2}$ are the orientations of the symmetry axes of statically deformed nuclei, and $C_{1,2}$ are the stiffness parameters, which can be calculated within the liquid drop model. The diffuseness parameter b of the proximity potential [14] was taken equal to 1 fm for all nuclei except for light projectiles such as ^{12}C and ^{16}O, for which it was chosen as 1.1 fm. Calculating the proximity forces we also took into account a change in the surface curvature of deformed nuclei. Nuclear radii were calculated with $r_0=1.16$ fm. In the case of zero deformations $\beta_{1,2}=0$ this potential yields the Coulomb barriers which are very close to the Bass barriers. To reduce the number of variables we assume that the deformation energies of two nuclei are proportional to their masses, i.e., $C_1\beta_1^2/C_2\beta_2^2=A_1/A_2$, and we may use only one deformation parameter $\beta=\beta_1+\beta_2$.

A characteristic topographical landscape of the total (Coulomb, nuclear, and deformation) potential energy of the nucleus-nucleus interaction in the (r,β) space is shown in Fig. 2(a). The interaction potential of spherical nuclei ($\beta=0$) and potential energy along the ridge of the multidimensional barrier [dotted line in Fig. 2(a)] are shown in Figs. 2(b) and 2(c), respectively. There are two characteristic points on the potential energy surface, namely, the barrier of the interaction potential of two spherical nuclei B_0, which is very close to the corresponding Bass barrier, and the saddle point B_S, which is much lower than the Bass barrier. The difference B_0-B_S becomes greater and greater with increasing the masses of the interacting nuclei. It should be noted that in both the points, as well as along the entire ridge of the multidimensional barrier, the two nuclei are not in contact yet. The line of contact of the two nuclei [$r=R_1(\beta_1)+R_2(\beta_2)$] is shown by the dashed line in Fig. 2(a). The big grey-shaded arrow schematically shows the incoming flux, which overcomes the barrier at different values of the dynamic deformation. A quantum and classical analysis of this process made for a model system can be found in Ref. [9].

In order to determine the quantum penetrability of such a barrier one needs to solve a multidimensional Schrödinger equation. However, approximating the radial dependence of the barrier by a parabola [see Fig. 2(b)], one can use the usual Hill-Wheeler formula [15] with the barrier height modified to include a centrifugal term for the estimation of the quantum penetration probability of the one-dimensional potential barrier. Taking into account now the multidimensional character of the realistic barrier, we may introduce the "barrier distribution function" [16] $f(B)$ in order to determine its total penetrability

$$T(E,l)=\int f(B) \times\frac{1}{1+\exp\left(\frac{2\pi}{\hbar\omega(l)}\left[B+\frac{\hbar^2}{2\mu R_B^2(l)}l(l+1)-E\right]\right)}dB. \tag{3}$$

Here $\hbar\omega_B$ is defined by the width of the parabolic barrier, R_B defines the position of the barrier, and the barrier distribution function satisfies the normalization condition $\int f(B)dB=1$. At an accurate measurement of the capture cross section $\sigma_{\rm capt}(E)$ this function can be determined experimentally [5]. In other cases we may rely only on available experimental experience and the theoretical analysis of model systems. Here we use an asymmetric Gaussian approximation of this function

$$f(B)=N\begin{cases}\exp\left(-\left[\frac{B-B_m}{\Delta_1}\right]^2\right), & B<B_m,\\ \exp\left(-\left[\frac{B-B_m}{\Delta_2}\right]^2\right), & B>B_m,\end{cases} \tag{4}$$

where $B_m=(B_0+B_S)/2$. In the case of spherical colliding nuclei B_0 is the height of the barrier at zero dynamic deformation, whereas for deformed nuclei B_0 means the height of the Coulomb barrier calculated at $\theta_{1,2}=\pi/2$ (side-by-side orientation). B_S is the height of the saddle point (see Fig. 2).

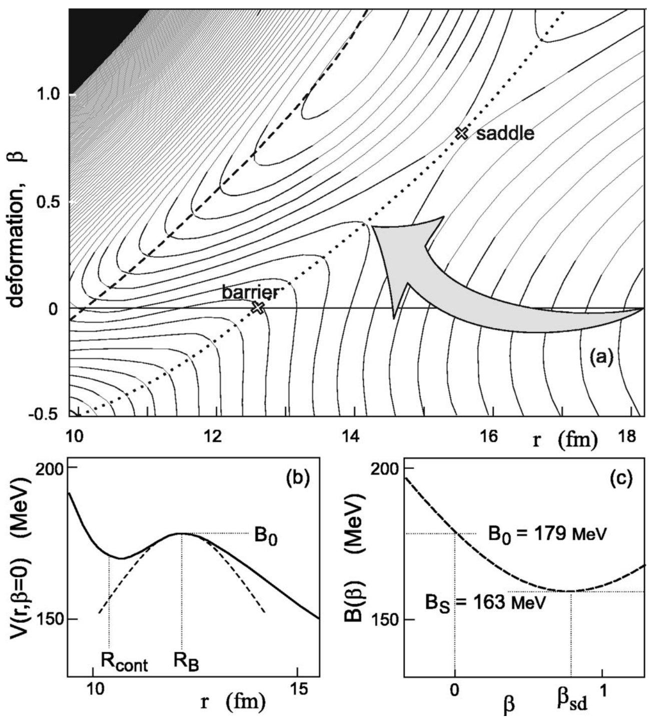

FIG. 2. Potential energy of $^{48}Ca+^{208}Pb$. The proximity potential is used for the nuclear interaction ($r_0=1.16$ fm, $b=1.0$ fm), and the standard stiffness parameters are used for the deformation energy. (a) Landscape of the potential energy surface. The saddle point and the potential barrier of spherical nuclei ($\beta=0$) are shown by crosses. The ridge of the barrier is shown by the dotted line, whereas the dashed line corresponds to the contact distance of the two nuclei. The incoming flux is shown schematically by the grey-shaded arrow. (b) Interaction potential of spherical nuclei and its parabolic approximation (dashed line) in the vicinity of the barrier. (c) Potential energy at the ridge of the two-dimensional barrier, i.e., along the dotted line passing through the saddle point [see Fig. 2(a)].

$N(\Delta_1,\Delta_2)$ is the normalization coefficient, $\Delta_2=(B_0-B_S)/2$. Experiments (see, e.g., Ref. [5]) and the theoretical analysis show that the value of Δ_1 is, as a rule, less than the value of Δ_2 and in all the cases considered below it was taken equal to 2 MeV.

Using this approach we calculated the capture cross sections for the three reactions shown in Fig. 1 (the solid lines). An additional decrease in the $^{48}Ca+^{244}Pu$ capture cross section at above barrier energies as compared with its geometrical limit could be explained by a much more shallow potential pocket and, thus, by a much less value of the critical angular momentum. For this reaction we used $L_{cr}=45$, whereas for the $^{16}O+^{208}Pb$ and $^{48}Ca+^{208}Pb$ reactions we did not need to use any restrictions on the angular momentum in the entrance channel at near-barrier energies.

