Extracting the spectral function of ^{4}He from a relativistic plane-wave treatment

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The spectral function of ^{4}He is extracted from a plane-wave approximation to the $(e,e'p)$ reaction using a fully relativistic formalism. We take advantage of both an algebraic "trick" and a general relativistic formalism for quasifree processes developed earlier to arrive at transparent, analytical expressions for all quasifree $(e,e'p)$ observables. An observable is identified for the clean and model-independent extraction of the spectral function. Our simple relativistic plane-wave calculations provide baseline predictions for the recently measured, but not yet fully analyzed, momentum distribution of ^{4}He by the A1 Collaboration from Mainz. Yet in spite of its simplicity, our approach predicts momentum distributions for ^{4}He that rival some of the best nonrelativistic calculations to date. Finally, we highlight some of the challenges and opportunities that remain, both theoretically and experimentally, in the extraction of quasifree observables.

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I. INTRODUCTION

Electron scattering from nuclei is a common and powerful tool for studying the structure of nuclei. The method relies on our superior understanding of quantum electrodynamics (QED) and the relative ease by which QED may be applied to a variety of processes, at least in the one-photon-exchange approximation. In inclusive (e,e') electron scattering all nuclear-structure information is contained in two dynamical quantities: the longitudinal and transverse response functions. The longitudinal response is sensitive to the distribution of charge in the nucleus, while the transverse response samples the distribution of currents and magnetization. Measurement of these quantities in the quasielastic region is expected to be particularly clean as the reactive content of the reaction is dominated by quasifree proton knockout. If so, "reduced" longitudinal and transverse response functions, obtained from the full nuclear responses by dividing out the corresponding single-nucleon form factor, should be equal. Yet a quenching of the longitudinal response relative to the transverse one of 14% in ^{4}He and 50% in ^{208}Pb has been reported from a quasielastic (e,e') electron-scattering measurement [1]. Indeed, from a recent global analysis of the world data on quasielastic electron scattering from ^{4}He this quenching appears to be even larger, approaching 40% [2]. A similar (20–40%) quenching in ^{4}He has also been reported in the semiexclusive $(e,e'p)$ reaction at quasielastic kinematics [3]. In order to explain the longitudinal/transverse (L/T) discrepancy a variety of scenarios have been proposed. These include medium modifications to vacuum polarization [4], nucleon "swelling [5], and Brown-Rho scaling [6]. Yet most of these explanations attributed the discrepancy to the quenching of the longitudinal response, one of the long-standing problems in nuclear physics. However, this view has recently been put into question. An analysis of world data on inclusive quasielastic electron scattering on medium-mass nuclei seems to indicate that the presumed quenching of the longitudinal response is absent after all [7]. Yet the issue continues to be controversial: A recent analysis seems to have reestablished the quenching of the longitudinal response, at least in medium and heavy nuclei [8]. Fortunately, the situation in light nuclei seems to be under better control, primarily due to the existence of exact Green's function Monte Carlo calculations of the inclusive Euclidean responses in ^{3}He and ^{4}He [2,9]. While the analytic continuation of these theoretical responses into real time is difficult, the opposite is not true: Accurate experimental Euclidean responses are now available from existent high-quality data. Two of the main conclusions drawn from these comparisons are as follows: (a) The quenching in the L/T ratio in ^{4}He is generated as a consequence of a substantial enhancement of the transverse response due to two-body mechanisms rather than a quenching of the longitudinal response (two-body effects seem to have a small impact on the longitudinal response), and (b) the L/T ratio decreases significantly in going from ^{3}He to ^{4}He [2]. While it is undeniable that much progress has been made, a considerable effort continues to be devoted to the understanding of the mass- and momentum-transfer dependence of the L/T ratio both in the inclusive (e,e') as well as in the semiexclusive $(e,e'p)$ reactions (see below).

The appeal of the $(e,e'p)$ reaction is due to the perceived sensitivity of the process to the nucleon momentum distribution. Interest in this reaction has stimulated a tremendous amount of experimental work at electron facilities such as NIKHEF, MIT/Bates, and Saclay, who have championed this effort for several decades. While it is undeniable that this reaction involves the best understood theory in all of physics (QED), many uncertainties remain due to the strongly interacting character of the many-body system. It is hoped that with the advent of modern electron-scattering facilities, such as the Thomas Jefferson National Accelerator Facility (JLab)

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and Mainz, some of the remaining open questions will be answered. Indeed, in an attempt to elucidate the mass- and momentum-transfer dependence of the L/T "anomaly" discussed earlier, a systematic study of the longitudinal and transverse response functions from ^{3}He and ^{4}He is being conducted at the Mainz Microton (MAMI) facility by the A1 collaboration [10–15]. Their extraction of "experimental" spectral functions and of momentum distributions relies on a plane-wave-impulse-approximation (PWIA). In such an approximation the $(e,e'p)$ cross section is proportional to the nucleon spectral function times an off-shell electron-proton cross section (σ_{ep}). Experimental analyses of this reaction employ, almost exclusively, the de Forest's $cc1$ prescription for σ_{ep} with both nucleon form factors unmodified from their free-space form [16].

Stimulated by this new experimental thrust, we report here relativistic plane-wave-impulse-approximation (RPWIA) calculations of the $(e,e'p)$ cross section in the quasielastic region. Our motivation for such a study is fourfold. First, we employ an established RPWIA formalism, first introduced in Ref. [17] and recently extended to the kaon-photoproduction reaction [18,19] for the study of the $(e,e'p)$ reaction in the quasielastic region. Second, we use this formalism to compute the spectral function of ^{4}He in anticipation of the recently measured, but not yet fully analyzed, A1 collaboration data from Mainz [11–15]. Third, we take advantage of the L/T separation at Mainz to introduce what we regard as the cleanest physical observable from which to extract the nucleon spectral function. Lastly, we highlight some of the challenges and opportunities that remain in the calculation of quasifree observables.

