Description of the identical superdeformed bands and $\Delta I=4$ bifurcation in the $A\sim130$ region

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Within the supersymmetry scheme including many-body interactions, identical superdeformed (SD) bands and $\Delta I=4$ bifurcation in $A\sim130$ mass region are investigated systematically. Quantitatively good results for the γ-ray energy (E_γ) spectra, dynamical moments of inertia, and energy differences $\Delta E_\gamma-\Delta E_\gamma^{\rm ref}$ are obtained. This shows that the supersymmetry approach is quite powerful in describing not only generic rotational properties, but also $\Delta I=4$ bifurcation and identical SD bands simultaneously. Meanwhile, the $\Delta I=4$ bifurcation may come from the perturbation with SO(5) [or SU(5)] symmetry on the rotational symmetry.

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I. INTRODUCTION

Since the observation of the rotational band in the second well of the energy surface of ^{132}Ce [1], more than 40 superdeformed (SD) bands have been established in the $A\sim130$ nuclei (for a compilation see Ref. [2]). Meanwhile it has been discovered that there exist some fascinating phenomena, such as the identical bands $\{^{133}\mathrm{Ce}(2),\ ^{136}\mathrm{Nd}(1)\}$, and $\{^{132}\mathrm{Ce}(1),\ ^{133}\mathrm{Ce}(1)\}$ [3,4], and the $\Delta I=4$ bifurcation [or the $\Delta I=2$ staggering in γ-ray energy (E_γ)] in the SD bands ^{131}Ce(1), ^{132}Ce(1), and ^{133}Ce(1,2) [5], even though the staggering sequence is not as regular as that in ^{149}Gd(1) [6] and in ^{194}Hg(1) [7,8]. Many attempts have been made to describe the bands and understand the underlying physics. The single particle configurations and the dynamical moments of inertia of the SD bands have then been investigated in the self-consistent cranked approaches (see, for example, Refs. [3,9,10]). The $\Delta I=4$ bifurcation in the band ^{132}Ce(1) has been interpreted as a result of band crossing in the projected shell model [11]. On the other hand, an algebraic approach on the basis of the interacting boson model [12] has been developed [13–16], with which the SD bands of the even-even nuclei in $A\sim190$ and 150 mass regions have been described well. With the approach being extended to supersymmetry scheme [17–19], the SD bands of the odd-A nuclei in $A\sim150$ mass region have been described systematically [20]. Meanwhile, the $\Delta I=2$ staggering in E_γ in the SD bands ^{149}Gd(1), ^{194}Hg(1), and ^{148}Gd(6), the identical SD bands in ^{152}Dy, ^{151}Tb, ^{151}Dy, ^{153}Dy, ^{153}Dy, and those in ^{191}Hg, ^{193}Hg, ^{193}Tl have also been reproduced well [20–22]. Furthermore, the identical SD bands with $\Delta I=4$ bifurcation, ^{149}Gd(1)-^{148}Gd(6)-^{148}Eu(1), have been reproduced quantitatively [23] too. However, systematic study of the identical SD bands and the $\Delta I=4$ bifurcation in $A\sim130$ mass region in the algebraic approach is still lacking. With the approach proposed in Ref. [23], we will investigate the identical nature of and the $\Delta I=4$ bifurcation in the SD bands in $A\sim130$ mass region in this paper.

The paper is organized as follows. Following this introduction, we describe the formalism of the approach briefly in Sec. II. In Sec. III, we present the calculation and the obtained results. Meanwhile some discussions are included. Finally, a summary and some remarks are given in Sec. IV.

II. FORMALISM

Experimental data show that superdeformed bands exhibit quite good rotational characteristics. The dynamical symmetry group chains utilized to label the states should be the ones ending with SO(3). Based on the fact that normally deformed nuclear states in even-even, odd-A, and odd-odd nuclei can be described well in a unified way in supersymmetry scheme [17], we hypothesize that the SD states can be classified with the supersymmetric group chain

$$\begin{array}{ccccccc} \mathrm{U}(m,n)\supset & \mathrm{U}_B(m)\otimes & \mathrm{U}_F(n)\supset\cdots\supset & \mathrm{SO}_{B+F}(3)\otimes & \mathrm{SU}_F(n')\supset & \mathrm{Spin}\ (3), \\ [N] & [N_B]_m & [N_F]_n & L & S & I \end{array}$$

where m is determined by the constituent of the bosons, n is fixed by the single particle configuration of the fermion(s), and n' by the total pseudospin. $N=N_B+N_F$ is the total number of particles with N_B and N_F the boson and fermion numbers, respectively, L is the effective-core angular momentum that is composed of the angular momentum L_B of the bosonic core and the pseudo-orbital angular momentum L_F of the fermion, S is the total pseudospin, and I is the total spin of the nucleus.

