Anharmonicities of giant dipole excitations

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(Received 26 July 2001; published 16 November 2001)

The role of anharmonic effects on the excitation of the double giant dipole resonance is investigated in a simple macroscopic model. Perturbation theory is used to find energies and wave functions of the anharmonic oscillator. The cross sections for the electromagnetic excitation of the one- and two-phonon giant dipole resonances in energetic heavy-ion collisions are then evaluated through a semiclassical coupled-channel calculation. It is argued that the variations of the strength of the anharmonic potential should be combined with appropriate changes in the oscillator frequency, in order to keep the giant dipole resonance energy consistent with the experimental value. When this is taken into account, the effects of anharmonicities on the double giant dipole resonance excitation probabilities are small and cannot account for the well-known discrepancy between theory and experiment.

DOI: 10.1103/PhysRevC.64.064605 PACS number(s): 24.30.Cz, 21.60.−n, 25.70.−z, 25.75.−q

The double giant dipole resonance (DGDR) has attracted considerable interest in the last decade. Several experiments to measure the DGDR cross section using relativistic heavy ion beams have been performed [1–6]. Comparison with the predictions of the harmonic oscillator model has clearly demonstrated a systematic discrepancy. The experimental values for the DGDR cross sections exceed the theoretical predictions by a considerable amount. One of the attempts to explain these differences was made by Bortignon and Dasso [7], using a macroscopic anharmonic oscillator model. These authors found that with a small anharmonic perturbation of the r^4 type one can reproduce both the experimentally observed DGDR excitation energy (which only marginally differs from that obtained in the harmonic approximation) and the DGDR cross section for the ^{208}Pb+^{208}Pb collision at 640*A* MeV. They reached a similar conclusion for the ^{136}Xe+^{208}Pb collision at 700*A* MeV, where a much greater discrepancy from the harmonic model appears [2]. The purpose of this paper is to point out that this model does not lead to the enhancement found in Ref. [7], if proper renormalization of the oscillator frequency is performed in order to guarantee that the theoretical giant dipole resonance (GDR) excitation energy is kept at the experimental value.

The model of Refs. [7,8] is based on the following Hamiltonian:

$$H = H_0 + F(x,y,z;t), \tag{1}$$

where H_0 is the anharmonic oscillator describing the intrinsic motion of the projectile,

$$H_0 = \frac{1}{2D}(p_x^2+p_y^2+p_z^2) + \frac{C}{2}(x^2+y^2+z^2) + \frac{B}{4}(x^2+y^2+z^2)^2, \tag{2}$$

where D is the mass parameter, C is the oscillator strength and, B is the strength of the anharmonicity. Here, we take the mass parameter to be the reduced mass for the motion of the protons against the neutrons,

$$D = \frac{NZ}{A} m_0,$$

where m_0 is the average nucleon mass. The beam is assumed to be parallel to the x axis and the coupling interaction F is derived from the Lienard-Wiechert potential [10] in the projectile frame

$$\phi(x,y,z,t) = \frac{Z_T e \gamma}{[\gamma^2(x-vt)^2+(y-b)^2+z^2]^{1/2}}, \tag{3}$$

were $Z_T e$ is the charge of the target, b is the impact parameter, and γ is the Lorentz factor, $\gamma = 1/\sqrt{1-(v/c)^2}$.

To be specific, we study the ^{208}Pb+^{208}Pb collision at 640A MeV. We first solve the Schrödinger equation for the intrinsic motion, described by H_0. For this purpose it is convenient to recast the intrinsic Hamiltonian into the following equivalent form:

$$H_0 = \hbar\omega\left[\frac{1}{2}(\pi^2+\rho^2) + \beta\rho^4\right]. \tag{4}$$

In the above, the commonly used variable transformations

$$\rho_i = \sqrt{\frac{D\omega}{\hbar}} r_i; \quad \pi_i = \frac{p_i}{\sqrt{D\hbar\omega}}, \tag{5}$$

have been made, where r_i and p_i stand for the components of the position and momentum operators, respectively. The oscillator frequency is given by

$$\hbar\omega = \hbar\sqrt{\frac{C}{D}}, \tag{6}$$

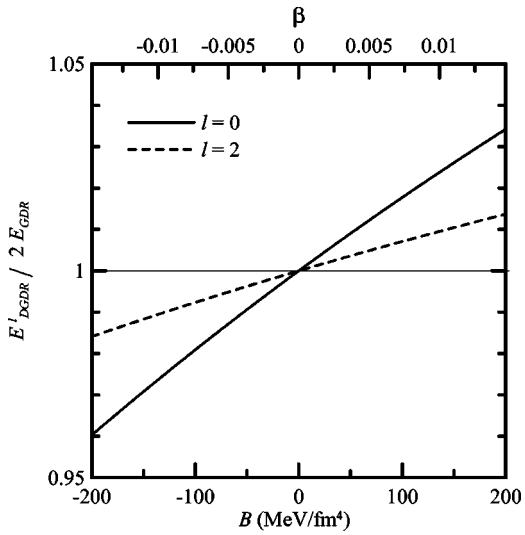

FIG. 1. The ratio $E^l_{DGDR}/(2\,E_{GDR})$ vs the anharmonicity parameter B, for ^{208}Pb. The solid line is for $l=2$ and the dashed line for $l=0$. The reduced mass for the oscillation of protons against neutrons is used for the mass parameter D.