There is a vast amount of literature on $(e,e'p)$ reaction in the quasifree region. Most relevant to our present discussion is the one pertaining to fully relativistic calculations [20–32]. An extensive set of these relativistic studies has been conducted by the "Spanish" group of Udias and collaborators [22–28]. These studies have shown that the many subtleties intrinsic to the relativistic approach challenge much of the "conventional wisdom" developed within the nonrelativistic framework and that, as a result, a radical revision of ideas may be required. Relativistic effects originating from medium modifications to the lower components of the Dirac spinors and from the negative-energy part of the spectrum seem to play an important role in the quasifree process. Indeed, the much debated issue of short-range correlations at large missing momenta [33–35] can now be attributed, at least in part, to contributions arising from the negative-energy states [25,36].

The power of the theoretical approach employed here lies in its simplicity. Analytic expressions for the response of a mean-field ground state may be provided in the plane-wave limit. The added computational demands placed on such a formalism, relative to that from a free on-shell proton, are minimal. The formalism owes its simplicity to an algebraic trick, first introduced by Gardner and Piekarewicz [17], that enables one to define a "bound" (in direct analogy to the free) nucleon propagator. Indeed, the Dirac structure of the bound nucleon propagator is identical to that of the free Feynman propagator. As a consequence, the power of Feynman's trace techniques may be employed throughout the formalism.

The paper has been organized as follows. In Sec. II some of the central concepts and ideas of the semiexclusive $(e,e'p)$ reaction are reviewed. Special emphasis is placed on defining the bound-state propagator and the simplifications that this entails in the plane-wave limit. In Sec. III we present our results for ^{4}He and discuss a (fairly) model-independent method for extracting the nucleon momentum distribution. Finally, a summary and conclusions are presented in Sec. IV.

II. FORMALISM

In Refs. [18,19] a general formalism has been developed for the study of a variety of quasifree processes in the relativistic plane-wave impulse approximation (RPWIA). This formalism is now applied to the $(e,e'p)$ reaction in a mean-field approximation to the Walecka model [37]. Although the use of a mean-field approach for a nucleus as small as ^{4}He is questionable, we allow ourselves this freedom in order to establish a baseline against which more sophisticated approaches may be compared.

Following a standard procedure, an expression for the unpolarized differential cross section per target nucleon for the $(e,e'p)$ reaction is derived. We obtain

$$\left(\frac{d^5\sigma}{dE'_e d\Omega_{\mathbf{k}'} d\Omega_{\mathbf{p}'}}\right)_{\rm lab} = \frac{4\alpha^2}{Q^4}\frac{|\mathbf{k}'|}{|\mathbf{k}|}|\mathbf{p}'||\mathcal{M}|^2. \tag{1}$$

In the above expression $\mathbf{k}$, $\mathbf{k}'$, and $\mathbf{p}'$ denote the linear momentum of the incoming electron, outgoing electron, and knocked-out proton, respectively. The four-momentum transfer is defined in terms of the energy loss $(\omega = E_e - E'_e)$ and the three-momentum transfer $(\mathbf{q}=\mathbf{k}-\mathbf{k}')$ as $Q^2=\mathbf{q}^2-\omega^2$. The transition matrix element $\mathcal{M}$ is given in a relativistic mean-field picture by

$$|\mathcal{M}|^2 = l^{\mu\nu}W_{\mu\nu}, \tag{2a}$$

$$l^{\mu\nu} = [k'^{\mu}k^{\nu} + k^{\mu}k'^{\nu} - g^{\mu\nu}(k\cdot k')], \tag{2b}$$

$$W^{\mu\nu} = \frac{1}{4(2j+1)}\sum_{s'm}[\overline{\mathcal{U}}(\mathbf{p}',s')j^{\mu}\mathcal{U}_{\alpha m}(\mathbf{p})] \times [\overline{\mathcal{U}}(\mathbf{p}',s')j^{\nu}\mathcal{U}_{\alpha m}(\mathbf{p})]^* = \frac{1}{4}{\rm Tr}[(\not{p}'+M)j^{\mu}S_{\alpha}(\mathbf{p})j^{\nu}]. \tag{2c}$$

Here $\mathcal{U}(\mathbf{p}',s')$ is the free Dirac spinor for the knocked-out proton, normalized according to the conventions of Bjorken and Drell [38], while $\mathcal{U}_{\alpha m}(\mathbf{p})$ is the Fourier transform of the relativistic spinor for the bound proton. Note that α denotes the collection of all quantum numbers necessary to specify the single-particle orbital, except for the magnetic quantum number (m) which is indicated explicitly. We have also introduced a "bound-state propagator"

$$S_\alpha(\mathbf{p}) \equiv \frac{1}{2j+1} \sum_m \mathcal{U}_{\alpha m}(\mathbf{p}) \overline{\mathcal{U}}_{\alpha m}(\mathbf{p}), \tag{3}$$

normalized according to

$$\int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}[\gamma^0 S_\alpha(\mathbf{p})] = \int \frac{d^3p}{(2\pi)^3} \mathcal{U}^\dagger_{\alpha m}(\mathbf{p}) \mathcal{U}_{\alpha m}(\mathbf{p}) = 1. \tag{4}$$

Here j is the total angular momentum quantum number and $2j+1$ is the multiplicity of protons in the struck shell. It follows from simple kinematical arguments that the missing momentum $\mathbf{p} \equiv \mathbf{p}' - \mathbf{q}$ is, in a mean-field picture, identical to the momentum of the struck proton. It is the possibility of mapping the nucleon momentum distribution that makes the $(e,e'p)$ reaction so appealing.

