Quantum collisions of finite-size ultrarelativistic nuclei

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We show that the boost variable, the conjugate to the coordinate rapidity, which is associated with the center-of-mass motion, encodes the information about the finite size of colliding nuclei in a Lorentz-invariant way. The quasielastic forward color-changing scattering between the quantum boost states rapidly grows with the total energy of the collision and leads to an active breakdown of the color coherence at the earliest moments of the collision. The possible physical implications of this result are discussed.

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I. INTRODUCTION

It is commonly accepted that on the scale of the strong interaction, which is responsible for nuclear integrity and compactness, the large nuclei have a macroscopically finite size and a well-defined boundary.[1] This size can be physically measured in the rest frame of a nucleus, and it undergoes the Lorentz contraction in the moving frame without any physical limitations (as is required by special relativity). In this paper, we suggest to take this fact as a guideline, and explore the consequences of the finite size of the nuclei for the *quantum process* of their collision at ultrarelativistic energies. Of these consequences, the most important is the change of the symmetry: The incoming nuclei are prepared in a homogeneous space having a given energy and momentum. The fixed space-time point of the first interaction corrupts the initial symmetry, and enforces a different choice of the conserved quantum numbers for the later stages. Of the ten symmetries of the Poincaré group, only rotation around the collision z axis, Lorentz transformation along it, and the translations in the transverse x and y directions survive. Therefore, it is profitable to choose, in advance, the set of normal modes that have the symmetry of the localized initial interaction and carry quantum numbers adequate to this symmetry. These quantum numbers are the transverse components $\vec{p}_t$ of momentum and the boost, $\nu = p^0 z - p^z t$, of the particle (which is associated with the center-of-mass motion and replaces the component p^z of its momentum).

These geometric considerations can be reinforced by the quantum mechanical arguments. Indeed, from the perspective of an external observer, the first thing that happens during the collision is a precise measurement, by means of the strong interactions, of the collision coordinate within a very short time interval. Therefore, statistically, by the uncertainty principle, the secondaries with any conceivable momentum p_z can be detected after collision. This is a well-known scheme of the Heisenberg microscope. The higher resolution we want to achieve, the larger must be the energy resources of the microscope. In the textbook example of the electron probed by the photon, the electron receives energy from the hard photon. In nuclear collisions, both the kinetic energy of the nuclei and the energy of the compression of the Lorentz-contracted nuclei are used for the purpose of a precise measurement of the coordinate. An internal observer that penetrates the future of the collision with the nuclei will see a violently expanding matter around him. The two viewpoints perfectly complement each other. The short scales of primary interaction provide a sufficient motivation to use the wedge dynamics that describes the fields inside the future domain of the "wedge" $\tau^2 = t^2 - z^2 > 0$, and employs the "proper time" τ as a Hamiltonian time of the evolution and the coordinate rapidity η as a longitudinal coordinate [1,2]. The infamous rapidity plateau persistently observed in high-energy nuclear collisions strongly supports this picture.

The approach advocated in this paper explicitly incorporates the macroscopic finite size of the interacting objects into the quantum theory of the earliest stage of the collision. We assume that there is no measurable gluon fields outside the large stable nuclei. Consequently, the time moment and the z coordinate, along the collision axis, of the first interaction are defined with the accuracy of at least ~ 0.01 fm, which is both the size of a Lorentz-contracted individual nucleon and the characteristic scale of color correlation in the z direction before the collision. The full size of the Lorentz-contracted gold nucleus at the energy ~ 100 GeV per nucleon is ~ 0.1 fm. We show, that despite an almost infinite Lorentz contraction and the quantum nature of the interaction process, the information about the finite size of the incoming nuclei does not fade away. It remains clearly identifiable in terms of the properly chosen Lorentz-invariant variable, *the boost*, which is associated with the center-of-mass motion. Thus, it is possible to describe the collision of the two nuclei staying on the same physical ground in any reference frame, either in the reference frame of one of the nuclei, or in the laboratory frame where both nuclei move almost at the speed of light.

The fact that nuclei have finite size is intimately connected with the gauge nature of the strong interactions. Therefore, when addressing the problem of interaction of the two compact nuclei, we must refer to the properties of the

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[1]By the finite size, we mean the size that is measured by means of the strong interaction of two nuclei. If the primary interaction were electromagnetic (as is in the ep or eA processes), then the whole concept of a finite size would become doubtful.

vector gauge fields.[2] The colored sources of these fields must be located inside the nuclei and they can be physically resolved only after the two nuclei overlap. This is the only assumption we make regarding the internal structure of a nucleus. By all means, location of a material object inside a nucleus implies that its center of mass should move with this nucleus without crossing its boundary. Therefore, before the collision, it is natural to characterize such an object by its center of mass, i.e., by its boost ν. The valence quarks are the first candidates to be considered in this manner. In this sense, we follow the idea of McLerran-Venugopalan model [3] in the form given by Kovchegov and Mueller [4]. However, we do not try populate the nuclei with the wee partons. We think that they are *gradually created* in the course of collision [5,6].

The framework of wedge dynamics also offers a unique opportunity to avoid various technical problems encountered when the moving at the speed of light nuclei, $V_z = \pm c$, are taken as the first approximation [3,5]. This state cannot be reached as a continuous limit of $V \to c$ and a significant effort has been made to smooth out the singular behavior of quantum fields at $V = c$ [6–8]. The wedge form of Hamiltonian dynamics is free of this difficulty. Furthermore, the gauge $A^\tau = 0$ of the wedge dynamics can be fixed completely. Hence, the transverse and longitudinal fields are well separated and the gluon propagators of wedge dynamics have no spurious poles that can stimulate a singular behavior of scattering amplitudes [2]. In this framework, one can use the same dynamics and the same gauge for the description of both incoming nuclei and the products of their reaction [5,6], thus avoiding all glitches of the ''on-line'' changing the gauge and redefinition the states [9].

Below, we concentrate on a specific interaction in the expanding system that emerges in ultrarelativistic nuclear collisions. It is mediated by the longitudinal part of the gluon field.[3] It seems to be the leading one at the earliest moments of the collision of the two nuclei, and to result in the intensive color exchanges even in quasielastic subprocesses. Eventually, these exchanges must cause an active breakdown of the fragile color coherence of the colliding nuclei and stimulate intense color radiation. The rate σ_1 of these color exchanges between the quantum boost states appears to be large at the earliest moments of the collision, and it grows as $\ln^2 E$ with the total energy E. This major result of this paper is given by Eq. (3.27). The $\ln^2 E$ dependence of the rate on the total energy of the collision resembles the one obtained in the early 1960s estimate on the maximal rate at which the total cross section may grow with the energy. It is known as the Froissart bound, and a close dependence is indeed observed in the proton-proton collisions.

Originally, the Froissart bound was derived in the scope of the axiomatic field theory, a powerful approach based on the most general requirements, like Lorentz invariance, causality, unitarity, completeness of the basis of physical states and the cluster decomposition principle (see Ref. [10] for the details). Since the perturbation theory (usually in a given order) can lead to an anomalously large total cross section (and thus to apparently violate unitarity) it is said that the perturbative total cross section requires unitarization. Recently, this problem received a vigorous attention in connection with the evolution equations for large nuclei at low x_F [9,11]. A physical protection from an excessive growth of cross section due to collinear problems was offered in Ref. [6]. From this standpoint, one can infer that the result (3.27) of this paper indeed complies with the unitarity. Though this issue has to be studied in more details, we suggest a plausible simple physical argument below.

The axioms of unitarity and completeness clearly are not truly independent. Discussion of any issue related to unitarity requires that the spaces of the initial and final states are completely specified. Physically, this means that the measurement is not accomplished until its products are analyzed. What the particular states are, depends on the detectors that resolve these states. In nuclear collisions, one cannot rely on the conventional ''external'' distant detectors. The role of the detectors for the earliest subprocesses (which only very tentatively can be viewed as the independent acts of scattering) is played by the subsequent interactions. The next-to-best thing one can do is to try to answer the following question. Let the fields excited at the beginning of the collision be expanded over a system of states characterized by some quantum numbers. Let two such states interact. What is the rate of these interactions? The answer will be related to the two main problems of ultrarelativistic heavy ion collisions.

[2]Addressing the issue of interaction of finite-size nuclei, one should keep in mind the source of the major difference between QED and QCD phenomena. The local gauge symmetry of QED can be extended to a global gauge symmetry that generates the conserved gauge-invariant global quantum number, the electric charge, which can be sensed at a distance. The proper field of an electric charge is the main obstacle for the definition of its size. On the other hand, the radiation field of QED appears as a result of the changes in the extended proper fields of accelerated charges, and one can physically create such an object as a front of electromagnetic wave. In QCD, the local gauge invariance of the color group cannot be extended to is global version that would correspond to a gauge-invariant conserved charge. Hence, we can readily define the size of the colorless nucleus, but we cannot create a front of color radiation in the gauge-invariant vacuum. Both these properties of QCD work for us. They allow one to use the Lorenz contraction to localize the primary domain of the collision and thus, to impose the classical boundary conditions on the color fields at later times. The existence of the collective propagating quark and gluon modes at the later times is the conjecture that has to be verified by the study of heavy ion collisions.

[3]The division of the gauge field into the longitudinal and transverse parts can be done only with respect to the property of propagation: transverse fields are emitted and then propagate being limited in space-time by the light-cone boundaries, while the longitudinal fields are simultaneous (in terms of the Hamiltonian time) with their sources. In QCD, this scheme can be practically implemented only in the framework of perturbation theory, which is assumed throughout this paper.

First, the known rate of the primary interaction will help to estimate the entropy production. At this point, the explicit knowledge of the final states is imperative, because the entropy is the number of the excited degrees of freedom. Second, it will be directly connected to the total cross section. Indeed, if the fields change their colors during the time $\sim 1/E$ with sufficient probability, then the nucleons will lose their coherence and fall apart. A new composition of hadrons will be created with the probability one, and it does not really matter how the interacting states are chosen. This argument has been tested long ago: the total cross section of the e^+e^- annihilation into hadrons coincides with the cross section of the process $e^+e^-\to q\bar{q}$. One of the recently studied examples is the interaction of the eikonalized quarks or gluons [12]. In this paper, for the same purpose, we consider the "natural" states of the wedge dynamics, deliberately leaving the key question of *what* interacts at the very beginning of the collision open. We find that, because the states of wedge dynamics carry internal currents in the coordinate rapidity direction, there exists a specific contact interaction of these currents, which grows when $\tau\to 0$ and leads to the amplitude of interaction, proportional to $\ln E$. (The contact term in the gluon propagator has been singled out in the course of the complete fixing of the gauge $A^\tau=0$, and its main effect is confined to the nearest vicinity of the light wedge, $\tau=0$, where the boundary conditions that fix the gauge are imposed.) If the QCD indeed falls under a jurisdiction of the axiomatic field theory (which by no means is self-evident), then our perturbative result, which exactly reaches the Froissart bound, may point to the major physical mechanism that triggers the scenario of ultrarelativistic heavy ion collisions.

The paper is organized as follows. In Sec. II we introduce the variables of wedge dynamics and clarify the physical meaning of the boost in classical and quantum contexts. In Sec. III we use the boost states to estimate the amplitude of forward scattering with color transfer at the earliest moments of the collision, paying attention to the contact interaction in the expanding system. In Appendix A, we demonstrate that the contact term has no counterparts, and that the standard Coulomb-type terms are still there in the propagator. They are somewhat modified, just in a way that one could expect on purely physical grounds. In Appendix B we show, that the contribution of the other terms into the forward scattering amplitude is subleading.

II. CLASSICAL AND QUANTUM PARTICLES IN WEDGE DYNAMICS

In this section, we address the basic connection between the classical and quantum aspects of the interaction of compact relativistic objects, in order to prepare the stage for a more involved analysis of the interaction picture. First, we discuss the role of the classical Lorentz boost as a natural variable which, by its origin, is closely related to the finite size. Second, we review the meaning of the boost as a quantum number, and establish its connection with the classical boost. Finally, we show that the genuinely classical distribution of the boosts in stable nuclei before the collision plays a role as the initial data for the primary quantum interactions between nuclei.