In spite of the rather good agreement between the calculated and experimental capture cross sections, we should realize that some uncertainty nevertheless remains in choosing the parameters ($b,r_0,C_{1,2},L_{cr}$) defining the multidimensional potential barrier and the capture cross section. In particular, the stiffness parameters $C_{1,2}$ calculated within the liquid drop model are not compatible in many cases with experimental properties of nuclear quadrupole excitations. The role of the neutron exchange is also not clear. It means that Eq. (3), which is qualitatively understood and proved, could be used also as an empirical formula with the fitting parameters, initial choice of which could be done as described above. Thus, in the cases of fusion of very heavy nuclei and especially in symmetric fusion reactions, in which the corresponding experimental data are not available, the accuracy of our today's predictions of the capture cross sections in the sub-barrier energy region is not better than an order of magnitude. At above-barrier energies this accuracy is much better, even if we do not know the value of L_{cr}, because only low angular momenta (much lower than L_{cr}) finally contribute to the cross section of the ER formation (see below).

III. THE STAGE OF COMPOUND NUCLEUS FORMATION

After overcoming the Coulomb barrier the nuclei come to the point of contact [the dashed line in Fig. 2(a)] and the further evolution of the system strongly depends on the masses of the touching nuclei and on their deformation at this moment. In the case of a strongly asymmetric combination, the system is transformed into the compound nucleus configuration with a high probability. This occurs in the synthesis of heavy elements when the charge of one of the nuclei is lower or of the order of 15, which corresponds to the so-called "hot fusion" when the compound nucleus excitation energy appears to be very high (several tens of MeV) and the probability of the compound nucleus survival in the cooling process is very low. On the other hand, in such asymmetric combinations it is impossible to synthesize superheavy elements. The reason is that there are no sufficiently long-lived nuclei with a $Z>98$, of which a suitable target could be prepared. In the case of less asymmetric nuclear combinations, the system may evolve with a high probability directly into the exit fission channel without forming a compound nucleus, which means that the so-called process of "fast fission" or quasifission takes place [17]. The probability of such a process should be definitely even higher if the nuclei in touch initially have a prolate deformation.

Since at sub-barrier collision energies the nuclei practically have zero kinetic energy at the moment of contact, a further evolution of the nuclear system is mainly determined by the character of its multidimensional potential energy. This, in its turn, is determined by collective degrees of freedom playing the major role in the process of the evolution. It is exactly the correct choice of these degrees of freedom and a further derivation of the potential energy and solving the corresponding transport equations that pose the main problem in the description of the process of the compound nucleus formation in the competition with quasifission. Unfortunately, this problem has not been solved so far, and for the estimation of the probability of the compound nucleus formation P_{CN} some rather simplified approaches are used.

In this connection, one may single out two mutually exclusive approaches to the description of the evolution of the nuclear system starting from the moment at which the two colliding nuclei touch each other and up to the moment of formation of a spherical compound nucleus or the moment of decay into two almost equal heavy fragments (quasifission process). In the first approach [18–22] it is assumed that the two touching nuclei instantly and completely lose their individualities and can be treated as one strongly deformed mononucleus which evolves in the multidimensional space of deformations into a spherical compound nucleus or goes into fission channels. In practice one has to restrict in use of only a few collective degrees of freedom defining the shape of the nuclear system and neglect the shell structure of the nuclei, i.e., their individuality, playing an important role at low excitation energies and especially at the initial moment of contact. Quite recently more realistic calculations within such an approach were made [23] using the Langevin equation and taking into account the shell-effects in the three-dimensional potential energy.

An opposite approach has been proposed in Refs. [24–26]. Here, the two nuclei having passed the Coulomb barrier reach the point of contact and after that remain in this position keeping entirely their individualities (i.e., g.s. masses) and shapes. Only the nucleon transfer causes subsequent evolution of the ''dinuclear system.'' Compound nucleus formation means complete transfer of all the nucleons from the light nucleus to the heavier one. This process competes with the nucleon transfer from the heavy nucleus to the lighter one, resulting in a subsequent separation of the two nuclei (the quasifission process).

The truth seems to be somewhere in the middle. It is improbable that during the whole evolution of the system, starting from the point of touch of the two nuclei and up to the moment of formation of an almost spherical compound nucleus, all the nucleons are strictly divided into two groups, namely, the nucleons belonging only to one nucleus and moving only in the volume of that nucleus, and those belonging to the other nucleus and also remaining within its volume. As was shown in Ref. [27] the probability of the valence nucleon collectivization starts to increase immediately after overcoming the Coulomb barrier, and after the contact of the nuclear surfaces it rapidly reaches a value close to unity. Later all the valence nucleons are moving in the volume of both nuclei, whereas the internal nucleons with lower energies remain in the volumes of the original nuclei. It means, that the concept of the ''dinuclear system,'' in which two touching nuclei keep their individualities during the whole process of compound nucleus formation [24–26], seems to be very simplified. The process of instantaneous nucleon collectivization and formation of one strongly deformed mononucleus at the moment of contact of the two colliding nuclei also looks unlikely to take place. However, the concept of the deformed mononucleus seems very suitable at the final stage of the compound nucleus formation, when the number of shared nucleons is rather large.

A new mechanism of the compound nucleus formation and quasifission was proposed in Refs. [27,28]. It was assumed that a certain number of shared nucleons appears when two nuclei get in contact. These nucleons move within the whole volume occupied by the nuclear system and belong to both nuclei forming something similar to a neck. Henceforth the number of such collectivized nucleons increases whereas the number of nucleons belonging to each particular nucleus decreases. The compound nucleus is formed at the instant when all the nucleons find their place in the volume of that nucleus. The inverse process of nucleon decollectivization brings the system to the fission channels.

In spite of the rather reasonable preliminary results obtained in Refs. [23,28] both for the P_{CN} values and mass distributions of quasifission fragments in reactions leading to the formation of superheavy nuclei, some uncertainty still remains in these simplified calculations. Therefore the problem has to be considered more thoroughly. In this connection, the necessity of performing measurement of the probability of compound nucleus formation becomes rather pressing. In such experiments, which are in progress [13], the yield of all fission fragments and the yield of fission fragments with close masses (for example, $A_{CN}/2\pm 20$) have been measured for asymmetric heavy ion fusion reactions, assuming that the former provides us with the capture cross section and the latter with the fusion cross section. Comparing them we can make conclusions about the probability of the compound nucleus formation P_{CN}. A detailed analysis of these data is still to be made. However, as preliminary calculations have shown [23,28], the quasi-fission process can also contribute to the yield of symmetric fission fragments in such reactions as, for example, $^{48}\mathrm{Ca}+^{248}\mathrm{Cm}$. It means, that direct measurement of the fusion cross sections (keeping in mind compound nucleus formation) also encounters some difficulties, and additional efforts (experimental and theoretical) are needed to overcome the uncertainty in the estimation of P_{CN} in reactions leading to the formation of superheavy nuclei.