We now invoke an algebraic trick first introduced in Ref. [17] to simplify the expression for the hadronic tensor $W^{\mu\nu}$. This technique is useful in quasifree processes as it enables one to cast the bound-state propagator of Eq. (3) into a form identical in structure to that of the free Feynman propagator. That is,

$$S_\alpha(\mathbf{p}) = (\not{p}_\alpha + M_\alpha), \tag{5}$$

where we have defined mass- and four-momentum-like $[p^\mu_\alpha \equiv (E_\alpha, \mathbf{p}_\alpha)]$ quantities according to

$$M_\alpha = \left(\frac{\pi}{p^2}\right) [g^2_\alpha(p) - f^2_\alpha(p)], \tag{6a}$$

$$E_\alpha = \left(\frac{\pi}{p^2}\right) [g^2_\alpha(p) + f^2_\alpha(p)], \tag{6b}$$

$$\mathbf{p}_\alpha = \left(\frac{\pi}{p^2}\right) [2 g_\alpha(p) f_\alpha(p) \hat{\mathbf{p}}]. \tag{6c}$$

Moreover, they satisfy the ''on-shell relation''

$$p^2_\alpha = E^2_\alpha - \mathbf{p}^2_\alpha = M^2_\alpha. \tag{7}$$

In these expressions $g_\alpha(p)$ and $f_\alpha(p)$ are the Fourier transforms of the upper and lower components of the bound-state Dirac spinor, respectively [17]. Using this form of the bound-state propagator the hadronic tensor simplifies to

$$W^{\mu\nu} = \frac{1}{4} \mathrm{Tr}((\not{p}' + M) j^\mu (\not{p}_\alpha + M_\alpha) j^\nu). \tag{8}$$

The obvious similarity in structure between the free and bound propagators results in an enormous simplification: powerful trace techniques developed elsewhere may now be employed here to compute all $(e,e'p)$ observables. Although the focus of this paper is the unpolarized cross section [Eq. (1)], the formalism may be extended without difficulty to the case in which the electron, the outgoing proton, or both, are polarized. Yet, in order to automate this straightforward but lengthy procedure, we rely on the FEYNCALC 1.0 [39] package with MATHEMATICA 2.0 to calculate all the necessary traces. For a general electromagnetic current operator for the proton, the output from these symbolic manipulations is transparent enough so that the sensitivity of the cross section to the various quantities in the problem may be assessed. Indeed, such a simplification will prove useful later in identifying the optimal observable from which to extract the spectral function. It is important to note, however, that this enormous simplification would have been lost had distortions been included in the formalism. Even so, the plane-wave approach discussed here, and used in most experimental extractions of the spectral function, is qualitatively useful. Moreover, if the main effect of distortions is to induce an overall suppression of the cross section without affecting significantly the distribution of strength, the plane-wave formalism provides solid quantitative predictions for a variety of spin observables [18,19].

Yet an important open question remains: What constitutes a suitable form for the nucleon electromagnetic current? A ubiquitous form given in the literature is

$$j^\mu(q) = F_1(q^2)\gamma^\mu + i F_2(q^2) \sigma^{\mu\nu} \frac{q_\nu}{2M}. \tag{9}$$

While this form is certainly general, as only two form factors are required to fully specify the electromagnetic current for an on-shell nucleon, the form is not unique. Indeed, many other forms—all of them equivalent on-shell—may be used. For example, through a Gordon decomposition of the current one arrives at

$$j^\mu(q) = (F_1 + F_2)\gamma^\mu - F_2 \frac{(p' + p)^\mu}{2M}. \tag{10}$$

However, as soon as one of the nucleons goes off its mass shell, an off-shell choice must be made. This decision is crucial, as various on-shell equivalent choices may yield vastly different results. This off-shell ambiguity remains one of the most serious obstacles in the field. Several attempts have been made in the literature to overcome this hurdle. Perhaps the most celebrated treatment is due to de Forest who uses physical constraints, such as current conservation, to reduce this ambiguity [16]. He imposes this condition on the two forms of the electromagnetic current given above [Eqs. (9) and 10] and produces what are known in the literature as the $cc2$ and the $cc1$ forms, respectively. Although noteworthy, this effort does not resolve the ambiguity. For example, there is no unique way to impose current conservation; one may eliminate either the time component or the longitudinal component of the three-vector current [25]. Alternatively, one may adopt some guiding principle, such as vector-meson dominance, to go off the mass shell. Here we adopt the ''natural'' choice by simply extrapolating off the mass shell the $cc2$ form, without imposing further constraints on the single-nucleon current.

