Collectivity of double giant resonances in extended RPA theories

Mitsuru Tohyama
Kyorin University School of Medicine, Mitaka, Tokyo 181-8611, Japan
(Received 31 July 2001; published 16 November 2001)

A comparison among extended random phase approximation (RPA) theories is made by calculating the strength functions of double phonon states of isovector dipole and isoscalar quadrupole giant resonances in ^{40}Ca using the small amplitude limit of the time-dependent density-matrix theory (STDDM). STDDM includes all the elements of one-body and two-body amplitudes and provides us with a quite general framework of extended RPA theories which consider the coupling of one-body amplitudes to two-body amplitudes. The results are compared with those obtained by using time-dependent versions of the second RPA (SRPA) and other extended RPA theories. It is found that SRPA overestimates collectivity of the isovector mode and underestimates that of the isoscalar mode. From a comparison between STDDM and other extended RPA theories, it is concluded that the inclusion of particle-hole–particle-hole amplitudes is necessary to obtain appropriate collectivity of double phonon states of giant resonances.

DOI: 10.1103/PhysRevC.64.067304 PACS number(s): 21.60.Jz, 24.10.Cn, 24.30.Cz

The double phonon states of giant resonances have become the subject of a number of recent experimental and theoretical investigations [1]. In the case of giant resonances, the random phase approximation (RPA) has extensively been used as a standard microscopic theory to study basic properties of giant resonances [2]. The second RPA (SRPA) [3–6] in which RPA is extended to include two-body amplitudes may be such a microscopic theory for double phonon states of giant resonances as RPA for giant resonances. Although SRPA has frequently been used to study the damping of giant resonances [5,6], numerical application of SRPA to double giant resonances has been rare: Only a work based on a schematic model has been reported [7]. Therefore, the applicability of SRPA and other extended RPA theories to the problem of double giant resonances has not been thoroughly investigated yet. The aim of this paper is to make a comparison among extended RPA theories including SRPA by calculating the strength functions of a double giant dipole resonance (DGDR) and a double quadrupole resonance (DGQR) in ^{40}Ca. We use the small amplitude limit of the time-dependent density-matrix theory as a basic extended RPA theory. The time-dependent density-matrix theory (TDDM) is an extended version of the time-dependent Hartree-Fock theory formulated by truncating the well-known Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy for reduced density matrices [8,9]. TDDM has been applied to the two phonon states of giant resonances [10,11], and it has been shown that both DGDR and DGQR in TDDM are highly harmonic. The small amplitude limit of TDDM (STDDM) [12] is a quite general framework as compared with other extended RPA theories in the sense that all the elements of one-body and two-body amplitudes are considered. We first calculate the strength functions of DGDR and DGQR using STDDM and then compare them with those obtained by using time-dependent versions of SRPA and other extended RPA theories [13,14]. We will point out that a problem of collectivity inherent to SRPA which has been known for low-lying two-phonon states [13,15] also exists in the double phonon states of giant resonances and demonstrate that the inclusion of particle-hole–particle-hole amplitudes is necessary to obtain appropriate collectivity of the double giant resonances.

We begin with presenting the equations of motion in STDDM. When the Hartree-Fock (HF) ground state is assumed in the formulation of STDDM, it consists of the following coupled equations for a one-body amplitude $x_{\alpha\alpha'}$ and a two-body amplitude $X_{\alpha\beta\alpha'\beta'}$ [12],

$$i\hbar\dot{x}_{\alpha\alpha'}=\sum_{\lambda\lambda'}[\langle\alpha\lambda|v|\alpha'\lambda'\rangle_A f_{\alpha'}-\langle\alpha\lambda|v|\alpha'\lambda'\rangle_A f_\alpha]x_{\lambda'\lambda}+\sum_{\lambda\lambda'\lambda''}[X_{\lambda\lambda'\alpha'\lambda''}\langle\alpha\lambda''|v|\lambda\lambda'\rangle-X_{\alpha\lambda'\lambda\lambda''}\langle\lambda\lambda''|v|\alpha'\lambda'\rangle], \quad (1)$$