and the dimensionless strength β is related to B as

$$B=\left[\frac{4(\hbar\omega)^3D^2}{\hbar^4}\right]\beta. \tag{7}$$

In Fig. 1, we show the ratios $E^{l=0}_{DGDR}/(2\,E_{GDR})$ and $E^{l=2}_{DGDR}/(2E_{GDR})$ as a function of B, in the same range as chosen in Ref. [7]. In this range, the anharmonicity can be treated using first-order perturbation theory to great accuracy ($\sim$2%). The GDR and DGDR energies, to first order in β, are given by

$$E_{GDR}(\beta)=\hbar\omega(1+5\beta), \tag{8}$$

$$E^{l=0}_{DGDR}(\beta)=2\hbar\omega(1+7.5\beta), \tag{9}$$

$$E^{l=2}_{DGDR}(\beta)=2\hbar\omega(1+6\beta). \tag{10}$$

Figure 1 is equivalent to that shown in Ref. [7] and our results are essentially identical to theirs.

The reduced transition matrix elements can also be easily calculated to first order in the parameter β. We find

$$\langle GDR\|E1\|GS\rangle=e\left(\frac{S_1}{\hbar\omega}\right)^{1/2}(1-2.5\,\beta),$$

$$\langle DGDR,l=0\|E1\|GDR\rangle=e\left(\frac{S_1}{\hbar\omega}\right)^{1/2}\sqrt{\frac{2}{3}}(1-5\,\beta),$$

$$\langle DGDR,l=2\|E1\|GDR\rangle=e\left(\frac{S_1}{\hbar\omega}\right)^{1/2}\sqrt{\frac{10}{3}}(1-3.5\beta),$$

where e is the absolute value of the electron charge and S_1 is given by the energy-weighted sum rule,

$$S_1=\frac{9}{4\pi}\frac{\hbar^2}{2m_0}\frac{NZ}{A}.$$

The energy-weighted sum rule for transitions from the ground state and from the GDR are satisfied to first order in the parameter β, using the above energies and reduced matrix elements.

In order to maintain $E_{GDR}(\beta)$ at the experimental value, namely, $E_{GDR}(\beta)=E^{\rm exp}_{GDR}$ (13.4 MeV, in the present case), the oscillator frequency must be renormalized as β is changed. The resulting renormalized frequency, from Eq. (8), is

$$\hbar\omega(\beta)=\frac{E^{\rm exp}_{GDR}}{(1+5\,\beta)}. \tag{11}$$

Note that in the B range of Fig. 1, the dimensionless parameter varies in the range $-0.014<\beta<0.014$ that yields $1.08\,E^{\rm exp}_{GDR}>\hbar\omega(\beta)>0.93\,E^{\rm exp}_{GDR}$. Whereas our oscillator frequency is a function of the anharmonicity parameter, in Ref. [7] it is kept constant at the harmonic value, $\hbar\omega(\beta=0)=E^{\rm exp}_{GDR}$. This difference does not affect the ratio $E^l_{DGDR}/(2\,E_{GDR})$ shown in Fig. 1, since the oscillator frequency cancels out in this case [see Eqs. (8)–(10)]. When the renormalized frequency is used in both the GDR and DGDR energies and matrix elements, the sum rules for transitions from the ground state and from the GDR are still satisfied. However, use of the renormalized frequency substantially changes the excitation probability of the DGDR, as will be shown below.

The calculation of electromagnetic excitation probabilities and cross sections is performed with the code RELEX [9], based on the Winther and Alder theory [10]. With this code, we perform a full coupled-channels calculation of the electromagnetic excitation of the GDR and DGDR. Similar results (about 10% larger) would be obtained when perturbation theory is used for the collision dynamics [11]. In Fig. 2, we show the enhancement of the DGDR excitation probability relative to its harmonic value as a function of B for the impact parameter $b=30$ fm. We find that for $B\sim-100$ MeV/fm^4 (which in this case corresponds to $\beta\sim-0.7\times10^{-2}$) the overall enhancement is 6%. For purposes of comparison, we have also performed calculations using a constant frequency ($\hbar\omega=13.4$ MeV in this case). We then obtain an enhancement of 35%, as shown by the dashed in line in Fig. 2, in agreement with Ref. [7] (see their Fig. 1).