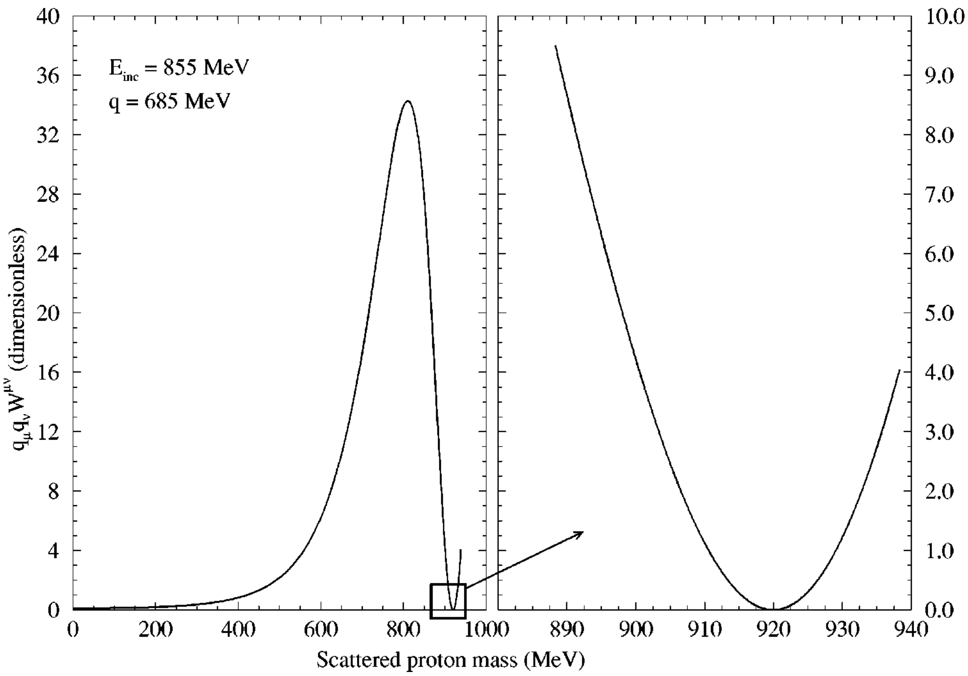

FIG. 1. The gauge variance term $q_\mu q_\nu W^{\mu\nu}$ for ^{4}He as a function of the scattered proton mass calculated in parallel kinematics for an incident photon energy of $E_{\rm inc}=855$ MeV and a momentum transfer of $q=685$ MeV. The right panel is a magnification of the boxed area in the left panel.

III. RESULTS

As de Forest has done in the past, we now attempt to impose some approximate form of gauge invariance. Yet rather than concentrating on the nucleon current, we focus directly on the nuclear responses. First, however, we address some important issues in this regard. For any mean-field treatment of the $(e,e'p)$ reaction to be gauge invariant, the mean-field potential for the bound proton must be identical to the distorting potential for the emitted proton. This represents a challenging task. Indeed, mean-field approximations to the nuclear ground state give rise to real, local and energy-independent potentials that are in contradiction to the complex and energy-dependent potentials that are needed to describe the propagations of the outgoing proton. Thus, present-day calculations of $(e,e'p)$ observables are presented with a dilemma. Calculations that use the same (real and energy-independent) mean fields to generate both the bound single-particle wave function and the distorted wave satisfy gauge invariance but miss some of the important physics, such as absorption, which is known to be present in the outgoing channel. On the other hand, calculations that incorporate the correct physics via a phenomenological optical potential are known to violate current conservation [28]. We offer here no solution to this complicated problem. Rather, we impose gauge invariance ''ad hoc'' by adjusting the effective nucleon mass of the emitted proton so that the ''gauge-variance'' term, $q_\mu q_\nu W^{\mu\nu}$, be minimized. This procedure, with perhaps its unexpected outcome, is displayed in Fig. 1. It shows that by decreasing the proton mass by about 20 MeV, one can restore gauge invariance in the calculation $q_\mu q_\nu W^{\mu\nu}=0$. Although by no means fundamental, this ''poor-man'' distortion ensures the conservation of gauge invariance without compromising the clarity of the formalism.

The essence of the experimental extraction of the spectral function is based on a nonrelativistic plane-wave result [40]:

$$S(E,\mathbf{p})=\frac{1}{p'E_p'\sigma_{eN}}\frac{d^6\sigma}{dE_e'd\Omega_{\mathbf{k}'}dE_p'd\Omega_{\mathbf{p}'}}. \qquad (11)$$

However, this procedure is problematic. First, the quasifree cross section [the numerator in Eq. (11)] suffers from the off-shell ambiguity; different on-shell equivalent forms for the single-nucleon current yield different results. Second, the problem gets compounded by the use of an elementary electron-proton cross section (σ_{eN}) evaluated at off-shell kinematics [16]. Finally, the projection of the bound-state wave function into the negative-energy sector as well as other relativistic effects spoil the assumed factorization of the cross section derived in the nonrelativistic limit [25].

Insights into the role of relativistic corrections, particularly those concerned with negative-energy states, may be gained by introducing the completeness relation in terms of free (plane-wave) spinors:

$$\sum_s [\mathcal{U}(\mathbf{p},s)\overline{\mathcal{U}}(\mathbf{p},s)-\mathcal{V}(\mathbf{p},s)\overline{\mathcal{V}}(\mathbf{p},s)]=1. \qquad (12)$$

Naively, one would expect that the projection of a positive-energy bound state into a negative-energy plane-wave state would be vanishingly small. This, however, it is not the case [36]. At the very least one must recognize that the positive-energy plane-wave states, by themselves, are not complete. Moreover, it has been shown that the projection of the bound-state spinors into the negative-energy states dominate at large missing momenta and may mimic effects perceived as ''exotic'' from the nonrelativistic point of view, such as an asymmetry in the missing-momentum distribution [17] or short-range correlations [36]. Indeed, Caballero and collaborators have confirmed that these contributions can have a significant effect on various observables, especially at large missing momenta [25].

