Parametric resonance at the critical temperature in high energy heavy ion collisions

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Parametric resonance in soft modes at the critical temperature (T_c) in high-energy heavy ion collisions is studied in the case when the temperature (T) of the system is almost constant for a long time. By dividing the fields into three parts, a zero mode (condensate), soft modes, and hard modes, and assuming that the hard modes are in thermal equilibrium, we derive the equation of motion for soft modes at $T=T_c$. Enhanced modes are extracted by comparing with the Mathieu equation for the condensate oscillating along the sigma axis at $T=T_c$. It is found that the soft mode of π fields at about 174 MeV is enhanced.

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I. INTRODUCTION

It is expected that a new phase of matter will be formed in high energy heavy-ion collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC). It is called "quark-gluon plasma" (QGP), and in it the chiral symmetry restoration occurs. Many signals of the chiral symmetry restoration have been proposed, but there is no decisive one. One of the proposed signals is the disoriented chiral condensate (DCC) which is a misalignment phenomena in the chiral space. The time development of the chiral condensate has been studied in terms of the DCC, and the possibility of soft mode enhancement by parametric resonance was suggested [1]. Parametric resonance in the last stage of high-energy heavy-ion collisions was discussed [1–7], and amplified modes were extracted. A parametric resonance is expected even in the chiral phase transition because the oscillation of the chiral condensate (momentum $k=0$) may amplify nonzero ($k\neq 0$) modes. The motion of the condensate must be investigated in order to reveal this phenomena. As indicated in Ref. [8], the motion of the chiral condensate in high-energy heavy-ion collisions is expected to be almost along the sigma axis in the linear σ model. The oscillation of the condensate is just that of the sigma condensate. There may be a chance of the amplification of the fields by parametric resonance if the sigma field oscillates for a long time.

The study of parametric resonance at finite temperature in a chiral phase transition has never been examined, while that at zero temperature [1–7] has been performed. Parametric resonance may occur even when the temperature is not zero if the condensates moves periodically like a sine function. The motion of the condensates can be described by the equation of motion with the effective potential if the system is not far from thermal equilibrium. The effective potential depends on the temperature of the system, which is a function of (proper) time. If the temperature is constant for a long time, the condensate will oscillate as it oscillates at zero temperature. The periodicity of the motion of the condensate depends on the effective potential at finite temperature. It requires that the temperature must be almost constant in the period of the oscillation of the condensate.

It is expected that the temperature may be almost constant as a function of time at the critical temperature (T_c) because of the large difference of entropy density between QGP and hadron phases. In particular, it may take some 10 fm for the phase transition to finish if the phase transition from quarks and gluons to hadrons is of the first order [9]. A similar time dependence of the temperature may also occur in the second-order chiral phase transition of a high-energy heavy-ion collision. The parametric resonance by the oscillation of the sigma condensate will occur at or near T_c in such a case. A similar time dependence of the temperature is also expected when the cooling of the system is slow. If the period of one-dimensional scaling [10] is long enough, the temperature is only slowly decreasing as a function of time in the last stage of the expansion.

In this paper, we discuss the possibility of parametric resonance at or near T_c in the framework of the linear σ model and extract the amplified modes assuming that the temperature is constant at T_c for a long time. The paper is organized as follows. In Sec. II, the equation of motion is derived in the case when the temperature is constant at $T=T_c$. In Sec. III, the amplified modes and the time scale of the amplification are extracted by comparing with the Mathieu equation. The time scale is explicitly shown by solving the Mathieu equation numerically. Section IV is assigned to conclusions.

II. EQUATION OF MOTION FOR SOFT MODES

The linear σ model is a useful tool to describe the motion of the condensate and soft modes below or near the critical temperature. The Lagrangian is

$$\mathcal{L}=\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{\lambda}{4}(\phi^2-v^2)^2+H\phi_0, \tag{1}$$

where $\phi=(\phi_0,\phi_1,\phi_2,\phi_3)=(\sigma,\vec{\pi})$, $\phi^2=\Sigma_{j=0}^{3}\phi_j^2$, and H is the explicit symmetry-breaking term. The field ϕ is divided into three parts: zero mode (ϕ_{jc}, the condensation), soft modes (ϕ_{js}), and hard modes (ϕ_{jh}):