In Fig. 3(a), we show the enhancement in the impact-parameter integrated DGDR cross section (solid line) vs B, for the same system. In the cross section calculations, impact parameters up to 200 fm are taken into account and a lower cutoff at 15 fm is used to eliminate nuclear effects. The full line in Fig. 3(a) represents the result of the present work, in which an enhancement of only 4% is obtained for $B=-100$ MeV/fm^4. The dashed line, obtained using a fixed value of the oscillator frequency, yields an enhancement of the DGDR cross section of 22% for the same value of B. The GDR cross section ratio $\sigma_{GDR}(B)/\sigma_{GDR}(B=0)$ obtained with fixed GDR energy, shown as a solid line in Fig. 3(b), is

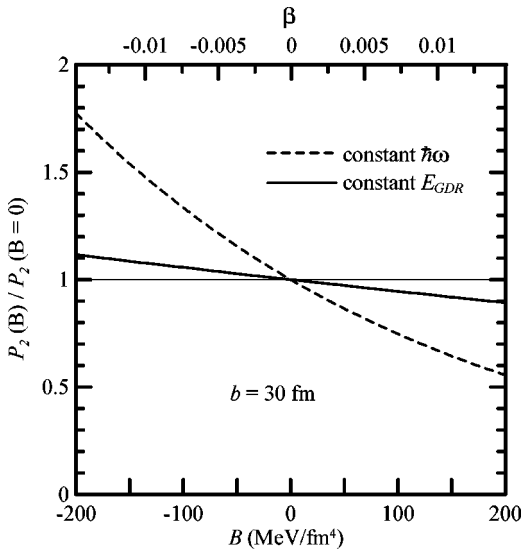

FIG. 2. The enhancement in the excitation of the DGDR in the collision of ^{208}Pb+^{208}Pb at 640A MeV for the impact parameter $b=30$ fm. The solid line represents the results of the present calculation while the dashed line corresponds to a constant oscillator frequency.

close to one over the entire range of B values but is slightly less than one for large, negative anharmonicities (about -0.5% at $B=-100$ MeV/fm^4). This small deviation is due to the increase in the population of the DGDR at these values of B and the correponding depopulation of the GDR. The GDR cross section ratio obtained with fixed oscillator frequency is shown as a dashed line in Fig. 3(b). In this case, we find the GDR cross section to be enhanced by about 10% at $B=-100$ MeV/fm^4. The enhancement of 10% in the GDR cross section of Fig. 3(b) is clearly responsible for the large enhancement of 22% in the DGDR cross section of Fig. 3(a) at $B=-100$ MeV/fm^4.

The above conclusions do not change noticeably when the calculations are extended to other systems, such as ^{136}Xe $+^{208}$Pb at 700A MeV. The microscopic study of Ref. [12] established that the anharmonicity parameter scales as A^{-1} with the mass number. Thus, if $B=-100$ MeV/fm^4 represents a reasonable value for ^{208}Pb, then for ^{136}Xe a corresponding value would be $B=-150$ MeV/fm^4. In Fig. 4, we display the results of calculations for this system as a function of the anharmonicity parameter B in Fig. 4. The solid line in the figure again shows the results of calculations in which the oscillator frequency is varied to maintain the GDR energy constant, while the dashed line represents the results of calculations in which the oscillator frequency is maintained fixed. Similar to the previous case, we find the enhancement of the DGDR cross section to be greatly reduced when the GDR resonance energy is maintained at a fixed value. As can be seen in Fig. 4, at $B=-150$ MeV/fm^4, the DGDR cross section is enhanced by 62% when the oscillator frequency is maintained constant, but is enhanced by less than 10% when the GDR energy is maintained at its physical value.

Before ending we comment briefly on the connection between the Bortignon-Dasso model used in this paper and