A. Introducing the variables

The wedge form of relativistic dynamics works inside the future domain of the hyperplane $t=z=0$ (light wedge) were the two finite-size ultrarelativistic objects touch each other for the first time. The natural coordinates inside this domain are parametrized by the proper time τ and the rapidity coordinate η

$$t=\tau\cosh\eta,\quad z=\tau\sinh\eta. \tag{2.1}$$

In terms of these variables, the action for a classical particle is

$$S=\int L\,d\tau=-m\int ds=-m\int\sqrt{1-v^2}\,d\tau$$
$$=-m\int d\tau\sqrt{1-\tau^2\dot{\eta}^2-\dot{\vec{r}}^2}, \tag{2.2}$$

where $v^2\equiv\tau^2\dot{\eta}^2+\dot{\vec{r}}^2$ is the spatial velocity squared, and the dot means derivative over the (Hamiltonian) time τ.[4] The canonical momenta of this particle,

$$p_\eta\equiv\nu=\frac{\partial L}{\partial\dot{\eta}}=\frac{m\tau^2\dot{\eta}}{\sqrt{1-v^2}},\quad \vec{p}=\frac{\partial L}{\partial\dot{\vec{r}}}=\frac{m\dot{\vec{r}}}{\sqrt{1-v^2}}, \tag{2.3}$$

are conserved by virtue of the equations of motion. The Hamiltonian is of a standard relativistic form

$$H=\nu\dot{\eta}+\vec{p}\cdot\dot{\vec{r}}-L=\frac{m}{\sqrt{1-v^2}}, \tag{2.4}$$

which, after excluding the velocities, can be rewritten in terms of the canonical momenta

$$H=\sqrt{m^2+\vec{p}^2+\frac{\nu^2}{\tau^2}}. \tag{2.5}$$

The useful relations of geometric origin, which will be often referred to later on, are

[4]Following a tradition, we use the Greek indices for the four-dimensional vectors and tensors in the curvilinear coordinates (η is an exception, it always stands for the rapidity direction), and the Latin indices from a to d for the vectors in flat Minkowsky coordinates. We use Latin indices from r to w for the transverse x and y components $(r,\dots,w=1,2)$, and the arrows over the letters to denote the two-dimensional vectors, e.g., $\vec{k}=(k_x,k_y)$, $|\vec{k}|=k_t$. The Latin indices from i to n $(i,\dots,n=1,2,3)$ will be used for the three-dimensional internal coordinates $u^i=(x,y,\eta)$ on the hypersurface $\tau=\text{const}$.

$$p^{\eta}=-\frac{1}{\tau^2}p_{\eta}=-\left(\frac{\sinh\eta}{\tau}p^0-\frac{\cosh\eta}{\tau}p^3\right)$$
$$=-\frac{m_t}{\tau}\sinh(\eta-\theta),$$

$$H=\cosh\eta\, p^0-\sinh\eta\, p^3=m_t\cosh(\eta-\theta), \tag{2.6}$$

where $m_t^2=m^2+p_t^2$, $p^0=m_t\cosh\theta$, and $p^3=m_t\sinh\theta$ are the Cartesian momenta. Therefore, the boost

$$\nu=p_{\eta}=\tau m_t\sinh(\eta-\theta)=x^3p^0-x^0p^3\equiv p^0(z-V_zt) \tag{2.7}$$

is related to the center-of-mass coordinate. According to Eq. (2.5), the quantity ν/τ plays a role as a local longitudinal momentum.

The Hamilton-Jacobi equation for the classical action of a particle reads as

$$\frac{\partial S}{\partial\tau}+\sqrt{\frac{1}{\tau^2}\left(\frac{\partial S}{\partial\eta}\right)^2+\left(\frac{\partial S}{\partial\vec{r}}\right)^2+m^2}=0. \tag{2.8}$$

It allows for the separation of variables and has a solution

$$S=\nu\eta+\vec{p}\cdot\vec{r}-\int\sqrt{m_t^2+\frac{\nu^2}{\tau^2}}d\tau$$
$$=\nu\eta+\vec{p}\cdot\vec{r}-\sqrt{m_t^2\tau^2+\nu^2}+\nu\sinh^{-1}\frac{\nu}{m_t\tau}. \tag{2.9}$$

In a quantum context, this action serves as the phase of a semiclassical wave function, $\psi\sim e^{iS}$, with the quantum numbers ν and $\vec{p}$, either when $\nu\gg\tau m_t$ or when $\tau m_t\gg\nu$. An isolated solution with the not separated variables is

$$S=\vec{p}\cdot\vec{r}-m_t\tau\cosh(\eta-\theta). \tag{2.10}$$

It corresponds to a plane wave, and its parameter, the (momentum) rapidity θ, is not a canonical momentum.

B. Classical trajectories. The physical meaning of the boost ν

In order to understand the physical meaning of the boost variable ν, the canonical conjugate to the rapidity η, one has to figure out how it enters the classical equations of motion. According to the Jacobi theorem, the action $S(x_n,a_n)$, known as a function of coordinates x_n and arbitrary constants a_n, allows one to find an additional set of the conserved quantities, $\partial S/\partial a_n=b_n$. While the constants $a_n=\partial S/\partial x_n$ usually are the canonical momenta corresponding to the cyclic coordinates and are conserved due to the equations of motion, as in Eq. (2.9), the constants b_n appear to be the initial coordinates. Applying the Jacobi theorem to the action (2.9), and choosing the constants in such a way that at $\tau=0$ we have $x=x_0$, and that at $\tau\to\infty$ we have $\eta=\theta$, we obtain the equation of the particle trajectory

$$x(\tau)-x_0=\frac{p_x}{m_t^2}(\sqrt{\tau^2m_t^2+\nu^2}-|\nu|),$$
$$\eta(\tau)-\theta=-\sinh^{-1}\frac{\nu}{m_t\tau}. \tag{2.11}$$

Despite their unusual appearance, these two equations parametrize a straight line, as it should be for the free motion of a pointlike particle. Let us rewrite the second of equations (2.11) in two ways:

$$m_t\tau\sinh[\eta(\tau)-\theta]=\nu=zp^0-tp^z\to m_tz_*, \tag{2.12}$$

and

$$m_t\tau\cosh[\eta(\tau)-\theta]=tp^0-zp^z=\sqrt{\tau^2m_t^2+\nu^2}\to m_tt_*, \tag{2.13}$$

where the arrows point to the special choice of the reference frame with $\theta=0$.[5] Then the first of the equations (2.11) becomes

$$x(\tau)-x_0=\frac{p_x}{m_t}\left(\sqrt{\tau^2+\frac{\nu^2}{m_t^2}}-\frac{|\nu|}{m_t}\right)\to\frac{p_x}{m_t}(t_*-|z_*|), \tag{2.14}$$

obviously satisfying the required boundary condition at $\tau=0$. Now, it is easy to understand that the quantity ν/m_t is the τ-independent coordinate z_* of the particle in the co-moving frame. By the definition, this quantity is Lorentz invariant: the boost ν is the same in all Lorentz frames. The Cartesian form (2.14) of the trajectory is obviously continued to all quadrants of the tz plane. This classical definition of the boost is fairly operational but, as the reader may notice, it requires that the base world line (plane) from which the coordinate z_* is measured is explicitly chosen. For the two colliding nuclei, it is natural that the base lines (corresponding to the rapidities $\pm Y$) go through the point $t=z=0$, where the nuclei touch each other by their surfaces. If the nuclei have radius R and are built from the fragments of the (transverse) mass m_t, then the boosts for the right-moving nucleus will be in the range $-2m_tR<\nu<0$, and in the range $0<\nu<2m_tR$ for the left-moving one. There is no contradiction with quantum mechanics at this point, since the nuclei are *macroscopic* stable objects that can be kept under nondestructive control (in their co-moving reference frames) before the collision. Asymptotically, they have the well-defined rapidities $\theta=\pm Y$, which can be also measured classically, without any contradiction with the anticipated uncertainty

[5] We consider the physical design of the nucleus as almost static, and neglect the possible velocity V_*^z of the nuclear constituent in the nuclear rest frame. In any case, it cannot be large without undermining the alleged stability of the nucleus. The origin of the transverse mass may be different. It includes both the Lagrangian mass and the ‘‘adjoint mass’’ due to the transverse momentum. Inside a stable nucleus, the momenta most probably characterize the standing waves that are not likely to be too short, if the nucleus is in the ground state.

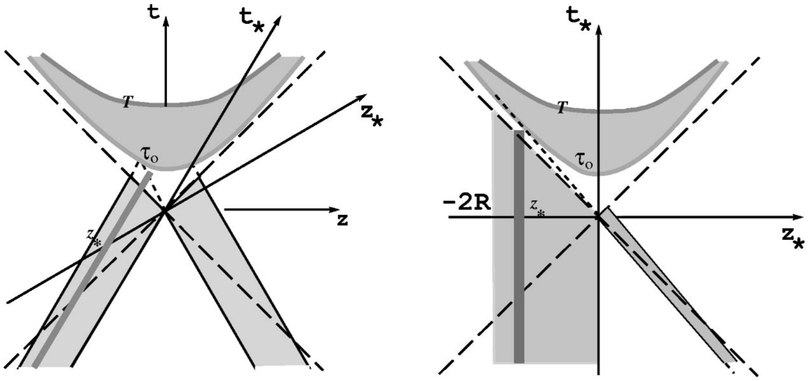

FIG. 1. Geometry of a nucleus-nucleus collision in the center-of-mass reference frame (left) and in the rest frame of one of the nuclei (right). The dark gray lines correspond to a semiclassical boost state in the right-moving nucleus before the collision.

relation, $\Delta\nu\Delta\eta \geqslant 1$. Indeed, the boosts $\nu \approx m_t z_*$ are measured *inside* the nuclei, while the measurements of the velocities of the nuclei is performed by external devices. Therefore, the boost variable is indeed perfectly suited for the description of the finite size objects. If the relative boosts of all constituents do not change in the course of the interaction, then the object remains unaltered in its (possibly new) rest frame (see Fig. 1).

As a matter of fact, the boosts provide an *invariant measure* of the distribution of the constituents of the compact objects. The picture of rectilinear trajectories holds outside the light wedge also. Therefore, the classically prepared distribution of the boosts is resolved as the distribution of the further interacting quantum states with the given boosts, when two such objects collide. Though Eq. (2.12) expresses the boost ν via the invariant m_t and distance z_*, in a quantum picture, the boost ν is an independent conserved additive quantum number.

For isolated pointlike (and thus, structureless) objects, the practical measurement of the boost requires that the rapidity $\eta(\tau)$ is measured at two time moments along the same trajectory. Then, solving the system of two equations (2.12), one finds ν and θ, the boost and the asymptotic rapidity of the particle. It is unrealistic to perform such measurements with sufficient accuracy in the asymptotic domain of the macroscopically large τ. Unlike the case of the macroscopic finite-size object, this kind of measurement does meet quantum-mechanical obstacles.

C. The boost ν in quantum context

The quantum-mechanical measurement of the boost ν is very similar to the measurement of a usual momentum and relies on the definition of the operator of the boost

$$\hat{\nu} = -i\frac{\partial}{\partial\eta}, \tag{2.15}$$

as the operator of translations in the η direction. Then it becomes evident, that a simultaneous measurement of coordinate η and momentum ν is limited by the uncertainty relation

$$\Delta\nu\,\Delta\eta \geqslant 1. \tag{2.16}$$

In the field-theory formulation, the boost operator is given by the generator of the Lorentz rotations in the tz plane. In the internal geometry of wedge dynamics, the boost operator is given by the $\tau\eta$ component of the energy-momentum tensor. The boost of the quantum field at the proper time τ

$$\nu = \int_{\tau=\mathrm{const}} T^{\tau\eta}(x)\,\tau d\eta d^2\vec{r} = \int d\Sigma_\mu M^{\mu 03}(x) \tag{2.17}$$

(where $M_{\mu\nu\lambda} = x_\nu T_{\mu\lambda} - x_\lambda T_{\mu\nu} + S_{\mu\nu\lambda}$ is the usual angular momentum tensor) is the integral of motion corresponding to the translation symmetry (Lorentz rotation) in η direction. The quantum states with the given boost ν are the eigenstates of the operator (2.15) and their eigenfunctions depend on η as $e^{i\nu\eta}$. The full wave function of a scalar particle with the boost ν and the transverse momentum $\vec{p}$ is the solution of Klein-Gordon equation with the separated variables τ, η, and $\vec{r}_t$,

$$\psi^{(+)}_{\vec{p},\nu}(x) = \frac{e^{-\pi\nu/2}}{2^{5/2}\pi} H^{(2)}_{-i\nu}(m_t\tau) e^{i\nu\eta + i\vec{p}\vec{r}}. \tag{2.18}$$

It is normalized on the hypersurfaces $\tau = \mathrm{const}$,

$$\int_{\tau=\mathrm{const}} \psi^{*}_{\theta',\vec{p}'}(x)\, i\frac{\overleftrightarrow{\partial}}{\partial\tau}\psi_{\theta,\vec{p}}(x)\,\tau d\eta d^2\vec{r} = \delta(\theta-\theta')\,\delta(\vec{p}-\vec{p}\,'). \tag{2.19}$$

This equation normalizes the measurements performed by an array of the detectors moving with all possible velocities. At any particular time of the Lorentz observer, this array even does not cover the whole space.