$$\phi_j=\phi_{jc}+\phi_{js}+\phi_{jh}. \tag{2}$$

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The bracket $\langle \mathcal{O} \rangle$, is used to describe $\mathcal{O}$ averaged over the hard modes. We apply the free-particle approximation for the hard modes. Consequently, $\langle \phi_{jh} \rangle$ and $\langle \phi_{jh}^3 \rangle$ are zero if the distribution of hard modes is thermal. Substituting Eq. (2) into Eq. (1) and taking the thermal average of hard modes, we obtain the following effective Lagrangians:

$$\langle \mathcal{L} \rangle = \langle \mathcal{L}_K \rangle + \langle \mathcal{L}_V \rangle, \tag{3a}$$

$$\langle \mathcal{L}_K \rangle = \frac{1}{2} \partial_\mu \phi_c \partial^\mu \phi_c + \frac{1}{2} \partial_\mu \phi_s \partial^\mu \phi_s + \partial_\mu \phi_c \partial^\mu \phi_s + \frac{1}{2} \langle \partial_\mu \phi_h \partial^\mu \phi_h \rangle, \tag{3b}$$

$$\langle \mathcal{L}_V \rangle = -\frac{\lambda}{4} (\phi_c^2 + \phi_s^2 + 2\phi_c \cdot \phi_s - v^2)^2 + H(\phi_{0c} + \phi_{0s}) - \frac{\lambda}{2} \langle \phi_h^2 \rangle (\phi_c^2 + \phi_s^2 + 2\phi_c \cdot \phi_s - v^2) - \frac{\lambda}{4} \langle (\phi_h^2)^2 \rangle - \lambda \sum_{j=0}^{3} (\phi_{jc} + \phi_{js})^2 \langle \phi_{jh}^2 \rangle, \tag{3c}$$

where the dot implies the inner product defined by $\phi_c \cdot \phi_s = \Sigma_{j=0}^3 \phi_{jc} \phi_{js}$.

If $\langle \mathcal{O} \rangle$ terms have no ϕ_c and $\partial \phi_c$ dependences, the Euler-Lagrange equation for ϕ_{jc} obtained from $\langle \mathcal{L} \rangle$ is

$$\Box \phi_{jc} + \Box \phi_{js} + \lambda (\phi_c^2 + \phi_s^2 + 2\phi_c \cdot \phi_s - v^2)(\phi_{jc} + \phi_{js}) - H\delta_{j0} + \lambda \langle \phi_h^2 \rangle (\phi_{jc} + \phi_{js}) + 2\lambda \langle \phi_{jh}^2 \rangle (\phi_{jc} + \phi_{js}) = 0. \tag{4}$$

An equation of the same form for ϕ_{js} is obtained if $\langle O \rangle$ terms have no ϕ_s and $\partial \phi_s$. Here it is assumed that $\langle \phi_{jh}^2 \rangle \overset{\text{def}}{=} \mathcal{F}(T)$ is j independent. The meaning of this assumption becomes apparent when the concrete expression of $\mathcal{F}(T)$ is obtained in Sec. III. We introduce the effective potential defined by

$$V(\phi, \phi_0; T) = \frac{\lambda}{4} (\phi_c^2 + 6\mathcal{F}(T) - v^2)^2 - H\phi_{0c}. \tag{5}$$

Note that the order of the phase transition described by this potential is second.

Since we are interested in the amplification of the soft mode with small amplitude, we first consider Eq. (4) with $\phi_{js} = 0$ for all j. This is the zeroth order equation of soft modes:

$$\Box \phi_{jc} + \partial V / \partial \phi_{jc} = 0. \tag{6a}$$

The equation for soft modes with small amplitudes is obtained by substituting Eq. (6a) into Eq. (4):

$$\Box \phi_{js} + \lambda (\phi_c^2 + \phi_s^2 + 2\phi_c \cdot \phi_s + 6\mathcal{F}(T) - v^2)\phi_{js} + \lambda (2\phi_c \cdot \phi_s + \phi_s^2)\phi_{jc} = 0. \tag{6b}$$

Equation (6b) is the equation of motion for ϕ_s, with a background field ϕ_c.