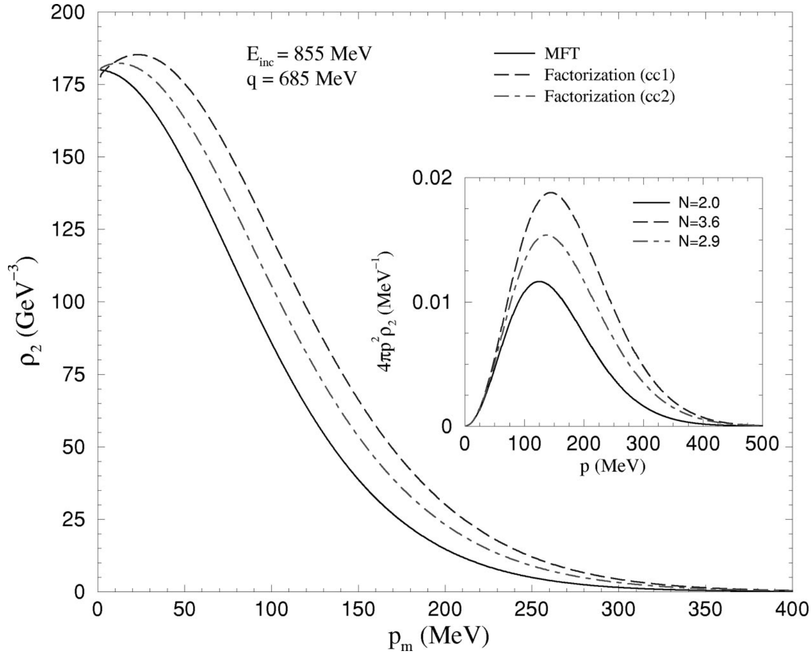

FIG. 2. The proton momentum distribution ρ_2 for ^{4}He as a function of the missing momentum calculated at an incident photon energy of $E_{\rm inc}=855$ MeV and a momentum transfer of $q=685$ MeV. The solid line is the relativistic mean-field calculation, while the dashed and dot-dashed lines display the momentum distribution extracted from a factorization approximation using the $cc1$ and $cc2$ prescriptions for σ_{eN}, respectively. The inset shows the corresponding integrands from which the shell occupancy may be extracted.

To "resolve" the off-shell ambiguity it has become ubiquitous in the field to use the de Forest $cc1$ prescription for evaluating the elementary cross section σ_{eN}—irrespective of the form of the electromagnetic current adopted to compute the quasifree cross section. This is the standard procedure used in comparing theoretical calculations of the spectral function to experiment. We may elect here to conform to tradition and use the de Forest $cc1$ prescription to compute σ_{eN} in Eq. (11), but at a cost. A price must be paid because of the inconsistency in using one prescription for evaluating the single-nucleon current σ_{eN} and a different one ($cc2$) to evaluate the quasifree cross section. To illustrate this point we display in Fig. 2 the proton momentum distribution defined by

$$\rho_2(\mathbf{p})=\int S(E,\mathbf{p})dE. \tag{13}$$

Note that the subscript "2" in ρ_2 stands for two-body breakup. The graph displays the "canonical" momentum distribution (solid line) obtained from the Fourier transform of the $1S^{1/2}$ proton wave function [see Eq. (6b)]. Note that this canonical momentum distribution has been normalized, as it is done experimentally, to the total number of protons in the shell (2 for the case ^{4}He). The other two curves were extracted from the quasifree cross section by adopting either the de Forest $cc1$ choice for σ_{eN} (dashed line) or the $cc2$ prescription (dot-dashed line). In both cases the quasifree cross section has been computed using the "vector-tensor" form of the electromagnetic current, as given in Eq. (9). The inset on the graph shows the integrand from which the occupancy of the shell may be computed. It is evident that the conventional $cc1$ prescription of de Forest greatly overestimates ρ_2 (it integrates to 3.6). We attribute this deficiency to the lack of consistency: The quasifree cross section has been evaluated using the $cc2$ form of the current, while the elementary amplitude uses the $cc1$ form. One can improve the situation by adopting the $cc2$ form in the evaluation of both. Yet significant differences remain; while the off-shell ambiguity has been reduced, it has not been fully eliminated. Moreover, the factorization assumption is only approximate, as it neglects the projection of the relativistic wave function onto the negative-energy spectrum and other relativistic effects.

While a consistent relativistic treatment seems to have spoiled the factorization picture obtained from a nonrelativistic analysis, and with it the simple relation between the cross-section ratio and the spectral function [Eq. (11)], the situation is not without remedy. Having evaluated all matrix elements of the electromagnetic current analytically in the plane-wave limit, the source of the problem can be readily identified. Upon evaluating the coincidence cross section, one learns that the off-shell ambiguity is manifested in the form of several ambiguous "kinematical" factors. For example, one must decide what value to use for the energy of the struck proton. Should it be the binding-energy of the struck proton or should it be the on-shell value? This is not an easy question to answer. Energy conservation demands that the energy be equal to the binding energy ($E_{\rm bin}=E'_p-\omega$), yet the equivalence between the various forms of the electromagnetic current is derived assuming the on-shell dispersion relation ($E_p=\sqrt{\mathbf{p}^2+M^2}$). This is one of the many manifestations of the off-shell ambiguity: Kinematical terms that are well defined for on-shell spinors become ambiguous off-shell. In Ref. [16] de Forest resolves the ambiguity, by fiat, using the on-shell choice. Perhaps a better option may be looking for an observable, that even though might be more difficult to isolate experimentally, it may display a weaker off-shell dependence than the unpolarized cross section. To do so we examine the various components of the hadronic tensor. We find, perhaps not surprisingly, that the longitudinal component of the hadronic tensor could be such a model-independent observable. Ignoring (for now) the