At large $\nu \gg 1$, and $\nu > m_t\tau$, which is relevant to the earliest stage, the asymptotic of this solution is semiclassical

$$\psi^{(+)}_{\vec{p},\nu}(x) \approx \frac{e^{i\pi/4}}{4\pi^2}\frac{e^{i\nu\eta+i\vec{p}\vec{r}}}{[m_t^2\tau^2+\nu^2]^{1/4}}\exp\Big[-i\sqrt{m_t^2\tau^2+\nu^2} + i\nu\sinh^{-1}(\nu/m_t\tau)\Big] \propto e^{iS}, \tag{2.20}$$

clearly indicating that at the small time τ the quantum particle with the finite boost ν continues to follow its classical trajectory, since its classical action is large. Indeed, the surface of the light wedge, everywhere except for its vertex, corresponds to $\eta\to\infty$.

The wave functions (2.18) are connected, by means of Fourier transform, with the plane-wave solutions

$$\omega^{(+)}_{\vec{k},\theta}(x)=\int_{-\infty}^{+\infty}\frac{d\nu}{(2\pi)^{1/2}i}e^{-i\nu\theta}\psi^{(+)}_{\vec{k},\nu}(x)$$
$$=\frac{1}{4\pi^{3/2}k_t}\exp[-ik_t\tau\cosh(\theta-\eta)+i\vec{k}\vec{r}]. \tag{2.21}$$

The saddle point of the Fourier transform (2.21) (or its inverse) is located at the value of ν (or θ) defined by the relation, $\nu=\tau m_t\sinh(\theta-\eta)$, corresponding to the classical definition (2.7) of the boost. One can easily see that these wave functions also are semiclassical with the action (2.10), and have a usual momentum (or the rapidity θ) as a quantum number. These states become localized in rapidity η at later times, $\tau m_t\gg 1$, and these states are most likely to be detected by the expanding collective system.

The key element of the suggested approach is that the Lorentz-invariant boost states, which are independently prepared in the two approaching nuclei, begin to interact as the quantum states only when the nuclei overlap. At this moment, the positions of the nuclei constituents (classical boosts, which describe the elementary constituents of the nuclei even outside the light wedge) are translated into the quantum numbers, which define the periodicity of the wave functions in the coordinate rapidity direction. It is evident that at the earliest times the distortion of the initial geometric picture should be only minimal. Therefore, it will be a sufficient approximation to study the transitions into other boost states, and we stay within this approximation until the end of this paper. The rate at which these early distortions develop appears to be quite large.

The dynamics of boost states preserves the *invariant information* about the finite size of the nuclei both in the laboratory frame when each of the two nuclei is contracted up to a negligible small size, and in the rest frame of one of the nuclei (target) when the second one (projectile) passes through it as a seemingly infinitely sharp shock front. One cannot assign a finite width to the moving in the x^+ direction front, neither in the z direction, nor in the x^- direction, without a conflict with the special relativity. On the other hand, in the framework of the wedge dynamics that operates with the boost states, it is safe to consider the limit of the infinite momentum frame at the end of the calculations.[6]

[6]This, however, leaves open the question of what is detected in high-energy collision. The answer crucially depends on what kind of the quantum-mechanical ensemble is involved in a particular measurement.

III. SCATTERING IN WEDGE DYNAMICS

The nuclei meet each other at the two-dimensional plane $t=z=0$, where the first interaction take place. This interaction resolves the nuclei constituents (e.g., the ‘‘partons,’’ or ‘‘color dipoles’’) with the boost $\nu\approx 0$, and excite the quantum states with the boost $\nu\approx 0$. The wave functions of these states do not depend on the rapidity coordinate η, and they evenly fill in the interior of the light wedge. At the same time, the two precursors, which are most likely to be the fronts of the propagating gluon field, begin their way in the lightlike directions, $t\pm z=0$, thus creating the physical boundaries of the light wedge, $\tau^2=t^2-z^2=0$. Passing through the nuclei, the precursors resolve the elements with the finite boosts, which are negative for the right-moving nucleus and positive for the left-moving one, and initiate a transient process of interaction between the nuclei. These interactions excite the quantum states with positive and negative boosts, which depend on η as $e^{i\nu\eta}$. In this way, the classical boosts, $\nu_{cl}=m_t z_*$, are transformed into the quantum numbers of the wave functions that have the period $2\pi/\nu$ in the η direction, and occupy the entire future domain of the point $t=z=0$. Before the collision the nuclei as a whole are the coherent states of QCD and their (color) coherence cannot be destroyed immediately. At $\tau\to+0$, the resolved boost states have the same phases they had in the nuclei: the decomposition of the nuclei in terms of the boost states is still a coherent superposition.[7] Furthermore, since the classical action of the states with the finite boosts is large, even the resolved partons continue to move along their rectilinear classical trajectories. The character of the further evolution crucially depends on the subsequent interactions. Below, we study the quasielastic forward scattering of the colored quarks prepared and detected in the given boost states. This scattering is mediated by the gluon field and results in the color exchange that alone is capable of destroying the coherence of the nuclear wave function.

The propagators of the gauge fields in wedge dynamics were studied in [1,2]. In Appendix A, we review their properties with the emphasis on the needs of the present study. The leading contribution comes from the spatially local ‘‘contact term’’ of the longitudinal part of the propagator. In order to give a flavor of its origin, we have to emphasize, that we study the phenomenon where the finite charge density is formed as a result of the interaction, and the proper fields of the gradually created and yet delocalized charges physically overlap with their sources. Thus, aiming at the dynamic picture, we have to give the priority to the currents, expressing the charge density $\rho(t)$ via the divergence of the

[7]The boundary condition $A_\eta(\tau=0)=0$ imposed on the gauge fields in the wedge dynamics, together with the gauge condition $A_\tau=0$, makes it impossible that the fields of precursors immediately modify the phases (rotate the color charges) along the lightlike planes $x^+=0$ and $x^-=0$. This property, which allows one to ‘‘switch on’’ the interaction between the nuclei without an artificial color-changing ‘‘shock wave,’’ is in contrast with the case of the null-plane dynamics with the gauges $A^\pm=0$.

current, $\partial_t \rho = -\nabla \cdot \mathbf{j}$, which eventually generates the contact term in the propagator. The effect of the evolving charge density $\rho(t)$ becomes fully included into the Hamiltonian $\mathcal{H}_{int} = \mathbf{j} \cdot \mathbf{A}$, which is the only form compatible with the *completely fixed* gauge $A^\tau = 0$. This evolution of the color charge density is the result of the interference between various partial waves, and it is not connected with the motion of the physically resolved pointlike color charges. Without an interaction, these partial waves would coherently sum and form the stable nuclei. Of those interactions that take place when the nuclei intersect, the most important are the ones that lead to the largest transition amplitudes.

An apparent complexity of the formulas in the wedge dynamics is caused by the curvature of the hypersurfaces of the constant τ. The hypersurface $\tau = +0$ is the one where the initial data are naturally set, and it has an infinite curvature. An explicit dependence of the internal metric on τ makes the vector differential operators more cumbersome and leads to an interplay between the longitudinal and transverse fields.

A. Choosing the observable

Wedge dynamics deals only with the fields that emerged from the localized collision of two macroscopic objects. This collision is considered as a precise measurement of the partons coordinates at the finite time moment $\tau \to +0$. Therefore, it is impossible to pose a formal scattering problem with the asymptotic initial states. Instead, we take an approach based on the calculation of the Heisenberg observable [5,13],

$$N(1',2') = \langle 1,2 | \hat{n}(1')(\hat{n}(2') - \delta_{1'2'}) | 1,2 \rangle = \langle 0 | a_2 a_1 S^\dagger a_{2'}^\dagger a_{1'}^\dagger a_{1'} a_{2'} S a_1^\dagger a_2^\dagger | 0 \rangle, \quad (3.1)$$

which is the inclusively measured number of pairs of the final state field excitations with quantum numbers $1' = (i_1', k_1')$ and $2' = (i_2', k_2')$ (k includes the transverse momentum and boost, i color). This observable is evolved from the initial state of the two interacting field excitations with quantum numbers $1 = (i_1, k_1)$ and $2 = (i_2, k_2)$. This quantity is closely related to the *total cross section*. Indeed, we assume that the measurement is an impulse process that freezes decomposition of the colliding nuclei in terms of the eigenfunctions of the corresponding operator. This decomposition can become incoherent only due to real interaction, which will either excite the new states, or just break the phase balance between the initial ones. All this will contribute to the probability that the initial state is altered, i.e., to the imaginary part of the forward scattering amplitude. The color exchanges take place at the earliest possible time $\tau_{min} \sim 1/\sqrt{s}$, and create a new color composition that must eventually (with the probability one) evolve into a new composition of hadrons. We emphasize that a particular choice of the basis of the interacting at $\tau > 0$ boost states is important only as long as we are interested in the rate at which the color coherence is broken. The color transfer between the boost states seems to be extremely intensive at the beginning of the collision. The geometry of the collective modes that will be the actual final states can be quite different [14].

Expression (3.1) is bilinear with respect to the evolution operator S and thus it cannot be treated according to the Feynman rules. For its evaluation one should use the so-called Schwinger-Keldysh technique [15] in the form adjusted for the calculation of inclusive amplitudes [13]. The evolution operator for the problem of evolution of the observable (3.1) is of a usual form

$$S = T \exp\left\{ i \int H_{int}(x) d^4x \right\} \quad (3.2)$$

with the Hamiltonian

$$H_{int}(x) = j^\mu(x) A_\mu(x) = j^\mu(x) \left[A_\mu^{[tr]}(x) + \int dz D_{\mu\nu}^{[long]}(x,z) j^\nu(z) \right], \quad (3.3)$$

where the second term in brackets is the longitudinal field $A_\mu^{[long]}(x)$. The propagator $D_{\mu\nu}^{[long]}(x,z)$ implicitly contains $\theta(x^0 - z^0)$. For the sake of definiteness, consider the fermion color current

$$j^\mu(x)_a = g \bar{\Psi}_i(x) t_{ij}^a \gamma^\mu \Psi_j(x), \quad (3.4)$$

and commute the final-state Fock operators with S and $S^\dagger$ using the commutators

$$a_i(k) S - S a_i(k) = \int dz \bar{\psi}_k^{(+)}(z) \frac{\delta S}{\delta \bar{\Psi}_i(z)},$$

$$S^\dagger a_i^\dagger(k) - a_i^\dagger(k) S^\dagger = \int dz \frac{\delta S^\dagger}{\delta \Psi_i(z)} \psi_k^{(+)}(z). \quad (3.5)$$

In this equation, $\psi_k^{(+)}(z)$ is the one-particle wave function from the decomposition of the field operator

$$\Psi_i(x) = \sum_k [a_i(k) \psi_k^{(+)}(x) + b_i^\dagger(k) \psi_k^{(-)}(x)]. \quad (3.6)$$

These commutations result in (disconnected pieces are omitted)

$$N(1',2') = \int dx_1 dx_2 dy_1 dy_2 \bar{\psi}_{k_2'}^{(+)}(y_2) \bar{\psi}_{k_1'}^{(+)}(y_1) \times \langle 0 | a_2 a_1 \frac{\delta^2 S^\dagger}{\delta \Psi_{i_2'}(x_2) \delta \Psi_{i_1'}(x_1)} \times \frac{\delta^2 S}{\delta \bar{\Psi}_{i_1'}(y_1) \delta \bar{\Psi}_{i_2'}(y_2)} a_1^\dagger a_2^\dagger | 0 \rangle \psi_{k_2'}^{(+)}(x_2) \psi_{k_1'}^{(+)}(x_1). \quad (3.7)$$