We would like to calculate the critical temperature. In a realistic case, the critical temperature cannot be defined exactly since H is not zero. Nevertheless, one can estimate the critical temperature T_c by requiring that one false minimum disappears at $T = T_c$:

$$(\overline{\phi}_{0c}^2 + 6\mathcal{F}(T_c) - v^2)\overline{\phi}_{0c} - H/\lambda = 0, \tag{7a}$$

$$\overline{\phi}_{nc} = 0 \quad (n = 1,2,3), \tag{7b}$$

where $\overline{\phi}_{jc}$ is the condensation which is the expectation value of the field ϕ_j at the minimum of the effective potential at $T = T_c$ [Eq. (7a)]. T_c can be obtained by the condition that two solutions of Eq. (7a) are the same:

$$\mathcal{F}(T_c) = \frac{v^2}{6} - \frac{1}{8} \left(\frac{4H}{\lambda} \right)^{2/3}. \tag{8}$$

We introduce the fluctuation fields $\delta\phi_{jc}$, defined by

$$\delta\phi_{jc} = \phi_{jc} - \overline{\phi}_{jc}. \tag{9}$$

Substituting Eq. (9) into Eq. (6a), and using Eq. (7a), we have

$$\Box(\delta\phi_{jc}) + \lambda[(\delta\phi)^2 + 2(\delta\phi) \cdot \overline{\phi}][\overline{\phi}_{0c} + (\delta\phi_{jc})] + (H/\overline{\phi}_{0c})(\delta\phi_{jc}) = 0. \tag{10}$$

Ignoring $O(\delta^2)$ and higher terms, we obtain

$$(\Box + m_j^2)(\delta\phi_{jc}) = 0, \tag{11}$$

where $m_0^2 \overset{\text{def}}{=} (H/\overline{\phi}_{0c}) + 2\lambda \overline{\phi}_{0c}^2$ and $m_n^2 \overset{\text{def}}{=} (H/\overline{\phi}_{0c})$ $(n = 1,2,3)$. It is easily found from Eq. (11) that the condensate oscillates around the minimum of the potential at T_c. In the same way, Eq. (6b) becomes

$$\Box \phi_{js} + (H/\overline{\phi}_{0c})\phi_{js} + \lambda(\phi_s^2 + 2\overline{\phi}_c \cdot \phi_s)\phi_{js} + 2\lambda[\overline{\phi}_c \cdot (\delta\phi_c) + \phi_s \cdot (\delta\phi_c)]\phi_{js} + \lambda(\delta\phi_c)^2 \phi_{js} + \lambda[2\overline{\phi}_c \cdot \phi_s + \phi_s^2 + 2(\delta\phi_c) \cdot \phi_s]\phi_{jc} = 0. \tag{12}$$

Since we are interested in the amplification by the oscillation of the condensate, we consider the small ϕ_s and discard $O(\phi_s^2)$ and $O(\phi_s^3)$ terms. The equation for ϕ_{js} [Eq. (12)] in such a case is

$$\Box \phi_{js} + \alpha_j \phi_{js} + \sum_{\substack{i=0 \\ (i \neq j)}}^{3} \beta_{ji} \phi_{is} = 0, \tag{13}$$

where

$$\alpha_j \overset{\text{def}}{=} H/\overline{\phi}_{0c} + 2\lambda[\overline{\phi}_c \cdot (\delta\phi_c) + (\overline{\phi}_{jc})^2 + 2\overline{\phi}_{jc}(\delta\phi_{jc})], \tag{14a}$$

$$\beta_{ji} \overset{\text{def}}{=} 2\lambda[\overline{\phi}_{jc}\overline{\phi}_{ic} + \overline{\phi}_{jc}(\delta\phi_{ic}) + \overline{\phi}_{ic}(\delta\phi_{jc})] \equiv \beta_{ij} \quad (j \neq i), \tag{14b}$$

and $O(\delta^2)$ terms have been ignored. Since $\overline{\phi}_{0c} \neq 0$ and $\overline{\phi}_{nc} = 0 (n=1,2,3)$ in the linear σ model, the coefficients α_j and β_{ji} have the following relations:

$$\alpha_1 = \alpha_2 = \alpha_3, \quad \beta_{12} = \beta_{13} = \beta_{21} = \beta_{23} = \beta_{31} = \beta_{32} = 0. \tag{15}$$