A systematic and painstaking analysis on the system and the successful description of the identical SD bands with $\Delta I=4$ bifurcation, ^{149}Gd(1)-^{148}Gd(6)-^{148}Eu(1), indicate that the boson part in the above dynamical symmetry group chain may be the SU(5) limit of the system including *s*, *d*, *g*, *p*, and *f* bosons [23]. Taking advantage of the spectrum generating principle, we know that every irrep of the SU(5) groups contributes a constant to the energies of all the states within the irrep. Consequently, all the irreps of the SU(5) groups and those of the parent groups contribute nothing to the relative excitation energies of the states in a band. The contribution of the bosons to the γ-ray energies in a SD band is thus, in fact, the one with the SO(5) symmetry. We get then

$$E=E_0(N_B,N_F)+B[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)]+C_L L(L+1)+C_S S(S+1)+C_I I(I+1), \quad (1)$$

where the $E_0(N_B,N_F)$ includes the contribution (a constant) of the irreps of the SU(5) groups and parent groups. The (τ_1,τ_2) is the irrep of the SO(5) group. With an effective aligned angular momentum i being introduced as $i=C_L S/(C_L+C_I)$, Eq. (1) can be written as

$$E=E_0(N_B,N_F)+B[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)]+CI'(I'+1), \quad (2)$$

where $I'=I-i$, $C=C_L+C_I$, and $E_0(N_B,N_F)$ is a little different from that in Eq. (1) since another constant is involved. Obviously, by adjusting the ratio C_L/C_I, one can get any value of the alignment *i*. In particular, taking $C_L=0$, one has the strong coupling limit (with $i=0$). If $C_I=0$, one gets the pseudospin decoupling limit (with $i=S$). For the irrep (τ_1,τ_2) of the SO(5) group, even though it can be generally fixed with the branching rules of the irrep reduction [24,25], it is not practical to do it because of the complexity of the single particle configurations and the shell structure of the SD states. Supposing that SD states are generated by the nontotally symmetric irrep $[2N-2,2,0,0]$ of the SU(5) dynamical symmetry (the normally deformed states are generated by the totally symmetric one $[2N,0,0,0]$), one can have the (τ_1,τ_2) in practical calculation as

$$(\tau_1,\tau_2)=\begin{cases}\left(\left[\frac{L}{2}\right],0\right), & \text{if } L=4k,\ 4k+1(k=0,1,2,\dots)\\ \left(\left[\frac{L}{2}\right]-1,2\right), & \text{if } L=4k+2,4k+3(k=0,1,2,\dots),\end{cases} \quad (3)$$

where $L=[I']$ is the integer part of the spin I'.

In light of the variable moment of inertia models [14–16,20–23,26–28], the parameter C in Eq. (2) should be angular momentum dependent. Extending the approach developed in Ref. [28] to include both the pairing and the antipairing effects, we propose that the energy of the states in a SD band can be given as

$$E=E_0(N_B,N_F)+B[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)]+\frac{C_0}{1+f_1 I'(I'+1)+f_2 I'^2(I'+1)^2}I'(I'+1). \quad (4)$$

It has been shown that involving the terms with parameters f_1,f_2 in such a way takes the many-body interactions into account [21]. Meanwhile, if f_1,f_2 take the value with different signs (i.e., $f_1<0,f_2>0$ or $f_1>0,f_2<0$), both the pairing and antipairing effects are included [14–16,20–23,28].