Here, the functional derivatives over Ψ act from the left, and the derivatives over $\bar{\Psi}$ act from the right. Next, we compute the functional derivatives retaining the terms up to the order g^2. This yields

$$
\begin{aligned}
N(1',2') = g^4 \int & dx_1 dx_2 dy_1 dy_2 \overline{\psi}^{(+)}_{k_2'}(y_2)\overline{\psi}^{(+)}_{k_1'}(y_1)\langle 0|a_{i_2}(k_2)a_{i_1}(k_1) \\
& \times T^{\dagger}[-\overline{\Psi}_{l_2}(x_2)\gamma^{\mu}\overline{\Psi}_{l_1}(x_1)\gamma^{\nu}A^{[tr]a}_{\mu}(x_2)A^{[tr]b}_{\nu}(x_1) + i\overline{\Psi}_{l_2}(x_2)\gamma^{\mu}D^{[long]ab}_{\mu\nu}(x_2,x_1)\overline{\Psi}_{l_1}(x_1)\gamma^{\nu} \\
& - i\overline{\Psi}_{l_1}(x_1)\gamma^{\mu}D^{[long]ba}_{\mu\nu}(x_1,x_2)\overline{\Psi}_{l_2}(x_2)\gamma^{\nu}]t^{a}_{l_2 i_2'}t^{b}_{l_1 i_1'}t^{a'}_{i_2' j_2}t^{b'}_{i_1' j_1} \\
& \times T[\Psi_{j_2}(y_2)\gamma^{\mu}\Psi_{j_1}(y_1)\gamma^{\nu}A^{[tr]a'}_{\mu}(y_2)A^{[tr]b'}_{\nu}(y_1) \\
& - i\gamma^{\mu}\Psi_{j_2}(y_2)D^{[long]a'b'}_{\mu\nu}(y_2,y_1)\gamma^{\nu}\Psi_{j_1}(y_1) \\
& + i\gamma^{\mu}\Psi_{j_1}(y_1)D^{[long]b'a'}_{\mu\nu}(y_1,y_2)\gamma^{\nu}\Psi_{j_2}(y_2)]a^{\dagger}_{i_1}(k_1)a^{\dagger}_{i_2}(k_2)|0\rangle \psi^{(+)}_{k_2'}(x_2)\psi^{(+)}_{k_1'}(x_1).
\end{aligned}
\tag{3.8}
$$

The calculations are accomplished as follows. The fermion operators are contracted with the remaining Fock operators of the initial state, producing the final-state wave functions, and making the final adjustment of the color indices. This can be done in two ways, which differ by a full interchange of the quantum numbers of the one-particle initial states. The vacuum average of the products of the transverse gluon field operators gives the transverse part of the T-ordered propagator $D^{[00]}(y_2,y_1)$ and of the $T^{\dagger}$- ordered propagator $D^{[11]}(x_2,x_1)$.[8] The two terms, with $D^{[long]}(y_2,y_1)$ and $D^{[long]}(y_1,y_2)$ cover two complementary domains, $y_2^0>y_1^0$ and $y_2^0<y_1^0$, respectively. Together, they form the longitudinal part of the T-ordered propagator $D^{[00]}(y_2,y_1)$. Finally, the transition probability can be cast in the form

$$
\begin{aligned}
N(1',2') = g^4 \Bigg| \int & dx_1 dx_2 [\overline{\psi}^{(+)}_{k_2}(x_2)\gamma^{\mu}\psi^{(+)}_{k_2'}(x_2)\overline{\psi}^{(+)}_{k_1}(x_1) \\
& \times \gamma^{\nu}\psi^{(+)}_{k_1'}(x_1)D^{[00]}_{\mu\nu}(x_2,x_1)t^{a}_{i_2 i_2'}t^{a}_{i_1 i_1'} \\
& - \text{the same}(k_1,i_1 \leftrightarrow k_2,i_2)]\Bigg|^2 .
\end{aligned}
\tag{3.9}
$$

B. Scattering of scalar quarks with the given boosts ν

The observable number of couples, $N(1',2')$, can be rewritten by introducing the full set of the intermediate states into Eq. (3.1),

$$
\begin{aligned}
N(1',2') &= \sum_X \langle 0|a_2 a_1 S^{\dagger} a^{\dagger}_{2'} a^{\dagger}_{1'}|X\rangle\langle X|a_{1'}a_{2'}S a^{\dagger}_1 a^{\dagger}_2|0\rangle \\
&= \sum_X |\langle X|a_{1'}a_{2'}S a^{\dagger}_1 a^{\dagger}_2|0\rangle|^2 .
\end{aligned}
\tag{3.10}
$$

In the lowest order of the perturbation theory there is no additional emissions and only the vacuum state $|X\rangle=|0\rangle$ contributes:

$$
N(1',2') = |M_{1,2\to 1',2'}|^2 . \tag{3.11}
$$

In the lowest order, the inclusive transition probability (3.10) is just the squared modulus of the matrix element depicted on Fig. 2.

1. Scattering amplitude

Consider the matrix element of the scattering amplitude

$$
M_{1,2\to 1',2'} = g^2 \int dx_1 dx_2 j^{\mu}_{k_2,k_2'}(x_1) j^{\nu}_{k_1',k_1'}(x_2) D^{[00]}_{\mu\nu}(x_1,x_2), \tag{3.12}
$$

where we exchange the spinor quarks for the scalar ones, and accordingly replace

$$
\begin{aligned}
j^{\mu}_{k,k'}(x) &= \overline{\psi}^{(+)}_{k}(x)\gamma^{\mu}(x)\psi^{(+)}_{k'}(x) \\
&\to g^{\mu\nu}(x)\overline{\psi}^{(+)}_{k}(x)\, i\overleftrightarrow{\partial}_{\nu}\psi^{(+)}_{k'}(x),
\end{aligned}
$$

using the states of scalar quarks with the quantum numbers $k=(\vec{k},\nu)$, transverse momentum and boost. In this case, the wave functions are of the form

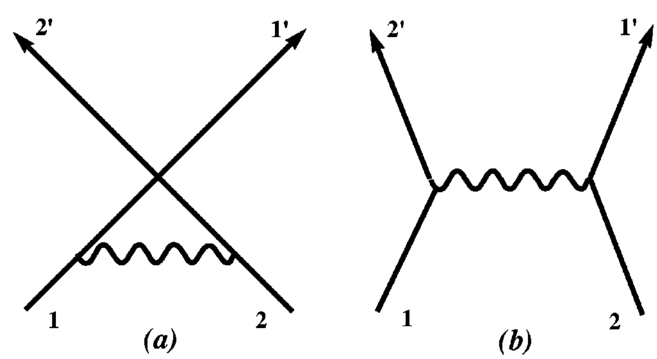

FIG. 2. Forward (a) and backward (b) amplitudes of the qq scattering.

[8]In this paper, we use the Keldysh-Schwinger formalism [15] in its modified form developed earlier with the view of application to the inclusive and transient processes. We employ the notation used in Refs. [5,6,13]. The indices of the field correlators with the Keldysh contour ordering of the field operators (e.g., $D_{[AB]}$) as well as the labels of their linear combinations (e.g., $D_{[ret]}$) are placed in square brackets.

$$\psi^{(+)}_{\vec{k},\nu}(x)=\frac{e^{-\pi\nu/2}}{2^{5/2}\pi}H^{(2)}_{-i\nu}(m_t\tau)e^{i\nu\eta+i\vec{k}\vec{r}},$$

$$\overline{\psi}^{(+)}_{\vec{k},\nu}(x)=\frac{e^{-\pi\nu/2}}{2^{5/2}\pi}H^{(1)}_{i\nu}(m_t\tau)e^{-i\nu\eta-i\vec{k}\vec{r}}. \tag{3.13}$$

Using the propagator in the mixed representation

$$D^{[00]}_{lm}(x_1,x_2)=\int\frac{d\zeta d\vec{q}}{(2\pi)^3}D^{[00]}_{lm}(\tau_1,\tau_2;\zeta,\vec{q})\times\exp[-i\zeta(\eta_1-\eta_2)-i\vec{q}(\vec{r}_1-\vec{r}_2)], \tag{3.14}$$

and integrating over the spatial coordinates, we obtain

$$\begin{aligned}M_{1,2\to1',2'}=&\frac{g^2}{2^7\pi}\delta(\nu_1+\nu_2-\nu_1'-\nu_2')\delta(\vec{k}_1+\vec{k}_2-\vec{k}_1'-\vec{k}_2')\\&\times\int_0^\infty\tau_1d\tau_1\int_0^\infty\tau_2d\tau_2H^{(1)}_{i\nu_1'}(m_1'\tau_1)H^{(2)}_{i\nu_1}(m_1\tau_1)\\&\times H^{(1)}_{i\nu_2'}(m_2'\tau_2)H^{(2)}_{i\nu_2}(m_2\tau_2)g^{ll}(\tau_1)g^{mm}(\tau_2)\\&\times(k_1+k_1')_l(k_2+k_2')_mD^{[00]}_{lm}(\tau_1,\tau_2;\zeta,\vec{q}),\end{aligned} \tag{3.15}$$

where $\zeta=\nu_1-\nu_1'=\nu_2-\nu_2'$, $\vec{q}=\vec{k}_1-\vec{k}_1'=\vec{k}_2-\vec{k}_2'$, and we introduced three-vectors, $g^{ll}(\tau)p_l=(-\vec{p},-\nu/\tau^2)$, as well as a short-hand notation, $m_i^2=m^2+\vec{k}_i^2$, $m_i'^2=m^2+\vec{k}_i'^2$.

Computing the transition amplitude (3.12), we will be interested in the states with the large boosts $|\nu|\gg1$, $\nu>m_t\tau$, $|\nu_1-\nu_2|\gg1$. In this case, the asymptotic of the Hankel functions reads as

$$\begin{aligned}\pi e^{-\pi\nu/2}H^{(2)}_{-i\nu}(m_t\tau)&=[\pi e^{-\pi\nu/2}H^{(1)}_{i\nu}(m_t\tau)]^*\\&\approx\frac{\sqrt{2\pi i}}{[m_t^2\tau^2+\nu^2]^{1/4}}\exp[-i\sqrt{m_t^2\tau^2+\nu^2}\\&\quad+i\nu\sinh^{-1}(\nu/m_t\tau)].\end{aligned} \tag{3.16}$$

We have mentioned already, that in this limit, we have $\psi^{(+)}_{\vec{k},\nu}(x)\propto\exp(iS_{cl})$, where S_{cl} is the classical action, found in Sec. II. In the limit of $|\nu_i|\gg m_i\tau$, we have

$$\begin{aligned}\nu\sinh^{-1}\frac{\nu}{m\tau}&=\nu\ln\left[\sqrt{\frac{\nu^2}{m^2\tau^2}+1}+\frac{\nu}{m\tau}\right]\\&\approx|\nu|\ln(2|\nu|)-\nu\ln(m\tau),\end{aligned}$$

and the product of the four Hankel functions in the integrand of Eq. (3.15) becomes

$$\begin{aligned}&\frac{4(m_1)^{-i\nu_1}(m_2)^{-i\nu_2}(m_1')^{i\nu_1'}(m_2')^{i\nu_2'}}{\pi^2|\nu_1\nu_2\nu_1'\nu_2'|^{1/2}}\\&\quad\times\exp[-i(|\nu_1|+|\nu_2|-|\nu_1'|-|\nu_2'|)]\\&\quad\times\exp\{-i[|\nu_1|\ln(2|\nu_1|)+|\nu_2|\ln(2|\nu_2|)-|\nu_1'|\ln(2|\nu_1'|)\\&\quad-|\nu_2'|\ln(2|\nu_2'|)]\}\left(\frac{\tau_1}{\tau_2}\right)^{-i\zeta}.\end{aligned} \tag{3.17}$$

The last factor here is the most significant for future analysis. The rest is just the phase factor.