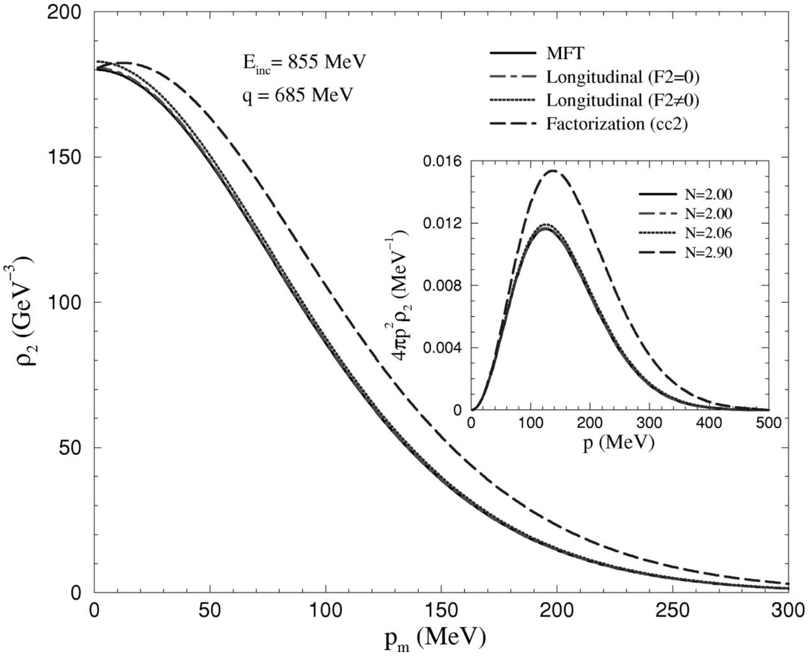

FIG. 3. The proton momentum distribution ρ_2 for ^{4}He as a function of the missing momentum calculated at an incident photon energy of $E_{\rm inc}=855$ MeV and a momentum transfer of $q=685$ MeV. The solid line is the relativistic mean-field calculation, while the dashed and dot-dashed lines display the momentum distribution extracted from the longitudinal response R_L without including (dot-dashed) and including (dotted) the contribution from the anomalous form factor F_2. Finally, the dashed curve is obtained by using the factorization approximation with the $cc2$ prescription for σ_{eN}. The inset shows the corresponding integrands from which the shell occupancy may be extracted.

anomalous part of the electromagnetic current, the Dirac-Dirac component of the longitudinal tensor [see Eq. (8)] becomes

$$W^{00}_{\rm DD}=F_1^2[M_\alpha M-p_\alpha\cdot p'+2E_\alpha E'_p]$$
$$=F_1^2[M_\alpha M+E_\alpha E'_p+\mathbf{p}_\alpha\cdot\mathbf{p}']. \tag{14}$$

This expression depends exclusively on p_α and p', which are unambiguous. Note that for scattering from a free on-shell nucleon the above expression becomes

$$W^{00}_{\rm DD}\underset{\rm free}{\longrightarrow}F_1^2[M^2-p\cdot p'+2E_pE'_p]$$
$$=F_1^2[M^2+E_pE'_p+\mathbf{p}\cdot\mathbf{p}']. \tag{15}$$

Also note, as a consequence of the lower component of the bound-state spinor $f_\alpha(p)$ being substantially smaller than the upper component $g_\alpha(p)$, that $|\mathbf{p}_\alpha|\ll E_\alpha$ while $M_\alpha\simeq E_\alpha$. This is true even though the lower-to-upper ratio f_α/g_α has been enhanced considerably in the nuclear medium relative to its free-space value. This is an important step towards isolating an observable sensitive to the spectral function. Indeed, if the longitudinal component of the hadronic tensor is computed in parallel ($\hat{\mathbf{p}}'=\hat{\mathbf{q}}$) kinematics, Eqs. (14) and (15) reduce to the following simple expressions:

$$W^{00}_{\rm DD}=F_1^2(E'_p+M)\left[\frac{\pi}{p^2}g^2_\alpha(p)\right]\left[1\pm\left(\frac{f_\alpha(p)}{g_\alpha(p)}\right)\left(\frac{|\mathbf{p}'|}{E'_p+M}\right)\right]^2, \tag{16a}$$

$$W^{00}_{\rm DD}|_{\rm free}=F_1^2(E'_p+M)\left[\frac{1}{2}(E_p+M)\right]\left[1\pm\left(\frac{|\mathbf{p}|}{E_p+M}\right)\times\left(\frac{|\mathbf{p}'|}{E'_p+M}\right)\right]^2. \tag{16b}$$

The $\pm$ sign in the above expressions corresponds to a missing momentum $\mathbf{p}$ either parallel or antiparallel to $\mathbf{p}'$. We observe that up to second-order corrections in the small (lower-to-upper) ratios, the hadronic tensor is proportional to the energylike (or masslike) quantity given in Eqs (6b). Yet this energylike quantity E_α is nothing but the Fourier transform of the bound-state nucleon density. Thus we conclude that, in a mean-field treatment, the nucleon spectral function is proportional to the longitudinal response. That is, $S(E,\mathbf{p})\propto W^{00}_{\rm DD}\propto E_\alpha$. Thus, the (Dirac-Dirac component of the) longitudinal hadronic tensor is, up to second-order corrections in the lower-to-upper ratios, proportional to the nucleon spectral function. Indeed, the nucleon momentum distribution may now be easily extracted from the longitudinal response. It becomes

$$\rho_2=2(2j+1)(E_p+M)(W^{00}_{\rm DD}/W^{00}_{\rm DD}|_{\rm free}). \tag{17}$$

The momentum distribution for ^{4}He is displayed in Fig. 3 using various methods for its extraction. The solid line gives the "canonical" momentum distribution, obtained from the Fourier transform of the $1S^{1/2}$ proton wave function [see Eq. (6b)]. The momentum distribution extracted from the longitudinal response as defined in Eq. (17) (dot-dashed line) is practically indistinguishable from the canonical momentum distribution. While it appears that a suitable observable has been found from which to extract the nucleon momentum distribution, it may be argued, and justifiably so, that $W^{00}_{\rm DD}$ is not a physical observable (as F_2 has been neglected). Hence,

