Decoupling chemical and thermal freeze outs in hydrodynamics

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(Received 18 December 2000; published 20 November 2001)

From an analysis of various types of data obtained in relativistic nuclear collisions, the following picture has emerged in thermal and hydrodynamical descriptions: as the fluid expands and cools, particles first undergo a chemical freeze out at $T_{ch.f.} \sim 160$–200 MeV, then a thermal freeze out at $T_{th.f.} \sim 100$–140 MeV. In this paper we show how to incorporate these separate freeze outs consistently in a hydrodynamical code via a modified equation of state (general case) or via a modified Cooper-Frye formula (particular case of $T_{ch.f.}$ close to $T_{th.f.}$ or few particle species undergoing early chemical freeze out). The modified equation of state causes faster cooling and may have sizable impact on the predicted values of observables.

DOI: 10.1103/PhysRevC.64.064906 PACS number(s): 25.75.−q, 12.38.Mh, 24.10.Nz, 24.10.Pa

I. INTRODUCTION

The behavior of strongly interacting matter under extreme conditions of pressure and temperature is the subject of the research programs at CERN (SPS) and Brookhaven (AGS and RHIC). Hydrodynamical and thermal models have been used extensively to describe data from these collisions and the following picture has emerged (see, e.g., [1,2]) from a study of collisions at SIS, AGS, and SPS energies, with a variety of targets and projectiles (for a compilation see, e.g., [3]). It is also expected to hold at RHIC and LHC energies. The dense and hot fluid expands and cools until chemical freeze out occurs for some species of particles. Namely, these particles stop having inelastic collisions so that their abundances are frozen. Therefore by studying the abundances of these chemically frozen particle species, one can learn the conditions at chemical freeze out. For example, at CERN, $T_{ch.f.} \sim 160$–200 MeV [4]. The fluid goes on cooling until thermal freeze out happens. Precisely, particles stop having elastic interactions and so the shape of their momentum distribution is fixed. Therefore by studying these spectra, one gets information about the conditions at thermal freeze out. For example at CERN, using various types of particles [5] or combining information about the spectrum of a single-particle species and its Bose-Einstein correlations [6,7], one extracts $T_{th.f.} \sim 100$–140 MeV. There are some deviations to this picture. For example, some particles like the Ω may undergo both freeze outs almost together and early (due to their small cross section).

Though this picture is simple and consistent with the data, its theoretical justification needs further scrutiny. First, in a way, this picture works too well as a statistical description seems to apply even to very elementary systems [8]. This has been debated a lot yet remains an open question. Second, in this description, it is assumed that particles make *sudden* freeze outs. For example, when they cross the 180 MeV temperature three-dimensional surface in the fluid they immediately stop interacting inelastically. In reality one expects that this should happen over a certain length. A formalism to account for finite freeze-out volumes and the subsequent new data interpretation have been presented in [9]. Third, in the freeze-out scenario, when computing particle distribution with the usual Cooper-Frye formula [10], there may be particles contributing negatively, corresponding to particles that are in the frozen out region and reentering the interacting region. Physically this should not happen, but in the calculation, it may. Ways to deal with this problem and how it affects particle data interpretation can be found in [11].

Leaving aside these problems for further studies, the objective of this paper is the following. In the past, cascade event generators (FRITIOF, VENUS, RQMD, ARC, etc.) were employed, in particular by experimental groups, to study their data. Recently, simple thermal and hydrodynamics-inspired models have been used increasingly. It seems useful therefore to start developing more sophisticated hydrodynamical codes [12] to extract physical information from the data. Though the data call for separate chemical and thermal freeze outs, no hydrodynamical code so far includes chemical and thermal freeze outs self-consistently. Namely, the effect of the early chemical freeze out on the fluid expansion is never taken into account. In this paper, we discuss how to incorporate separate chemical and thermal freeze outs in a hydrodynamical code and show that, in certain cases, this will have a sizable impact on the predicted values of observables.