$$\begin{aligned}i\hbar\dot{X}_{\alpha\beta\alpha'\beta'}=&-\sum_{\lambda}[(\overline{f}_\beta f_{\alpha'}f_{\beta'}+f_\beta\overline{f}_{\alpha'}\overline{f}_{\beta'})\langle\lambda\beta|v|\alpha'\beta'\rangle_A x_{\alpha\lambda}+(\overline{f}_\alpha f_{\alpha'}f_{\beta'}+f_\alpha\overline{f}_{\alpha'}\overline{f}_{\beta'})\langle\alpha\lambda|v|\alpha'\beta'\rangle_A x_{\beta\lambda}-(\overline{f}_\alpha\overline{f}_\beta f_{\beta'}\\&+f_\alpha f_\beta\overline{f}_{\beta'})\langle\alpha\beta|v|\lambda\beta'\rangle_A x_{\lambda\alpha'}-(\overline{f}_\alpha\overline{f}_\beta f_{\alpha'}+f_\alpha f_\beta\overline{f}_{\alpha'})\langle\alpha\beta|v|\alpha'\lambda\rangle_A x_{\lambda\beta'}]+\sum_{\lambda\lambda'}[(1-f_\alpha-f_\beta)\\&\times\langle\alpha\beta|v|\lambda\lambda'\rangle X_{\lambda\lambda'\alpha'\beta'}-(1-f_{\alpha'}-f_{\beta'})\langle\lambda\lambda'|v|\alpha'\beta'\rangle X_{\alpha\beta\lambda\lambda'}]+\sum_{\lambda\lambda'}[(f_{\alpha'}-f_\alpha)\langle\alpha\lambda|v|\alpha'\lambda'\rangle_A X_{\lambda'\beta\lambda\beta'}-(f_{\beta'}\\&-f_\alpha)\langle\alpha\lambda|v|\lambda'\beta'\rangle_A X_{\lambda'\beta\alpha'\lambda}+(f_{\beta'}-f_\beta)\langle\lambda\beta|v|\lambda'\beta'\rangle_A X_{\alpha\lambda'\alpha'\lambda}-(f_{\alpha'}-f_\beta)\langle\lambda\beta|v|\alpha'\lambda'\rangle_A X_{\alpha\lambda'\lambda\beta'}],\end{aligned} \quad (2)$$

where $f_\alpha=1$ (0) for occupied (unoccupied) single-particle states and $\bar{f}_\alpha=1-f_\alpha$, and the subscript A indicates that the matrix with it is antisymmetrized. The amplitudes $x_{\alpha\alpha'}$ and $X_{\alpha\beta\alpha'\beta'}$ in Eqs. (1) and (2) have no restriction on single-particle indices and consist of all possible components: For example, $x_{\alpha\alpha'}$ have one-particle (p)–one-hole (h), $1h$-$1p$, $1p$-$1p$, and $1h$-$1h$ components. Therefore, Eqs. (1) and (2) give a quite general coupling scheme between the one-body and two-body amplitudes. In the following, we point out some relation of STDDM with other extended RPA theories. SRPA has been formulated by using only the $1p$-$1h$ and $1h$-$1p$ components of the one-body amplitudes and the $2p$-$2h$ and $2h$-$2p$ components of the two-body amplitudes [3–6]. When only these components are kept in Eqs. (1) and (2), STDDM is equivalent to the time-dependent version of SRPA. It is well-known that in SRPA the $2p$-$2h$ amplitudes cannot couple to the $2h$-$2p$ ones [3–6]. It is necessary to include the $1p1h$-$1p1h$ components of $X_{\alpha\beta\alpha'\beta'}$ in addition to the $2p$-$2h$ and $2h$-$2p$ ones to make the $2p$-$2h$ components couple to the $2h$-$2p$ ones. This version of extended RPA has been proposed by Kanesaki *et al.* [13] for low-lying two-phonon states. It has been discussed [13,15] that the $1p1h$-$1p1h$ components of $X_{\alpha\beta\alpha'\beta'}$ are important to give appropriate collectivity to low-lying double-phonon states. Lauritsch and Reinhard [7] considered the coupling of the $2p$-$2h$ amplitudes to the $2h$-$2p$ ones in their application of SRPA to double giant resonances not by explicitly using the $1p1h$-$1p1h$ amplitudes but by renormalizing the residual interaction using a correlated RPA ground state. If the coupling to the one-body amplitude $x_{\alpha\alpha'}$ is neglected in Eq. (2), Eq. (2) describes correlations in two-body space with all the two-body amplitudes. Equation (2) without $x_{\alpha\alpha'}$ is equivalent to an extended RPA equation presented by Danielewicz and Schuck [14]. As will be discussed below, the coupling of $X_{\alpha\beta\alpha'\beta'}$ to $x_{\alpha\alpha'}$ is negligible in the case of the double giant resonances considered here.