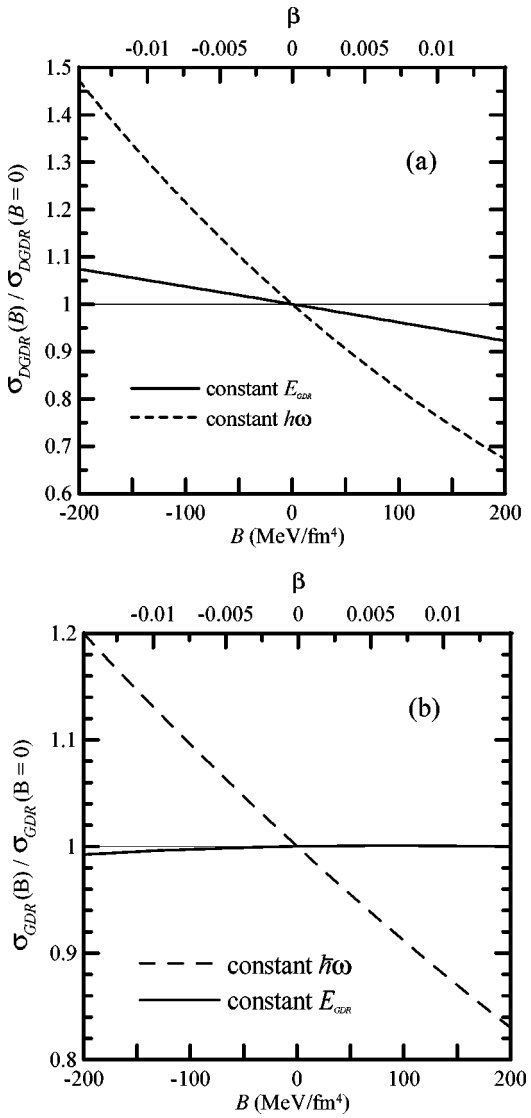

FIG. 3. Enhancement factor of the (a) DGDR and (b) GDR cross sections in the collision of ^{208}Pb+^{208}Pb at 640A MeV. The dashed lines correspond to the results obtained with fixed oscillator frequency, while the full lines correspond to a fixed E_{GDR}.

microscopic models [12–15] that aim to assess the importance of the anharmonic effects both on the spectrum and on the transition operator. Reference [14] finds, within the Lipkin model, small effects on the spectrum (which scale roughly as $1/A$). Hamamoto finds, within nuclear field theory, that the nonlinear effects in the one-phonon to two-phonon transition operator are also quite small and scale as $1/A$ [16]. As mentioned above, Ref. [12], through detailed microscopic calculations, finds that the anharmonic effects are indeed small and scale as $1/A$. The values of the parameter B in both the Bortignon-Dasso and present calculations are taken to be small enough to be in line with the microscopic findings but also with the experimentally observed DGDR excitation energies (although the enhancement of the DGDR cross section could be increased thorough an artificially large B, there is no choice for this parameter that would simultaneously explain the observed cross section

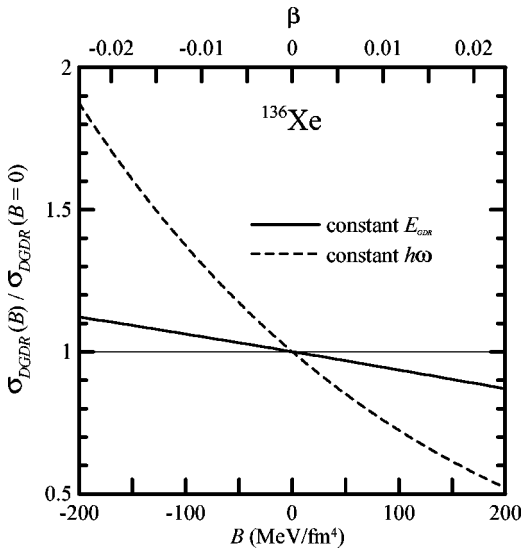

FIG. 4. Enhancement factor of the DGDR cross section in the collision of ^{136}Xe+^{208}Pb at 700*A* MeV. The dashed line corresponds to the results obtained with fixed oscillator frequency, while the full line corresponds to a fixed E_{GDR}.

enhancement and the only very small deviations of the DGDR excitation energy from the harmonic limit).

Another interesting point to mention is that the GDR has a width, which is considered neither by Bortignon and Dasso nor in the present calculation. The effect of the width of the GDR on the excitation of the DGDR has been recently studied within a harmonic picture [17]. The overall effect of the width, at the energies considered here is, to produce a slight increase in the DGDR cross section, although not enough to explain all the available data. It would certainly be of interest to extend the present calculation within the anharmonic model by coupling the oscillator to other degrees of freedom (which would generate the damping width).

In conclusion, we have investigated the effect of anharmonicities in the excitation of the DGDR in relativistic heavy-ion collisions, with the same macroscopic model used by Bortignon and Dasso [7]. We point out that variations of the anharmonicity strength must be accompanied by a renormalization of the oscillator frequency, in order to maintain the GDR energy at a value consistent with the experimental one. We have found that this condition strongly reduces the enhancement in the DGDR excitation probabilities and corresponding cross sections, so that they remain much below the experimental results.

This work was supported in part by DAAD/CAPES Cooperative Agreement No. 415-bra-probral/bu, CNPq and the MCT/FINEP/CNPq(PRONEX) under Contract No. 41.96.0886.00. D.T.P and L.F.C. acknowledge partial support from the Fundação Universitária José Bonifácio, and M.S.H. and B.V.C. acknowledge support from the FAPESP.

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