Finally let us mention that early universe and relativistic heavy ion collisions call for different treatments for the following reason. In the early universe, the expansion rate ($H \sim 10^4$ s^{-1}) at the QCD phase transition is many orders of magnitude smaller than for relativistic heavy ion collisions ($H \sim 10^{21}$–10^{23} s^{-1}). In both cases, a typical reaction rate for hadrons is $\Gamma = \sigma n v_{rel} \sim 1$ fm$^{-1} = 10^{23}$ s^{-1}. Therefore, while in the early universe chemical and thermal equilibrium between hadrons must have prevailed ($\Gamma \gg H$), this is not the case for relativistic heavy ion collisions (Γ may be $\sim H$). Moreover, in relativistic heavy ion collisions, we expect separate chemical and thermal freeze outs (i.e., at distinct temperatures) because inelastic collisions (responsible for chemical equilibrium) need higher center-of-mass energy to be operative in general, compared to elastic collisions (responsible for thermal equilibrium). This distinction needs not be made in the early universe (both rates Γ_{inel} and Γ_{el} being much larger than the expansion rate).

II. INCLUSION OF SEPARATE CHEMICAL AND THERMAL FREEZE OUTS IN THE HYDRODYNAMICAL EQUATIONS

To simplify the discussion, we use a known and simple hydrodynamical model, Bjorken one-dimensional boost-invariant model [13]. Before chemical freeze out, the fluid evolution is governed by the hydrodynamical equations

$$\frac{\partial \epsilon}{\partial t}+\frac{\epsilon+p}{t}=0, \tag{1}$$

$$\frac{\partial n_B}{\partial t}+\frac{n_B}{t}=0. \tag{2}$$

The last equation can be solved easily

$$n_B(t)=\frac{n_B(t_0)t_0}{t}. \tag{3}$$

The first equation must be completed by the choice of an equation of state $p(n_B,\epsilon)$, for the pressure as function of the net baryon density and energy density.

Also to simplify the discussion, we suppose that chemical or thermal freeze out occurs at some fixed temperature (as often assumed in the analysis of experimental data). Attempts to incorporate more physical freeze-out conditions have been carried out [14–17,12] and in principle might be incorporated in the scheme described below.

When the fluid temperature has decreased to some temperature $T_{ch.f.}$ (which corresponds to a certain time $t_{ch.f.}$), some particle species get their abundances frozen. To fix ideas, we suppose that Λ and $\bar{\Lambda}$ are in this situation. Then, in addition to the above hydrodynamical equations, we introduce separate conservation laws for these two types of particles for time $t>t_{ch.f.}$, namely,

$$\frac{\partial n_\Lambda}{\partial t}+\frac{n_\Lambda}{t}=0, \tag{4}$$

$$\frac{\partial n_{\bar{\Lambda}}}{\partial t}+\frac{n_{\bar{\Lambda}}}{t}=0. \tag{5}$$

These equations have solutions of the same form as Eq. (3) but with t_0 substituted by t_{ch}. Therefore what remains to be done is to solve the energy-momentum equation (1) with a *modified equation of state*, to account for the particles who make an early chemical freeze out.

We suppose that the fluid is a gas of noninteracting resonances. Then for particle species i, using an expansion in terms of modified Bessel functions [18] to allow the study of their limit more easily in Eqs. (9)–(11),

$$n_i=\frac{g_i m_i^2 T}{2\pi^2}\sum_{n=1}^{\infty}(\mp)^{n+1}\frac{e^{n\mu_i/T}}{n}K_2(nm_i/T), \tag{6}$$

$$\epsilon_i=\frac{g_i m_i^2 T^2}{2\pi^2}\sum_{n=1}^{\infty}(\mp)^{n+1}\frac{e^{n\mu_i/T}}{n^2}\times\left[3K_2(nm_i/T)+\frac{nm_i}{T}K_1(nm_i/T)\right], \tag{7}$$

$$p_i=\frac{g_i m_i^2 T^2}{2\pi^2}\sum_{n=1}^{\infty}(\mp)^{n+1}\frac{e^{n\mu_i/T}}{n^2}K_2(nm_i/T), \tag{8}$$

where m_i is the particle mass, g_i its degeneracy, and μ_i its chemical potential; the minus sign holds for fermions and plus for bosons. In principle each particle species i making early chemical freeze out has a chemical potential associated with it; this potential controls the conservation of the number of particles of type i. For particle species not making early chemical freeze out, the chemical potential is of the usual type, $\mu_i=B_i\mu_B+S_i\mu_S$, where μ_B (μ_S) ensures the conservation of baryon number (strangeness) and B_i (S_i) is the baryon (strangeness) number of particle of type i. So the modified equation of state depends not only on T and μ_B but also μ_Λ, $\mu_{\bar{\Lambda}}$, etc. (the notation "etc." stands for all the other particles making early chemical freeze out) [19]. This complicates the hydrodynamical problem; however, we can note the following.