In what follows, we compute the leading term corresponding to the contact part of the gluon propagator [see Eq. (A26) in Appendix A]:

$$[D^{[00]}_{\eta\eta}(\tau_1,\tau_2;\eta,\vec{r})]_{contact}=-\frac{|\tau_1^2-\tau_2^2|}{2}\delta(\eta)\delta(\vec{r}). \tag{3.18}$$

It is local in η and $\vec{r}$, and the modulus accounts for both terms with $D^{[long]}$ in Eq. (3.8). In this approximation, the matrix element (3.15) becomes

$$M_{1,2\to1',2'}=\frac{g^2}{2(2\pi)^3}\delta(\nu_1+\nu_2-\nu_1'-\nu_2')\times\delta(\vec{k}_1+\vec{k}_2-\vec{k}_1'-\vec{k}_2')\frac{(\nu_1+\nu_1')(\nu_2+\nu_2')}{4|\nu_1\nu_2\nu_1'\nu_2'|^{1/2}}e^{i\alpha}I, \tag{3.19}$$

where α is an inessential real phase. In the approximation given by Eq. (3.17) it absorbs all the dependence on the transverse momenta. Now, it remains to compute the integral

$$\begin{aligned}I&=\int_0^\infty\tau_1d\tau_1\int_0^\infty\tau_2d\tau_2g^{\eta\eta}(\tau_1)g^{\eta\eta}(\tau_2)\\&\quad\times[D^{[00]}_{\eta\eta}(\tau_1,\tau_2)]_{contact}\left(\frac{\tau_1}{\tau_2}\right)^{-i\zeta}\\&=\int_{\tau_0}^Td\tau_1\int_{\tau_0}^Td\tau_2\frac{|\tau_1^2-\tau_2^2|}{2\tau_1\tau_2}\left(\frac{\tau_1}{\tau_2}\right)^{-i\zeta},\end{aligned} \tag{3.20}$$

where the cutoffs are introduced in order to isolate the possible singular behavior. Computation is straightforward,

$$\begin{aligned}2I&=\int_{\tau_0}^T\tau d\tau\int_{\tau_0/\tau}^1dx[x^{i\zeta}+x^{-i\zeta}]\left(\frac{1}{x}-x\right)\\&=\frac{T^2}{\zeta^2+4}\left\{4\left(1+\frac{\tau_0^2}{T^2}\right)\frac{\sin[\zeta\ln(T/\tau_0)]}{\zeta}\right.\\&\quad\left.-2\left(1-\frac{\tau_0^2}{T^2}\right)(1+\cos[\zeta\ln(T/\tau_0)])\right\}.\end{aligned} \tag{3.21}$$

The cutoffs $\tau_{min}=\tau_0$ and $\tau_{max}=T$ in this formula are the external physical input. Making a choice for τ_0 and T it is useful to keep in mind that the interaction (3.18) is due to a nonstationary part of the longitudinal (Coulomb) field of the charges resolved at $\tau=0$. Similar cutoffs are needed in a stationary part. A proper choice leads to the Coulomb logarithm in the collision term in the QED plasma, and we follow this example.

The only field A_η that contributes the contact term (3.18) vanishes at $\tau=0$ and its effect on the charges resolved at $\tau=0$ cannot be instantaneous. Therefore, the lower limit τ_0 is related to the earliest time when the boost states belonging to the incoming nuclei are resolved by means of the strong interaction. Practically, this is the time that the two nuclei take to overlap completely. Therefore, this minimal time is defined by the velocities of the incoming nuclei in the laboratory frame, $\tau_0\sim 1/\sqrt{s}$. At this time, the stationary phase of partial waves (2.21), $\omega^{(+)}_{k,\theta}(x)$, corresponding to the particles with the given rapidities θ, are stretched over the widest rapidity interval $\Delta\eta\sim 2\ln(\sqrt{s}/m_\perp)$. This estimate coincides with the well-known kinematically allowed width $2Y$ of the rapidity plateau, $2Y\approx\ln(s/m^2_{char})$. (See Ref. [1] for further details.)

The upper limit T has to be set because at some time τ_{max} the picture of the independent collisions breaks up. The "final" state fields are not emitted into the free space any more (which affects even the QCD evolution equations [6]). Therefore, T corresponds to the time when subsequent interactions begin to erase the memory about the origin of the boost states from the compact nuclei. By this time, the system must develop collective interactions that result in the effective masses of the plasmonlike modes in a dense medium. It is clear that these masses can emerge only gradually [1,6]. An attempt to evaluate this gradual process in the scope of wedge dynamics has been undertaken in Refs. [6,14]. This calculation relies on the following physical mechanism: The low-p_t mode of the radiation field acquires a finite effective mass as a result of its forward scattering on the strongly localized (and formed earlier) particles with $q_t \gg p_t$. Regardless of what the exact value of this "screening mass" μ_D is, it seems reasonable to take $T\sim 1/\mu_D$, which is consistent with the semiclassical approximation, $T\mu_D\ll\nu$.

The two limits of the Eq. (3.21) are of special interest. Let $\sqrt{s}\to\infty$, while ζ is kept finite. Then

$$I\sim\frac{T^2}{\zeta^2+4}\{2\pi\delta(\zeta)-1\}, \tag{3.22}$$

the amplitude is strongly confined near the forward region and the corresponding cross section diverges.

Next, let us consider the physical limit of the forward scattering, $\zeta\to 0$, while keeping $\sqrt{s}$ finite. In this case, we have

$$I\sim T^2[\ln(T\sqrt{s})-1], \tag{3.23}$$

the inclusive amplitude is proportional to the maximal width of the rapidity plateau, $Y\propto\ln(\sqrt{s})$, which is the only geometric factor that can accompany the contact interaction (3.18). Its square naturally sets the upper bound for the scattering probability.

The second term in the forward scattering amplitude (3.9), which corresponds to the complete exchange of the two initial states (backward scattering), is obviously small. Indeed, this case corresponds to $\nu_1\approx\nu_2'$, and $\nu_2\approx\nu_1'$. In this case,

$$|\zeta|=|\nu_1-\nu_1'|\approx|\nu_1-\nu_2|\gg 1,$$

and the function (3.21) is small.

2. Scattering probability

Since we consider the processes that develop in the course of a single collision, the notion of the cross section is not well defined. In order to deal with the quantity that is as close as possible to the standard cross section, let us introduce the "normalization volume" $\Omega=\pi R^2 Y$, the product of the transverse area and the length of the rapidity interval over which the nuclei become expanded by the first measurement of the collision coordinates. The wave functions of all states begin to occupy this volume when the two nuclei have completely overlapped, i.e., by the time $\tau_{min}\sim 1/\sqrt{s}$. The wave functions $\psi_{\vec{k},\nu}$, given by Eq. (3.13), in the matrix element (3.9) thus acquire an additional factor $(2\pi)^{3/2}\Omega^{-1/2}$. The quantity $\rho=\Omega^{-1}$ will play the same role as the flux factor $j=1/ST=v_{rel}/V$ in the case of the standard $2\to n$ scattering (see, e.g. Ref. [16]). Multiplying the squared modulus of the matrix element (3.19) by the densities of the final states, $\Omega d^2\vec{k}'d\nu'/(2\pi)^{3/2}$, and replacing one of the δ functions by $\Omega/(2\pi)^3$, we arrive at the differential inclusive probability

$$dw=\frac{\delta(\nu_1+\nu_2-\nu_1'-\nu_2')\,\delta(\vec{k}_1+\vec{k}_2-\vec{k}_1'-\vec{k}_2')}{\Omega}\times\frac{\alpha_s^2}{2\pi}\,\frac{(\nu_1+\nu_1')^2(\nu_2+\nu_2')^2}{16|\nu_1\nu_2\nu_1'\nu_2'|}I^2 d^2\vec{k}_1'd\nu_1'd^2\vec{k}_2'd\nu_2'. \tag{3.24}$$

Dividing dw by the density $\rho=\Omega^{-1}$, we obtain the closest analog of the cross section that can be introduced in order to characterize a *single event*,

$$d\sigma_1=\delta(\nu_1+\nu_2-\nu_1'-\nu_2')\,\delta(\vec{k}_1+\vec{k}_2-\vec{k}_1'-\vec{k}_2')\times\frac{\alpha_s^2}{2\pi}\,\frac{(\nu_1+\nu_1')^2(\nu_2+\nu_2')^2}{16|\nu_1\nu_2\nu_1'\nu_2'|}I^2 d^2\vec{k}_1'd\nu_1'd^2\vec{k}_2'd\nu_2'. \tag{3.25}$$

Since I^2 has the dimension $[\mathrm{L}]^4$, the quantity σ_1 also has the dimension of area. The upper limit $\tau_{max}=T$ in Eqs. (3.20)–(3.23) can be estimated from the condition $\tau\mu_D\approx 1\ll\nu$, and is related to the formation of the (final) states as they are detected by the subsequent interactions at the later period of the evolution. In the limit of a nearly forward scattering, and integrating $d^2\vec{k}_2'd\nu_2'$ with the aid of the δ functions, we arrive at

$$\frac{d\sigma_1}{d^2\vec{k}_t' d\zeta} = \frac{\alpha_s^2}{2\pi}\frac{2}{9}\frac{(2\nu_1+\zeta)^2(2\nu_2-\zeta)^2}{16|\nu_1\nu_2(\nu_1+\zeta)(\nu_2-\zeta)|}\frac{1}{\mu_D^4}\left(\frac{2}{\zeta^2+4}\right)^2 \times\left\{\frac{\sin[\zeta\ln(\sqrt{s}/\mu_D)]}{\zeta} - \frac{1+\cos[\zeta\ln(\sqrt{s}/\mu_D)]}{2}\right\}^2. \tag{3.26}$$

In the limit of the forward scattering it becomes

$$\left[\frac{d\sigma_1}{d^2\vec{q}_t d\zeta}\right]_{\zeta\to 0} = \frac{\alpha_s^2}{8\pi}\frac{2}{9}\frac{1}{\mu_D^4}\ln^2\sqrt{\frac{s}{\mu_D}}, \tag{3.27}$$

where $\vec{q}_t \approx \vec{k}_t'$ is considered as the transverse momentum transfer. Our basic approximation implies that this transfer is small, $q_t < \mu_D$. The color trace

$$\frac{2}{9} = \frac{1}{3}\times\frac{1}{3}\times\left(\frac{6}{4}+\frac{2}{4}\right)$$

accounts for the processes with and without color transfer.

IV. SUMMARY

The main result of this paper is given by Eq. (3.27). The logarithmic character of the answer [the color-changing amplitude $\propto \alpha_s \ln(\tau_{min}/\tau_{max}) \approx \alpha_s \ln(\sqrt{s}/\mu_D)$] is due to the dimensionlessness of the rapidity and the boost variables, rather than due to the Coulomb nature of the interaction. This answer indicates that we may expect a massive breakdown of the color balance in the colliding nuclei at the earliest time $\tau \sim 1/\sqrt{s}$. The rate at which the intensity of this breakdown grows with the energy is proportional to $\ln^2 s$.

The key assumption that led to this result is the existence of a sharp boundary of the colliding nuclei. If this assumption is not correct, then there is no reason to consider the problem of the nuclear collision in the framework of wedge dynamics, and the whole picture of the collision will look differently. This would also undermine alternative approaches to the problem, such as the McLerran-Venugopalan model [3,4]. An immediate logical consequence of the finite size is the absence, inside the stable nuclei, of the finite color charge density, which could significantly fluctuate and produce the long-range fields. Only under this assumption could we safely discard the static component of the gauge field that would correspond to the finite charge density at $\tau = 0$ and consider the creation of color charges in the course of the nuclear collision as a transient process. The currents in rapidity direction, which we relied upon in our calculations, appear as a result of the phase shifts in the system of delocalized fields (and thus propagating with the phase velocity), rather than due to the motion of the resolved pointlike charges.

The wedge dynamics was conceived as a tool that is adequate for the earliest stage of the collision, where the initial color coherence becomes broken. It is not applicable to the ep DIS, where the electron probes the long-range electromagnetic fluctuations in the proton [5,6]. In its turn, the evolution equations that describe QCD fluctuations that accompany ep DIS do not seem to be relevant to the collisions of the finite-size nuclei. The primary breakdown of color coherence in a nuclear collision (in terms of the states of the wedge dynamics, it is indeed the earliest process) must result in color radiation that can exist only for a short period (in proper time), only before the fields begin to build up the collective modes of the expanding continuous media. Only these collective effects can bring in the scale (μ_D) to the entire process and serve as a feedback that limits the intensity of the primary emissions [6,14]. Later on, the dynamics of the process must become local on this scale.

The transient process of the plasma formation will come to its saturation at the moment when the growing with time (and density) effective masses of the collective modes begin to screen all emission, from the evolving sources, at the scales below the one given by the dynamically generated effective masses [6,14]. Being unable to radiate, these sources must pass through and form the receding nuclear remnants. Thus, it is likely that the total energy of the collision is responsible only for the time scale of the initial interaction and the full width of the rapidity plateau, while the parameters of the final state in the central rapidity region are universal and independent of the initial energy (above a certain threshold). Eventually, the total energy of the colliding nuclei is shared by the newly born matter and these receding remnants. It is not clear yet if the quark-gluon matter will have time to sufficiently thermalize and be described by a single parameter, the temperature. However, it seems unavoidable that the entropy created at the earliest moments must result in the *pressure*, which is the first thing we shall try to theoretically estimate. A success at this point will very much simplify the whole scenario by allowing incorporation of the hydrodynamic picture from a sufficiently early proper time.