IV. STATISTICAL DECAY OF WEAKLY EXCITED HEAVY NUCLEI

The survival probability of the excited compound nucleus $C(E^*,J)$ in the process of its cooling by means of neutron evaporation and γ emission in the competition with fission and emission of light charged particles $C\to B+xn+N\gamma$ can be calculated within the statistical model of atomic nuclei [29]. The partial decay widths of the compound nucleus for the evaporation of the light particle $a(=n,p,\alpha, \dots)$, emission of γ rays of multipolarity L, and the fission are given by

$$\Gamma_{C\to B+a}(E^*,J)=g^{-1}\int_0^{E^*-E_a^{\mathrm{sep}}}\sum_{l,j} T_{lj}(e_a) \times \sum_{I=|J-j|}^{I=J+j} \rho_B(E^*-E_a^{\mathrm{sep}}-e_a,I;\beta_2^{\mathrm{g.s.}})de_a, \tag{5}$$

$$\Gamma_\gamma^L(E^*,J)=g^{-1}\int_0^{E^*}\sum_{I=|J-L|}^{I=J+L} f_L(e_\gamma) \times e_\gamma^{2L+1}\rho_C(E^*-e_\gamma,I)de_\gamma, \tag{6}$$

$$\Gamma_f(E^*,J)=g^{-1}\frac{\hbar\omega_B}{T}(\sqrt{1+x^2}-x)\times\int_0^{E^*}T_{\rm fis}(e,J)\rho_C(E^*-e,J,\beta_2^{sd})de. \quad (7)$$

Here $g=2\pi\rho_C(E^*,J)$, $\rho_A(E^*,J)$ is the state density of the nucleus A with the excitation energy E^* and spin J, $T_{lj}(e_a)$ is the penetration probability of the Coulomb and centrifugal barriers by the light particle a emitted from the nucleus C. Assuming that the electric dipole radiation ($L=1$) dominates in high-energy γ emission, we may use the strength function [30]

$$f_{E1}=3.31\times10^{-6}(\mathrm{MeV}^{-1})\frac{(A-Z)Z}{A}\frac{e_\gamma\Gamma_0}{(E_0^2-e_\gamma^2)^2+(e_\gamma\Gamma_0)^2},$$

with the resonance energy

$$E_0=\frac{167.23}{A^{1/3}\sqrt{1.959+14.074A^{-1/3}}}$$

and $\Gamma_0\approx5$ MeV [31] for heavy nuclei.

For the fission width we use the Kramers correction, which takes into account the influence of nuclear viscosity η on the fission probability [32–34], $x=\eta/2\omega_0$. Here ω_0 and ω_B are the characteristic frequencies of parabolic approximations of the compound nucleus potential energy depending on the deformation near the ground state and near the saddle point of the fission barrier. The appearance of the temperature in the denominator of Eq. (7) is due to the fact that the Bohr-Wheeler formula for the fission width $g^{-1}\int_0^{E^*}T_{\rm fis}(e)\rho_C(E^*-e)de$ is proportional to $T\exp(-B_f/T)$ at high excitation energies, whereas correct asymptotic value should be proportional to $\omega_B\exp(-B_f/T)$ [33]. Note that this factor is not so important for the excitation energies considered below $E^*\sim10$–50 MeV ($T\sim0.7$–1.4 MeV). For very low excitation energies Eq. (7) seems to be not valid and then the standard Bohr-Wheeler formula is more appropriate.

Experimental and theoretical estimations of nuclear viscosity yield the values of η in a range of $(1$–$30)\times10^{21}$ s^{-1} [35] and show that viscosity increases with nuclear temperature T [36,37]. In our calculations we used the expression $\eta=(1+cT^2)\times10^{21}$ s^{-1} for nuclear viscosity with $c=1$ MeV^{-2}. $T_{\rm fis}(e,J)=(1+\exp\{-2\pi/\hbar\omega_B[e-B_{\rm fis}(E^*,J)]\})^{-1}$ is the penetrability of the fission barrier $B_{\rm fis}(E^*,J)=B_0(E^*,J)-(\hbar^2/2\,\Im_{\rm g.s.}-\hbar^2/2\,\Im_{sd})J(J+1)$ is the height of the fission barrier of the rotating nucleus, $\Im_{\rm g.s.,\ sd}=k\frac{2}{5}MR^2(1+\beta_2^{\rm g.s.,\ sd}/3)$ are the moments of inertia of the fissing nucleus in its ground state and at the saddle point, where $k\approx0.4$, $B_0=B_{LD}-\delta We^{-\gamma_DE^*}$, B_{LD} is the LDM fission barrier, δW is the shell correction energy calculated for the nucleus in its ground state (we ignore here the shell effects at the saddle point), and γ_D is the damping parameter describing the decrease of the influence of the shell effects on the energy level density with increasing the excitation energy of the nucleus. The value of this parameter is especially important in the case of super-heavy nuclei, the fission barriers of which are determined mainly just by the shell corrections to their ground states. In the literature one can find close but slightly different values of the damping parameter. We paid a special attention to the sensitivity of the calculated cross sections to this parameter.

For the state density, which is the main part of Eqs. (5)–(7), we used the formula [38]

$$\rho(E,J;\beta_2)=\mathrm{const}K_{\rm coll}(\beta_2)\frac{2J+1}{E^2}\exp\{2\sqrt{a[E-E_{\rm rot}(J)]}\}, \quad (8)$$

where $E=E^*-\delta$, $\delta=0$, Δ, or 2Δ for odd-odd, odd-even, and even-even nuclei, $\Delta=11/\sqrt{A}$ MeV, $K_{\rm coll}$ is the collective enhancement factor, and the level-density parameter

$$a=a_0\left[1+\delta W\frac{1-\exp(-\gamma_DE_{\rm int})}{E_{\rm int}}\right],$$

$E_{\rm int}=E-E_{\rm rot}(J)$, $E_{\rm rot}=(\hbar^2/2\,\Im_{\rm g.s.})J(J+1)$. The asymptotic parameter $a_0=0.073A+0.095B_S(\beta_2)A^{2/3}$ MeV^{-1} was taken from Ref. [39] with a dimensionless surface factor B_S from Ref. [40].

Rotational bands of deformed nuclei bring the main contribution to the collective enhancement in the level density. For spherical nuclei the collective enhancement is smaller and is caused by vibrational excitations. In Ref. [41] it was proposed to use $K_{\rm rot}=\Im_\perp T/\hbar^2$ for deformed nuclei and $K_{\rm rot}=1$ for spherical ones, where $T=\sqrt{E_{\rm int}/a}$ is the nuclear temperature and $\Im_\perp$ is the rigid body moment of inertia perpendicular to the symmetry axis. From the analysis of experimental data on the fission of near-spherical nuclei it was found in Ref. [42] that the "borderline" between deformed and almost spherical nuclei is somewhere at $|\beta_2|\approx0.15$. $K_{\rm rot}$ changes very sharply from the value of about 150 (at $T\sim1$ MeV) to 1 when this critical deformation is passed. This sharp change may be smoothed by the function $\varphi(\beta_2)=\{1+\exp[(\beta_2^0-\beta_2)/\Delta\beta_2]\}^{-1}$, where $\beta_2^0\approx0.15$ and $\Delta\beta_2\approx0.04$ [43]. Following Ref. [42] we assume that for spherical nuclei the disappearing rotational enhancement factor has to be replaced by a vibrational factor $K_{\rm vib}$. Its value (≈1–10) is much lower than $\Im_\perp T/\hbar^2$. It strongly depends on proton and neutron numbers and is not as clear as the value of $K_{\rm rot}$. Thus, it may be considered as an empirical information on the level density of spherical nuclei [42]. Here we use the following approximate formula for the collective enhancement factor, which smoothly changes from the large value $\Im_\perp T/\hbar^2$ for well deformed nuclei to the lower value $K_{\rm vib}$ for spherical nuclei (compare with Fig. 8 of Ref. [42])

$$K_{\rm coll}(\beta_2)=\frac{\Im_\perp T}{\hbar^2}\varphi(\beta_2)+K_{\rm vib}[1-\varphi(\beta_2)]. \quad (9)$$

In fact, the survival probability of a weakly excited compound nucleus depends only on the ratio Γ_n/Γ_f, i.e., roughly speaking, on the ratio $\rho_B(E-E_n^{\rm sep},\beta_2^{\rm g.s.})/\rho_C(E-B_{\rm fis},\beta_2^{sd})$. It means that the collective enhancement factor does not influence at all the survival probability of deformed compound nuclei because in this case $K_{\rm coll}(\beta_2^{\rm g.s.})\approx K_{\rm coll}(\beta_2^{sd})\approx \Im_\perp T/\hbar^2$ and they practically cancel each other. For spherical nuclei the ratio Γ_n/Γ_f is proportional to $K_{\rm vib}(\beta_2^{\rm g.s.})/K_{\rm rot}(\beta_2^{sd})$, i.e., the collective enhancement factor can here significantly reduce the survival probability, and the dependence of $K_{\rm coll}$ on the deformation plays an important role.