If $m_i-\mu_i\gg T$ (the density of type i particle is low) and $m_i\gg T$ (these relations should hold for all particles except pions and we checked them for various times and particle types), the following approximations can be used:

$$n_i=\frac{g_i}{2\pi^2}\sqrt{\frac{\pi}{2}}(m_iT)^{3/2}e^{(\mu_i-m_i)/T}\left(1+\frac{15T}{8m_i}+\frac{105T^2}{128m_i^2}+\cdots\right), \tag{9}$$

$$\epsilon_i=n_i m_i\left(1+\frac{3T}{2m_i}+\frac{15T^2}{8m_i^2}+\cdots\right), \tag{10}$$

$$p_i=n_iT. \tag{11}$$

We note that ϵ_i and p_i are written in term of n_i and T. Therefore we can work with the variables $T,\mu_B,n_\Lambda,n_{\bar{\Lambda}}$, etc., rather than $T,\mu_B,\mu_\Lambda,\mu_{\bar{\Lambda}}$, etc. The time dependence of $n_\Lambda,n_{\bar{\Lambda}}$, etc., is known as discussed already. So the modified equation of state can be computed from t, T, and μ_B.

The scheme presented above can easily be generalized to particles making chemical freeze out at different times (using different $t_{ch.f.}$) and particles doing chemical and thermal freeze outs together (whose contribution drop out of the equation of state). One can show that entropy is conserved even in the presence of an early chemical freeze out when the hydrodynamical equations are satisfied by a perfect fluid [19].

For illustration, we present results using in the equation of state the basic multiplets of resonances (pseudoscalar meson octet plus singlet, vector meson octet plus singlet, baryon octet, and baryon decuplet) and supposing that the early

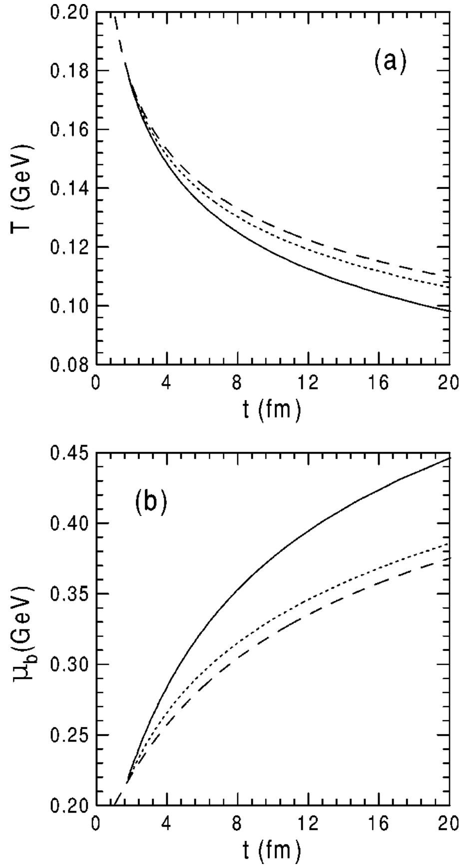

FIG. 1. μ_B and T as a function of time in the case where all particles have simultaneous freeze outs (dashed line) and (I) all strange particles in basic multiplets make an early chemical freeze out (solid line) and (II) all strange particles except K and K^* make an early chemical freeze out (dotted line).