To solve Eqs. (1) and (2) as an initial value problem, we assume that the motion of a double giant resonance is generated by a two-body operator $\hat{V}^2$ as

$$|\Psi(t=0)\rangle=e^{ik\hat{V}^2}|\Phi_0\rangle, \tag{3}$$

where $\hat{V}$ is either a one-body dipole operator or a quadrupole operator, k is a boosting parameter, and $|\Phi_0\rangle$ the ground-state wave function. The initial conditions for $x_{\alpha\alpha'}$ and $X_{\alpha\beta\alpha'\beta'}$ are determined by using the above boosted wave function. Assuming that $|\Phi_0\rangle$ is the HF ground-state wave function, we evaluate

$$x_{\alpha\alpha'}(t=0)=\langle\Psi(t=0)|a^+_{\alpha'}a_\alpha|\Psi(t=0)\rangle, \tag{4}$$

$$X_{\alpha\beta\alpha'\beta'}(t=0)=\langle\Psi(t=0)|a^+_{\alpha'}a^+_{\beta'}a_\beta a_\alpha|\Psi(t=0)\rangle. \tag{5}$$

At first order of k, the initial values for $X_{\alpha\beta\alpha'\beta'}$ are given by

$$\begin{aligned}X_{\mu\nu\rho\sigma}&=\langle\Psi|a^+_\rho a^+_\sigma a_\nu a_\mu|\Psi\rangle\\&=2ik\{\langle\mu|V|\rho\rangle\langle\nu|V|\sigma\rangle-\langle\mu|V|\sigma\rangle\langle\nu|V|\rho\rangle\},\end{aligned} \tag{6}$$

$$\begin{aligned}X_{\rho\sigma\mu\nu}&=\langle\Psi|a^+_\mu a^+_\nu a_\sigma a_\rho|\Psi\rangle\\&=-2ik\{\langle\rho|V|\mu\rangle\langle\sigma|V|\nu\rangle-\langle\rho|V|\nu\rangle\langle\sigma|V|\mu\rangle\},\end{aligned} \tag{7}$$

where ρ and σ refer to unoccupied single-particle states, and μ and ν refer to occupied ones. We choose $V=\tau_z z$ for the dipole operator and $V=z^2-(x^2+y^2)/2$ for the quadrupole operator. Other elements of the initial $X_{\alpha\beta\alpha'\beta'}$ vanish at first order of k. Similarly, nonvanishing initial values of $x_{\alpha\alpha'}$ are

$$\begin{aligned}x_{\mu\rho}&=\langle\Psi|a^+_\rho a_\mu|\Psi\rangle\\&=2ik\sum_\nu\langle\mu|V|\nu\rangle\langle\nu|V|\rho\rangle,\end{aligned} \tag{8}$$

$$\begin{aligned}x_{\rho\mu}&=\langle\Psi|a^+_\mu a_\rho|\Psi\rangle\\&=-2ik\sum_\nu\langle\rho|V|\nu\rangle\langle\nu|V|\mu\rangle.\end{aligned} \tag{9}$$