III. CALCULATION AND DISCUSSION

It is known that, for the SD bands in $A\sim130$ mass region, there exist identical bands {^{133}Ce(2), ^{136}Nd(1)} and the band ^{133}Ce(1) is identical to ^{132}Ce(1) in the region $\hbar\omega>0.6$ MeV [3]. The band ^{133}Ce(1) is believed to be built upon the one neutron orbital with respect to ^{132}Ce(1) [3] and the ^{133}Ce(2) is assigned to be built upon the neutron hole orbital against ^{136}Nd(1) [29,30]. To take into account the contribution of the microscopic configuration, the single particle energy ε_F that is usually rotational frequency dependent (referred to single particle routhian), should be included. Equation (4) should then be rewritten as

$$E=E_0'(N_B,N_F)+\varepsilon_F N_F+B[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)]+\frac{C_0}{1+f_1 I'(I'+1)+f_2 I'^2(I'+1)^2}I'(I'+1), \quad (5)$$

where $E_0'(N_B,N_F)$ is a constant for the states with boson number N_B and fermion number N_F.

To discuss the identical nature of the bands {^{133}Ce(2), ^{136}Nd(1)} and {^{132}Ce(1), ^{133}Ce(1)}, we calculate the γ-ray energy spectrum of the reference band ^{136}Nd(1) and ^{132}Ce(1) separately at first. After a nonlinear square fitting to the experimental data, we obtain the γ-ray energies with spin assignment $I_0=24$ for ^{132}Ce(1) and 19 for ^{136}Nd(1). The latter is just the experimentally determined one [30]. The calculated result is listed in Table I. The best fitted parameters are $B=0.001\,824$ keV, $C_0=7.936$ keV, $f_1=9.384\times10^{-6}$, $f_2=-5.142\times10^{-9}$ for ^{132}Ce(1) and $B=-0.000\,3120$ keV, $C_0=9.064$ keV, $f_1=4.257\times10^{-5}$, $f_2=-1.115\times10^{-8}$ for ^{136}Nd(1). With these parameters we evaluate the "supersymmetry part" of the γ-ray energies of ^{133}Ce(1) and ^{133}Ce(2) with $\varepsilon_F\equiv0$ and spin assignment $I_0=24.5$ and 22.5, respectively. The results are represented with label Cal.$_{\text{ps}}$ in Table I. It is definite that the calculated results in this sense do not agree with experimental data well, since the effect of the single particle configuration has not yet been taken into account. Theoretical calculations and experimental data analyses indicate that the SD bands ^{133}Ce(1) and ^{133}Ce(2) are built upon the $[530]\frac{1}{2}(\alpha=-)$ orbital of neutron particle and $[660]\frac{1}{2}$ orbital of neutron hole, respectively [3,29,30]. The pairing (including a quadrupole pairing interaction) and deformed self-consistent calculations [3] indicate that $\varepsilon_F(\hbar\omega)=-0.095(\hbar\omega)^2-0.215\hbar\omega-8.76$ MeV for the $[530]\frac{1}{2}(\alpha=-)$ orbital and $\varepsilon_F(\hbar\omega)=-0.157(\hbar\omega)^2$

TABLE I. Calculated γ-ray energies of the identical SD bands $\{^{132}$Ce(1), ^{133}Ce(1)$\}$ and $\{^{133}$Ce(2), ^{136}Nd(1)$\}$ in the pure supersymmetry scheme (referred as Cal.$_{ps}$) and that together with single particle energy of the fermion being considered (referred as Cal.$_f$) and the comparison with experiment data (taken from Ref. [2]).