chemical freeze out occurs at 180 MeV, a value typical for SPS energy more or less independently of the projectile [3]. We use for initial conditions $T_0 = \mu_{B0} = 200$ MeV, $\tau_0 = 1$ fm, so that $\mu_{B\,ch.f.} = 210$ MeV, which is in agreement with results for S or Pb at SPS [3]. In Fig. 1, we compare the behavior of T and μ_B as function of t, obtained from the hydrodynamical equations using the modified equation of state and the unmodified one. For the modified equation of state, we considered two scenarios: (I) all strange particles in the basic multiplets and (II) all strange particles except K and K^*'s make an early chemical freeze out. This is a conservative estimate; it is possible, for example, that the pions make an early freeze out [12]. Comparing scenarios I and II, we note that if more particles undergo early chemical freeze out, stronger effects for $T(t)$ and $\mu_B(t)$ are seen. We concentrate on I hereon. We see that the deviations between I and the unmodified equation of state case increase with time. In particular from this figure, if the thermal freeze out occurs at 110 MeV, the thermal freeze-out time is 13 fm for the modified equation of state and 20 fm for the unmodified one; the corresponding baryonic potentials are not very different, 405 and 375 MeV, respectively. If the thermal freeze out occurs earlier, say, at 140 MeV, the difference in the thermal freeze-out times would be much less. An immediate consequence of this is that the thermal freeze-out volume (in our case simply proportional to time) may be much smaller for the modified equation of state. We expect this conclusion to hold qualitatively even in the presence of transverse expansion: in this case, expansion is faster and the thermal freeze-out temperature is reached faster, so the thermal freeze-out times for the modified and unmodified equation of state are less different; however, there is a competing effect for the thermal freeze-out volumes: they now scale with higher powers of time. We also expect this conclusion to hold at RHIC energies, with T_{ch} still of order 180 MeV (it cannot be much higher since a transition to quark-gluon plasma is expected at about this temperature from lattice gauge simulations), but a lower value of $\mu_{B,ch}$ (less baryon stopping is expected at RHIC than SPS); the value of T_{th} may be a little smaller than at SPS [20,15] and $\mu_{B,th}$ will be higher than $\mu_{B,ch}$ (cf. Fig. 1). *We conclude that if the chemical and thermal freeze-out temperatures are very different* (in our simplified case, 180 and 110 MeV) *or if many particle species make an early chemical freeze out* (e.g., also pions), *it is important to take into account the effect of the early chemical freeze out on the equation of state* to make predictions for observables which depend on thermal freeze-out volumes—for example, particle abundances and eventually particle correlations. This is our main result.

III. PARTICULAR CASE WHERE $T_{ch} \sim T_{th}$ OR FEW PARTICLE SPECIES UNDERGO EARLY CHEMICAL FREEZE OUT

If the chemical and thermal freeze-out temperatures are not very different (say, 180 and 140 MeV) or if few particle species make the early freeze out, one can proceed as follows. One can use an unmodified equation of state in a hydrodynamical code and to account for early chemical freeze out of species i, when the number of type i particles was fixed, use a modified Cooper-Frye formula

$$\frac{Ed^3N_i}{dp^3} = \frac{N_i(T_{ch.f.})}{N_i(T_{th.f.})}\int_{S_{th.f.}} d\sigma_\mu p^\mu f(x,p). \qquad (12)$$

The second factor on the right hand side is the usual one and it gives the shape of the spectrum at thermal freeze out; the first factor is a normalizing term introduced such that upon integration on momentum p, the number of particles of type i is $N_i(T_{ch.f.})$. For illustration, we show results obtained with the hydrodynamical model HYLANDER-PLUS [21]. It provides a numerical solution of the relativistic hydrodynamical equations in 3+1 dimensions with axial symmetry (for details, see [21–24]). It gives a good description of single-particle-rapidity data, transverse-momentum spectra of h^-, π^-, p, $\bar{p}$, K^0, π^-/π^+, and pion correlation data at CERN energies, for an appropriate choice of the initial conditions, an equation of state incorporating a first-order phase transition, and a freeze-out temperature of 139 MeV. In Figs. 2 and 3, we show results obtained for Λ, $\bar{\Lambda}$ and Ξ, $\bar{\Xi}$, respectively (neglecting resonance decays), and data [25]. We see that both