In numerical applications shown below, we found that the coupling of $X_{\alpha\beta\alpha'\beta'}$ to $x_{\alpha\alpha'}$ are quite small both for DGDR and DGQR. Therefore, the above initial values of $x_{\alpha\alpha'}$ can practically be neglected in numerical calculations. The strength function of the double phonon states, defined by

$$S_2(E)=\sum_n|\langle\Phi_n|\hat{V}^2|\Phi_0\rangle|^2\delta(E-E_n), \tag{10}$$

is given by the Fourier transform of a time-dependent part of a two-body moment as

$$S_2(E)=\frac{1}{\pi k\hbar}\int_0^\infty V_2(t)\sin\frac{Et}{\hbar}dt, \tag{11}$$

where V_2 is given by

$$\begin{aligned}V_2(t)&=\langle\Psi(t)|\hat{V}^2|\Psi(t)\rangle-\langle\Phi_0|\hat{V}^2|\Phi_0\rangle\\&=\sum_{\alpha\alpha'}\langle\alpha|V^2|\alpha'\rangle x_{\alpha'\alpha}+\sum_{\alpha\beta\alpha'\beta'}\langle\alpha|V|\alpha'\rangle\langle\beta|V|\beta'\rangle\\&\quad\times\{X_{\alpha'\beta'\alpha\beta}-2f_\alpha\delta_{\alpha\beta'}x_{\alpha'\beta}\}.\end{aligned} \tag{12}$$

The terms with $x_{\alpha\alpha'}$ in the above equation have negligible contributions to the Fourier transformation in Eq. (11). The k dependence of $S_2(E)$ is negligible as long as k is sufficiently small. The energy-weighted sum rule (EWSR) for DGDR is given as [10]

$$\int_0^\infty ES_2(E)dE = \frac{1}{2}\langle\Phi_0|[\hat{V}^2,[H,\hat{V}^2]]|\Phi_0\rangle$$
$$= \frac{2\hbar^2}{m}\langle\Phi_0|\hat{V}^2|\Phi_0\rangle + 4(t_1+t_2)$$
$$\times\langle\Phi_0|\hat{R}_2\hat{V}^2|\Phi_0\rangle, \quad (13)$$

where H is the total Hamiltonian, m is the nucleon mass and $\hat{R}_2$ is the following two-body operator:

$$\hat{R}_2 = \sum_{i\in\text{protons}, j\in\text{neutrons}} \delta^3(\mathbf{r}_i - \mathbf{r}_j). \quad (14)$$

In Eq. (13), we assumed that H consists of a two-body interaction of the Skyrme type. The second term on the right-hand side of Eq. (13), the enhancement term, is due to the momentum dependence of the Skyrme force and has a contribution of about 30% to the total EWSR value [10]. Similarly, EWSR for DGQR is given as [11]

$$\int_0^\infty (ES_2E)dE = \frac{1}{2}\langle\Phi_0|[\hat{V}_2,[H,\hat{V}_2]]|\Phi_0\rangle$$
$$= \frac{2\hbar^2}{m}\langle\Phi_0|\hat{V}^2\hat{R}_1|\Phi_0\rangle, \quad (15)$$

where $\hat{R}_1$ is a one-body operator associated with the function $4z^2+x^2+y^2$. To be consistent with the derivation of Eqs. (1) and (2), we use the HF wave function for $|\Phi_0\rangle$ to evaluate the EWSR values. As in previous calculations [10,11], we use the Skyrme III force (SKIII) [16] as the effective interaction for a mean-field potential and also as the residual interaction. The spin-orbit force is neglected. To solve the coupled equations Eqs. (1) and (2), we use the $1s$, $1p$, $2s$, $1d$, $2p$, and $1f$ states for DGDR and the $1s$, $1p$, $2s$, $1d$, $2p$, $1f$, $3s$, $2d$, and $1g$ orbits for DGQR. Although the single-particle space is truncated, a giant dipole resonance (GDR) and a giant quadrupole resonance (GQR) have sufficient strength: The RPA calculations with the truncated single-particle space had shown that the fractions of the EWSR values depleted below 40 MeV are 93% for GDR [10] and 95% for GQR [11]. The integration in Eq. (11) is performed for a finite time interval of 750 fm/c. As a result, $S_2(E)$ has small fluctuations. To reduce the fluctuations in $S_2(E)$, we multiply $V_2(t)$ by a damping factor $e^{-\Gamma t/2}$ before performing the time integration. This corresponds to smoothing the strength function with a width Γ. We choose $\Gamma = 1$ MeV. Other calculational details are explained in our previous publications [10,11].