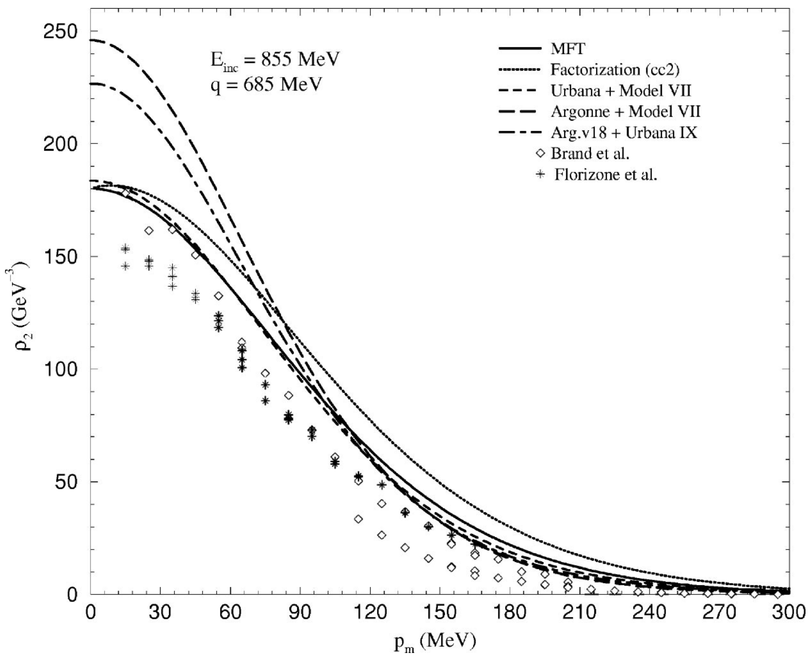

FIG. 4. A comparison between our relativistic calculations, nonrelativistic calculations reported elsewhere, and experimental data for the proton momentum distribution in ^{4}He. The solid line is our mean-field calculation while the dotted curve is our calculation using the factorization approximation at incident photon energy of $E_{\rm inc}=855$ MeV and a momentum transfer of $q=685$ MeV. The nonrelativistic calculations of Schiavilla *et al.* are included for both the Urbana (dashed) and the Argonne (long dashed) potentials as well as the calculations of Wiringa *et al.* (dashed-dotted). The NIKHEF data of van den Brand *et al.*, for two different kinematical settings as well as preliminary data of Florizone *et al.* (A1 Collaboration), which were measured at MAMI (Mainz), are also shown.

the merit of such an extraction may be put into question. To show that the above procedure is still robust, we display in the figure (with a dotted line) the momentum distribution extracted from the full longitudinal response, namely, one that also includes the anomalous component of the current. This result remains indistinguishable from the canonical momentum distribution. Although this behavior is general, it is most easily understood by limiting the discussion to the case of parallel kinematics. In this case the longitudinal response becomes equal to [17]

$$R_{\rm L}\equiv W^{00}=(E'_p+M)\left[\frac{\pi}{p^2}g_\alpha^2(p)\right]\Bigg[(F_1-\xi'_p\overline{q}F_2)\pm(\xi'_pF_1+\overline{q}F_2)\left(\frac{f_\alpha(p)}{g_\alpha(p)}\right)\Bigg]^2. \tag{18}$$

The contribution from the anomalous form factor F_2 to the longitudinal response is small because it appears multiplied by two out of three "small" quantities in the problem: the lower-to-upper ratio, $\xi'_p\equiv|\mathbf{p}'|/(E'_p+M)$, and $\overline{q}\equiv|\mathbf{q}|/2M$. Thus, up to second order corrections in these small quantities, the longitudinal response is given by

$$R_{\rm L}\simeq F_1^2(E'_p+M)\left[\frac{\pi}{p^2}g_\alpha^2(p)\right]\simeq F_1^2(E'_p+M)E_\alpha. \tag{19}$$

The last calculation displayed in Fig. 3 corresponds to a momentum distribution extracted from the factorization approximation using the $cc2$ form for the electromagnetic current (long dashed line). The momentum distribution extracted in this manner overestimates the canonical momentum distribution over the whole range of missing momenta and integrates to 2.9 rather than 2; this represents a discrepancy of 45%.

In summary, the longitudinal response appears to be a robust observable from which to extract the nucleon momentum distribution. Experimentally, one should proceed as follows: Perform a Rosenbluth separation of the $(e,e'p)$ cross section so that the longitudinal response ($R_{\rm L}\equiv W^{00}$) may be extracted. This expression should then be divided by the corresponding single-nucleon response. Up to a simple and unambiguous kinematical factor this yields, at least in the plane-wave limit, the nucleon momentum distribution:

$$\rho_2=2(2j+1)(E_p+M)\left(\frac{R_{\rm L}}{R_{\rm L}^{\rm free}}\right). \tag{20}$$

Note that up to second order corrections in various small quantities, this form is independent of the small components of the Dirac spinors and also of the negative-energy states. Moreover, it is also free of off-shell ambiguities. Indeed, we could have used the $cc1$ form of the electromagnetic current and the results would have remained unchanged. We regard the outlined procedure as much more robust than the conventional one given in Eq. (11) because the transverse component of the hadronic tensor is strongly dependent on the small components of the wave function and also sensitive to off-shell extrapolations [17].