Subsequent estimation of the total probability for the formation of a cold residual nucleus after the emission of x neutrons $C\rightarrow B+xn+N\gamma$ is usually performed within numerical calculations based on the analysis of the multistep decay cascade [44–46]. Here we use an explicit analytic expression for such probability, which directly takes into account the Maxwell-Boltzmann energy distribution of evaporated neutrons

$$
\begin{aligned}
P_{ER}(C\rightarrow B+xn)
&=\int_0^{E_0^*-E_n^{\rm sep}(1)}\frac{\Gamma_n}{\Gamma_{\rm tot}}(E_0^*,J_0)P_n(E_0^*,e_1)de_1\\
&\quad\times\int_0^{E_1^*-E_n^{\rm sep}(2)}\frac{\Gamma_n}{\Gamma_{\rm tot}}(E_1^*,J_1)P_n(E_1^*,e_2)de_2\cdots\\
&\quad\times\int_0^{E_{x-1}^*-E_n^{\rm sep}(x)}\frac{\Gamma_n}{\Gamma_{\rm tot}}(E_{x-1}^*,J_{x-1})P_n(E_{x-1}^*,e_x)\\
&\quad\times G_{N\gamma}(E_x^*,J_x\rightarrow {\rm g.s.})de_x. \qquad (10)
\end{aligned}
$$

Here $E_n^{\rm sep}(k)$ and e_k are the binding and kinetic energies of the kth evaporated neutron, $E_k^*=E_0^*-\Sigma_{i=1}^k[E_n^{\rm sep}(i)+e_i]$ is the excitation energy of the residual nucleus after the emission of k neutrons $P_n(E^*,e)=C\sqrt{e}\exp[-e/T(E^*)]$ is the probability for the evaporated neutron to have energy e, and the normalization coefficient C is determined from the condition $\int_0^{E^*-E_n^{\rm sep}}P_n(E^*,e)de=1$. The quantity $G_{N\gamma}$ defines the probability that the remaining excitation energy and angular momentum will be taken away by γ emission after the evaporation of x neutrons. It can be approximated by the expression

$$
G_{Ny}(E^*,J\rightarrow {\rm g.s.})=\prod_{i=1}^{N}\frac{\Gamma_\gamma(E_i^*,J_i)}{\Gamma_{\rm tot}(E_i^*,J_i)}, \qquad (11)
$$

where $E_i^*=E^*-(i-1)\langle e_\gamma\rangle$, $J_i=J-(i-1)$, $\langle e_\gamma\rangle$ is the average energy of a dipole γ quantum, and the number of γ quanta N is determined from the condition $E_N^*<B_{\rm fis}$, assuming that at energies lower than the fission barrier the fission probability is very small as compared to γ emission and $\Gamma_\gamma/\Gamma_{\rm tot}\approx 1$. Numerical calculations show that a choice of the average energy of the emitted γ quanta $\langle e_\gamma\rangle$ in the range of 0.1–1.0 MeV weakly influences the final results in all the cases except for the $0n$ fusion channel, the cross section of which is negligibly small in the reactions considered here.

V. ANALYSIS OF EXPERIMENTAL DATA AND DISCUSSION

In this section the approach described above is applied to the analysis of available experimental data on the synthesis of heavy fissile nuclei in order to find the borderlines of its applicability and the sensitivity of the calculated cross sections to the poorly determined quantities and parameters entering into the formulas. To avoid adjustment of the calculated and experimental data by simple varying of parameters, the same scheme of the calculation of $T(E,l)$ and $P_{ER}(C\rightarrow B+xn)$ described above was used in all the cases. In addition to the neutron evaporation, γ emission, and fission, the evaporation of protons and α particles was also taken into account in the calculation of the total decay width $\Gamma_{\rm tot}$ used in the neutron cascade. Experimental nuclear masses [47] were used for the determination of the separation energies of all the light particles. In the case of superheavy nuclei, the predicted masses [48] were used for that purpose. The fission barriers $B_{\rm fis}(A;E^*,J)$ of the formed nuclei are the most important and most vague parameters of the calculation. Theoretical estimations of the fission barriers for the region of superheavy nuclei are not yet very reliable and significantly differ from each other (see, e.g., Refs. [48–50]). To make the systematic analysis more consistent, the liquid drop fission barriers [51] and shell corrections [48] obtained within similar approaches were used in all the cases considered here.

A. Synthesis of heavy deformed nuclei

The capture cross sections $\sigma_{\rm capt}(E)$ and the production cross sections of the evaporation residues $\sigma_{ER}(E)$ in the reactions ^{16}O$+{}^{208}$Pb [11,52,53], ^{12}C$+{}^{238}$U [55], and ^{48}Ca$+{}^{204-208}$Pb [56] leading to the formation of rather deformed compound nuclei ($\beta_2^{\rm g.s.}>0.16$) are shown in Figs. 3–5. One can see that the standard approach (with sufficiently accurate calculation of all the quantities) describes satisfactorily the experimental data obtained in both the "cold" and "hot" asymmetric fusion reactions. Consideration of the dynamic deformation of nuclear surfaces allows reproducing correctly the capture cross section in the so-called "sub-barrier region," meaning the center-of-mass incident energies which are lower than the height of the Coulomb potential barrier of spherical nuclei B_0 or the height of the Bass barrier. In fact, as can be seen from Fig. 2, the barrier saddle point is located much lower than B_0 and thus, all energies above the saddle point B_S are "above-barrier" energies. Note that with increasing the mass of the projectile the dynamic deformation of nuclear surfaces acquires more and more importance, the difference B_0-B_S is only 4 MeV in the case of ^{16}O$+{}^{208}$Pb and about 17 MeV in the case of ^{48}Ca$+{}^{208}$Pb.