In Fig. 1 the strength function of DGDR calculated in STDDM (solid line) is compared with that in SRPA (dotted line). Since the coupling of $X_{\alpha\beta\alpha'\beta'}$ to $x_{\alpha\alpha'}$ is small, a time dependent version of the extended RPA equation in Ref. [14] would give a result similar to the STDDM one. The peak energy in SRPA is 1.2 MeV higher than that in STDDM, and the fractions of the EWSR value depleted below 60 MeV are 87% in STDDM and 112% in SRPA, respectively. The

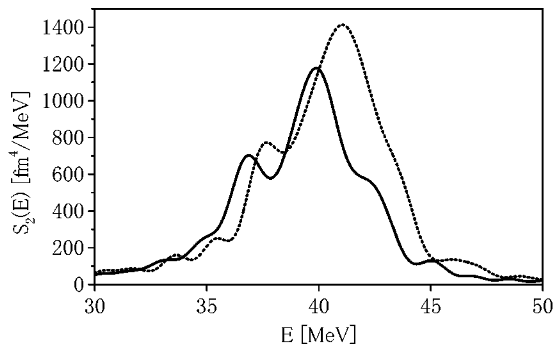

FIG. 1. Strength functions $S_2(E)$ of DGDR in ^{40}Ca calculated in STDDM (solid line) and SRPA (dotted line).

EWSR value in STDDM seems reasonable as compared with the RPA value for GDR (93%). Since the EWSR value in SRPA exceeds 100%, DGDR in SRPA has too high collectivity. This problem originates in the fact that the coupling of the $2p$-$2h$ amplitudes $X_{\rho\sigma\mu\nu}$ to the $2h$-$2p$ ones $X_{\mu\nu\rho\sigma}$ is missing in SRPA, though in RPA, the $1p$-$1h$ amplitude $x_{\rho\mu}$ couples to the $1h$-$1p$ one $x_{\mu\rho}$. In order to include such coupling of the two-body amplitudes, we should consider the $1p1h$-$1p1h$ amplitudes $X_{\rho\mu\sigma\nu}$ in addition to $X_{\rho\sigma\mu\nu}$ and $X_{\mu\nu\rho\sigma}$ as has been pointed out by Kanesaki *et al.* [13]. We performed a calculation using such a modified version of SRPA (MSRPA). We found that the strength function in MSRPA is almost identical to that in STDDM. Thus, the coupling of the $2p$-$2h$ amplitudes to the $2h$-$2p$ amplitudes which is bridged by $X_{\rho\mu\sigma\nu}$ plays a role in reducing collectivity of DGDR. The little difference between the STDDM and MSRPA results also indicates that no two-body amplitudes other than $X_{\rho\sigma\mu\nu}$, $X_{\mu\nu\rho\sigma}$, and $X_{\rho\mu\sigma\nu}$ are important.

In Fig. 2 the strength function of DGQR calculated in STDDM (solid line) is compared with that in SRPA (dotted line). The result in MSRPA differs little from that in STDDM and is not shown in Fig. 2. The peak in SRPA is 0.6 MeV higher than that in STDDM, and the fractions of the EWSR value depleted below 60 MeV are 91% in STDDM and 79% in SRPA, respectively. The EWSR value in STDDM seems reasonable for the truncated single-particle space. Since the EWSR value in SRPA is much smaller than the RPA value (95%), the collectivity of DGQR in SRPA seems to be too