In Fig. 4 a comparison is made between our results and nonrelativistic state-of-the-art calculations of the momentum distribution of ^{4}He. The solid line displays, exactly as in Fig. 3, the canonical momentum distribution. We see no need to include the momentum distribution extracted from the longitudinal response [Eq. (20)] as it has been shown to give identical results. In addition to our own calculation, we have also included the variational results of Schiavilla and collaborators [41], for both the Urbana [42] (dashed line) and the Argonne [43] (long-dashed line) potentials, with both of them using Model VII for the three-nucleon interaction. The variational calculation of Wiringa and collaborators [44–46] (dashed-dotted) has also been included; this uses the Argonne v18 potential [47] supplemented with the Urbana IX three-nucleon interaction [48]. Figure 4 also shows NIKHEF data by van den Brand and collaborators [49,50] as well as preliminary data from MAINZ by Florizone and collaborators [11,12] for three different kinematical settings. (Results in final form will be submitted shortly.) Comparisons to the preliminary Mainz data of Kozlov and collaborators [13–15] have also been made (although the data are not shown). These measurements are consistent, in the region where comparisons are possible, to the experimental data of both van den Brand and Florizone. Thus, high-quality data for the momentum distribution of ^{4}He is now available up to a missing momentum of about 200 MeV. We find the results of Fig. 4 quite remarkable. It appears that a simple relativistic mean-field calculation of the momentum distribution rivals—and in some cases surpasses—some of the most sophisticated nonrelativistic predictions. The mean-field calculations reported here, with the scalar mass adjusted to reproduce the root-mean-square charge radius of ^{4}He, provide a good description of the experimental data. Still, theoretical predictions of the momentum distribution overestimate the experimental data by up to 50–60%. Part of the discrepancy is attributed to distortion effects which are estimated at about 12% [11,51]. However, distortions are not able to account for the full discrepancy. We have argued earlier that an additional source of error may arise from the factorization approximation [see Eq. (11)] used to extract the spectral function from the experimental cross section. The use of an off-shell prescription, such as the $cc1$ prescription for σ_{eN}, combined with the in-medium changes in the lower-component of the Dirac spinors contaminate the extraction of the spectral function. One could estimate the source of the off-shell ambiguity by monitoring the variations in the spectral function as other on-shell equivalent forms for the single-nucleon current are used. While such an approach is useful for estimating a theoretical error, it is clearly not sufficient to eliminate it. We are confident that the approach suggested here, based on the extraction of the spectral function from the longitudinal response, is robust. While the method adds further experimental demands, as a Rosenbluth separation of the cross section is now required, the extracted spectral function appears to be weakly dependent on off-shell extrapolations and relativistic effects. If deviations between experiment and theory still persist, these may suggest physics beyond the baseline model, such as violations to the impulse approximation or to the independent particle picture.

IV. CONCLUSIONS

To summarize, we have calculated the spectral function of ^{4}He in a plane-wave approximation to the $(e,e'p)$ reaction using a fully relativistic formalism. We have taken advantage of an algebraic trick originally introduced by Gardner and Piekarewicz and of our recently developed relativistic formalism for quasifree processes to arrive at transparent, analytical results for the quasifree reaction. We have found that a simple relativistic mean-field calculation of the momentum distribution in ^{4}He rivals—and in some cases surpasses—some of the most sophisticated nonrelativistic predictions to date. These calculations attempt to provide theoretical support to the recently measured, but not yet fully analyzed, A1 collaboration data from Mainz. The final experimental reports are expected to be published shortly.

We have also demonstrated that a more robust procedure, relative to the conventional factorization prescription, exists for extracting the spectral function. This procedure uses the ratio of quasifree to single-nucleon longitudinal responses, rather than the ratio of cross sections, to isolate the momentum distribution. We have shown that the longitudinal ratio is fairly insensitive to off-shell ambiguities and to the negative-energy part of the spectrum, as both of these effects appear as second-order corrections to a ''canonical'' momentum distribution. This ceases to be true in the case of the ratio of cross sections because the transverse response is sensitive to both effects. While this procedure relies on a Rosenbluth (L/T) separation of the quasifree cross section, and thus presents the experimentalist with a more demanding task, the experimental field has evolved to such a level of maturity that L/T separations are now almost routine. Indeed, in a recent publication [52] a Rosenbluth separation of the ^{3}He$(e,e'p)$ cross sections was made in order to extract ''longitudinal'' and ''transverse'' spectral functions in the hope of resolving the anomaly in the longitudinal-transverse ratio alluded to in the introduction. We speculate that the sensitivity of the transverse response to more complicated dynamical processes might be partially responsible for the quenching of the longitudinal–transverse ratio.

Finally, although in this article we focused exclusively on the spectral function, the formalism presented here may be extended in a straightforward fashion to the calculation of spin observables in quasifree electroproduction processes. Indeed, we speculate that, because the ratio of quasifree cross sections are fairly insensitive to distortion effects, spin observables may be a more fruitful testing ground for our relativistic plane-wave model. Moreover, our formalism may be easily extended to neutrino-induced reactions. It has been suggested that a measurement of the ratio of neutral to charge-changing neutrino-nucleon scattering may provide a clean signature of the strange-quark content of the nucleon [53]. This measurement is believed to be free from most of the uncertainties, such as radiative corrections, that hinder the parity-violating electron scattering program. Yet neutrino experiments suffer from very low counting rates. To remedy this situation neutrino experiments employ large quantities of nuclear targets (such as organic scintillators) that provide both the target and the detection medium. Thus neutrinos interact, not only with the free protons in the target, but also with protons and neutrons bound to nuclei; hence, one must compute quasifree $(\nu,\nu' p)$ and $(\nu,\mu^- p)$ cross sections. (Of course, one must integrate the quasifree cross section over the undetected outgoing neutrino.) Therefore, the relativistic plane-wave formalism presented here is ideally suited, after including an additional axial-vector term in the single-nucleon current, to predict ratios of quasifree neutrino-nucleus cross sections in the quasifree region.

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