The decay properties of nobelium isotopes produced in the reactions ^{48}Ca$+{}^{204-208}$Pb are already very close to the properties of superheavy nuclei. The liquid drop part of the fission barrier is here about 1.2 MeV, and $B_{\rm fis}$ is determined mainly by the shell effects. Thus, the role of the shell correction and its damping with increasing the excitation energy can be studied quite accurately. It is very important that we have to describe simultaneously the experimental excitation

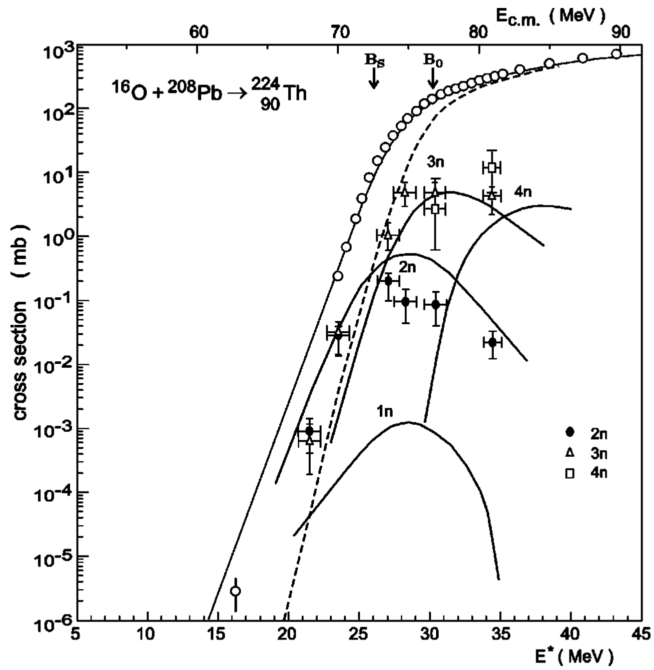

FIG. 3. Capture cross section (○) and formation cross sections for the evaporation residues in the $^{16}O+^{208}Pb$ reaction. Experimental data on the capture cross sections are from Ref. [11], the lowest point at $E^*=16$ MeV is from Ref. [52]. Experimental data on the cross sections in the xn channels are from Ref. [53]. The dashed curve shows the capture cross section calculated without dynamic deformations of the nuclei (see Sec. II). Positions of the Coulomb barrier at zero deformation and at the saddle point are shown by the arrows.

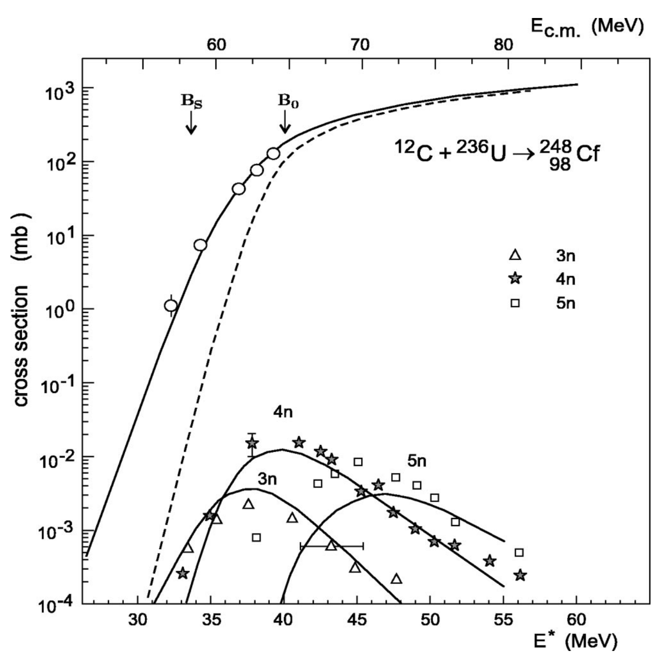

FIG. 4. Capture cross sections [54] and cross sections for the formation of evaporation residues in the $3n$, $4n$, and $5n$ channels [55] in the $^{12}C+^{236}U$ reaction. The other notations are the same as in Fig. 3.

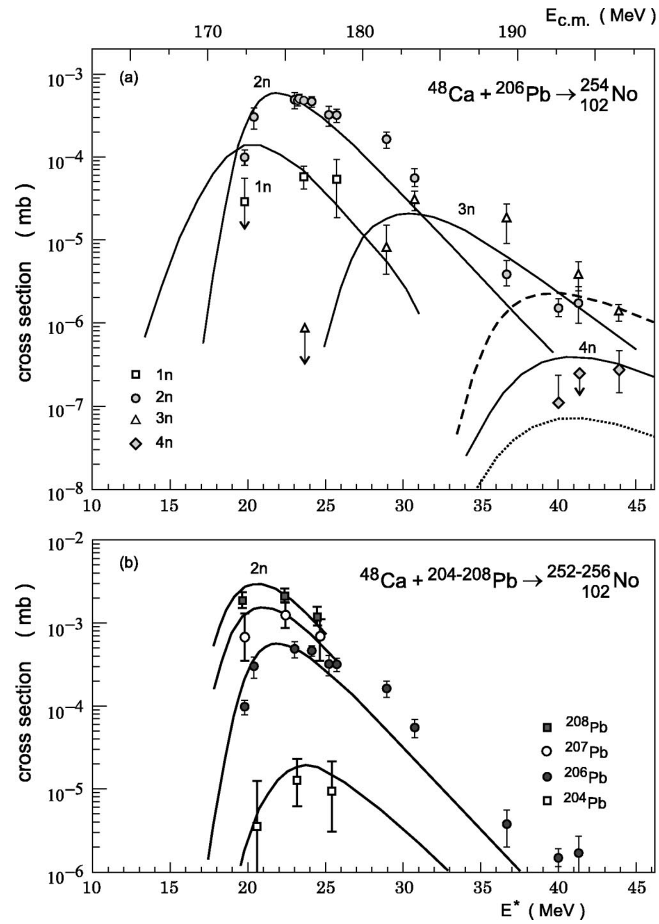

FIG. 5. (a) Cross sections of the ER production for different xn channels in the $^{48}Ca+^{206}Pb$ reaction (a) and for the $2n$ channel in the fusion reaction of ^{48}Ca with different isotopes of lead (b). The experimental data are from Ref. [56]. The solid lines correspond to the calculations with the damping factor $\gamma_D=0.061$ MeV^{-1} [57], the dashed and dotted curves for the $4n$ channel are calculated with $\gamma_D=0.05$ and 0.08 MeV^{-1}, respectively.

functions of the ER production in several xn channels for all these reactions. The decay widths of all nobelium isotopes $^{250-256}$No have to be calculated and used simultaneously in the neutron evaporation cascades when we calculate $\sigma_{ER}^{xn}(E)$. It significantly narrows the possibility to change free all the parameters. As an example, in Fig. 5(a) the dependence of the cross section in the $4n$ evaporation channel on the damping factor γ_D is shown. Sometimes this factor is used as a free fitting parameter. By changing this parameter it is very easy to fit the value of σ_{ER} for a given reaction. However, if there are experimental data in a wide energy range for different evaporation channels, this parameter can be fixed much better. Our calculations show that for heavy nuclei the values of γ_D^{-1} are in the range of 14–18 MeV. In the calculations of the fission barriers of nobelium isotopes we used the shell corrections to their ground states proposed in Ref. [48] and found that those barriers along with experimental values of neutron separation energies reproduce sufficiently well the corresponding survival probabilities.