	^{132}Ce(1)				^{133}Ce(1)			^{136}Nd(1)				^{133}Ce(2)	
Spin	Expt.	Cal.	Spin	Cal.$_{ps}$	Expt.	Cal.$_f$	Spin	Expt.	Cal.	Spin	Cal.$_{ps}$	Expt.	Cal.$_f$
24	769.61(10)	741.88	24.5	757.63	748.30(11)	748.92	19	656.5(2)	654.08				
26	808.55(5)	804.93	26.5	820.74	809.13(3)	811.55	21	716.8(2)	721.38	22.5	771.67	720.32(9)	721.40
28	864.85(5)	868.42	28.5	884.32	872.86(5)	875.23	23	795.1(2)	788.29	24.5	837.96	785.43(10)	786.12
30	928.80(5)	932.16	30.5	948.19	937.27(9)	938.62	25	856.8(3)	854.44	26.5	904.11	854.22(7)	853.92
32	994.63(5)	996.67	32.5	1012.87	1003.03(7)	1003.10	27	918.6(3)	920.56	28.5	969.84	920.29(7)	920.94
34	1060.32(5)	1061.75	34.5	1078.17	1067.56(6)	1067.90	29	982.8(3)	986.26	30.5	1035.95	986.92(8)	985.99
36	1127.27(6)	1128.03	36.5	1144.74	1132.26(6)	1134.26	31	1049.6(2)	1052.47	32.5	1102.16	1052.19(7)	1050.41
38	1194.72(6)	1195.34	38.5	1212.41	1198.39(10)	1201.72	33	1117.5(2)	1118.81	34.5	1169.48	1118.23(8)	1118.23
40	1263.63(6)	1264.43	40.5	1281.94	1266.60(6)	1270.38	35	1186.0(3)	1186.42	36.5	1237.66	1184.33(9)	1184.49
42	1334.56(7)	1335.15	42.5	1353.18	1337.40(8)	1341.39	37	1255.3(4)	1254.96	38.5	1307.94	1253.19(8)	1254.41
44	1408.34(9)	1408.40	44.5	1427.06	1411.39(8)	1414.69	39	1325.2(4)	1325.80	40.5	1380.13	1323.86(10)	1323.06
46	1485.67(10)	1484.06	46.5	1503.48	1488.63(9)	1490.15	41	1398.8(4)	1398.64	42.5	1455.73	1397.89(11)	1396.65
48	1566.70(10)	1563.23	48.5	1583.53	1570.09(8)	1569.57	43	1476.3(4)	1475.14	44.5	1534.62	1475.51(9)	1472.48
50	1651.49(12)	1645.85	50.5	1667.19	1654.70(12)	1652.20	45	1558.2(4)	1555.09	46.5	1618.65	1557.29(11)	1554.21
52	1740.29(14)	1733.21	52.5	1755.77	1743.04(14)	1739.71	47	1643.5(5)	1640.47	48.5	1707.83	1642.98(21)	1636.83
54	1832.64(17)	1825.39	54.5	1849.37	1833.42(56)	1832.18	49	1732.6(7)	1731.25	50.5	1804.42	1731.25(21)	1728.98
56	1926.50(17)	1923.93	56.5	1949.56	1927.50(120)	1930.41	51	1815(1)	1829.81	52.5	1908.71	1821.65(22)	1825.12
58	2023.50(20)	2029.08	58.5	2056.63		2035.81	53			54.5	2023.47	1910.00(20)	1930.00
60	2119.00(25)	2142.73	60.5	2172.52		2149.94	55			56.5	2149.44		2045.56

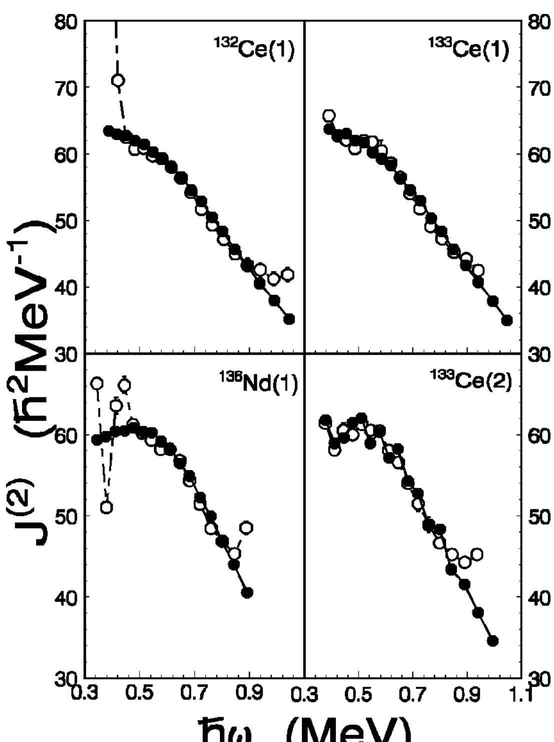

FIG. 1. Calculated result (solid line with filled circles) of the dynamical moments of inertia of the identical SD bands $\{^{132}$Ce(1), ^{133}Ce(1)$\}$ and $\{^{136}$Nd(1), ^{133}Ce(2)$\}$ and the comparison with experiment. The experimental data are taken from Ref. [2] and shown as open circles with error bar.