As stated in Sec. I, the condensate moves and oscillates almost along the sigma axis. Then $\delta\phi_{nc} \sim 0$ for $n=1,2$, and 3. Consequently all β's any zero. In such a case, Eq. (13) is diagonalized:

$$\Box\phi_{0s} + [m_0^2 + 6\lambda\overline{\phi}_{0c}(\delta\phi_{0c})]\phi_{0s} = 0, \tag{16a}$$

$$\Box\phi_{ns} + [m_n^2 + 2\lambda\overline{\phi}_{0c}(\delta\phi_{0c})]\phi_{ns} = 0. \tag{16b}$$

The solution of Eq. (11) for $j=0$ is

$$\delta\phi_{0c} = -B\cos(m_0 t + \theta), \tag{17}$$

where B and θ in Eq. (17) are determined by the initial condition of $\delta\phi_{0c}$. This solution, [Eq. (17)], is substituted into Eqs. (16a) and (16b) and the transformation from t to $\xi \overset{\text{def}}{=} (m_0 t + \theta)/2$ is applied. The equations of motion now become

$$\left\{ \frac{d^2}{d\xi^2} + \frac{4}{m_0^2}(\vec{k}^2 + m_0^2) - \frac{24\lambda\overline{\phi}_{0c}B}{m_0^2}\cos(2\xi) \right\} \phi_{0s}(\xi;\vec{k}) = 0, \tag{18a}$$

$$\left\{ \frac{d^2}{d\xi^2} + \frac{4}{m_0^2}(\vec{k}^2 + m_n^2) - \frac{8\lambda\overline{\phi}_{0c}B}{m_0^2}\cos(2\xi) \right\} \phi_{ns}(\xi;\vec{k}) = 0, \tag{18b}$$

where $\phi_{0s}(\xi;\vec{k})$ and $\phi_{ns}(\xi;\vec{k})$ are the Fourier transformation of $\phi_{0s}(\xi;\vec{x})$ and $\phi_{ns}(\xi;\vec{x})$, respectively.

III. AMPLIFIED MODES

Equations (18a) and (18b) are just Mathieu equations. To investigate the amplified modes, we define the following quantities:

$$A_\sigma = \frac{4}{m_0^2}(\vec{k}^2 + m_0^2), \quad A_\pi = \frac{4}{m_0^2}(\vec{k}^2 + m_n^2). \tag{19}$$

The amplified modes are obtained from the above coefficients by the help of a knowledge of Mathieu equation. The amplified modes of σ field for nonzero modes ($k \neq 0$) corresponds to $A_\sigma = 9,16,\ldots$ because $A_\sigma > 4$ apparently, while that of π field corresponds to $A_\pi = 1,4,9,16,\ldots$. ($A_\pi = 1$ is not satisfied for some parameters of the linear σ model.) Then the masses of σ and π fields are needed to determine such modes. The condensate ($\overline{\phi}_{0c}$) at T_c is $(4H/\lambda)^{1/3}$, which is easily found from the definition of T_c. Therefore, the masses are obtained if λ and H are given. The amplified modes corresponding to $A_\sigma, A_\pi = 1,4,9,16$ are shown in Table I for $\lambda = 20$ and $H^{1/3} = 119$ MeV, which generate $m_\pi = 135$ MeV, $m_\sigma = 600$ MeV, and $f_\pi = 92.5$ MeV at $T=0$ for $v = 87.4$ MeV.

TABLE I. Amplified modes in σ and π fields for various A_σ and A_π.

A_σ, A_π	k_σ (MeV)	k_π (MeV)
1	–	174.0
4	0	440.1
9	521.9	682.7
16	808.5	920.5

The amplification of π fields is determined by A_π and the factor ($2Q_\pi$) in the presence of cosine in the Mathieu equation. This factor is given by

$$2Q_\pi = \frac{8\lambda\overline{\phi}_{0c}B}{m_0^2}, \tag{20}$$

where B is the amplitude of zero mode introduced in Eq. (17). Its numerical value for the previous parameters of the linear σ model is about 0.051 $(\text{MeV}^{-1}) \times B$. For example, $2Q_\pi$ is -1.53 for $B = -30$ MeV. The solution $w(\xi)$ of the Mathieu equation for $w(0)=1, dw/d\xi(0)=0$, has the property $w(\xi+\pi) = e^{i\nu\pi}w(\xi)$, where ν is called "the characteristic exponent" which has a relation

$$\cos(\nu\pi) = w(\pi). \tag{21}$$

The time period π in ξ corresponds to about 2.65 fm in t. It is obtained that $|e^{i\nu\pi}| \sim 3.1$ [11] for $A_\pi = 1$ and $-2Q_\pi/A_\pi = 1.53$. The numerical solutions for $B = \mp 30$ MeV for $w(0)=1, dw/dt(0)=0$ are shown in Fig. 1. $\omega(\xi=\pi)$ is about -1.71, which corresponds to $|e^{i\nu\pi}| \sim 3.1$. It is found that the fields π are strongly amplified in a few (five or more) fm.