Our preliminary estimates show that the boost states of wedge dynamics do not effectively scatter with large transverse momentum transfer. Further analysis is necessary to verify this estimate, which (being correct) could explain the absence of high-p_t jets observed in the first available RHIC data. The jets are not strongly quenched, they can well be absent at all. Does this mean that perturbative QCD is totally unrelated to the ultrarelativistic heavy ion collisions? We do not think so. It just has to be used in a different way than in ep DIS or pp collisions. The major source of this difference has been first outlined in Ref. [6]: in nuclear collisions, the final states that saturate the unitary cut in the ladders that correspond to QCD evolution equations cannot be saturated by quark and gluon states in free space. In this paper, we point to the fact that the initial states can be different from the free massless wee partons with the given light-cone momenta. They can well be the boost states of valence quarks that are explicitly confined inside the finite-size nuclei before the collision.

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APPENDIX A: THE GLUON PROPAGATOR

In order to study the interaction of the two charged states with the given boosts ν, one needs to have an explicit form of the gauge field propagator. Particularly, since the boosts are additive and obey the conservation law, one needs to know what the quanta of radiation that carry the boost quantum numbers are? It is also necessary to know the form of the proper (longitudinal) fields produced by the charged particles. In this section, we present a detailed analysis of the gluon propagator in wedge dynamics, which has been derived in Ref. [2]. The main purpose is to carefully trace the origin of the new contact term. At first glance, it may look abnormal since it neither shows up in the field of a moving static charge, nor has it any properties associated with the propagation. We want to show that all Coulomb-type terms still exist in the propagator. They are somewhat modified, in a way that one could expect on purely physical grounds. Namely, the Coulomb fields vanish outside the future domain of the point where the charge was created. Our analysis indicates that the other parts of the propagator cannot hide anything similar to the exclusive contact part that is solely responsible for the final result, Eq. (3.27), of this paper.

1. The field of a static source

The field of a static source in wedge dynamics is found [2] when one solves the linearized (Maxwell) equations of motion without the external current, imposing the gauge condition $A^\tau = 0$. An additional boundary condition, which allows one to fix the gauge completely, is $A_\eta(\tau=0)=0$. In fact, this condition brings nothing new, since the hypersurface $\tau=0$ is lightlike, and the τ and η directions are degenerate there. In this way, one finds *three* modes, of which two, $V^{(TE)}_{\nu\vec{k}}(x)$ and $V^{(TM)}_{\nu\vec{k}}(x)$ are the transverse fields. The modes $V^{(TE)}$ and $V^{(TM)}$ are normalized according to a usual definition of the scalar product in the functional space of the solutions of the Maxwell equations

$$(V,W) = \int_{-\infty}^{\infty} d\eta \int d^2\vec{r}\, \tau g^{ik} V_i^* i \overleftrightarrow{\partial}_\tau W_k, \tag{A1}$$

and satisfy the Gauss law without the charge. The third mode $V^{(stat)}$ has zero norm, and its definition is accomplished with the aid of Gauss law with the *static* source. (In the absence of any currents, the source can be only static.) The electric and magnetic fields of this mode are

$$E_l^{[stat]}(\tau,\vec{r},\eta) = \int \frac{d\nu d^2\vec{k}}{(2\pi)^3} \frac{e^{i\nu\eta + i\vec{k}\vec{r}}}{ik_t^2} \begin{bmatrix} k_r \tau^{-1} s_{1,i\nu}(k_t\tau) \\ \nu k_t^2 \tau s_{-1,i\nu}(k_t\tau) \end{bmatrix}_l \rho(\vec{k},\nu), \tag{A2}$$

$$B_l^{[stat]}(\tau,\vec{r},\eta) = \int \frac{d\nu d^2\vec{k}}{(2\pi)^3} \frac{e^{i\nu\eta + i\vec{k}\vec{r}}}{k_t^2} \begin{bmatrix} k_y \\ -k_x \\ 0 \end{bmatrix}_l \nu \dot{s}_{-1,i\nu}(k_t\tau)\rho(\vec{k},\nu), \tag{A3}$$

where $s_{m,i\nu}(x)$ is the Lommel function, a solution of the inhomogeneous Bessel equation with x^{m-1} as the external source,

$$f'' + \frac{1}{x} f' + \left(1 + \frac{\nu^2}{x^2}\right) f = x^{m-1}.$$

There exists an extremely important relation between the two Lommel functions,[9]

$$s_{1,i\nu}(k_t\tau) + \nu^2 s_{-1,i\nu}(k_t\tau) = 1. \tag{A5}$$

First of all, it is necessary in order to verify that the electric field of a static charge distribution (A2) indeed satisfies Gauss law[10]

$$\frac{1}{\tau}\partial_\eta E_\eta + \tau \partial_r E_r = \tau j^\tau = \rho. \tag{A6}$$

Second, it is precisely the unit on the right side of Eq. (A5) that will give rise to the contact term in the full propagator.

The Fourier component of the vector potential of the static field is

$$A_l^{[stat]}(\vec{k},\nu;\tau) = \frac{\rho(\vec{k},\nu)}{(2\pi)^3 i k_t^2} \begin{bmatrix} k_r Q_{-1,i\nu}(k_t\tau) \\ \nu Q_{1,i\nu}(k_t\tau) \end{bmatrix}_l, \tag{A7}$$

where we introduced the functions

$$Q_{m,i\nu}(x) = \int_0^x x^m s_{-m,i\nu}(x) dx.$$

In spite of an unusual appearance, this is nothing else but Coulomb's law in the framework of wedge dynamics. In order to see this explicitly, let us consider the system of point-like charges located at the points $\vec{r}_i$ in the transverse plane and moving with rapidities θ_i,

$$\rho = \tau j_\tau = \sum_i q_i \delta(\eta - \theta_i)\delta(\vec{r} - \vec{r}_i). \tag{A8}$$

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[9] It is useful to keep in mind the integral representation

$$s_{1,i\nu}(k_t\tau) = 1 - \frac{\nu}{\sinh \pi\nu} \int_0^\pi \cos(k_t\tau \sin\phi) \cosh \nu\phi d\phi, \tag{A4}$$

which indicates that the functions $s_{1,i\nu}(x)$ and $\nu^2 s_{-1,i\nu}(x)$ are regular at $\nu=0$.

[10] In terms of the physical components, $\mathcal{E}^m = \sqrt{-g} g^{mn} E_n = \sqrt{-g} g^{mn} \partial_\tau A_n$, the Coulomb law reads exactly as in Cartesian coordinates, $\partial_m \mathcal{E}^m = \rho$.

For a single charge, the explicit form of the electric field components is

$$E_l^{[stat]}(\tau,\vec{r},\eta)=\frac{q}{4\pi}\begin{bmatrix}\vec{r}\cosh(\eta-\theta)\\ \tau^2\sinh(\eta-\theta)\end{bmatrix}_l\frac{\theta(\tau-r_t)}{R^3}+\begin{bmatrix}\vec{r}/r_t^2\\ \tanh(\eta-\theta)\end{bmatrix}_l\delta(\tau-r_t),\qquad \text{(A9)}$$

where $R=[\vec{r}^2+\tau^2\sinh^2(\eta-\theta)]^{1/2}$ is the distance between the points $(\vec{0},\theta)$ and $(\vec{r},\eta)$ in the internal geometry of the surface $\tau=\text{const}$. On can obtain the first term in this formula taking the usual (gauge-independent) expression for the electric field of the moving charge, transforming it to the new coordinates, and multiplying it by the $\theta(\tau-r_t)$, which eliminates the field outside the light cone of the point where the charge had emerged. The second term (with the light-cone δ function) corresponds to the wave front that accompanies the process of the charge creation at $\tau=0$.[11]

Since the electric field is $E_l=\partial_\tau A_l$, the vector potential is recovered by means of integration,

$$A_l(\tau)=\int_0^\tau E_l(\tau')d\tau'\to\int_{r_t}^\tau E_l(\tau')d\tau'.\qquad \text{(A10)}$$

Now, when r_t is taken as the actual lower limit, the result of the integration explicitly coincides with the Fourier transform of Eq. (A7). The Fourier transform of the Lommel functions appears to be discontinuous in an exactly relativistic way (the details of its calculation are in the following section).

One may ask how the Coulomb mode could be found from Maxwell's equations of motion that do not include Gauss's law. The answer is simple and natural: the Coulomb field outside the static charge distribution must satisfy the equations of motion for a free field.

There are two surprises connected with the static solutions of the wedge dynamics. First, *the source is static if it expands in such a way that its physical component* $\mathcal{J}^\tau=\tau j^\tau(\tau,\eta,\vec{r})$ *does not depend on* τ. Indeed, the charge conservation has its physical form, $\partial_\mu\mathcal{J}^\mu=0$, only in terms of the physical components $\mathcal{J}^\mu=\sqrt{-g}g^{\mu\nu}j_\nu$ of the electric current. The second surprise is the light-cone boundary of the static field in Eq. (A9).

Finally, let us consider the conservation of the charge of a fundamental field in full QCD. Now, the equation of charge conservation reads as

$$\partial_\mu\mathcal{J}_a^\mu+\mathrm{g}f^{abc}A_\mu^b\mathcal{J}_c^\mu=0.\qquad \text{(A11)}$$

Let only the j_a^τ component of the current differ from zero. Then for the charge $Q^a=\int\tau d\eta d^2\vec{r}j^\tau$, we have

$$\partial_\tau Q_a+\mathrm{g}f^{abc}A_\tau^bQ_c=0.\qquad \text{(A12)}$$

Since the gauge condition is $A_\tau=0$, we conclude that $Q_a=\text{const}$. In the framework of wedge dynamics, the notion of static charge is well defined even if the individual charges move with respect to each other (in a specific way). A similar result can be obtained in the system with the Hamiltonian time $t=x^0$ with the gauge condition $A^0=0$. If all (color) charges are at rest, their proper static field does not "rotate" their color. However, this will not be the case if we chose a different gauge condition, e.g., $A^3=0$ or div$\mathbf{A}=0$, which would require that $A^0\neq0$.

Finally, it is easy to understand, that since the proper gluon field of the static fundamental color charge does not affect the charge itself, this gluon field cannot be a carrier of the color charge. This fully agrees with the fact, that the norm of the Coulomb mode equals zero, because its field is real (contrary to the complex fields of the transverse modes that represent gluons). An additional reason to pay special attention to the static field configuration is that the field corresponding to the charge density $\rho(\tau=0)$ is an isolated exceptional static field. It was necessary to describe it in detail in order to have a reference point for a more involved analysis of the fields created by the charged currents.

2. The full longitudinal field

The gluon propagator, which we review and analyze in some detail below, was found as a (retarded) response function between the potential and the current for the linearized (Maxwell) equations of motion. The potential is represented as a sum of three terms,

$$A=A^{[tr]}+A^{[L]}+A^{[inst]}=A^{[tr]}+A^{[long]}.$$

The second and the third terms constitute the longitudinal (in a sense of the Gauss's law) field. The goal of this somewhat technical analysis is to demonstrate that the longitudinal part of this propagator indeed includes a new contact term. At the same time we want to show, that the standard Coulomb fields are still present in the propagator, almost unchanged and are modified only by the relativistic causal boundaries that one would expect to appear for the fields of the emerging charges.

The transverse part of the retarded propagator is trivial. It is built from the partial solutions of the homogeneous wave equations,

$$A_l^{[tr]}(x_1)=\int d^4x_2\,\theta(\tau_1-\tau_2)\Delta_{lm}^{(tr)}(x_1,x_2)j^m(x_2),\qquad \text{(A13)}$$

where

$$\Delta_{lm}^{(tr)}(x,y)=-i\int_{-\infty}^{\infty}d\nu\int d^2\vec{k}\sum_{\lambda=TE,TM}[V_{\nu\vec{k};l}^{(\lambda)}(x)V_{\nu\vec{k};m}^{(\lambda)*}(y)-V_{\nu\vec{k};l}^{(\lambda)*}(x)V_{\nu\vec{k};m}^{(\lambda)}(y)],\qquad \text{(A14)}$$

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[11]This is not a true radiation. The real Coulomb mode $A_l^{[stat]}$ is *orthogonal* to the complex propagating modes $V_l^{(TE)}$ and $V_l^{(TM)}$.

which can be easily recognized as the Riemann function of the original homogeneous hyperbolic system. The Riemann function solves the boundary value problem for the evolution of the free radiation field. It is obtained immediately as a bilinear expansion over the full set of solutions of the homogeneous system.