$-1.38\hbar\omega - 8.48$ MeV for the $[660]\frac{1}{2}(\alpha=-)$ orbital. Adding the corresponding single particle energy to the "supersymmetry part," we get finally the E_γ's of SD bands ^{133}Ce(1) and ^{133}Ce(2), respectively. The obtained results are listed in Table I. The induced dynamical moments of inertia of these SD bands are illustrated in Fig. 1.

The table and the figure manifest that both the γ-ray energies and the dynamical moments of inertia of the identical

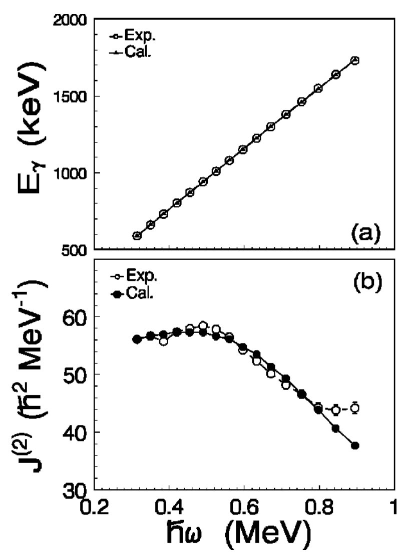

FIG. 2. Calculated result of the E_γ spectrum [panel (a)] and dynamical moment of inertia [panel (b)] of the SD bands ^{131}Ce(1) and the comparison with experiment. The experimental data are taken from Ref. [2].

SD bands $\{^{132}\mathrm{Ce}(1),\ ^{133}\mathrm{Ce}(1)\}$, and $\{^{133}\mathrm{Ce}(2),\ ^{136}\mathrm{Nd}(1)\}$ have been reproduced well simultaneously in the supersymmetry approach with the frequency dependent single particle energy being involved. It indicates that the supersymmetry scheme is quite powerful in describing the phenomenon of identical bands of SD states. The identical SD bands may then be interrelated with the supersymmetry.

Experimental data indicate that there exists $\Delta I=4$ bifurcation in the SD bands $^{131}\mathrm{Ce}(1)$, $^{132}\mathrm{Ce}(1)$, and $^{133}\mathrm{Ce}(1,2)$, even though the staggering sequence is not so regular as that in $^{149}\mathrm{Gd}(1)$ [6] and in $^{194}\mathrm{Hg}(1)$ [7,8]. To examine the capability of our approach to describe the detailed characteristics of SD bands, we calculate the energy differences ΔE_γ between two consecutive γ-ray transitions of these SD bands as a function of rotational frequency after subtraction of a smooth reference expressed in notation of Cederwall's [7]

$$\Delta E_\gamma(I)-\Delta E_\gamma^{\mathrm{ref}}=\frac{3}{8}\left\{E_\gamma(I)-\frac{1}{6}[E_\gamma(I-2)+E_\gamma(I+2)-E_\gamma(I-4)-E_\gamma(I+4)]\right\}.$$

Since the band $^{131}\mathrm{Ce}(1)$ does not have a corresponding twin, we evaluate the E_γ spectrum with a freely nonlinear square fitting. The obtained E_γ spectrum and the deduced dynamical moment of inertia and the comparison with experimental data are illustrated in Fig. 2. The calculated results of $\Delta E_\gamma-\Delta E_\gamma^{\mathrm{ref}}$ of the bands $^{131}\mathrm{Ce}(1)$, $^{132}\mathrm{Ce}(1)$, and $^{133}\mathrm{Ce}(1,2)$ are presented in Fig. 3. The figure indicates that the $\Delta I=4$ bifurcation in the four SD bands has been described quite well except for the phase shift at high rotational frequency. To demonstrate the significance of the interaction with the SO(5) [or SU(5)] symmetry, we have also evaluated the E_γ spectra with $B\equiv 0$. The deduced $\Delta E_\gamma-\Delta E_\gamma^{\mathrm{ref}}$ of the bands do not exhibit any staggering. It indicates that the $\Delta I=4$ bifurcation cannot be reproduced if the Hamiltonian does not involve an interaction with the SO(5) [or SU(5)] symmetry. Then one can easily recognize that the interaction with the SO(5) [or SU(5)] symmetry plays a pivotal role in generating the $\Delta I=4$ bifurcation.