There are several amplified modes in the Mathieu equations [Eqs. (18a) and (18b)]. However, modes larger than k_Λ cannot actually be amplified modes because we assume that hard modes are thermal. Therefore, the cutoff (k_Λ) between soft and hard modes is important in order to know whether the amplified modes extracted from Eqs. (18a) and (18b) belong to soft modes or not. In this paper, k_Λ is determined as follows. Since $\mathcal{F}(T)$ is a function of k_Λ, the latter is obtained from Eq. (8), in which T_c is related to k_Λ. That is, k_Λ is fixed if T_c is given. The density operator of j field for hard modes $[\rho_h^j(T)]$ is assumed as

$$\rho_h^j(T) = \exp\left(-T^{-1} \sum_{|\vec{k}| \geq k_\Lambda} \omega_j a_j^\dagger(\vec{k}) a_j(\vec{k}) \right) \Big/ \mathrm{Tr}\left[\exp\left(-T^{-1} \sum_{|\vec{k}| \geq k_\Lambda} \omega_j a_j^\dagger(\vec{k}) a_j(\vec{k}) \right) \right], \tag{22}$$

where $\omega_j^2 = \vec{k}^2 + m_j^2$. We find

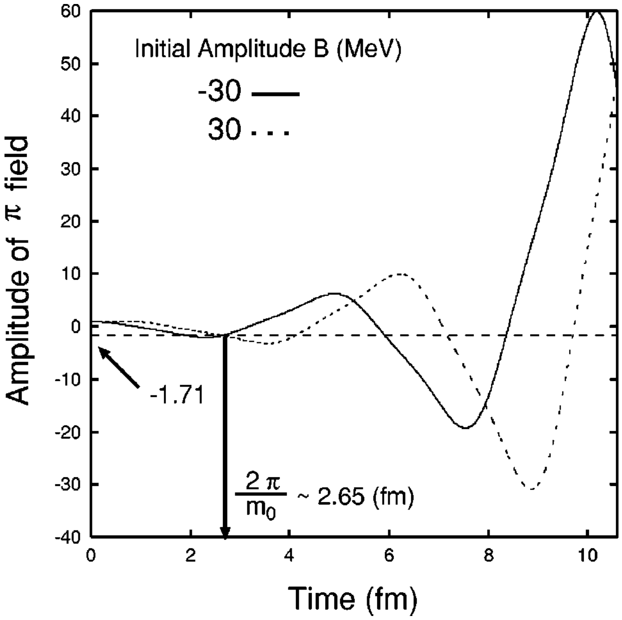

FIG. 1. The solution $w(t)$ of the Mathieu equation for π fields with $A_{\pi}=1$ with the initial amplitude $B=\mp 30$ MeV and $w(0)=1$ and $dw/dt(0)=0$. The thick and dotted lines are for $B=-30$ and 30 MeV, respectively. The dashed line is for -1.71, which is the value of $\cos(\nu\xi)$ at $\xi=\pi$. The arrow with the string "$2\pi/m_0$" shows the time corresponding to $\xi=\pi$.

$$\begin{aligned}\mathcal{F}(T) &\equiv \langle \phi_{jh}^2(x)\rangle \\ &= \mathrm{Tr}(\rho_h^j(T)\phi_{jh}^2(x)) \\ &= \frac{1}{2V}\sum_{|\vec{k}|\geqslant k_\Lambda}\omega_j^{-1}+\frac{1}{V}\sum_{|\vec{k}|\geqslant k_\Lambda}\frac{1}{[\exp(\omega_j(\vec{k})/T)-1]\omega_j(\vec{k})}.\end{aligned} \tag{23}$$