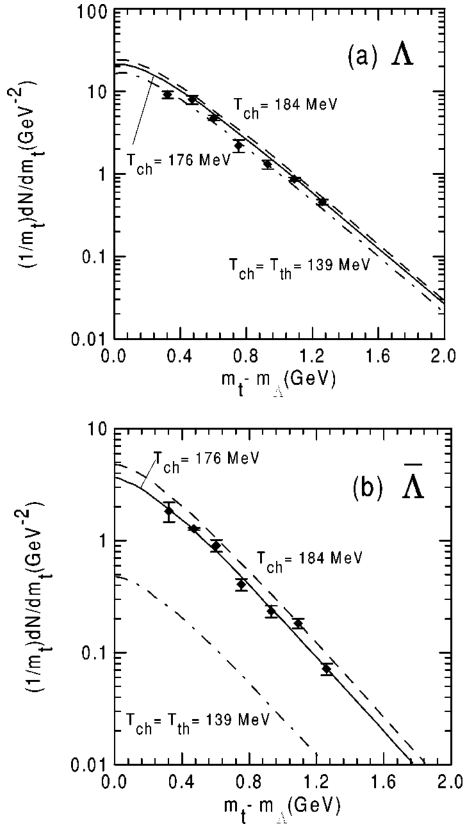

FIG. 2. Results from HYLANDER-PLUS for the transverse momentum spectra of Λ and $\bar{\Lambda}$ compared with data [22] for simultaneous freeze outs at $T_{ch.f.} = T_{th.f.} = 139$ MeV (dash-dotted line) as well as separate freeze outs at $T_{ch.f.} = 176$ MeV (solid line) or $T_{ch.f.} = 184$ MeV (dashed line) and $T_{th.f.} = 139$ MeV.

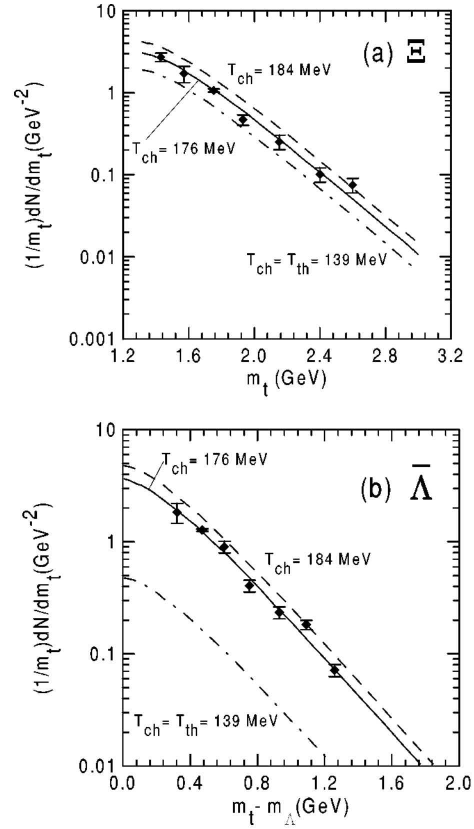

FIG. 3. Same as Fig. 2 but for Ξ and $\bar{\Xi}$.

their shapes and abundances can be reproduced for $T_{ch.f.} = 176$ MeV and $T_{th.f.} = 139$ MeV, while simultaneous freeze outs at $T_{ch.f.} = T_{th.f.} = 139$ MeV would yield the correct shapes but too few particles. Therefore, results with HYLANDER-PLUS and a modified Cooper-Frye formula (12) support the separate freeze-out pictures. However, in this code, $T_{th.f.}$ is fixed to 139 MeV while, as already mentioned, some data seem to imply lower thermal freeze-out temperatures. In the next generation of hydrodynamical codes, it is desirable to consider a wider range of $T_{th.f.}$.

IV. CONCLUSION

In summary, we showed how to incorporate separate chemical and thermal freeze outs in a hydrodynamical code via a modified equation of state in Sec. II (general case) or via a modified Cooper-Frye formula in Sec. III (particular case of $T_{ch.f.}$ close to $T_{th.f.}$ or few particle species undergoing early chemical freeze out). The modified equation of state causes faster cooling and may have sizable impact on the predicted values of observables.

ACKNOWLEDGMENTS

This work was partially supported by FAPESP (2000/04422-7,2000/05769-0) and CNPq (proc. 300054/92-0).

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