The name of the instantaneous part is motivated by its explicit form

$$A_l^{[inst]}(\vec{k},\nu;\tau_1)=\frac{\rho(\vec{k},\nu,\tau_1)}{(2\pi)^3 ik_t^2}\begin{bmatrix} k_r Q_{-1,i\nu}(k_t\tau_1) \\ \nu Q_{1,i\nu}(k_t\tau_1)\end{bmatrix}_l, \tag{A15}$$

the *potential* $A_l^{[inst]}$ is simultaneous with the charge density $\rho=\tau j_\tau$. Formally, it can be obtained by adding the time dependence to the charge density in the expression for the static potential (A7). However, this form is inconvenient as long as we have to use $A_\mu j^\mu = A_l j^l$ as the basic form of the interaction Hamiltonian. Therefore, we have to eliminate the charge density ρ completely, and replace it by the spatial components j^n of the current. The replacement follows an evident prescription

$$\rho(\tau_1,\nu,\vec{k})-\rho(0,\nu,\vec{k})=\int_0^{\tau_1}d\tau_2\frac{\partial\rho}{\partial\tau_2}$$
$$=-i\int_0^{\tau_1}\tau_2 d\tau_2[k_s j^s(\tau_2,\nu,\vec{k}) + \nu j^\eta(\tau_2,\nu,\vec{k})]. \tag{A16}$$

The effect of the initial charge density $\rho_0=\rho(0,\nu,\vec{k})$ would correspond to the clearly visible static pattern in the longitudinal part of the field. In the framework of perturbative QCD, this pattern is not active, since it cannot transmit the color charge. Furthermore, as we have argued previously, in nuclear collisions, the initial density of the color charges at $\tau=0$ is zero. This leads to

$$A_l^{[inst]}(\tau_1,\nu,\vec{k})=-\int_0^{\tau_1}\frac{\tau_2 d\tau_2}{(2\pi)^3 k_t^2} \times\begin{bmatrix} k_r Q_{-1,i\nu}(k_t\tau_1) \\ \nu Q_{1,i\nu}(k_t\tau_1)\end{bmatrix}_l\begin{bmatrix} k_s \\ \nu\end{bmatrix}_m j^m(\tau_2,\nu,\vec{k}). \tag{A17}$$

In this form, the three remaining (in the gauge $A^\tau=0$) spatial components A_l of the vector potential are expressed via the spatial components of the current.

The dynamical longitudinal field $A^{(L)}$ is of the form

$$A_l^{[L]}(\tau_1,\nu,\vec{k}) =\int_0^{\tau_1}\frac{\tau_2 d\tau_2}{(2\pi)^3 k_t^2}\begin{bmatrix} k_r \\ \nu\end{bmatrix}_l\begin{bmatrix} k_s Q_{-1,i\nu}(k_t\tau_2) \\ \nu Q_{1,i\nu}(k_t\tau_2)\end{bmatrix}_m j^m(\tau_2,\nu,\vec{k}). \tag{A18}$$

It also does not allow for the bilinear expansion with two temporal arguments. Its electric and magnetic field is simultaneous with the current j^m also. In what follows immediately, we intend to single out the contact part of the propagator, which shows up only in the $D_{\eta\eta}$ component and connects A_η with j_η.

In order to set the stage, it is instructive to start with the electric and magnetic fields of these two modes, $E_m=\mathring{A}_m$, $\mathcal{E}^m=\sqrt{-g}g^{mn}\mathring{A}_n$, and $\mathcal{B}^m=-(2\sqrt{-g})^{-1}e^{mln}F_{ln}$. Since the potential $A_l^{[L]}$ is the three-dimensional gradient, we immediately see that $\mathcal{B}_l^{[L]}=0$. (Note, that $A_l^{[L]}$ is the gradient of a *time-dependent* function, and thus is not a pure gauge.) Starting from the expression for $A^{[inst]}$, and using the relation [2],

$$Q_{-1,i\nu}(k_t\tau)-Q_{1,i\nu}(k_t\tau)=-\frac{\tau}{\nu^2}\frac{\partial}{\partial\tau}s_{1,i\nu}(k_t\tau) =\tau\frac{\partial}{\partial\tau}s_{-1,i\nu}(k_t\tau), \tag{A19}$$

we obtain by a straightforward calculation that

$$\mathcal{B}_l^{[inst]}(\tau,\nu,\vec{k}) =\int_0^{\tau}\frac{\tau_2 d\tau_2\nu}{(2\pi)^3 ik_t^2}\begin{bmatrix} k_y \\ -k_x \\ 0\end{bmatrix}_l\begin{bmatrix} k_x \\ k_y \\ \nu\end{bmatrix}_m \dot{s}_{-1,i\nu}(k_t\tau)j^m(\tau_2,\nu,\vec{k}), \tag{A20}$$

i.e., the longitudinal part of the magnetic field has only the azimuthal component (the magnetic field circulates around the current flowing in the η direction), which is natural for the distribution of charges that experience expansion in z direction. Note, that the magnetic field exists even when ρ is τ independent.

In the same way, we compute the electric fields

$$E_l^{[L]}(\tau,\nu,\vec{k})=\frac{\tau}{(2\pi)^3 k_t^2}\begin{bmatrix} k_r \\ \nu\end{bmatrix}_l\begin{bmatrix} k_s Q_{-1,i\nu}(k_t\tau) \\ \nu Q_{1,i\nu}(k_t\tau_2)\end{bmatrix}_m j^m(\tau,\nu,\vec{k}), \tag{A21}$$

and

$$E_l^{[inst]}(\tau,\nu,\vec{k})=\frac{-i}{(2\pi)^3 k_t^2}\left\{\begin{bmatrix} k_r Q_{-1,i\nu}(k_t\tau) \\ \nu Q_{1,i\nu}(k_t\tau)\end{bmatrix}_l\dot{\rho}(\tau,\nu,\vec{k}) +\begin{bmatrix} k_r\tau^{-1}s_{1,i\nu}(k_t\tau) \\ k_t^2\nu\tau s_{-1,i\nu}(k_t\tau)\end{bmatrix}_l\rho(\tau,\nu,\vec{k})\right\}. \tag{A22}$$

Once again, in the static limit, $j^m=0$, and $\dot{\rho}=0$; thus, $E^{[L]}=0$ and only the second term in $E^{[inst]}$ survives and becomes the previously found $E^{[stat]}$. Notice that the time integration in expressions for potentials looks as retarded, $\tau_1>\tau_2$. This has nothing to do with causal (and the only one meaningful) retardation. This inequality is due to the boundary conditions imposed on A_l (to fix the gauge) when A_l is being rebuilt from E_l, which is simultaneous with the sources. The same inequality appears when we shall rebuild the charge density $\rho(\tau)$ via $j^m(\tau)$ at the previous time.

Now, leaving the vanishing effect of $\rho(\tau=0)$ aside, we can move to the fields produced by the currents. We want to present the propagator in its general tensor form, which implies that

$$A_l^{[long]}(x_1)=\int d^4x_2 D_{lm}^{[long]}(x_1,x_2)j^m(x_2).$$

Let us begin with the electric fields produced by the component j^η of the current:

$$E_\eta^{[L]}(\tau_1,\nu,\vec{k}|j^\eta)=\frac{1}{(2\pi)^3}\left[\frac{\tau_1^2}{2}-\int_0^{\tau_1}s_{1,i\nu}(k_t t)t dt\right]\times\tau_1 j^\eta(\tau_1,\nu,\vec{k}),\tag{A23}$$

$$E_\eta^{[inst]}(\tau_1,\nu,\vec{k}|j^\eta)=\frac{1}{(2\pi)^3}\left\{-\tau_1[1-s_{1,i\nu}(k_t\tau_1)]\times\int_0^{\tau_1}\tau_2 d\tau_2 j^\eta(\tau_1,\nu,\vec{k})-\left[\frac{\tau_1^2}{2}-\int_0^{\tau_1}s_{1,i\nu}(k_t t)t dt\right]\times\tau_1 j^\eta(\tau_1,\nu,\vec{k})\right\}.\tag{A24}$$

We see, that the $E_\eta^{[L]}$ cancel out the second term in $E_\eta^{[inst]}$, originating, in its turn, from the term with $\dot{\rho}$ in Eq. (A22). In this way, we obtain the full form of the η component of the longitudinal field

$$E_\eta^{[long]}(\tau,\nu,\vec{k}|j^\eta)=[-\tau+\tau s_{1,i\nu}(k_t\tau)]\int_0^\tau\frac{\tau_2 d\tau_2}{(2\pi)^3}j^\eta(\tau_2,\nu,\vec{k}),$$

$$A_\eta^{[long]}(\tau_1,\nu,\vec{k}|j^\eta)=\int_0^{\tau_1}\frac{\tau_2 d\tau_2}{(2\pi)^3}\left[\frac{\tau_2^2-\tau_1^2}{2}-\int_{\tau_2}^{\tau_1}s_{1,i\nu}(k_t t)t dt\right]\times j^\eta(\tau_2,\nu,\vec{k}),\tag{A25}$$

where the first term is independent of ν and $\vec{k}$, and yields the contact part of the propagator, which (in the coordinate representation) reads as

$$D_{\eta\eta}^{[contact]}(\tau_1,\tau_2;\eta_1-\eta_2;\vec{r}_1-\vec{r}_2)=-\frac{\tau_1^2-\tau_2^2}{2}\delta(\eta_1-\eta_2)\delta(\vec{r}_1-\vec{r}_2).\tag{A26}$$

The first line of Eq. (A24) clearly illustrates its origin: We started in Eq. (A22) with the product $\nu s_{-1,i\nu}(k_t\tau)\rho(\tau,\nu,\vec{k})$. Then, since $\rho(\tau)$ is developed dynamically, we expressed ρ via $\partial_\eta j^\eta\to\nu j^\eta$, gaining an extra power of ν. This allows us to use the relation between two Lommel functions, Eq. (A5), and replace (in fact, after integrating by parts) $\nu^2 s_{-1,i\nu}\to 1-s_{-1,i\nu}$, which is equivalent to a straightforward account for the Gauss law. The ν- and k_t-independent unit gives Eq. (A26).[12]

The second integral term in Eq. (A25) can also be Fourier transformed into the coordinate representation. We want to do that here, in order to verify that the contact term is not singled out artificially and that it is not canceled by something hidden in the second term. To compute the integrals from the function $s_{1,i\nu}$ it can be conveniently decomposed in the following way:

$$s_{1,i\nu}(x)=S_{1,i\nu}(x)-h_{i\nu}(x),$$

$$h_{i\nu}(x)=\frac{e^{-\pi\nu/2}}{2}\frac{\pi\nu/2}{\sinh(\pi\nu/2)}[H_{i\nu}^{(1)}(x)+H_{-i\nu}^{(2)}(x)].\tag{A27}$$

The function $h_{i\nu}(x)$ obeys the homogeneous Bessel equation and thus can describe only the field outside the domain of the source influence. In the course of calculations, we use the following integral representation for the Hankel functions:

$$e^{-\pi\nu/2}H_{\mp i\nu}^{\binom{2}{1}}(k_t\tau)=\frac{\pm i}{\pi}\int_{-\infty}^{\infty}\exp[\mp ik_t\tau\cosh\theta]e^{\pm i\nu\theta}d\theta.\tag{A28}$$

The Lommel function $S_{1,i\nu}$ has a similar representation

$$S_{1,i\nu}(x)=x\int_0^\infty\cosh u\cos\nu u e^{-x\sinh u}du,\tag{A29}$$

which allows one to compute the integral $d\nu$ exactly,

$$\int_{-\infty}^{\infty}S_{1,i\nu}(k_t\tau)e^{i\nu\eta}d\nu=\pi k_t\tau\cosh\eta e^{-k_t\tau\sinh|\eta|},\tag{A30}$$