The first term is the vacuum contribution, and the second is the thermal contribution. The vacuum contribution can be discarded in the following calculation, since it is the well-known infinity which is removed by the redefinition of the energy including the contribution from the soft modes (subtraction of the vacuum energy.) Equation (8) is rewritten by the massless particle approximation and the integration of the angle:

$$\frac{1}{\pi^2}\int_{k_\Lambda/T_c}^{\infty} du\,\frac{u}{\exp(u)-1}=\frac{1}{3}\left(\frac{v}{T_c}\right)^2-\frac{1}{4T_c^2}\left(\frac{4H}{\lambda}\right)^{2/3}. \tag{24}$$

This approximation is valid near T_c, because the masses of σ and π mesons become small. These masses are zero at $T=T_c$ in the chiral limit. Note that $\langle O\rangle$ terms have no dependence on $\partial\phi_0, \phi_0, \partial\phi_s$, and ϕ_s because $\mathcal{F}(T)$ is independent of j in the massless particle approximation.

Table II shows k_Λ obtained by solving Eq. (24) for various T_c's with $\lambda=20$, $H^{1/3}=119$ MeV, and $v=87.4$ MeV. T_c is about 123 MeV in the chiral limit. It has been estimated from lattice QCD and some other methods. It is between 140 and 190 MeV in lattice QCD [12,13]. Since k_Λ is above 190 MeV except for $T_c\leqslant 130$ MeV, we conclude that $k_\pi \sim 174$ MeV is an amplified mode at least when T_c is adjusted above 130 MeV. Other modes are irrelevant, since these modes belong to hard modes. ($k_\pi\sim 440.1$ MeV may be a soft mode in this sense.)

TABLE II. The relation between T_c and k_Λ.

T_c (MeV)	k_Λ (MeV)
89.5	0
130	150.5
140	192.3
150	235.7
160	280.5
170	326.8
180	379.3

The amplified mode of the π field is not directly related to the observed modes since the mass of a π meson at $T\neq 0$ is different from that at $T=0$. Then we must know the effect of the difference of two masses. In quantum field theory, the relation between these two masses can be described by a Bogoliubov transformation. We can estimate the effect by evaluating the coefficients of this transformation. The amplified mode is not changed essentially, because the difference of the masses between zero temperature and critical temperature is small enough for π fields in this model. The peak coming from the parametric resonance at $T=T_c$ will be found near $k_\pi\sim 174$ MeV if the peak is not smeared out by scattering, absorption, and so on.

IV. CONCLUSIONS

The parametric resonance at critical temperature in high-energy heavy-ion collisions is studied in the case when the temperature of the system is constant at T_c for a long time. We consider the case in which the condensate oscillates along the sigma axis at the critical temperature of the second-order phase transition in the framework of the linear σ model.

The enhancement of a soft mode at about 174 MeV in the π field is found at various T_c's above 140 MeV. Other modes are irrelevant as amplified modes because these modes belong to the hard modes which are assumed to be in thermal equilibrium in the present study. Conversely, there is no enhanced soft mode in the σ field. The amplified mode (174 MeV) at $T=T_c$ is softer than that (for example, about 265 MeV in the one-dimensional expansion case [5]) at $T=0$ in the π field because the difference between sigma and pion masses at $T=T_c$ is smaller than that at $T=0$. It takes a short time for the soft modes to be amplified. This implies that an amplification of the soft mode by the parametric resonance at T_c is possible in real collisions at high energies. The enhancement by parametric resonance in the present study may be observed experimentally if the smearing effects are weak enough.

It has been pointed out that there may be a parametric resonance at zero temperature in the last stage of heavy-ion collisions. Then the pion momentum distribution caused by the parametric resonance at finite temperatures may be the initial distribution of the subsequent parametric resonance at zero temperature. If so, some peaks which correspond to the enhanced modes may appear in the final momentum distribution.

The explicit symmetry-breaking term ($H\phi_0$) plays important roles for the parametric resonance at $T=T_c$, because this term makes the condensate $\overline{\phi}_{0c}$ nonzero and generates masses at $T=T_c$. The condensates ($\overline{\phi}_{jc}$) and masses (m_j) at T_c are exactly zero in the chiral limit $(H=0)$. Note that Eqs. (16a) and (16b) are valid when $O(\phi_s^2)$ and $O(\phi_s^3)$ terms in Eq. (12) are negligible. The resonance structure may change in small-H cases.

We have not shown the strength of the peak in the present study. If we wish to obtain it, we must know the time interval in which the temperature is (almost) constant and consider the evolution of the QGP. The nonlinear effects and back reaction are also ignored in this investigation. These problems will be answered in the future studies.

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