In the theoretical viewpoint of our approach, the state with $L=[I]=4k$ $(4k+1)$ in the SD bands corresponds to the irrep $(2k,0)$ of the SO(5), and the one with $L=[I]=4k+2$ $(4k+3)$ relates to the $(2k,2)$. It means that the SD bands with level sequence $\Delta I=2$ involve both the totally symmetric irrep $(\tau,0)$ and the nontotally symmetric irrep $(\tau,2)$ of the SO(5) simultaneously. With the definition of γ-ray energy $E_\gamma(I)=E(I)-E(I-2)$, one can readily obtain that the contribution of the term with the SO(5) symmetry to E_γ is $E_\gamma^{SO(5)}([I]=4k+2,4k+3)=6B$ and $E_\gamma^{\mathrm{SO(5)}}([I]=4k,4k+1)=(4[I/2]-4)B$. Therefore the term with the SO(5) symmetry makes the E_γ staggering even though the $|B|$ may be very small. To show this point more explicitly, we evaluate the energy spectrum of the SU(5) dynamical symmetry with Eq. (2) and $N=5$ as an example. The calculated result indicates that the spectrum generated by the totally symmetric irrep $[2N,0,0,0]$ does really not exhibit any

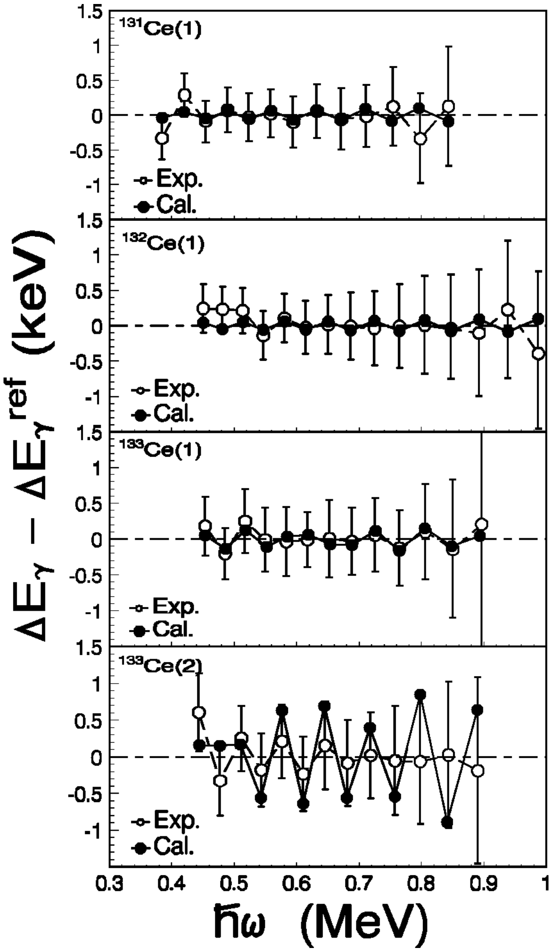

FIG. 3. Calculated results of the γ-ray energy differences $\Delta E_\gamma-\Delta E_\gamma^{\mathrm{ref}}$ of the SD bands $^{131}\mathrm{Ce}(1)$, $^{132}\mathrm{Ce}(1)$, and $^{133}\mathrm{Ce}(1,2)$ and the comparison with experimental data (taken from Ref. [2]).