Quite recently, new and very important experimental data on the mechanism of formation of the ^{256}No nucleus in the

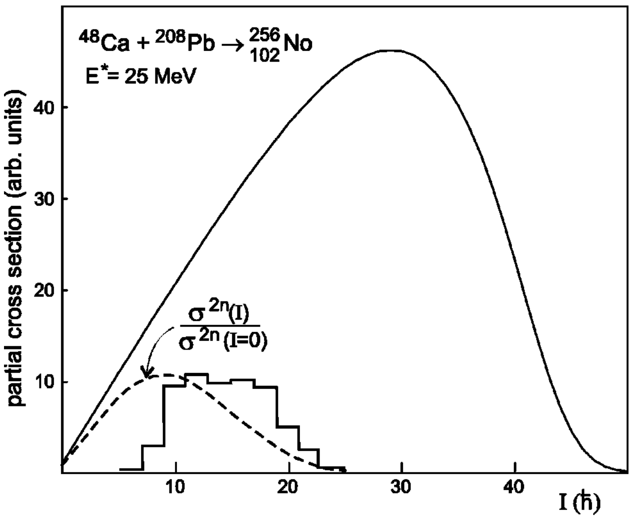

FIG. 6. Partial capture cross sections [$\sigma_{\mathrm{capt}}(l)/\sigma_{\mathrm{capt}}(l=0)$, solid line] and spin distribution of ^{254}No nuclei produced in the $2n$ evaporation channel of the ^{48}Ca$+^{208}$Pb fusion reaction at 25 MeV initial excitation energy. The experimental data (histogram) are from Ref. [58].

fusion reaction ^{48}Ca$+^{208}$Pb have been obtained in Ref. [58], in which the energy and spin distributions of the survived compound nucleus have been measured. In Fig. 6 the experimental data are compared with the theoretical calculation of the spin distribution of ^{254}No nuclei produced in the $2n$ evaporation channel at an excitation energy of 25 MeV. Note that this distribution is much more narrow than the initial distribution of the excited compound nucleus (the solid line in Fig. 6), which is defined by the fusion barrier in the entrance channel. In that case, less than half of the partial waves contributing to the capture cross section bring a contribution to the evaporation residue cross section, i.e., the survival probability $P_{ER}(C\rightarrow B;E^*,l)$ is also strongly dependent on the angular momentum and cannot be factored out from the sum over partial waves in (1) as it is very often done for approximate estimations of the synthesis cross sections.

B. Synthesis of heavy spherical nuclei

Nuclei close to the "island of stability" are predicted to be more or less spherical [48,49]. It means that the collective enhancement factor (9) should play an important role in their survival probability (see discussion in Sec. IV). To study more accurately this problem we applied our approach to the description of the synthesis of almost spherical thorium isotopes in the reaction ^{86}Kr$+^{136}$Xe [59], see Fig. 7. We do not have experimental data on the capture cross section for this reaction and it was calculated as described in Sec. II (the dash-dotted line in Fig. 7). Disregarding the deformation dependence of K_{coll}, i.e., assuming $\varphi(\beta_2)=1$ in Eq. (9), we obtained the results (shown by the dotted curves in Fig. 7), which exceed the experimental data for the $5n$ evaporation channel by about 2 orders of magnitude. Using an appropriate formula for the smoothing function $\varphi(\beta_2)$ (see Sec. IV) and $K_{\mathrm{vib}}=10$ for all thorium isotopes (which is very close to

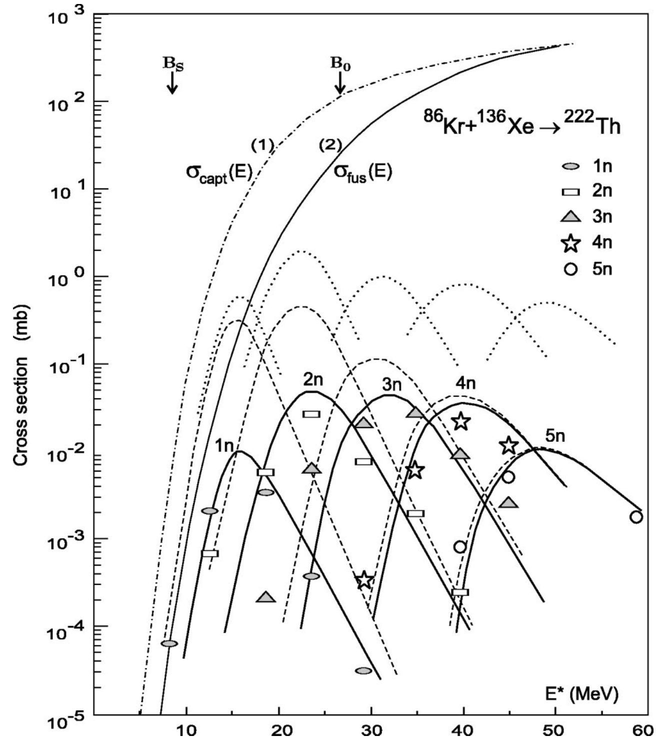

FIG. 7. Cross sections for the formation of evaporation residues in the $1n$-$5n$ channels in the ^{86}Kr$+^{136}$Xe fusion reaction [59]. Curves (1) and (2) show the capture and fusion cross sections, respectively. The dotted lines correspond to the calculations with $\varphi(\beta_2)=1$ in Eq. (9) and with $P_{CN}=1$. The dashed lines are obtained with the appropriate use of the collective enhancement factor and the solid lines show the final calculations (see the text).

the value found in Ref. [42]) we can describe quite well the evaporation residue cross sections for above barrier energies (the dashed curves in Fig. 7). Thus, the collective enhancement factor (its rotational as well as vibrational parts) really plays a very important role in the synthesis of heavy spherical nuclei.

The calculated cross sections in the $1n$ and $2n$ evaporation channels still overestimate experimental data. This near-barrier hindrance effect in the fusion of symmetric heavy nuclei is well known [60] and is caused by a close location of two nuclear configurations, namely, two nearly symmetrical touching nuclei and the saddle (or scission) configuration of the same compound nucleus in the fission channel. The extra-push model based on the liquid-drop potential energy with one-body dissipation of kinetic energy [18–20] and the surface friction model [61] were proposed to explain this dynamical hindrance effect. Empirically this effect can be simulated by increasing the fusion barrier and by broadening the barrier distribution function [60]. The latter has already been taken into account in our approach by the expression (4) (see the arrows in Fig. 7). In fact, there is no fusion extra barrier in the entrance channel, only some "intrinsic barrier" on the way from the point of contact to the compound nucleus configuration [28]. Calculation of the probability of the compound nucleus formation is a very difficult problem (see discussion in Sec. III). Here we assume that two colliding nuclei have almost a zero kinetic energy in the point of

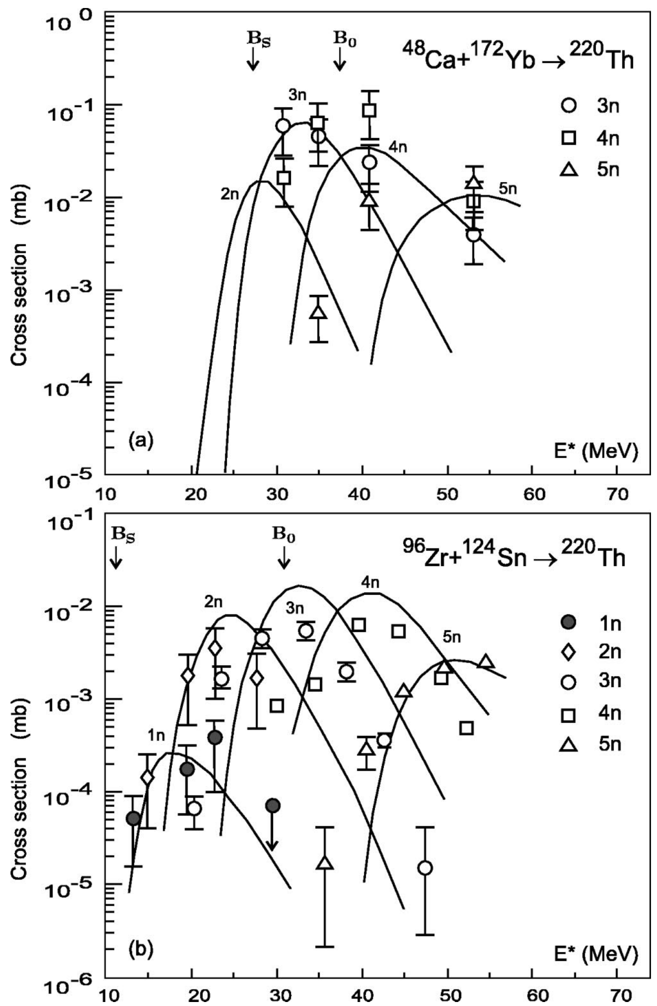

FIG. 8. Cross sections for formation of evaporation residues in the fusion reactions ^{48}Ca+^{172}Yb (a) and ^{96}Zr+^{124}Sn (b). Experimental data are from Ref. [60].