[12]One may wonder, why the same type contact term does not show up in other dynamics (and gauges, such as $A^0=0$). The propagators of these gauges are constructed in such a way that the translation invariance and the possibility of a simple momentum representation are preserved. The price for this apparent simplicity is the spurious poles in the propagator without a physically motivated prescription to handle these poles. These poles reflect an intrinsic uncertainty in the way one can approach the limit of the static field. In order to fix the gauge $A^0=0$ completely, one has to impose some boundary condition on the gauge fields at some time t, thus corrupting the translation invariance and gaining additional terms in the propagator, which, in fact, are of the same origin as the contact term in the gauge $A^\tau=0$. At large τ_1 and τ_2, and locally in the coordinate rapidity η (when the curvature of the hypersurface of the constant τ becomes negligible), the gauge $A^\tau=0$ can be approximated locally by the gauge $A^0=0$ [2], provided the boundary conditions at $\tau=0$ are released. In this domain, the contribution of the contact term is suppressed by the two small curvature factors $g^{\eta\eta}(\tau_1)g^{\eta\eta}(\tau_2)=\tau_1^{-2}\tau_2^{-2}$. Therefore, if a usual scattering process between the asymptotic states takes place at large τ, wedge dynamics will treat it according to the standard scattering theory.

and from Eq. (A28) it follows

$$\int_{-\infty}^{\infty} d\nu e^{i\nu\eta} h_{i\nu}(k_t\tau) = \int_{-\infty}^{\infty} d\,\theta \frac{\sin[k_t\tau\cosh\theta)]}{\cosh^2(\theta+\eta)} = \int_{-\infty}^{\infty} d\,\theta \frac{\sin[k_t\tau\cosh(\theta-\eta)]}{\cosh^2\theta}. \tag{A31}$$

Next we may write the full Fourier transforms. From Eq. (A30), we have

$$\int \frac{d^2\vec{k}}{(2\pi)^3} e^{i\vec{k}\vec{r}} \int_{-\infty}^{\infty} S_{1,i\nu}(k_t\tau) e^{i\nu\eta} d\nu = -\frac{\tau\cosh\eta}{4\pi}\nabla_\perp^2 \int_0^\infty J_0(kr)\exp[-k_t\tau\sinh|\eta|]dk = -\frac{\tau\cosh\eta}{4\pi}\nabla_\perp^2\left[\frac{1}{(\vec{r}^2+\tau^2\sinh^2\eta)^{1/2}}\right]. \tag{A32}$$

Starting from Eq. (A31), we continue by introducing $k_z = k_t\sinh\theta$ and $k_0 = k_t\cosh\theta = |\mathbf{k}|$ and changing $d^2\vec{k}d\theta$ for the three-dimensional integration $d^3\mathbf{k}$. With $t=\tau\cosh\eta$, $\mathbf{r} = (x,y,\tau\sinh\eta)$, this leads to

$$\int \frac{d^2\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \int_{-\infty}^{\infty} d\nu e^{i\nu\eta} h_{i\nu}(k_t\tau) = -\nabla_\perp^2 \int \frac{d^2\vec{k}}{2i(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \times \int_{-\infty}^{\infty} \frac{dk_z}{k_0^3}[e^{ik_0t-ik_zz} - e^{-ik_0t+ik_zz}] = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{|\mathbf{k}|^3}\sin k_0 t = -\frac{\nabla_\perp^2}{4\pi}\left[\theta(r^2-\tau^2)\frac{\tau\cosh\eta}{(r^2+\tau^2\sinh^2\eta)^{1/2}} + \theta(\tau^2-r^2)\right]. \tag{A33}$$

Adding Eqs. (A32) and (A33) we indeed find that the $\eta\eta$ component of the longitudinal propagator vanishes at the distances r_t exceeding τ, i.e., outside the light cone of the position of the current which creates the field. Finally,

$$E_\eta^{[long]}(\tau,\eta_1,\vec{r}_1|j^\eta) = \int d\eta_2 d\vec{r}_2 \int_0^\tau \frac{\tau_2 d\tau_2}{(2\pi)^3} j^\eta(\tau_2,\eta_2,\vec{r}_2) \times \left\{-\tau\delta(\eta)\delta(\vec{r}) - \frac{\nabla_\perp^2}{4\pi} \times\left[\theta(\tau-r_t)\left(\frac{\tau^2\sinh\eta}{(r_t^2+\tau^2\sinh^2\eta)^{1/2}}-1\right)\right]\right\}, \tag{A34}$$

where, $\eta=\eta_1-\eta_2$ and $\vec{r}=\vec{r}_1-\vec{r}_2$.

The first (contact) term in this formula is indeed very special. It is not limited by the light-cone boundary. The second term, does have these boundaries, which are just imposed on the Coulomb-type fields rewritten in terms of the natural coordinates of wedge dynamics. It also includes the radiation fields propagating along the light cone $\tau=r_t$. Therefore, only this term can interfere with the radiation fields of the transverse modes. This is clear evidence that the cancellation between the contact term and the nonlocal parts of the propagator is impossible. As it was demonstrated in Ref. [2], the transverse electric field is governed by a usual relativistic wave equation. Integrating Eq. (A34) over τ from zero to τ_1, we recover the potential, and the $\eta\eta$ component of the propagator,

$$D_{\eta\eta}^{[long]}(\tau_1,\tau_2;\eta,\vec{r}) = -\frac{\tau_1^2-\tau_2^2}{2}\delta(\eta)\delta(\vec{r}) - \frac{\nabla_\perp^2}{4\pi} \times \int_{\tau_2}^{\tau_1} t\,dt\,\theta(t-r_t)\left[\frac{t\cosh\eta}{R(t)}-1\right]. \tag{A35}$$

The remaining components of the propagator $\Delta_{lm}^{[long]}$ are

$$D_{rs}^{[long]}(\tau_1,\tau_2;\eta,\vec{r}) = \frac{\partial_r\partial_s}{4\pi}\int_{\tau_2}^{\tau_1}\frac{dt}{t}\theta(t-r_t)\left[\frac{t\cosh\eta}{R(t)}+1\right], \tag{A36}$$

$$D_{r\eta}^{[long]}(\tau_1,\tau_2;\eta,\vec{r}) = \frac{\partial_r}{4\pi}\left\{\theta(t-r_t) \times\left[\int_0^{\tau_1}\frac{r_t^2\sinh\eta\,dt}{R^3(t)} - \int_{\tau_2}^{\tau_1}\frac{t^2\sinh\eta\,dt}{R^3(t)}\right] + \tanh\eta\int_0^{\tau_2}\delta(t-r_t)dt\right\} = D_{\eta r}^{[long]}(\tau_2,\tau_1;-\eta,\vec{r}), \tag{A37}$$

where $R(t)=[r_t^2+t^2\sinh^2\eta]^{1/2}$. The propagator identically vanishes at $r_t>\tau$, and the derivatives of the step function are confined to the light cone corresponding to the transient radiation that accompanies the creation of the color charges.

APPENDIX B: SUBLEADING TERMS IN FORWARD SCATTERING OF THE BOOST STATES

In the limit of the forward scattering, the general formula (3.15) can be rewritten in the following form:

$$\begin{aligned}M_{1,2\to 1',2'}=&\frac{g^2}{2^7\pi}\delta(\nu_1+\nu_2-\nu_1'-\nu_2')\,\delta(\vec{k}_1+\vec{k}_2-\vec{k}_1'-\vec{k}_2')\\&\times\int_0^\infty \tau_1 d\tau_1\int_0^\infty \tau_2 d\tau_2 H^{(1)}_{i\nu_1'}(m_1'\tau_1)H^{(2)}_{i\nu_1}(m_1\tau_1)\\&\times H^{(1)}_{i\nu_2'}(m_2'\tau_2)H^{(2)}_{i\nu_2}(m_2\tau_2)\\&\times\Bigg[\frac{\nu_1+\nu_1'}{\tau_1^2}D^{[00]}_{\eta\eta}(\tau_1,\tau_2;\zeta,\vec{q})\frac{\nu_2+\nu_2'}{\tau_2^2}\\&-q^r D^{[00]}_{rs}(\tau_1,\tau_2;\zeta,\vec{q})q^s\\&+\frac{\nu_1+\nu_1'}{\tau_1^2}D^{[00]}_{\eta s}(\tau_1,\tau_2;\zeta,\vec{q})q^s\\&-q^r D^{[00]}_{r\eta}(\tau_1,\tau_2;\zeta,\vec{q})\frac{\nu_2+\nu_2'}{\tau_2^2}\Bigg],\end{aligned}\tag{B1}$$

where we took the initial transverse momenta $\vec{p}_1=\vec{p}_2=0$, and correspondingly, the final state momenta, $\vec{p}_2'=-\vec{p}_1'=\vec{q}$. By its design, the full T-ordered propagator $D^{[00]}_{lm}$ is a sum of the longitudinal part and two terms originating from the transverse electric $V^{(TE)}$ and transverse magnetic $V^{(TM)}$ modes of the radiation field

$$D^{[00]}=D^{[00,long]}+D^{[00](TE)}+D^{[00](TM)}.$$

The $\eta\eta$ component of the longitudinal part of the propagator can be read out from the Eq. (A25),

$$D^{[long]}_{\eta\eta}(\tau_1,\tau_2;\zeta,\vec{q})=\frac{1}{2\pi}\left[-\frac{\tau_1^2-\tau_2^2}{2}-\int_{\tau_2}^{\tau_1}s_{1,i\zeta}(q_t t)t\,dt\right],\tag{B2}$$

where the first term on the right has already been used in Eq. (3.20) to obtain the main estimate (3.27). The second term yields

$$I_1=\int_{\tau_0}^{T}\frac{d\tau_1}{\tau_1}\int_{\tau_0}^{T}\frac{d\tau_2}{\tau_2}\left(\frac{\tau_1}{\tau_2}\right)^{-i\zeta}\mathrm{sgn}(\tau_1-\tau_2)\int_{\tau_2}^{\tau_1}s_{1,i\zeta}(q_t t)t\,dt.\tag{B3}$$

The behavior of the Lommel function in the limit $\zeta\to 0$ can be found from the integral representation (A4)

$$s_{1,i0}(q_t\tau)=\frac{1}{\pi}\int_0^\pi[1-\cos(k_t\tau\sin\phi)]d\phi=1-J_0(q_t\tau).\tag{B4}$$

Expanding the Bessel function $J_0(q_t t)$ at small q_t, and integrating, we arrive at

$$\begin{aligned}2I_1=\frac{q_t^2T^4}{\zeta^2+16}\Bigg\{&\left(1+\frac{\tau_0^4}{T^4}\right)\frac{\sin[\zeta\ln(T/\tau_0)]}{\zeta}\\&-\left(1-\frac{\tau_0^4}{T^4}\right)\frac{1+\cos[\zeta\ln(T/\tau_0)]}{4}\Bigg\}.\end{aligned}\tag{B5}$$

This term vanishes in the limit of the forward scattering, $q_t\to 0$.

The rs component of the longitudinal field propagator

$$D^{[long]}_{rs}(\tau_1,\tau_2;\zeta,\vec{q})=\frac{1}{2\pi}\frac{q_r q_s}{q_t^2}\int_{\tau_2}^{\tau_1}s_{1,i\zeta}(q_t t)\frac{dt}{t}\tag{B6}$$

brings in the term

$$\begin{aligned}I_2=q_t^2\int_{\tau_0}^{T}\tau_1 d\tau_1\int_{\tau_0}^{T}\tau_2 d\tau_2\left(\frac{\tau_1}{\tau_2}\right)^{-i\zeta}\\\times\mathrm{sgn}(\tau_1-\tau_2)\int_{\tau_2}^{\tau_1}s_{1,i\zeta}(q_t t)\frac{dt}{t}.\end{aligned}\tag{B7}$$

At $\zeta\to 0$ and at small q_t, it can be represented as the integral

$$2I_2=\frac{q_t^4T^6}{8}\int_{\tau_0/T}^{1}y^5dy\int_{\tau_0/Ty}^{1}[x^{i\zeta}+x^{-i\zeta}](1-x^2)dx,\tag{B8}$$

which also vanishes in the limit of the forward scattering, $q_t\to 0$.

The contribution of the components $D^{[long]}_{r\eta}$ and $D^{[long]}_{\eta s}$ into the matrix element (B1), as well as of all components $D^{(TM)}_{lm}$ of the transverse propagator is estimated exactly in the same way. All these components are defined as the integrals, from zero to τq_t, of the functions that are regular at the origin. Therefore, at small q_t, all these terms have at least one factor q_t^2 and vanish in the limit of the forward scattering.

The only exception from this scheme is the piece connected with the transverse part $D^{(TE)}$ of the propagator. This part is the bilinear form $D^{(TE)}_{rs}$ that has only rs components and includes the projector $\delta_{rs}-q_r q_s/q_t^2$. Since this projector is orthogonal to the vector $\vec{q}_t$, the contribution of this mode to the matrix element (B1) identically vanishes.

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