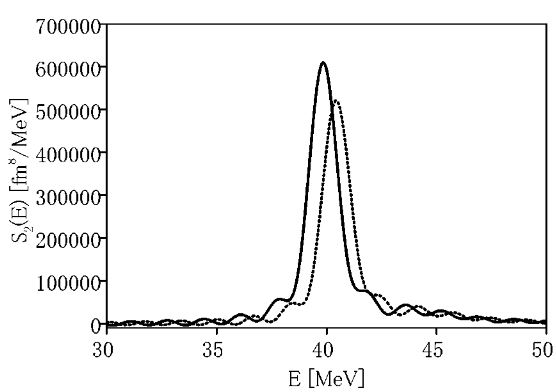

FIG. 2. Strength functions $S_2(E)$ of DGQR in ^{40}Ca calculated in STDDM (solid line) and SRPA (dotted line).

low. In the case of DGQR, the comparison between the SRPA and MSRPA results indicates that the coupling of the $2p$-$2h$ amplitudes to the $2h$-$2p$ amplitudes which is bridged by the amplitudes $X_{\rho\mu\sigma\mu}$ plays a role in enhancing collectivity. The coupling of the $1p$-$1h$ amplitudes to the $1h$-$1p$ ones has a similar effect in RPA. We calculated the strength functions of the single giant resonances, i.e., GDR and GQR using RPA and the Tamm-Dancoff approximation (TDA). In TDA, there is no coupling of the $1p$-$1h$ amplitudes to the $1h$-$1p$ ones. We found that TDA gives more collective GDR and less collective GQR than RPA does.

In summary, the strength functions of DGDR and DGQR in ^{40}Ca were calculated using STDDM. The obtained strength functions were compared with the results in extended RPA theories including SRPA. It was found that SRPA overestimates the collectivity of DGDR and underestimates that of DGQR and that the inclusion of the $1p1h$-$1p1h$ amplitudes is important to obtain appropriate collectivity of the double giant resonances. Thus, it is concluded that the $2p$-$2h$, $2h$-$2p$, and $1p1h$-$1p1h$ amplitudes are the most important two-body amplitues to be considered in the application of extended RPA theories to double giant resonances.

[1] P. Chomaz and N. Frascaria, Phys. Rep. **252**, 275 (1995); T. Aumann, P. F. Bortignon, and H. Emling, Annu. Rev. Nucl. Part. Sci. **48** (1998) and references therein.
[2] G. Bertsch and S. Tsai, Phys. Rep., Phys. Lett. **18C**, 125 (1975).
[3] H. Suhl and N. R. Werthamer, Phys. Rev. **122**, 359 (1961).
[4] J. Sawicki, Phys. Rev. **126**, 2231 (1962); J. Da Providencia, Nucl. Phys. **61**, 87 (1965).
[5] C. Yannouleas, M. Dworzecka, and J. J. Griffin, Nucl. Phys. **A397**, 239 (1983).
[6] S. Drożdż, S. Nishizaki, J. Speth, and J. Wambach, Phys. Rep. **197**, 1 (1990).
[7] G. Lauritsch and P. G. Reinhard, Nucl. Phys. **A509**, 287 (1990).
[8] S. J. Wang and W. Cassing, Ann. Phys. (N.Y.) **159**, 328 (1985); W. Cassing and S. J. Wang, Z. Phys. A **328**, 423 (1987).
[9] M. Gong and M. Tohyama, Z. Phys. A **335**, 153 (1990).
[10] M. Tohyama, Phys. Lett. B **484**, 231 (2000).
[11] M. Tohyama, Nucl. Phys. **A657**, 343 (1999).
[12] M. Tohyama and M. Gong, Z. Phys. A **332**, 269 (1989).
[13] N. Kanesaki, T. Marumori, F. Sakata, and K. Takada, Prog. Theor. Phys. **49**, 181 (1973); **50**, 867 (1973).
[14] P. Danielewicz and P. Schuck, Nucl. Phys. **A567**, 78 (1994).
[15] T. Tamura and T. Udagawa, Nucl. Phys. **53**, 33 (1964).
[16] M. Beiner, H. Flocard, Nguyen Van Giai, and P. Quentin, Nucl. Phys. **A238**, 29 (1975).