contact and their ability to overcome the ‘‘intrinsic barrier’’ and form a compound nucleus depends mainly on the excitation energy of the system. It means that the probability to form a compound nucleus P_{CN} may be approximated by the expression

$$P_{CN}(E^*)=\frac{P_0}{1+\exp\left(\frac{E_0-E^*}{\Delta}\right)}, \tag{12}$$

where E_0 is the critical excitation energy depending on the fusing nuclei, Δ is the width, which we put to be equal to the width Δ_1 of the barrier distribution function (4), and P_0 is the asymptotical (above-barrier) fusion probability. The latter is found to be 1 in the formation of not so heavy nuclei [60], but it is definitly less than 1 in the formation of superheavy elements.

Taking $E_0=30$ MeV and $P_0=1$ we calculated the fusion cross section of the considered reaction [curve (2) in Fig. 7] and the cross sections of all the evaporation channels (the solid curves in Fig. 7). It is important to note that the factor

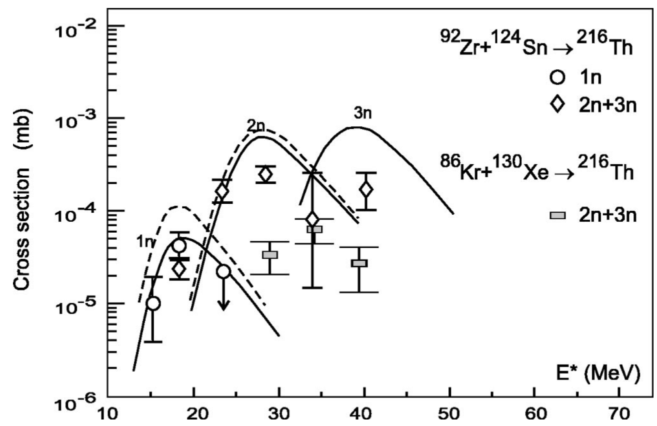

FIG. 9. Cross sections for formation of evaporation residues in the fusion reactions ^{92}Zr+^{124}Sn [60] (solid lines) and ^{86}Kr+^{130}Xe [59] (dashed lines).

P_{CN} does not influence here the ER cross sections at above-barrier energies (the $4n$ and $5n$ channels) and, thus, our conclusion about the collective enhancement factor.

The ground state deformations of thorium isotopes change gradually from $\beta_2^{\text{g.s.}}=0.164$ for ^{224}Th to almost a zero value for ^{216}Th. All these isotopes enter the same evaporation cascade for a given fusion reaction and were synthesized in different target-projectile combinations. By calculating the decay properties of these isotopes on the same basis (see Sec. IV) and describing simultaneously all available experimental data it is possible to make a more definite conclusion about the correct treatment of the collective enhancement factor (9).

In Fig. 8 the calculated cross sections for ER formation are compared with the experimental data for the asymmetrical ^{48}Ca+^{172}Yb and symmetrical ^{96}Zr+^{124}Sn fusion reactions leading to the same compound nucleus ^{220}Th. In the first case we used $P_{CN}=1$ and in the second case the value $E_0=33$ MeV for the critical excitation energy, which is close to the value for synthesis of ^{222}Th. As can be seen, agreement with the experimental data is quite satisfactory taking into account the absence of any other adjustable parameters.

Figure 9 shows results of our calculations for the synthesis of ^{216}Th in two different reactions. For both reactions we chose $E_0=43$ MeV, which is by 10 MeV higher than for the synthesis of ^{220}Th. In the synthesis of ^{216}Th the cross sections for the ER formation at 35 MeV excitation energy (the $2n+3n$ channels) are smaller by more than two orders of magnitude as compared to the synthesis of ^{222}Th (Fig. 7). Nevertheless, the experimental data for the ^{92}Zr+^{124}Sn reaction are reproduced rather well. However, for the less symmetric fusion reaction ^{86}Kr+^{130}Xe the calculated cross sections significantly overestimate the experiment data. It means that there is some additional hindrance effect in the fusion dynamics of this reaction which is still not clear.

C. Synthesis of superheavy nuclei

All the above mentioned factors bring considerable uncertainty into the calculations of the formation cross sections of superheavy nuclei. For a number of asymmetric fusion reactions leading to the formation of superheavy nuclei the total yield of fission fragments has been measured experimentally [13]. Thus, we can calculate accurately enough the corresponding capture cross sections. For heavy symmetric combinations similar information is not available and results of the calculations of $\sigma_{\rm capt}$ may be rather dispersed. In that case the potential pocket is very shallow (if any), L_{cr} is very low, dynamic deformations can be very large, and height of the fusion barrier itself is poorly determined.

The mechanism of superheavy compound nucleus formation is also quite specific. The macroscopic fission barrier is absent here, and the shell effects are the only stabilizing factor preventing the superheavy nucleus to decay spontaneously. However, the shell correction rapidly decreases with increasing the deformations, and the saddle point configuration is very close to the ground state configuration of the superheavy nucleus. For heavy spherical nuclei ($Z\sim114$, $N\sim180$) the area in the configuration space of collective variables, which corresponds to formation of a compound nucleus, becomes very small, and the probability of evolution of the weakly excited system of two touching heavy nuclei to this area is much lower compared to the probability of evolution of the system to the nearby located quasifission channels.

Apart from the very uncertain probability P_{CN} of the compound nucleus formation we encounter numerous difficulties also with the calculation of the decay properties of superheavy nuclei. The fission barriers of these nuclei are unknown and their theoretical calculations lead to values differing by about 2 MeV for nuclei with $Z>112$ [48–50]. Moreover, as mentioned above, the macroscopic component of the fission barriers of these nuclei is close to zero and their survival is governed largely by the shell effects, which, in turn, strongly depend on excitation energy and, perhaps, on angular momentum of the compound nucleus. As a result, we obtain very large uncertainty in the calculation of the ER cross sections for these nuclei.

The calculated cross sections for the formation of isotopes of element 114 in the ^{48}Ca$+^{244}$Pu fusion reaction at near-barrier energies are shown in Fig. 10. The capture cross section was calculated within the approach described in Sec. II, and it reproduces the corresponding experimental data [13] well if we take $L_{cr}=45$. If we assume, following [13], that the yield of nearly symmetric fission fragments with $A_f=A_{CN}/2\pm20$ (the solid circles in Fig. 10) provides us with the fusion cross section, then the probability of the compound nucleus formation in this reaction is about 10^{-1} for excitation energies of 25–40 MeV. Using this P_{CN} value in Eq. (1), the value of $K_{\rm vib}=10$, and the shell corrections from Ref. [48], we obtained the ER cross sections in the $2n$–$4n$ channels shown by the dashed curves in Fig. 10.