energy staggering. However, the one generated by the nontotally symmetric irrep $[2N-2,2,0,0]$, from which the irreps $(\tau_1,\tau_2)=(2N-2,2)$, $(2N-2,0)$, $(2N-4,2)$, $(2N-4,0)$, ... of the SO(5) group emerge, holds the staggering definitely. Figure 4 displays a part of the obtained energy spectrum (to show the $\Delta I=2$ staggering more obviously, we take $B>C$). The figure manifests apparently that the simultaneous appearance of the totally and nontotally symmetric irreps of the SO(5) group, which is governed by the branching rules of the irrep reduction, generates the band with $\Delta I=2$ staggering in E_γ (in other word, the $\Delta I=4$ bifurcation) naturally. Meanwhile one nucleus holding the SU(5) symmetry can have more than one SD bands with the $\Delta I=2$ staggering. In the present calculation, the best fitted parameter $|B|$ is so small (less than $10^{-3}C_0$) that the term with the SO(5) symmetry can be regarded as a perturbation on the rotational symmetry. Then the emergence of the rotational band with $\Delta I=2$ staggering is the inherent property of the rotational system being perturbed by an interaction with the SO(5) symmetry. The present calculation provides thus an evidence that the $\Delta I=4$ bifurcation in SD band may result from the perturbation that holds the SO(5) [or SU(5)] symmetry on the rotation. By the way, comparing the best fitted

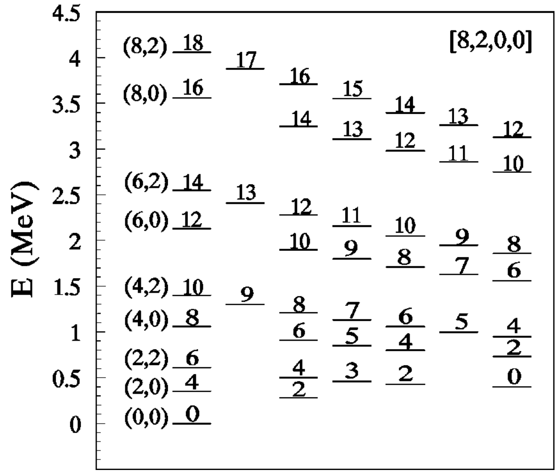

FIG. 4. A part of the energy spectrum generated by the irrep $[2N-2,2,0,0]$ of the SU(5) symmetry with $N=5$. The parameters are taken as $E_0=0$, $B=25$ keV, and $C=5$ keV. The labels on the left side of the levels denote for the irreps of the SO(5) group.

parameter B in the present calculation with those for the SD bands in $A\sim150$ and 190 mass region [15,20,21], one can know that the $|B|$ for the SD bands in $A\sim130$ region is only about 20% of those for the other SD bands with $\Delta I=2$ staggering. It means that the perturbation for the SD bands in $A\sim130$ mass region is much weaker than that for the other regions. As a consequence, the staggering in $A\sim130$ region is neither so obvious nor regular as that in $A\sim150$ region.

IV. SUMMARY AND REMARKS

In summary, with the supersymmetry scheme including many-body interactions, the identical superdeformed bands $\{^{132}$Ce(1), ^{133}Ce(1)$\}$ and $\{^{133}$Ce(2), ^{136}Nd(1)$\}$ and the $\Delta I=4$ bifurcation in the SD bands ^{131}Ce(1), ^{132}Ce(1), and ^{133}Ce(1,2) are investigated systematically. The calculated results show that, with the single particle energy being considered simultaneously, the supersymmetry approach reproduces the identical SD bands $\{^{132}$Ce(1), ^{133}Ce(1)$\}$ and $\{^{133}$Ce(2), ^{136}Nd(1)$\}$ well. It indicates that the supersymmetry with many-body interactions is probably a source of the identical SD bands. At the same time, the $\Delta I=4$ bifurcation has also been described well. Since the interaction with SO(5) [or SU(5)] symmetry is in fact a perturbation on the rotation ($|B|\ll C_0$), the present calculation provides a clue that the $\Delta I=4$ bifurcation may result from the perturbation with SO(5) [or SU(5)] symmetry on the rotational symmetry.

Finally it is worth to mention that, since the SO(5) and SU(5) symmetry is the symmetry of a five-dimensional space, and other investigations have shown that the SD states in $A\sim190$ mass region can be described well in the framework of a quantal Hamiltonian with five collective quadrupole coordinates [31], the mean field to generate the SD states may not be the usual three-dimensional space field, but the five-dimensional superspace field spanned by the five collective quadrupole coordinates. Such a superspace may possess the orthogonal rotational symmetry SO(5), even the unitary symmetry SU(5).

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