The calculated value of σ_{ER}^{4n} overestimates by a factor of 40 the experimental cross section for the formation of the ^{288}114 evaporation residue measured in Ref. [2]. There can be several reasons for such an overestimation. First, as discussed in Sec. III, the quasifission process may also contribute to the yield of nearly symmetric fission fragments. Thus, the probability of the compound nucleus formation may be less than 10^{-1} for this reaction. It is almost evident for the low energies ($E^*\leqslant30$ MeV), at which the experimental cross section for the ER formation was found to be less than 1 pb [2] (see also the microscopic calculation of P_{CN} for that case [28], which gives a sharp decrease of the compound nucleus formation at $E^*<30$ MeV). Second, the fission barrier $B_{\rm fis}=8.9$ MeV of the compound nucleus ^{292}114, which was obtained with the shell corrections from Ref. [48], could be too high. Using the fission barrier $B_{\rm fis}=6.8$ MeV proposed in Ref. [49], we obtained the ER cross sections shown by the solid curves in Fig. 10. As can be seen, the difference between the two sets of calculations is about 2 orders of magnitude. Anyhow, from the analysis of the experimental data on the fusion-fission reaction ^{48}Ca$+^{244}$Pu $\rightarrow^{292}$114 we may certainly conclude that the fission barriers of the nuclei with $Z=114$, $A\sim290$ are not negligibly low even at excitation energy of 35 MeV, and at low excitation energies they are not lower than the barriers of nuclei with $Z=102$, $A\sim254$.

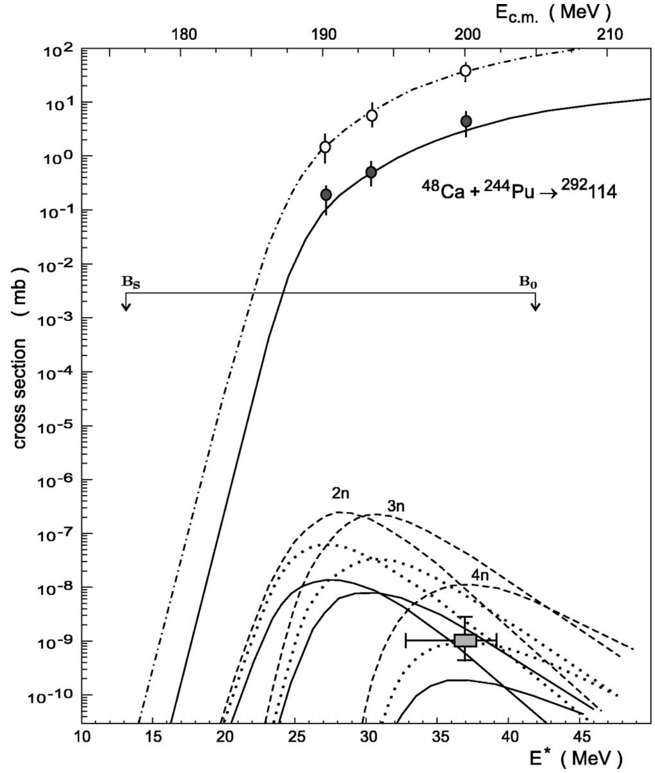

FIG. 10. Capture cross section (the dash-dotted line) and cross sections for formation of evaporation residues in the ^{48}Ca$+^{244}$Pu reaction. Experimental data on the capture cross sections (the open circles) are from Ref. [13]. The solid circles correspond to the cross sections of the fusion-fission reaction leading to the formation of nearly symmetric fission fragments with $A_f=A_{CN}\pm20$ [13]. The cross sections for the $2n-4n$ evaporation channels were calculated with the shell corrections taken from Ref. [48] (the dashed curves) and with the fission barriers from Ref. [49] (the solid curves), in both cases $K_{\rm vib}=10$. The dotted curves correspond to the cross sections calculated with $K_{\rm vib}=5$ and with the shell corrections from Ref. [48]. The experimental point for the formation of the ^{288}114 nucleus in the $4n$ evaporation channel is from Ref. [2].

We found also that the yield of almost spherical isotopes of element 114 is very sensitive to the value of the collective enhancement factor in the state density. Using the value $K_{\text{vib}}=5$ (justified for closed shell nuclei with $Z=114$) we obtained the ER cross sections (the dotted curves in Fig. 10), which are less by about one order of magnitude than those for $K_{\text{vib}}=10$.

VI. CONCLUSION

In the framework of a unified approach a systematic analysis of the experimental data on the near-barrier fusion reactions leading to the formation of heavy (including superheavy) evaporation nuclear residues has been made. The aim of such an analysis was to understand how well we can describe the whole process, what factors and quantities bring major uncertainty into the calculated cross sections and how accurately the cross sections for formation of the superheavy elements can be calculated. A new semiphenomenological approach which takes into account the coupling of the relative motion with the nuclear surface deformations has been used for the description of the initial reaction stage. In terms of this approach a good description of the experimental capture cross sections in the asymmetric fusion reactions became possible, including the region of sub-barrier energies. Special attention was paid to the role of the collective enhancement factor in the state density, whose role in the synthesis of deformed nuclei is negligible but becomes important in the survival of weakly excited heavy spherical nuclei. Numerical estimations have been made concerning the dependence of the calculated formation cross sections of heavy evaporation residues on a number of quantities, the values of which are poorly determined today.

The performed analysis makes it possible to conclude that at present we are capable of calculating and predicting the values of formation cross sections of superheavy elements with $Z>112$ with an accuracy of two orders of magnitude at best. There are two main reasons for that. The first one is the lack of a satisfactory quantitative model for the calculation of the probability of the compound nucleus formation, which in the synthesis of superheavy elements may vary over a wide range depending on the excitation energy and the chosen combination of target and projectile nuclei. The second one is that the formation cross section of superheavy nuclei is very sensitive to the value of the fission barrier, which can be calculated today with an accuracy not better than 2 MeV. This corresponds to two orders of magnitude in the value of the calculated cross section. In reactions of "hot synthesis," the formation cross sections of superheavy nuclei in the channels with the evaporation of several neutrons turn out to be sensitive to the value of the damping parameter γ_D, a change of which by only 10% leads to a change in the cross section by a factor of 10. However, the values of this parameter given in literature differ by more than 10%. In the synthesis of heavy spherical nuclei with $Z=114{,}116$ and $N\sim184$ a collective enhancement factor of the state density, decreasing the survival probability of these nuclei, also plays an important role. In connection with the above mentioned, accurate theoretical predictions (or experimental measurements, if possible) of the values $\beta_2^{\text{g.s.}},\beta_2^{sd.}$ and K_{vib} for the nuclei in this region acquire great importance. The collective enhancement factor practically does not influence the survival probability of heavy deformed nuclei (see Sec. IV) and thus we cannot study its role using the available experimental data on the cold synthesis of superheavy nuclei with $100<Z<112$. For this reason a more thorough theoretical and experimental study of decay properties of more light fissile spherical nuclei with $N\sim126$ and $Z\sim90$ is also of much importance. There is no doubt that spectroscopic measurements for heavy nuclei could give us much more realistic data on the parameters of the state density formula.

ACKNOWLEDGMENTS

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